Public Financing with Financial Frictions and Underground Economy*

Andrés Erosa†
Universidad Carlos III de Madrid

Luisa Fuster‡
Universidad Carlos III de Madrid

Tomás R. Martinez§
Universidade de Brasília and Universitat Pompeu Fabra

Abstract

What are the aggregate effects of informality in a financially constrained economy? We develop and calibrate an entrepreneurship model to data on matched employer-employee from both formal and informal sectors in Brazil. The model distinguishes between informality on the business side (extensive margin) and the informal hiring by formal firms (intensive margin). We find that when informality is eliminated along both margins, aggregate output increases 9.3%, capital 14.7%, TFP 5.4%, and tax revenue 37%. The output and TFP increases would be much larger if informality were only eliminated on the extensive margin, a result that supports the view that the informal economy can play a positive role in an economy with financial frictions. Finally, we find that the output cost of financing social security in our baseline model is about twice as large as the one in an economy with no frictions.

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†Universidad Carlos III de Madrid, Department of Economics, Calle Madrid 126, 28903 Getafe, Madrid, Spain. E-mail: aerosa@eco.uc3m.es. Phone: (34) 916249600.

‡Universidad Carlos III de Madrid, Department of Economics, 28903 Getafe, Spain. E-mail: lfuster@eco.uc3m.es.

§Universidade de Brasília, Universitat Pompeu Fabra, and Barcelona Graduate School of Economics, Departament d'Economia i Empresa, 08005 Barcelona, Spain. Email: tomas.rodriguez@upf.edu.

1 Introduction

Large informal economies and underdeveloped financial markets are distinguishing features of most developing countries.[1] In this paper, we develop a quantitative theory (and calibrate it to Brazilian microdata) to assess how informality affects capital accumulation, occupational choices, and resource allocation in an economy *with* financial frictions. Moreover, we assess how informality and financial frictions affect the ability of the government to raise taxes and, in particular, the costs of financing a pay-as-you-go social security system.

In our framework, informality acts as a size-dependent policy by allowing unproductive entrepreneurs to avoid taxation when using little capital and labor. Financial frictions reduce the scale of operation of high productivity entrepreneurs that lack sufficient resources to operate at their optimal scale. The effects of informality and financial frictions, on the one hand, reinforce each other in creating a competitive advantage for low productivity entrepreneurs, distorting occupational choice and the allocation of capital and labor across entrepreneurs. On the other hand, informality allows financially constrained entrepreneurs to operate at lower costs, speed up the accumulation of capital, and relax borrowing constraints. But the benefits of informality may come at a cost if entrepreneurs in the informal economy are subject to tighter borrowing constraints. In sum, whether the interaction between financial frictions and informality improves or worsens resource allocation in the economy is a quantitative question.

Central to our quantitative findings is the distinction between two margins of informality that we borrowed from Ulyssea (2018): (i) the *extensive margin* represents the entrepreneurial decision of whether to register the business to operate formally or to avoid paying taxes and regulation costs by operating the business in the underground economy; (ii) the *intensive margin* corresponds to the extent to which entrepreneurs, who have registered their business and attain formal status, hire some workers "off the books" to avoid fully complying with their mandatory contributions to the social security system. While the informality literature has focused on the extensive margin alone, the intensive margin of informality is empirically relevant as most informal workers in Brazil are hired by formal businesses. Moreover, we find that the effects of informality on capital accumulation and resource allocation critically depend on financial frictions and that the effects caused by the interaction between informality and financial frictions vary substantially depending on the

[1] For instance, in Brazil, around 70% of businesses and 35% of workers are informal. Similarly, in Mexico, around 60% of workers are in the informal sector. Both countries are characterized by low financial development when compared to advanced economies. In Brazil, domestic credit to private sector GDP is around 66%, while in Mexico is around 32%. In comparison, the US domestic credit to private sector GDP is 188%. Data from World Bank Development Indicators, 2015.

relative importance of the two margins of informality.

Our analysis proceeds in three steps. In the first part of the paper, we use matched-employee data to document the key facts on informality in Brazil. Following Ulyssea (2018) we document that informality in Brazil is pervasive (both along the intensive and extensive margin): About 70% of business and 35% of workers are informal. Out of the total informal workers, roughly 70% are located in formal firms. We provide new evidence on the large differences between formal and informal entrepreneurs in the use of capital, investment, debt, and value-added. Conditional on the size of the establishment and industry, the value-added by formal businesses is a factor of 2.3 the one by informal businesses. Differences in capital and debt are a factor of 5 and 6.

In a second step, we build a theory of occupational choice, financial frictions, and informality along the intensive and extensive margin. The government collects taxes on payroll and sales and administers a pay-as-you-go social security system. Our setting is ideal to study the interactions between the different micro-distortions with tax policy: the collateral constraint distorts the marginal products of capital across producers, while a size-dependent policy on labor - informality - reallocate resources from large to smaller producers. This results in an economy where production units differ along their scale and in their capital to labor ratio, and therefore in their tax liability. The model is calibrated to match Brazilian data on the shares of formal businesses, informal paid workers, and informal-paid workers hired by formal businesses. Moreover, the calibration targets moments on the size distribution of formal and informal businesses, the relative differences in value-added, debt, and capital intensities across businesses in the formal and informal sectors.

In a third step, we use the model to evaluate the effect of financial frictions and informality in the allocation of resources and public financing. We find that informality in Brazil is quite costly. Whereas one would expect that higher taxation of economic activity caused by the elimination of informality should depress economic activity, we find that the elimination of the informal economy leads to a substantial increase in both output (9.3%) and the fraction of taxes collected per unit of production (25%). The key to this result is that the combination of informality with financial frictions generates a competitive advantage for the operation of small businesses, thereby leading to a large misallocation of productive resources. As a result, in the presence of financial frictions, the elimination of informality reduces the mass of entrepreneurs more than a half (from 0.169 to 0.073), increases aggregate capital by 14.3%, and rises TFP by 5.4%. The improved allocation of productive resources explains why output rises by 9.3% despite the increase in effective taxation. On the contrary, in the absence of financial frictions, the increase in effective taxation caused by the elimination of informality *reduces* output by 1%.

Our results point to important interactions between financial frictions and informality. The gains from removing financial frictions in the baseline economy are larger than in an economy with no informality (both along the intensive and extensive margins): 38% versus 25% for output, 43% versus 23% for capital, and 25% versus 19% for TFP. More importantly, the benefits of eliminating financial frictions in an economy without informality along the intensive margin *are even larger*: 46% for output, 53% for capital, and 30% for TFP. Financial frictions distort resource allocation the most in an economy with informal businesses (extensive margin of informality) *and* in which formal businesses cannot hire informal workers (no intensive margin of informality).

Why financial frictions have such strong effects on an economy with informal businesses but no intensive margin of informality? The effects of the intensive and extensive margins of informality depend on the extent of financial frictions differently. The extensive margin of informality allows entrepreneurs to avoid taxation at the cost of operating at a small scale, acting like a size-dependent policy that interferes with the efficient allocation of resources. Financial frictions constrain the scale of operations that entrepreneurs can attain, increasing the likelihood that high productivity businesses choose to operate in the informal economy and amplifying the misallocation of resources caused by the extensive margin of informality. As a result, the negative consequences of financial frictions and business informality on resource allocation reinforce each other. The intensive margin of informality reduces labor costs (payroll taxes) faced by all entrepreneurs in the formal sector. Since credit-constrained entrepreneurs tend to rely more heavily on the use of labor, they benefit more strongly from the reduction in labor costs. In an economy with financial frictions, the intensive margin of informality speeds up capital accumulation by entrepreneurs facing tight borrowing constraints, relaxing credit constraints over the life cycle, increasing capital accumulation, and improving resource allocation. In sum, the intensive margin diminishes the negative consequences of financial frictions on macroeconomic outcomes.

Motivated by the disproportional distortionary effect of the payroll taxes on borrowed constrained entrepreneurs, we further inspect the effect of a revenue-neutral reform of social security that eliminates the payroll tax and replaces the lost tax revenue with an increase in the sales tax rate. We find that substituting the payroll tax by the sales tax in a financially constrained economy decreases informality and has positive effects on the allocative efficiency of the economy: output, capital, and TFP increase by 3.5%, 2.4%, and 3.1% relative to the baseline economy. In an economy without financial frictions, however, payroll taxation is a better instrument to finance pensions than sales taxes. In this case, the elimination of the payroll tax leads to *negative* changes in all of the macroeconomic variables (-10.7% in output, -27% in capital, -2.7% in TFP). Our results are consistent with the view of Itskhoki and Moll

(2019), who advocate for low labor taxation when entrepreneurs face tight constraints in the use of capital.

The large distortionary effects of payroll taxes make the social security system dramatically more costly to finance in our baseline economy than in an economy with no financial frictions. While the elimination of the pension system leads to an increase in output of 19.5% and of government revenue of 11% in the baseline economy, in the economy without financial friction output increases by half while government revenue *decreases* by 10%.

Overall, our results point to the importance of the interaction between financial frictions and informality on both margins for a complete and unbiased assessment of how changes in policies and institutions impact macroeconomic variables.

Literature. We contribute to different strands of the literature. Broadly, we are connected to the literature that studies aggregate consequences of informality.[2] In recent work, Ulyssea (2018) uses a model of heterogeneous firms to evaluate the result of different formalization policies on output, TFP, and welfare. An important contribution of Ulyssea (2020) is to consider informal hiring by formal firms, the "intensive margin" of informality. Incorporating the intensive margin into the model produces new insights: policies that decrease firms' informality might not decrease labor informality, and lower informality may not be associated with welfare gains. By incorporating financial frictions and an occupational choice, we deliver additional insights based on the incentives to self-finance and the different margins of informality. Other works have used different approaches to study informality. Meghir et al. (2015) analyze the firm productivity distribution through the lens of a wage-posting model. In the equilibrium model studied by de Paula and Scheinkman (2010), the incentives produced by value-added taxes increase informality across the supply chain. Prado (2011) uses cross-country data to calibrate a static industry model with tax, imperfect enforcement, and entry costs.

