Functional Programming in Haskell

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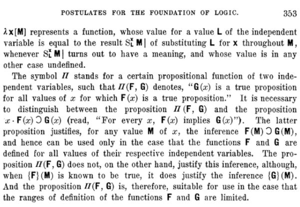
POSTULATES FOR THE FOUNDATION OF LOGIC. 353

$\lambda \mathbf{x}[\mathbf{M}]$ represents a function, whose value for a value $\mathbf{L}$ of the independent variable is equal to the result $S^{\mathbf{x}}_{\mathbf{L}}\mathbf{M}|$ of substituting $\mathbf{L}$ for $\mathbf{x}$ throughout $\mathbf{M}$, whenever $S^{\mathbf{x}}_{\mathbf{L}}\mathbf{M}|$ turns out to have a meaning, and whose value is in any other case undefined.

The symbol Π stands for a certain propositional function of two independent variables, such that $\Pi(\mathbf{F}, \mathbf{G})$ denotes, "$\mathbf{G}(x)$ is a true proposition for all values of x for which $\mathbf{F}(x)$ is a true proposition." It is necessary to distinguish between the proposition $\Pi(\mathbf{F}, \mathbf{G})$ and the proposition $x \cdot \mathbf{F}(x) \supset \mathbf{G}(x)$ (read, "For every x, $\mathbf{F}(x)$ implies $\mathbf{G}(x)$"). The latter proposition justifies, for any value $\mathbf{M}$ of x, the inference $\mathbf{F}(\mathbf{M}) \supset \mathbf{G}(\mathbf{M})$, and hence can be used only in the case that the functions $\mathbf{F}$ and $\mathbf{G}$ are defined for all values of their respective independent variables. The proposition $\Pi(\mathbf{F}, \mathbf{G})$ does not, on the other hand, justify this inference, although, when $\{\mathbf{F}\}(\mathbf{M})$ is known to be true, it does justify the inference $\{\mathbf{G}\}(\mathbf{M})$. And the proposition $\Pi(\mathbf{F}, \mathbf{G})$ is, therefore, suitable for use in the case that the ranges of definition of the functions $\mathbf{F}$ and $\mathbf{G}$ are limited.

Excerpt from "Postulates for the foundation of Logic" by A. Church, 1932

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Introduction to the Lambda calculus

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Introduction to the Lambda Calculus

- The lambda calculus was developed in the 1930s by Alonzo Church (1903–1995), one of the leading developers of mathematical logic.
- The lambda calculus was an attempt to formalise functions as a means of computing.

Significance to computability theory

- A major (really *the* major) breakthrough in computability theory was the proof that the lambda calculus and the Turing machine have exactly the same computational power.
- This led to *Church's thesis* — that the set of functions that are effectively computable are exactly the set computable by the Turing machine or the lambda calculus.
- The thesis was strengthened when several other mathematical computing systems (Post Correspondence Problem, and others) were also proved equivalent to lambda calculus.
- The point is that the set of effectively computable functions seems to be a fundamental reality, not just a quirk of how the {Turing machine, lambda calculus} was defined.

Significance to programming languages

- The lambda calculus has turned out to capture two aspects of a function:
  - A mathematical object (set of ordered pairs from domain and range), and
  - An abstract black box machine that takes an input and produces an output.
- The lambda calculus is fundamental to denotational semantics, the mathematical theory of what computer programs mean.
- Functional programming languages were developed with the explicit goal of turning lambda calculus into a practical programming language.
- The ghc Haskell compiler operates by (1) desugaring the source program, (2) transforming the program into a version of lambda calculus called *System F*, and (3) translating the System F to machine language using *graph reduction*.

Abstract syntax of lambda calculus

- We will work with the basic lambda calculus "enriched" with some constants and primitive functions (strictly speaking, that is not necessary).
- The language has constants, variables, applications, and functions.

```
exp
  = const
  | var
  | exp exp
  | \ var -> exp
```

Variables

- Each occurrence of a variable in an expression is either *bound* or *free*
  - In $\backslash x \rightarrow x+1$, the occurrence of x in $x+1$ is *bound* by the $\backslash x$.
  - In $y * 3$, the occurrence or y is *free*. It must be defined somewhere else, perhaps as a global definition.
- In general, an occurrence of a variable is bound if there is some enclosing lambda expression that binds it; if there is no lambda binding, then the occurrence if free.

We need to be careful: the first occurrence of a is free but the second occurrence is bound.

```
a + (\ a -> 2^a) 3  -- >  a + 2^3
```

Being free or bound is a property of an *occurrence* of a variable, not of the variable itself!

Conversion rules

- Computing in the lambda calculus is performed using three *conversion rules*.
- The conversion rules allow you to replace an expression by another ("equal") one.
- Some conversions simplify an expression; these are called *reductions*.

Alpha conversion

- Alpha conversion lets you change the name of a function parameter consistently.
- But you can't change free variables with alpha conversion!
- The detailed definition of alpha conversion is a bit tricky, because you have to be careful to be consistent and avoid "name capture". We won't worry about the details right now.

```
(\x -> x+1) 3
(\y -> y+1) 3
```

Beta conversion

- Beta conversion is the "workhorse" of lambda calculus: it defines how functions work.
- To apply a lambda expression an argument, you take the body of the function, and replace each bound occurrence of the variable with the argument.

```
(\x -> exp1) exp2
```

is evaluated as $exp1[exp2/x]$

Example:

```
(\x -> 2*x + g x) 42
```

is evaluated as $2 * 42 + g\ 42$

Eta conversion

- Eta conversion says that a function is equivalent to a lambda expression that takes an argument and applies the function to the argument.

```
(\x -> f x)
```

is equivalent to f

Example (recall that $(*3)$ is a function that multiplies its argument by 3)

```
(\x -> (*3) x)
```

is equivalent to $(*3)$

Try applying both of these to 50:

```
(\x -> (*3) x) 50
```

is the same as $(*3)\ 50$

Removing a common trailing argument

There is a common usage of Eta conversion. Suppose we have a definition like this:

```
f x y = g y
```

This can be rewritten as follows:

```
f = \x -> (\y -> g y)
f = \x -> g = f x = g
```

Thus the following two definitions are equivalent:

```
f x y = g y
f x = g
```

In effect, since the last argument on both sides of the equation is the same (y), it can be “factored out”.

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