Moreover, our paper relates to the large literature that investigates how the misallocation of resources across heterogeneous produces can account for the large cross-country income differences in the data.[3] In particular our paper relates to a large literature assessing the role of financial frictions in models of entrepreneurship (Midrigan and Xu (2014), Buera et al. (2011), Moll (2014), Erosa (2001) and Allub and Erosa (2019)). We were not the first to study the relationship between financial development and informality. In Ordóñez (2014) and Franjo et al. (2019), the probability of detection is a function that depends on the capital hired by the entrepreneur. This distorts the capital decision of informal firms but

[2]For a survey on the current state of the literature, see Ulyssea (2020).

[3]See Restuccia and Rogerson (2008), Guner et al. (2008) and García-Santana and Pijoan-Mas (2014). For a recent survey see Restuccia and Rogerson (2017).

not formal firms. D'Erasmo and Moscoso Boedo (2012) explicitly model firms' bankruptcy procedures in equilibrium with the credit market. Antunes and Cavalcanti (2007) uses a static occupational choice model where formal firms have (imperfectly) access to finance. These papers abstract from the large number of informal workers employed at formal firms and its importance for self-financing in the presence of financial frictions

There is a large literature analyzing the effects of tax evasion on public finances. Although the literature spans over theoretical and empirical approaches (Slemrod and Yitzhaki (2002), Slemrod (2019)), the work on aggregate effects is somewhat limited. A notable exception is Di Nola et al. (2021). They build an occupational choice model in which entrepreneurs can misreport part of their income to study distributional welfare. Their focus is on personal income tax evasion, while our work differentiates between payroll and sales taxes allowing us to assess the effect of distinct tax policies.

Finally, there is a large literature studying the effects of social security on capital accumulation and labor supply (see, for instance, Attanasio et al. (2007), Imrohorolu et al. (1995), Conesa and Krueger (1999), Fuster (1999), and Fuster et al. (2007)). To the best of our knowledge, this literature abstracts from how the financing of social security affects resource allocation across heterogeneous entrepreneurs. McKiernan (2021) and Tkhir (2020) model social security in the presence of an informal sector but their focus is on the worker's labor supply decision, while ours is on occupational choice and resource allocation across entrepreneurs.

2 Empirical Evidence

This section discusses the empirical evidence on the main stylized facts regarding firms, informality, and financial frictions. To carry on our empirical analysis, we make use of several Brazilian data sets. The main data comes from the ECINF (*Pesquisa de Economia Informal Urbana*), a cross-sectional survey of non-agricultural businesses. The survey is nationally representative for small urban businesses (up to 5 employees) and it was conducted by the Brazilian Bureau of Statistics in 1997 and 2003. The data cover detailed information on the business characteristics (revenue, capital, credit), and workers' characteristics - including the owner and non-paid labor. Because of its structure, it provides a unique opportunity to understand the relationship between productivity, credit, and hiring decisions of informal production units.

Although ECINF gives a good representation of the characteristics of the informal businesses, where the average size is 1.15 and 97% of the businesses have two workers or less, the size cap of five workers is too small to provide a good representation of the true size dis-

tribution of the formal sector. Hence, we use multiple data sets to supplement the ECINF. The formal firm size distribution comes from RAIS, which is an administrative matched-employer employee dataset that covers the universe of formal firms. Unfortunately, RAIS does not provide any information on informal firms nor informal workers. Therefore, we supplement it with two surveys: PNAD (*Pesquisa Nacional por Amostra de Domicílios*) and PME (*Pesquisa Mensal de Emprego*). PNAD is a nationally representative household survey and PME is a monthly rotational panel of workers that covers the six largest metropolitan areas in Brazil. Both provide valuable individual-level information such as the total share of informal workers, the share of entrepreneurs in the economy, and the share of informal workers among large businesses. To keep the data comparable, we look at data from 2003 and maintain the same sample selection whenever possible.[4] Our definition of informality is the usual: a firm is formal when it possesses a tax identification number, and a worker is formal when the labor contract is registered in her worker's booklet - a document that records all formal employment relationships and ensures that workers are entitled to receive all social security benefits.

2.1 Formal Firms and Informal Workers

Many empirical facts about the informal economy have been documented using microdata from a variety of countries. La Porta and Shleifer (2014) suggests that informal firms employ few workers, have low value-added per employee, and pay low wages relative to their formal counterparts. Ulyssea (2018) confirms this evidence in Brazil, but adds that formal and informal firms coexist in narrowly defined industries and share common support in the productivity distribution. Regarding worker characteristics, La Porta and Shleifer (2014) reports that managers of informal firms are, on average, less educated than the ones of formal firms. Yet, there are no clear differences between the human capital of the other employees. This is perhaps surprising since a well-known stylized fact is that informal workers are on average less educated than formal workers.[5] Table 1 confirms that, in Brazil, the share of informal firms decreases with firm size. While the fraction of informal businesses among businesses with one worker is around 90%, that fraction is 30% for businesses with 5 workers. Moreover, the size distribution of informal firms is highly concentrated, with 97% of all informal firms employing two workers or less (including the owner).

Although the most used definition of informality relies on whether the business is formally

[4]The sample is selected to be all privately owned firms, including own-account workers, in urban areas.

[5]However, both facts are fully consistent with each other when we consider that a large fraction of informal workers are employed by formal firms, especially that informal workers employed by formal firms are on average low educated workers.

Table 1: Share of Informal Firms and Informal Workers by Firm Size

Size	Share Inf. Firms	Share Inf. Workers in Formal Firms	Cum. Informal
1	0.930	-	0.898
2	0.657	0.476	0.972
3	0.449	0.463	0.988
4	0.344	0.373	0.994
5	0.296	0.262	0.998
6	0.311	0.317	1.000
7	0.069	0.165	1.000
All (≤ 7)	0.868	0.322	

Notes: Size includes paid employees plus business owners. Share of informal workers in formal firms includes paid employees only. Cum. Informal denotes the cumulative distribution of informal firms. Source: ECINF 2003.

registered with the tax authorities, recently, the literature has focused on formal firms that can be "partially" informal by hiring informal workers. The hiring of informal workers by formal firms, sometimes referred to as the "intensive" margin of informality, potentially accounts for a large share of the informal employees. In Mexico, around 47% of all informal workers are employed in a formal firm (Samaniego de la Parra (2017)), while in Peru, 32% of the informal workers in manufacturing are located in a formal business (Cisneros-Acevedo (2019)).[6] In the context of financial frictions, the intensive margin of informality helps productive but constrained firms to speed up capital accumulation and grow larger without the size constraints imposed by belonging to the informal sector.

Since one needs to know the formality status of both the firm and the worker, knowing the exact extent of the intensive margin is challenging in many countries. Table 1 indicates that, in small Brazilian firms, 32.2% of the informal employment is by formal businesses. Furthermore, the gradient of the intensive margin of informality is decreasing in size. While formal businesses with at least two workers hire almost 50% of workers informally, formal businesses with five workers hire only half of that. As argued by Ulyssea (2018), given that ECINF only covers small firms, the share of informal employment at formal firms in the economy is likely much larger than 32.2%. Table 2 presents the employment share by each pair of worker and firm formality status using the household survey PNAD. First, out of 22% of informal workers in 2012, almost 14% were employed by formal firms. This means that formal firms account for 62% of the total informal employment.[7] Second, similarly to

[6]Moreover, López and Torres (2020) provide evidence that smaller Mexican establishments pay, on average, lower social security contributions for their paid employees.

[7]The formality status of the employer is asked only in the updated PNAD, which started rolling in 2012.

ECINF, the employment share of informal workers decreases in larger firms. Yet, even in firms with more than 50 employees, 7.5% of the total employment is informal. A possible explanation for this fact is that hiring too many informal workers increases the probability of being detected, hence, the marginal worker in a large firm is likely to be formal.

Table 2: Employment Share by Worker and Firm Informality Status and Firm Size

Worker-Firm Status	≤ 5	≥ 6 and ≤ 10	≥ 11 and ≤ 50	≥ 51	All Firms
Formal Worker in Formal Firm	42.48	69.99	82.95	91.36	78.02
Informal Worker in Formal Firm	25.76	20.35	13.79	7.54	13.80
Informal Worker in Informal Firm	31.75	9.66	3.27	1.11	8.18
Total Employment Share	17.84	13.85	19.72	48.59	100.00

Notes: Employment share by worker and firm formality status and firm size. Urban paid employees in private firms only. Size is defined by the number of paid employees. Source: PNAD-C 2012.

2.2 Informality, Capital and Debt

In this section, we further explore the relationship between informality, capital, and debt. On the one hand, in a world with financial frictions, informality can alleviate the burden of high taxes and allow financially constrained firms to operate. On the other hand, a registered business often has access to better credit conditions as banks may require some form of managerial supervision such as well-developed business plans or accounting books.[8] Using the World Bank Enterprise Surveys, La Porta and Shleifer (2014) shows that access to financing is the most important obstacle to do business for both formal and informal firms. Nevertheless, while 43.8% of informal businesses report financing as the most important issue, just 18.5% of formal businesses argue the same. ECINF directly asked the source of the loan to the entrepreneurs who asked for credit. While 73.6% of the formal firms used public or private banks instead of other loan sources such as friends and family, the same share for informal firms is only 53% (see Appendix Table A.3).

On top of the anecdotal evidence, Appendix Table A.4 displays summary statistics of our ECINF sample conditional on the characteristics of the entrepreneur. On average, formal firms have higher profits, revenues, and costs than informal firms. Also, they employ almost five times more capital, hold six times more debt, and invest two times more. Aggregate

Because the share of informal workers decreased 13 p.p. from 2003 to 2012 (see Appendix Table A.2), the number of informal workers at formal firms is presumably higher in 2003. In Appendix A.2, we argue that it can be as high as 75.9%.

[8]In general, a registered entrepreneur has better loan conditions such as friendlier repayment structure, higher credit limits, and different default options.

debt to output (considering only small firms) is 43% in the formal sector, while just 31% in the informal sector. The aforementioned differences are explained, to a large extent, by the fact that formal firms are larger and possibly operate in different sectors than informal firms. Hence, to account for possible differences across sectors, Table 3 exhibits the partial correlations of debt, capital, and investment with the formality status conditional on size, industry and value-added per worker.

Table 3: Partial Correlations of Debt, Capital and Investment with Formality Status

VARIABLES	(1) log(Debt)	(2) log(Capital)	(3) log(Investment)
Informal	-0.538***	-0.658***	-0.505***
	(0.0760)	(0.0500)	(0.0902)
log(VA p/ worker)	0.455***	0.789***	0.673***
	(0.0276)	(0.0164)	(0.0359)
Observations	7,856	32,797	7,696
R-squared	0.414	0.615	0.584
Size FE	Yes	Yes	Yes
Industry FE	Yes	Yes	Yes
State FE	Yes	Yes	Yes

Notes: Size is define as number of paid workers plus business owners. Industry dummies are at 4-digit level. Only firms with positive values of debt, capital and investment are included. Robust standard errors in parentheses. $^{***}p < 0.01$, $^{**}p < 0.05$, $^{*}p < 0.1$. Source: ECINF 2003.

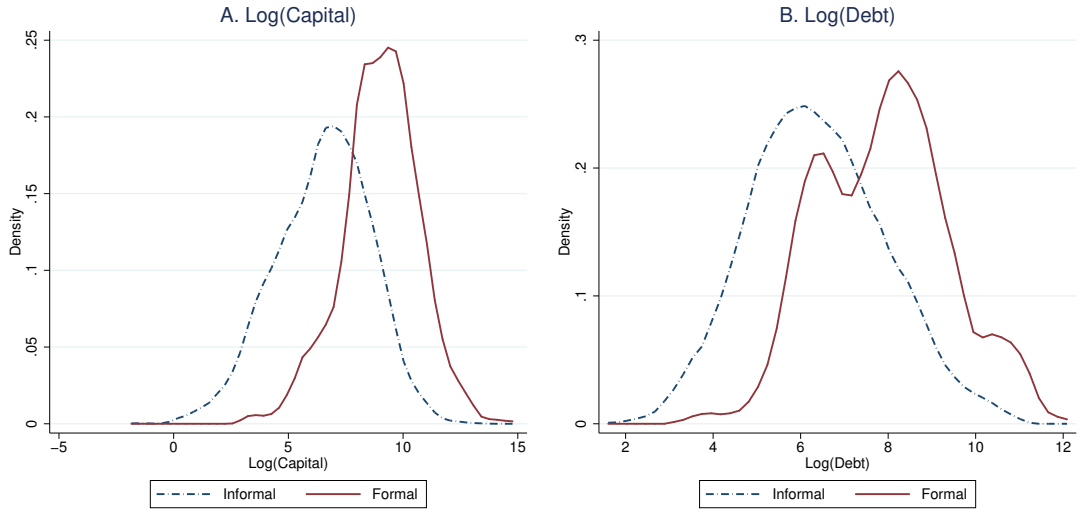
After differences in the number of workers, sector of activity, and value-added are taken into account, an informal business still holds 53.8% less debt, 65.8% less capital, and invest 50.5% less than a formal business. Although this can be seen as evidence that the informal sector faces stronger frictions in the financial market than the formal sector, we cannot argue that there is a direct causal relationship.

First, one should expect some degree of selection across sectors based on initial capital. For instance, an entrepreneur with low asset levels might not have enough economies of scale to operate in the formal sector, and instead, will decide to produce informally. Second, entrepreneurs possibly self-select based not only on assets but also on their expectation of business success. An entrepreneur who believes she has a successful and large business will select into the formal sector, as opposed to an entrepreneur who wants to operate on a small scale. Finally, firms in the informal sector might endogenously accumulate less capital to

avoid detection.[9]

Figure 1 shows the distribution of capital and debt in both formal and informal (conditional on industry) for entrepreneurs with less than one year of operation. We focus on entrants to abstract for capital accumulation post-entry. While the average entrant in the formal sector hold more debt and employ more capital, the distributions display a large common support across sectors, illustrating that entrepreneurs with similar asset levels may self-select into different sectors.

Figure 1: Distribution of Capital and Debt of Entrants



Notes: Smoothed densities of firms with less than one year old, and positive capital and debt by formal and informal. Log capital and debt are conditional on industry. Kernel function is Epanechnikov with bandwidth of 0.22. Source: ECINF 2003.

In sum, the selection into entrepreneurship and the decision of whether to operate formally or informally is affected by a host of factors. On the one hand, business informality may be discouraged by size restrictions and worse credit market conditions in the informal sector. On the other hand, the desire to evade taxes and avoid entry costs into the formal sector may raise business informality. These tradeoffs depend on the level of assets of entrepreneurs, their productivity, and their expected future productivity. When formal entrepreneurs can hire workers off-the-books, informality on the extensive margin may be reduced at the cost of an increase of informality along the intensive margin. All of these considerations imply that the evaluation of government policies is far from trivial. In the next section, we develop

[9]For example, Ordóñez (2014) argues that physical equipment and large structures make business activities more difficult to hide. Therefore, the low capital-labor ratio of the informal sector would be explained by the way taxes are enforced in developing countries, rather than financial friction.

a quantitative theory of entrepreneurship and credit market frictions that acknowledges: (i) plausible levels of informal hiring by formal businesses, (ii) possible heterogeneity in the degree of financial frictions across sectors, and (iii) an overlapping distribution of capital and debt in both sectors.

3 Model

We study an economy characterized by a large number of informal firms and informal workers, frictions in the financial markets, and a social security system. The framework builds on Ulyssea (2018) and extends it in two fundamental dimensions. First, we model capital accumulation and financial frictions. Second, there is an occupational choice decision: households decide whether to work for the market wage or to become an entrepreneur in the formal (f) or the informal sector (i).

These extensions are important for the focus of our paper. By modeling capital, we can examine how informality - jointly with financial frictions - affects capital accumulation decisions and the allocation of capital across sectors and entrepreneurs. Furthermore, by including an endogenous occupational choice on top of the informality decision, the model allows us to understand how the entrepreneurship rate is affected by changes in the economic environment. Since most of the entrepreneurs at the margin are small and informal, the entrepreneurship decision is potentially responsive to policies targeting informality.

3.1 Environment and Preferences

Time is discrete and the economy is in a steady-state competitive equilibrium. The economy is populated by a continuum of households that transit stochastically through two stages in their life: A working stage and a retirement stage. During the working stage, households make occupational choice decisions and are heterogeneous in their assets and the productivity of their entrepreneurial idea. Every period with probability π_z individuals keep the same business idea or, with probability $1-\pi_z$, they draw a new idea from a fixed distribution Γ_z.

A working-age individual faces a retirement shock every period with probability ρ_r. During the retirement stage, which lasts for T periods, individuals collect pensions, make consumption and savings decisions until they die with zero assets. When an individual dies, she is replaced by a newborn individual with zero assets and an initial idea drawn from Γ_z. The size of the population is normalized so that the mass of individuals in the working stage is 1.

3.2 Production Technology

Each period there is a unique output good y that can be consumed or invested. The output can be produced by establishments in the formal entrepreneurial sector (f), in the informal entrepreneurial sector (i), or the corporate sector (c). An establishment with productivity z in sector $j \in \{c, f, i\}$ produces output according to the following production function:

$$y = zq_j(k, l), \tag{1}$$

where (z, k, l) represents the productivity, capital, and labor of the establishment. The function q_j, which is allowed to vary with the establishment sector, is twice differentiable, strictly increasing, and strictly concave.

Entrepreneurial businesses. Each entrepreneur owns a unique entrepreneurial business, whose productivity is determined by the quality of her entrepreneurial idea z. Entrepreneurs supply inelastically their own labor $\bar{l}$ to their businesses.[10] Following Moll (2014), Buera and Shin (2013), and Midrigan and Xu (2014), the capital used by an entrepreneur with a units of assets, in sector $j \in \{f, i\}$, is limited by the collateral constraint:

$$k \leq \lambda_j a, \qquad \lambda_j \geq 1 \text{ and } a \geq 0. \tag{2}$$

Intuitively, λ controls the degree of credit frictions faced by the entrepreneur, where the limiting case $\lambda = \infty$ corresponds to a perfect capital market, and $\lambda = 1$ corresponds to the situation where all capital has to be self-financed. The degree of credit friction is allowed to differ across entrepreneurs in the formal and informal sectors.

Informal entrepreneurs do not pay payroll taxes nor sales taxes. Therefore, given factor prices, w and r, the profit function of an informal entrepreneur with assets a and entrepreneurial idea z is:

$$\begin{aligned} \pi_i(a, z; w, r) = \max_{k, l_i \geq 0} \; & zq_i(k, l) - (r + \delta)k - wl_i + (1 + r)a - c_i, \\ \text{s.t.} \quad & k \leq \lambda_i a, \\ & l = l_i + \bar{l}, \end{aligned} \tag{3}$$

where c_i is the fixed cost of operation in the informal sector and l_i the labor hired by the

[10] In the data, 89.8% of the informal entrepreneurs do not employ paid labor. In the next section, we set $\bar{l}$ to one.

entrepreneur. Informal entrepreneurs cannot hire formal workers.

Formal entrepreneurs pay payroll and sales taxes and are subject to a fixed cost of operation. We allow formal entrepreneurs to hire informal workers and avoid part of their payroll taxes. As argued before, explicitly modeling the intensive margin of informality is important as it can alleviate credit frictions for formal firms and it is empirically relevant in developing economies. Hiring informal workers, however, is not free of cost. Firms are subject to inspections and may suffer fines for labor law violations. Intuitively, the higher is the number of informal workers, the higher is the likelihood that the firm is caught and the monetary cost of the fine. Therefore, the cost of hiring informally, $\tau(l_i, l_f)$, is modeled as a convex and increasing function of the number of informal workers, $\partial\tau(\cdot)/\partial l_i > 0$, but possibly decreasing in the number of formal workers (consequently in the size of the firm), $\partial\tau(\cdot)/\partial l_f \leq 0$.[11] Profits of an entrepreneur with assets a and entrepreneurial idea z operating in the formal sector are given by:

$$\begin{aligned} \pi_f(a, z; w, r) = \max_{k, l_i, l_f \geq 0} & (1-\tau_y) z q_f(k, l) - (r+\delta)k - w(l - \bar{l}) - \tau_{ss} w l_f \\ & - w\tau(l_i, l_f) + (1+r)a - c_f, \\ \text{s.t.} \quad & k \leq \lambda_f a, \\ & l = l_i + l_f \geq \bar{l}, \end{aligned} \tag{4}$$

where l denotes total labor input (including entrepreneur's own labor), l_i and l_f are the number of informal and formal labor input, k is the capital input, τ_{ss} is the payroll tax used to finance social security, τ_y the sales tax, and c_f a fixed cost of operation incurred by formal entrepreneurs. As in Ulyssea (2018), we assume formal and informal workers are perfect substitutes in production. Since formal and informal employees perform the same tasks, there is no wage difference between the two types of workers so that total wage disbursements are given by $w(l - \bar{l})$.[12] Formal entrepreneurs choose the mix between formal and informal labor that minimizes total labor costs.

Corporate firms. The corporate sector is composed of a large number of establishments that are heterogeneous in their productivity and are owned by a representative mutual fund.[13]

[11] The convex cost function acts as a reduced form for the expected cost of being caught and receive a fine. It effectively imposes a limit on informal hiring.

[12] Also, since we abstract from non-wage benefits perceived by formal workers, there is no compensating wage differential.

[13] Although the literature on entrepreneurship typically abstracts from the corporate sector, a handful number of papers include it in their models (for instance, Quadrini (2000) and De Nardi and Cagetti (2006)).

The distribution of productivities z_c across corporate establishments is described by a fixed distribution Γ_{z_c}. Corporations cannot engage with any informal activity but are not subject to financial frictions. They accumulate capital and are owned by a representative mutual fund that distributes dividends to households. The value of a corporate firm solves:

$$\begin{aligned} V_c(z_c) = \max_{\{k_t,l_t\}_{t=1}^{\infty}} & \sum_{t=1}^{\infty} \left(\frac{1}{1+r}\right)^t d_t, \\ \text{s.t.} \quad & x_t = k_{t+1} - (1-\delta)k_t, \\ & d_t = (1-\tau_y) z_c q_c(k_t, l_t) - w l_t - w\tau_{ss} l_t - c_c - x_t \end{aligned} \tag{5}$$

where c_c is the fixed cost of operation of corporate establishments and d_t stands for the dividends distributed. In steady state (constant prices and taxes), the value of a firm with productivity z_c is

$$V_c(z_c) = \frac{d^*(z_c)}{r} \tag{6}$$

where $(k^*(z_c),\, l^*(z_c))$ solves the problem in (5), given constant factor prices and tax policies, and d^* represents period dividends under the optimal production and investment plan. Note that d^* and $V_c(z_c)$ are increasing in z_c. Given the presence of a fixed cost of operation, there is a threshold value $\overline{z}_c$ such that the value of a firm is positive for all $z > \overline{z}_c$.

Let $\overline{M}$ be the mass of corporations. In every period, the aggregate dividends paid by the representative mutual fund are

$$D = \overline{M} \int_{\overline{z}_c}^{\infty} d^*(z_c) d\Gamma_{z_c}. \tag{7}$$

Finally, in equilibrium, the rate of return of investing in the mutual fund should be equal to the rate of return in deposits r. Denoting the price of one share of the mutual fund by P and normalizing the total number of shares to one, gives the following no-arbitrage condition:

$$\frac{P+D}{P} = 1 + r \Rightarrow P = D/r. \tag{8}$$

The introduction of the nonentrepreneurial sector comes with two advantages. First, financial frictions depress the demand for capital and the interest rate. By modeling corporations, the equilibrium interest rate will be positive and bounded away from zero. Second, the entrepreneurial decision introduces non-convexities that may generate steps in the aggregate excess demand functions. The corporate sector mitigates this problem by introducing additional demand for capital and labor.

3.3 Household Problem

We start with the problem of a household recently retired from the labor market. A newly retired household with initial assets a_0 and pension benefit b solves the following deterministic problem:

$$V_r(a_0; b) = \max_{\{c_t, a_t\}_{t=1}^{T}} \sum_{t=1}^{T} \beta^{t-1} u(c_t), \tag{9}$$
$$\text{s.t.} \quad c_t + a_t = b + a_{t-1}(1+r),\ a_0 \text{ given.}$$

The state of a household in the working stage is given by her assets a, an entrepreneurial idea z, and her initial occupation (the occupational choice is a dynamic decision). The household chooses how much to consume, save, and the occupational choice they will start in the next period.

The entrepreneurship decision is costly and depends on whether the entry is into the formal or informal sector. To enter in the formal (informal) sector a household must pay an entry cost c_e^f (c_e^i). The differential between the entry costs of the formal and informal sector, $c_e^f - c_e^i > 0$, captures the costs of registering and complying with the regulations necessary to operate a formal business. Let $W_j(a, z)$ be the value of a worker with assets a that chooses to implement the business idea z in the sector $j = \{i, f\}$. This value satisfies the following equation:

$$W_j(a, z) = \max_{c, a'} u(c) + \beta \left[(1-\rho_r) V_j(a', z) + \rho_r V_r(a') \right], \tag{10}$$
$$\text{s.t.} \quad c + a' + c_e^j = w + (1+r)a,$$

where V_j represents the value of an entrepreneur in the sector j and V_r is the value of retirement defined in (9). The value of a worker that chooses to remain a worker next period is given by

$$W_w(a, z) = \max_{c, a'} u(c) + \beta \left[(1-\rho_r) \left(\pi_z W(a', z) + (1-\pi_z) \int W(a', z') d\Gamma_{z'} \right) + \rho_r V_r(a') \right],$$
$$\text{s.t.} \quad c + a' = w + (1+r)a. \tag{11}$$

Note that in the next period the worker might get a new business idea with probability π_z. The value of a worker is the outer envelope over the value of the three occupational choices:

$$W(a, z) = \max\{W_w(a, z), W_f(a, z), W_i(a, z)\} \tag{12}$$

At each period, the entrepreneur decides whether to close her business or continue operating. If an entrepreneur decides to exit, she will become a worker next period with a new business idea drawn from Γ_z. For simplicity, we assume that a business cannot directly transit between informal and formal status, an assumption that is consistent with the fact that the vast majority of formal businesses start as formal upon being created and that formal businesses cannot choose to become informal.[14] Finally, with probability $1-\pi_z$, the entrepreneur is forced to shut down the business (e.g. the business idea dies). In this case, she will become a paid worker and draw a new business idea.

The value of an entrepreneur of type $j = \{i, f\}$ is

$$V_j(a,z) = \max\{V_j^j(a,z), V_j^w(a,z)\}, \tag{13}$$

where V_j^j is the value function of an entrepreneur that stays operating and V_j^w of an entrepreneur that decides to exit and become a worker in the next period. In recursive form, these value functions are given by

$$\begin{aligned} V_j^j(a,z) =& \max_{c,a'} u(c) + \beta(1-\rho_r)\left[\pi_z V_j(a',z) + (1-\pi_z)\int W(a',z')d\Gamma_{z'}\right] + \beta\rho_r V_r(a'), \quad (14)\\ \text{s.t.} \quad & c + a' = \pi_j(a,z), \\ V_j^w(a,z) =& \max_{c,a'} u(c) + \beta(1-\rho_r)\int W(a',z')d\Gamma_{z'} + \beta\rho_r V_r(a'), \quad (15)\\ \text{s.t.} \quad & c + a' = \pi_j(a,z), \end{aligned}$$

where $\pi_i(a,z)$ and $\pi_f(a,z)$ are the indirect profit functions defined in (3) and (4).

3.4 Government Budget and Market Clearing Conditions

The social security system is assumed to pay a fixed pension benefit to all retired households. The excess of government tax revenue (from all sources) over pensions payments is spent on consumption of a public good (G). The public good G does not affect the marginal utility of private consumption and thereby has no consequences on household decisions.

Denote by F the invariant measure of households across states (a,z,j) when production takes place. The output of a type j entrepreneur, net of the fixed cost of operation, can be written as function of the state of the entrepreneur and its optimal production plan according to $y(a,z,j) = zq_j(k(a,z,j), l(a,z,j)) - c_j$. The output (net of the operating fixed cost) of a

[14] La Porta and Shleifer (2014) provides evidence that on average, among 14 Latin American countries, more than 90 percent of formal businesses are registered upon creation.

corporate establishment with productivity z is written as $y_c(z) = zq_c(k(z), l(z)) - c_c$. In a steady state equilibrium the following market clearing conditions hold:

$$\sum_{j=\{i,f\}} \int_{(a,z)} l(a,z,j)dF(a,z,j) + \overline{M} \int_{\bar{z}_c} l_c(z)d\Gamma_{z_c} = 1; \tag{16}$$

$$\sum_{j=\{i,f\}} \int_{(a,z)} k(a,z,j)dF(a,z,j) + P = \sum_{j=\{w,i,f\}} \int_{(a,z)} adF(a,z,j) + A_{ret} \tag{17}$$

$$\sum_{j=\{w,i,f\}} \int_{(a,z)} c(a,z,j)dF(a,z,j) + C_{ret} + \delta K + G = \sum_{j=\{i,f\}} \int_{(a,z)} y(a,z,j)dF(a,z,j) + \dots$$
$$\cdots + \overline{M} \int_{\bar{z}} y_c(z)d\Gamma_{z_c} - \int_{(a,z)} (c_e^f I_w^f(a,z) + c_e^i I_w^i(a,z))dF(a,z,w), \tag{18}$$

where C_{ret} and A_{ret} denote aggregate consumption and savings of all retired households, K represents the aggregate stock of capital among all establishments in the economy, G government spending, and $I_w^j(a,z)$ an indicator function that is equal to one when the worker decides to be an entrepreneur in sector $j \in \{i,f\}$ in the next period. Equation (16) states that the sum of labor demand across all establishments equals the mass of households in the working stage, which is normalized to 1.[15] Equation (17) states that the sum of the capital across entrepreneurs and the equilibrium value of corporations (P) should be equal to aggregate savings of retired and non-retired households.[16] The final condition says that the sum of aggregate consumption, investment, and government expenditures is equal to the aggregate supply of output net of operating fixed cost and entry cost.

4 Baseline Economy

We now fully specify our baseline economy. First, we specify and motivate the functional forms chosen for the analysis in the paper. Second, we explain our calibration strategy and present the calibration results for the Brazilian economy. We also discuss the performance of our baseline economy along non-targeted dimensions.

[15] Recall that the entrepreneurs' labor supply, $\bar{l}$, is set to 1.

[16] Since capital in the corporate sector is internally accumulated by firms, it does not appear in the market-clearing condition for capital.

4.1 Functional Forms

Before proceeding to the calibration of the model economy, we first specify the functional forms that characterize the model economy.

Preferences. We assume a log utility, $u(c) = \log(c)$. The utility function of public goods is not specified as it is inconsequential for the analysis in the remainder of the paper.

Entrepreneurial ideas. Entrepreneurial ideas are assumed to be drawn from a Pareto distribution, with c.d.f

$$\Gamma_z(x) = \begin{cases} 1 - (\frac{z_0}{x})^{\xi} & x \geq z_0 \\ 0 & x < z_0, \end{cases} \tag{19}$$

where z_0 is the minimum possible entrepreneurial value and ξ governs the tail of the distribution.[17]

Cost of hiring informal workers. The cost function faced by formal entrepreneurs when hiring informal workers is an extension of the one considered by Ulyssea (2018). An entrepreneur that uses l_i informal labor and l_f formal labor incurs the resource cost

$$\tau(l_i, l_f) = \tau_{1,f}(l_i)^2 \left(\frac{l_i}{l_i + l_f}\right)^{\omega} \qquad \omega \geq 0, \tag{20}$$

which is assumed to be reduced form for the expected costs of being detected by the government. These costs are assumed to increase in number of informal workers and, if $\omega > 0$ to decrease with the total number of workers hired by the entrepreneur. Formal entrepreneurs choose the optimal mix between formal and informal workers to minimize total labor costs. Equating the marginal cost of formal and informal workers yields the following relationship between the number of informal workers and total employment:[18]

$$\ln(l_i) = \frac{1}{1+\omega} \ln\left(\frac{\tau_{ss}}{\tau_{1,f}(2+\omega)}\right) + \frac{\omega}{1+\omega} \ln(l_i + l_f). \tag{21}$$

The parameter ω controls how the number of informal workers rises with firm size. Conditional on the size of the firm, larger values of ω are associated with more informal workers. Note that under the extreme case where $\omega = 0$, the cost function is exactly the one as in

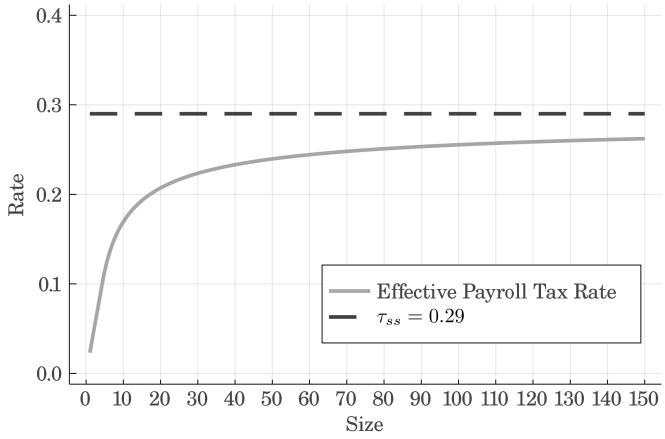
[17] Since we discretized the distribution when solving the model numerically, the effective c.d.f is truncated. For more details see Appendix C.

[18] Appendix B.1 provides details of derivations.

Ulyssea (2018). In this case, all formal firms hire at most a fixed number l_i^* of informal workers, and the first l_i^* workers are always informal. Note that the selected functional form has one convenient property. Even though the number of informal workers is increasing in firm size (if $\omega > 0$), the fraction of informal workers is decreasing.[19] The empirical relationship between the size of an establishment and the number of informal workers implied by Equation (21) will be exploited in the calibration of ω.

Since the fraction of informal labor decreases with firm size, larger firms pay relatively more payroll taxes per worker. This implies that the intensive margin of informality acts as a size-dependent policy that implicitly subsidizes small firms. Figure 2 plots the effective payroll tax rate for different firm size. Small firms hire little formal labor and most of their additional labor cost comes from the resource cost $\tau(l_i, l_f)$. As firms grow larger, the relative number of formal workers increases so that the fraction of total labor costs accounted by payroll taxes dominate the resource cost of informal workers. In the limit, the effective payroll tax rate converges to the actual payroll tax.

Figure 2: Effective Payroll Tax



Notes: The figure plots the Payroll Tax Rate (τ_{ss}) against the Effective Payroll Tax Rate. The Effective Payroll Tax Rate is calculated using the baseline calibrated values of Table 4.

Production function. The production function is assumed to take the form

$$y = z \left(k^{\alpha_j} l^{1-\alpha_j}\right)^{\theta_j}, \tag{22}$$

[19] Our calibration implies that an establishment with 10, 100, or 1000 workers hires 6.4, 18.4, and 52.1 informal workers respectively.

where $\theta_c = \theta_f \geq \theta_i$ and $\alpha_c = \alpha_f \geq \alpha_i$. We remark that allowing for establishments in the informal economy to operate with a (relatively) low span of control (θ) and low capital intensity (α) allows the model economy to match important aspects of the data.[20]

Discussion. We find it useful to end this section with a discussion of how our baseline model works. These insights will be useful to develop some intuition on the calibration of the model economy. Note that the capital used by formal and informal establishments satisfy:

$$(1-\tau_y)MPK_f = (1-\tau_y)\alpha_f\theta_f y_f/k_f = r+\delta+\mu_f, \tag{23}$$

$$MPK_i = \alpha_i\theta_i y_i/k_i = r+\delta+\mu_i, \tag{24}$$

where μ_f and μ_i represent the Lagrange multipliers associated to the borrowing constraint faced by formal and informal entrepreneurs. These two expressions yield

$$\frac{k_f/y_f}{k_i/y_i} = (1-\tau_y)\ \frac{r+\delta+\mu_i}{r+\delta+\mu_f}\ \frac{\alpha_f\theta_f}{\alpha_i\theta_i} \tag{25}$$

The capital to output ratio of small formal businesses in Brazil (with less that 5 workers) is about a factor of 1.32 the one of informal firms (see Table A.4). Equation (25) shows that in our model economy this ratio can be expressed as the product of three terms. The first term is less than one since $\tau_y > 0$. The second term will tend (on average across establishments) to be less than 1 as small formal businesses are more likely to be borrowing constrained than informal businesses ($\mu_f > \mu_i$). Hence, the calibration of the baseline economy will set $\theta_f\alpha_f > \theta_i\alpha_i$ in order to match the fact that formal firms have a higher capital to output ratio than informal businesses of similar size.

Now consider the labor demand decision of formal and informal entrepreneurs. The marginal worker is chosen so that

$$(1-\tau_y)MPL_f = (1-\tau_y)(1-\alpha_f)\theta_f y_f/l_f = w(1+\tau_{ss}), \tag{26}$$

$$MPL_i = (1-\alpha_i)\theta_i y_i/l_i = w, \tag{27}$$

where, for simplicity, we set $\omega = 0$ in the cost function (see 20) and assumed that the marginal worker of the formal entrepreneur is formal.[21]

[20] We interpret the differences in the production technology across sectors as the evasion cost to produce in the informal sector. Alternatively, one could follow Ordóñez (2014) and micro-found the probability of detection in the informal sector as a size-dependent policy in capital.

[21] Assuming that the marginal worker of the formal entrepreneur is informal and $\omega > 0$ does not change the result. In this case, instead of τ_{ss}, equation (26) would have $\partial\tau(\cdot)/\partial l_i$ which is also a positive term.

Combining these expressions yield an expression for the ratio of gross output (including fixed costs of operation) per worker of a formal to informal business:

$$\frac{y_f/l_f}{y_i/l_i} = \frac{1+\tau_{ss}}{1-\tau_y} \frac{(1-\alpha_i)\theta_i}{(1-\alpha_f)\theta_f}. \tag{28}$$

Value added per worker in establishments of type j can be expressed as,

$$\frac{VA_j}{l_j} = \frac{y_j - c_j}{l_j} = \frac{y_j}{l_j}(1 - c_j/y_j) \tag{29}$$

Combining the last two expressions yields:

$$\frac{VA_f/l_f}{VA_i/l_i} = \frac{1+\tau_{ss}}{1-\tau_y} \frac{(1-\alpha_i)\theta_i}{(1-\alpha_f)\theta_f} \left(\frac{1-c_f/y_f}{1-c_i/y_i}\right) \tag{30}$$

Hence, given the span of control parameters (θ_f, θ_i), the calibration will set $\alpha_f > \alpha_i$ to account for the fact that the value added (conditional on the number of workers) is 2.3 times higher for formal when compared to informal entrepreneurs.

4.2 Parameter Values Set Exogenously

The model period is set to a year. The retirement probability is chosen so that the expected working life of a household corresponds to 40 years ($\rho_r = 1/40$). Retired households live for 16 years ($T = 16$).

Entrepreneurs. The parameters of the production function of formal entrepreneurs are set to standard values, $\alpha_f = 0.3$ and $\theta_f = 0.90$. The corresponding parameters for the production function of informal entrepreneurs will be calibrated internally. The depreciation rate of capital is $\delta = 0.06$. The labor services supplied by entrepreneurs in their businesses is normalized to 1 ($\bar{l} = 1$), so the owner of the businesses is interpreted to supply the same labor as an additional worker. This also implies that aggregate labor supply is equal to the unity and does not change with the share of entrepreneurs in the economy.

The persistence of entrepreneurial ideas is set to a value of $\pi_z = 0.90$, a standard value in the literature. Moreover, this value is roughly consistent with the average business tenure in Brazil which is around 10 years (see Table A.4).

The entry cost of informal entrepreneurs is set to zero, which means that entry into formal entrepreneurship is, effectively, the only dynamic occupational choice in our baseline economy.[22]

[22]We have also calibrated the model economy allowing for positive entry costs of informal businesses but

Based on equation (21), ω is recovered from the slope of the regression of the number of informal workers on firm size and a constant using the ECINF data. Note that, conditional on τ_{ss}, the estimated constant suggests a value for $\tau_{1,f}$. Nevertheless, since the sample covers only small business, it is unlikely that the coefficients jointly match well the aggregate share of informal workers at formal firms. Hence, our strategy involves to fix the estimated value of ω ($\omega = 0.8454$), and calibrate $\tau_{1,f}$ to match the aggregate data.

Taxes. The taxes are assigned their statutory values, specifically $\tau_y = 0.2925$ and $\tau_{ss} = 0.29$.[23] Following the OECD Pension Statistics, the pension replacement rate is set to 70% of the equilibrium wage.

Corporate sector. Productivity in the corporate sector, z_c, is Pareto distributed with a location parameter z_{cmin} and tail parameter ξ_c. These are set to be $z_{cmin} = 2$ and $\xi_c = 3$. We assume that corporations are subject to a relatively large fixed cost operation ($c_c = 5$), so that establishments in the corporate sector are large. Given these parameters, the mass of corporate firms $\overline{M}$ determines the aggregate market valuation of corporate firms (see (8)).

4.3 Parameter Values Set by Solving the Model Economy

The remaining 12 parameters are chosen to minimize a loss function that consists of the square deviations between some selected model statistics and their data counterparts. In particular, we pin down the parameters of the production function of informal entrepreneurs (θ_i and α_i), the mass of corporate firms $\overline{M}$, the discount factor β, the location and tail parameter of the distribution of entrepreneurial ideas (z_0 and ξ), the fixed cost of operation of formal and informal businesses (c_f and c_i), the entry cost of formal businesses c_e^f, the parameter governing the cost of hiring informal workers by formal businesses ($\tau_{1,f}$), and the parameters on the collateral constraint faced by formal and informal entrepreneurs (λ_f and λ_i).

Although the equilibrium outcomes will be jointly determined by all of the parameters, it is useful to discuss how each of the parameters connects with some moments of interest. The discount factor, β, affects the equilibrium rate of return on capital and hence the K/Y ratio among formal businesses. The parameter θ_i is used to match the ratio of K/Y between (small) formal and informal businesses, as it determines the capital intensity of informal firms. Similarly, α_i is used to pin down the ratio of value-added between formal and informal

the estimation implied negligible entry costs without a noticeable improvement in the fit of the data targets. Hence, for simplicity, we eliminated this parameter from the baseline estimation of the model economy.

[23] For a discussion of the tax values see Ulyssea (2018).

businesses (with up to 5 employees). As discussed in Section 4.1, the lower α_i relative to α_f, the higher will be the ratio of value-added between formal to informal businesses (conditional on operating fixed cost and (θ_f, θ_i)). The mass of corporate firms, $\overline{M}$, is directly related to the stock market valuation of corporations to GDP and has a first-order effect on the equilibrium interest rate. The parameters λ_i and λ_f determine the credit to output ratio of informal and informal entrepreneurs. The entry cost c_e^f affects the share of formal businesses in the economy. The parameter $\tau_{1,f}$ determines the mass of informal workers in formal establishments. The parameters z_0 and ξ, together with the fixed operating costs c_f and c_i, determine the size distribution of formal and informal establishments. Besides, the fixed cost of operation of informal entrepreneurs affects the profitability of informal businesses and, hence their mass and the labor force employed by them.

With these connections in mind, our calibration targets the following moments in the Brazilian data:

1. The share of 35% of informal paid workers among total paid workers.
2. The share of informal paid workers hired by formal businesses of 70%.
3. The fraction of formal businesses of 0.30.
4. A capital to output ratio of 1.38 among formal entrepreneurs with less than 6 workers.
5. A credit to output ratio of 0.43 among formal entrepreneurs.
6. A capital to output ratio of 1.04 among informal entrepreneurs with less than 6 workers.
7. A credit to output ratio of 0.31 among informal entrepreneurs.
8. A value added per worker ratio between formal and informal firms (with less than 6 workers) of 2.3.
9. The size distribution of formal establishments.
10. The size distribution of informal establishments.
11. The value of the stock market to GDP of 40%.[24]

The totality of the calibration targets are listed in Table 4. Appendix A.2 explains how the data targets were obtained.

[24]The stock market to GDP in Brazil was 32% in 2003 and 43% in 2004. We target 40% to avoid business cycle fluctuations.

4.4 Calibration Results

The model economy accounts reasonably well for the targeted moments. Table 4 presents the calibration results (parameter values, targets, and model moments). We now describe how the calibrated parameters help to attain the desired targets.

Table 4: Calibration Results: Baseline Economy

Parameters	Values	Target	Model	Data
θ_i	0.653	Share of Informal Workers	0.349	0.350
$\tau_{1,f}$	0.023	Share of Informal Workers in Formal B.	0.713	0.700
c_e^f	0.089	Share of Formal Firms	0.274	0.300
β	0.931	K/Y Formal (≤ 5)	1.388	1.380
α_i	0.162	K/Y Informal	1.039	1.040
λ_f	1.490	Credit/GDP Formal (≤ 5)	0.440	0.431
λ_i	1.506	Credit/GDP Informal	0.315	0.311
		VA Ratio Formal to Informal (≤ 5)	1.800	2.317
$\overline{M}$	0.625×10^{-13}	Stock Market Value to GDP	0.414	0.400
		Formal Size: ≤ 5	0.775	0.701
z_0	1.351	Formal Size: 6 - 10	0.113	0.141
ξ	7.698	Formal Size: 11 - 20	0.055	0.083
c_f	0.243	Formal Size: 21 - 50	0.035	0.048
c_i	0.635	Informal Size: ≤ 2	0.888	0.957
		Informal Size: ≤ 5	1.000	0.998

The model captures relative well that most businesses in Brazil are informal. The fraction of formal businesses is 0.27 in the model economy relative to 0.30 in the data. The share of informal paid workers among paid workers is 0.35 in the model and data. Moreover, formal businesses hire about 71% of paid informal workers. The model captures that informality is pervasive in the Brazilian economy, both along the intensive and extensive margin of informality.

The model is consistent with the fact that conditional on size there are important differences between formal and informal businesses. First, the ratio of value-added between formal to informal businesses (with less than 6 employees) is 1.8 relative to 2.3 in the data. Second, informal businesses are much less capital intensive than formal businesses: The capital to output ratio is 1.04 for the former and 1.40 for the latter. These ratios in the data are 1.04 and 1.38. To account for these observations, the model implies that informal businesses have a low span of control ($\theta_i = 0.65$) and a low capital share ($\alpha_i = 0.16$) relative to formal businesses ($\theta_f = 0.90$ and $\alpha_f = 0.30$). The model accounts for the fact that the credit to output ratio of formal businesses, conditional on size, is higher than that of informal businesses,

even though λ_f and λ_i are about the same. The fact that formal businesses are more capital intensive than informal businesses is important for the relatively high borrowing of formal businesses.

The model implies that informal businesses tend to be much smaller than formal businesses. While all informal businesses have less than 5 workers, only 76% of formal businesses have less than 5 workers (70% in the data). In the model, the fraction of firms with more than 20 workers is about 14%, relative to 16% in the data.

The stock market value of corporations in the model economy is about 41% of GDP, which is consistent with the data target. This is attained with a relatively low fraction of firms $\overline{M} = 0.6 \times 10^{-13}$ and with an equilibrium return on capital of 3.1%. The model is calibrated so that corporations are large: There are no firms with less than 20 workers, and most corporations have more than 250 workers.

4.5 Model Performance in Non-targeted Dimensions

In this section, we discuss how the model performs on non-targeted moments of the economy. Table 5 shows how the model fares along key macroeconomic dimensions. The baseline economy implies a high rate of entrepreneurship, a feature of the Brazilian data. While the model implies that 24% of the working-age population are entrepreneurs, in the data this statistic is about 32%. A notorious characteristic of emerging economies is their low labor share of the national income relative to developed economies. The model replicates well this feature of the Brazilian economy. It predicts the labor share to be roughly 50%, while in the data is around 48%.[25]

Aggregate tax revenue. Table 5 also shows aggregate tax revenue as a fraction of GDP, both for social security contributions (including other payroll taxes) and for sales tax. The sum of these two revenue sources accounts for around 64% of the total government revenue and 56% of the federal government revenue (see Table A.5). A fundamental question of this paper is how informality impacts the government's capacity to finance a social security system. This requires a good model performance concerning the aggregate contribution to social security relative to GDP. The aggregate revenue from social security contributions and

[25] We define labor share as the share of labor compensation of employees (wage payments) over gross domestic product. Since this does not include own-account workers nor entrepreneur's income, it usually serves as a lower bound for the estimate of the labor share in developing economies. We decide to use this measure since it gives a clear mapping of the data into the model. Another way to measure labor share in economies with high rates of entrepreneurship is to include the labor share of income of self-employed individuals. This requires to assume that self-employed individuals use the same proportion of capital and labor as the rest of the economy. In the case of Brazil, once we make this adjustment the labor share increases to 0.530.

Table 5: Model Performance along Selected Macroeconomic Moments

Variable	Model	Data
Fraction of Entrepreneurs	0.240	0.322
Labor Share	0.502	0.480
Social Sec. Contribution/GDP	0.061	0.065
Sales Tax/GDP	0.252	0.168
(Sales Tax + Income Tax)/GDP	0.252	0.236
Employment Share by Firm Size		
Micro ($size \leq 5$)	0.186	0.148
Small ($5 < size \leq 10$)	0.064	0.086
Large ($size > 10$)	0.750	0.766

Notes: Labor share is the wage payments on national income (does not include self-employed income). Social Security Contribution includes payroll taxes plus SS contribution (Table A.5). Sales Tax includes federal, state and local government taxes. Employment Share by firm size includes both formal and informal paid workers and is calculated using the 2003 PME. Sources: PNAD (2003), PME (2003), Penn World Table 8.0, and IMF Government Finance Statistics (2006).

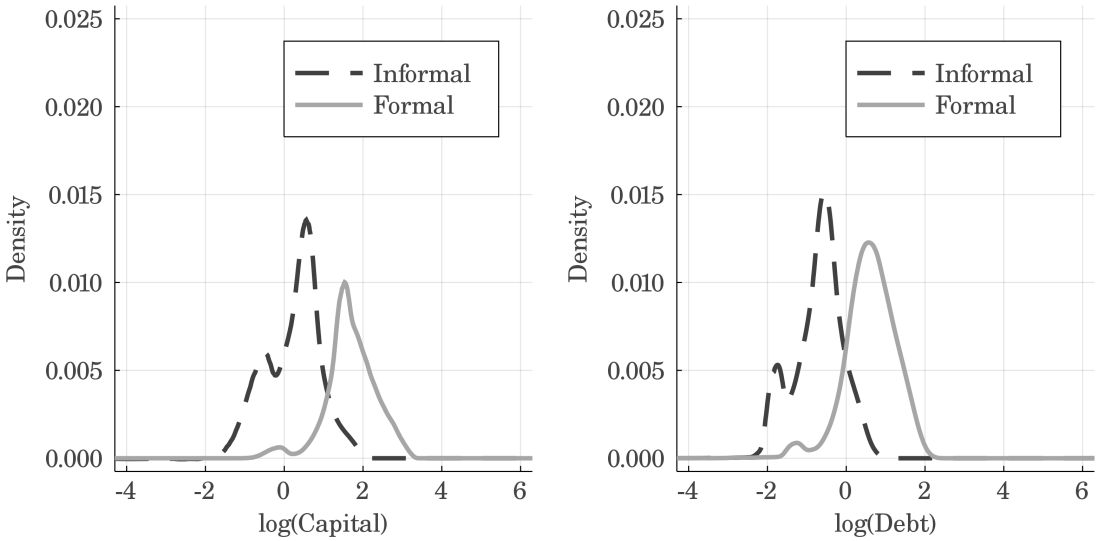
payroll taxes is 6.1% in the model and 6.4% in the data. The fact that the model matches the data quite closely is reassuring for our investigation of the financing of social security in Brazil. Regarding the sales tax, the model predicts that the aggregate revenue to GDP is about 25.2% compared to 16.8% in the data. Since our model economy abstracts from income taxes and informal entrepreneurs are likely to evade income taxes (on top of sales taxes), we believe it is reasonable to view the value-added tax in our model economy as representing both income and sales taxes. Under this interpretation, the predictions of our theory are well aligned with the data since the tax revenue from sales and income taxes amount to 24% of GDP in Brazil.

Employment share by firm size. The model is calibrated to match the firm size distribution of the formal and informal sector. One question is whether the two entrepreneurial sectors, together with the corporate sector, imply the correct distribution of workers among different business size. Table 5 shows the employment share by firm size implied by the model relative to the data from Table 2.

The model matches the fact that in Brazil most paid workers are hired by large firms. Although the model slightly overstates the employment fraction in small firms (18.6% relative to 14.8% in the data), it correctly predicts that large firms account for about 75% of the paid employees both in the model and in the data.

Distributions of capital and debt. In Section 2, we documented that the support of the distribution of capital and debt of small businesses of the formal and informal sector overlaps. Figure 3 replicates the same picture in the model.

Figure 3: Distribution of Capital and Debt: Model



Notes: The figure plots the model invariant distribution of capital (left panel) and debt (right panel) for firms with less than five workers (including the entrepreneur). The distribution is smoothed using a local linear regression with 5% smoothing span.

A question posed in the empirical section is whether the overlapping distribution arises due to differences in collateral constraint, selection, or both. Here we attempt to shed light on this issue. We remark that, in the baseline economy, the estimated parameters of the collateral constraint are roughly the same in both sectors ($\lambda_f = 1.49$ and $\lambda_i = 1.50$). The fact that some informal firms use more capital than some formal firms - despite capital intensity being higher in the formal sector - points to the coexistence of credit-constrained formal entrepreneurs with unconstrained informal entrepreneur. The reason is that high-productivity entrepreneurs self-select into the formal sector in the hope of accumulating capital and, eventually, overcoming their borrowing constraints. In contrast, unconstrained low-productivity entrepreneurs operate at their optimal scale in the informal sector and have higher access to credit than more productive entrepreneurs in the formal sector.

4.6 Understanding Informality in the Baseline Economy

The baseline economy mimics informality in Brazil along the extensive and intensive margins. Since informality in the baseline economy is affected by entry costs and financial frictions, we

can evaluate their role in understanding informality along each of the two margins. Moreover, we find it useful to validate our model by comparing our results with empirical studies.

Table 6 reports how informality is affected by eliminating entry costs and financial frictions in the baseline economy. The removal of entry costs diminishes the mass of informal entrepreneurs from 0.17 to 0.12. It also increases the fraction of paid workers hired as informal workers from 0.35. to 0.38. Hence, the changes in the extensive and intensive margin of informality have opposite signs, making the overall reduction in informality small. The most important change in the occupational structure is a reallocation of entrepreneurs from the informal to the formal economy, keeping the entrepreneurship rate roughly constant. This result contrasts with the findings in Ulyssea (2018). He finds a large effect of entry costs on the entrepreneurship rate in an economy with no financial friction and no occupational choice.

In line with our results, previous empirical studies found positive but small effects of the reduction in the entry cost on the entry of formal entrepreneurs and on overall informality. For instance, Kaplan et al. (2011) and Bruhn (2011) exploit a reform that simplifies business registration procedures in selected industries in Mexico and found that the reform increased the number of registered businesses by 5%. Other studies in developing countries found similar results (Bruhn and McKenzie, 2014). In Brazil, Monteiro and Assunção (2012) found that bureaucratic simplification for small firms increased formal licensing in the retail sector by 13 percentage points (with no effects in the other sectors). Although their results are large relative to other studies, the reform studied in their paper impact mostly ongoing red tape and tax bureaucracy, which likely maps more closely to the cost of operating businesses than entry costs. Finally, we point out that none of the empirical papers addressed a full elimination of the entry costs, which in the model encompasses not only registration costs but also technological differences across sectors. Under this view, our model experiment is likely more extreme than the empirical studies. Nevertheless, it is reassuring that informality is far from being accounted for by entry costs alone in our model.

The elimination of financial frictions reduces the fraction of informal entrepreneurs from 0.164 to 0.014 and informality among paid workers from 35% to 28%. The elimination of financial frictions leads to a reallocation of resources towards productive entrepreneurs. This effect increases the equilibrium wage rate and diminishes the number of low productivity entrepreneurs in operation. The intensive margin of informality also diminishes since high productivity entrepreneurs are less likely to hire informal workers. The elimination of financial frictions causes a decrease in the entrepreneurship rate of 13 percentage points.

Our results emphasize the importance of financial frictions in accounting for informality. Although the cross-country relationship between financial development, entrepreneurship

Table 6: Occupational Choice

	Baseline Economy	No Entry Costs	No Financial Frictions
Informal Ent.	0.169	0.121	0.014
Formal Ent.	0.07	0.135	0.095
Paid Workers	0.761	0.743	0.891
Inf. among Paid Workers	34.9%	38.2%	28.2%

Notes: The table displays the changes of removing entry costs ($c_e^f = 0$) and financial frictions $\lambda_f = 100$ relative to the baseline economy.

and informality is well-documented, evidence on the causal link between financial development and informality is still scarce. Rajeev and Gupta (2019) exploit a large expansion in banking infrastructure in India and find that financial access shifts workers from informal entrepreneurship into formal employment. In particular, they find that moving to a district with twice as many branches decreases the micro-entrepreneurship in about 6.4 p.p. (relative to the mean of 18%). In Brazil, Catão et al. (2009) find that sectors that rely more on external finance experienced higher rates of employment formalization through the large supply-side driven expansion of credit.[26] The increase in formalization was largely driven by an employment shift from small self-employment entrepreneurs to large firms.[27] In general, the large effects of financial reforms corroborate the results of the model.

5 Quantitative Experiments

We now assess the macroeconomic effects of informality in Brazil, the role played by the interaction of financial frictions and informality, and the costs of funding social security.

5.1 Assessing the Effects of Informality in Brazil

We find that informality in Brazil is quite costly. Table 7 summarizes all the finds. Whereas one would expect that higher taxation of economic activity caused by the elimination of

[26] In particular, they found that a 10% increase in aggregate credit to firms over GDP increased formalization by 6.5 percentage points in the most financially dependent sector relative to the least financially dependent sector.

[27] Moreover, in 2005, there was a large change in the Brazilian bankruptcy law. A few empirical studies documented that the law change had a positive impact on debt and investment of formal firms (Araujo et al. (2012), Ponticelli and Alencar (2016)). We are not aware of any causal study that analyzes empirically the effect of the law on the informality rates.

informality should depress economic activity, we find that the elimination of the informal economy leads to a substantial increase in both output (9.3%) and the fraction of taxes collected per unit of production (25%). The key to this result is that the combination of informality with financial frictions generates a competitive advantage for the operation of small businesses, thereby leading to a large misallocation of productive resources. As a result, in the presence of financial frictions, the elimination of informality reduces the mass of entrepreneurs more than a half (from 0.169 to 0.073), increases aggregate capital by 14.3%, and rises TFP by 5.4%. The improved allocation of productive resources explains why output rises by 9.3% despite the increase in effective taxation. On the contrary, in the absence of financial frictions, the increase in effective taxation caused by the elimination of informality *reduces* output by about 1%.

The effects of policies that target informality along the extensive or the intensive margin have quite distinctive effects in our baseline economy with financial frictions. The extensive margin of informality gives a competitive advantage to unproductive entrepreneurs who tend to operate on a small scale, reinforcing the negative effects of financial frictions on resource allocation. As a result, its elimination leads to the largest output gains (11.6%). The intensive margin of informality allows productive entrepreneurs, who desire to operate on a large scale and tend to be borrowing constrained, to hire some workers-off the books. It acts as a subsidy that helps undo the negative impact of credit constraints on productive entrepreneurs. This mechanism explains why the elimination of the intensive margin of informality has disastrous effects on macroeconomic variables: Output decreases by 9.4%, capital by 11.4%, and TFP by 6.4%.

Our results highlight the importance of the substitution between the two margins of informality and the response of occupational choices for evaluating the impact of formalization policies. We find that when the intensive margin is shut down, the mass of informal entrepreneurs rises from 0.17 to 0.28 because of two channels. First, many formal entrepreneurs in the baseline economy shift to the informal economy when they cannot hire workers off the books. Second, in general equilibrium, the decrease in the wage rate discourages working for a wage. As a result, informality remains high because there is a large number of informal entrepreneurs and a low number of formal paid workers. Similarly, shutting down informality on the extensive margin leads to an increase in informality on the intensive margin as the fraction of paid workers hired informally rises from 35% in the baseline economy to about 40%. In sum, shutting down informality requires confronting both margins of informality.

We find that the intensive and extensive margins of informality have quite different effects on government tax revenue. Moreover, the interaction between the two margins makes their joint effect on tax revenues different from the sum of their individual effects. Shutting down

Table 7: Effects of Informality in the Baseline Economy

	No Informality	No Extensive Margin	No Intensive Margin
Panel 1: Change in Macroeconomic Aggregates (%)			
Agg. Output	9.3%	11.6%	-9.4%
Agg. Capital	14.3%	14.1%	-11.4%
TFP	5.4%	7.7%	-6.4%
Tax Rev./GDP	25.5%	10.6%	-8.7%
Panel 2: Occupational Choice			
Informal Ent.	0.000	0.000	0.281
Formal Ent.	0.073	0.192	0.006
Paid Workers	0.927	0.808	0.713
Inf. among Paid Workers	0.0%	39.9%	22.6%
Panel 3: Change in Government Tax Revenue (%)			
S.S. Tax Rev.	81.5%	0.0%	8.8%
Sales Tax Rev.	26.4%	29.1%	-23.6%
Total Tax Rev.	37.1%	23.4%	-17.3%

Notes: The table displays the changes of removing informality relative to the baseline economy. The baseline economy has a fraction of 0.169 informal entrepreneurs, 0.07 formal entrepreneurs, and 0.761 paid workers. The Informality among Paid Workers is equal to 34.9% in the baseline economy.

informal businesses (extensive margin of informality) rises government tax revenue by 23% whereas shutting down the intensive margin of informality depresses government tax revenue by 17% (Panel 3 in Table 7). However, the elimination of both margins of informality leads to an increase of government revenue of 37%, which is a factor of 6 higher than the sum of the individual effects of the two margins.

Policies that eliminate the intensive margin of informality depress government revenues. They lead to a large increase in the number of informal businesses (from 0.17 in the baseline economy to 0.28) that negatively affects tax enforcement. Moreover, in our economy with financial frictions, when formal entrepreneurs are unable to hire informal workers output decrease by 9.4% leading to a reduction in the sales tax revenue of more than 23%.[28] Policies that eliminate informal businesses surprisingly do not increase the social security revenue,

[28] The 9% increase in the social security revenue cannot overturn the decrease in government revenue from sales taxes, as the social security tax represents a small share of the government tax revenue in our baseline economy.

which is what these policy recommendations aim to do. The reason is that this policy increases the share of paid informal workers from 35% in the baseline economy to 40%. This result underscores the importance of modeling both margins of informality jointly: Reducing informality along one margin may lead to an increase of informal paid workers through the other margin.

Shutting down informality along *both* margins leads to a large increase in the aggregate government tax revenue (37%). Shutting down informal businesses increases sales tax revenue because these entrepreneurs do not pay sales taxes. Moreover, when entrepreneurs cannot shift their production into the informal economy, shutting down the hiring of informal workers by formal employers leads to an increase in the revenue from payroll taxation. As a result, and differently from the previous cases considered, shutting down both margins of informality increases tax revenue from *both* sales and social security taxation.

Summary of key findings. We highlight the following findings. We find that the elimination of informality leads to large output and TFP gains (9.3% and 5.4%) despite the increase in effective taxation (tax revenue per unit of output rises 25%), a result that underscores that in an economy with financial frictions informality is quite costly. The two margins of informality affect quite differently macroeconomic outcomes. In an economy with financial frictions, policies that eliminate the intensive margin have pervasive effects on output and tax revenue (output decreases 9.4% and TFP 6.4%). Hiring some workers off-the-books allows small but productive businesses to outgrow borrowing constraints and to operate at a more efficient scale. On the contrary, policies that eliminate business informality (extensive margin) lead to the largest output and TFP gains (11.6% and 7.7%). Finally, the two margins of informality also have distinctive effects on government tax revenue.

5.2 Interactions between Financial Frictions and Informality

One of the contributions of our paper is to study the effects of financial frictions in a model economy that features informality along the intensive and extensive margins. We show that these margins interact quite differently with financial frictions by assessing the effects of eliminating financial frictions under different scenarios on informality (with and without an extensive/intensive margin of informality). Table 8 presents the key findings.

We find that the smallest output and TFP gains of eliminating financial frictions are attained in the economy with no extensive margin of informality (25.3% and 18.5%). The reason is that financial frictions play an important role in accounting for the high mass of informal entrepreneurs in the baseline economy. Since informal businesses enhance the

misallocation of productive resources, the gains from removing financial frictions become substantially smaller when the extensive margin is not operative.

Table 8 shows that the highest output and TFP gains of eliminating financial frictions are attained in the economy with no intensive margin of informality but with informal businesses (46% and 30%). The reason is that the intensive margin of informality helps improve resource allocation by relaxing credit constraints faced by productive entrepreneurs. Moreover, when informality along the extensive margin is present, these effects are amplified because the intensive margin diminishes the likelihood that productive entrepreneurs operate in the informal economy.

When the two margins of informality are present, the output and TFP gains from removing financial frictions are 38% and 25%. These gains are higher than the ones in the economy with no extensive margin of informality, pointing that the presence of informal businesses amplifies the costs of financial frictions. The gains are lower than the ones in the economy with no intensive margin, underscoring that the ability to hire workers off-the-books reduces the negative effects of both financial frictions and of informality along the extensive margin.

Table 8: Eliminating Financial Frictions in Alternative Economies

	Baseline Economy	Economy with No Informality	Ec. No Informal Businesses	Ec. No Informal Paid Workers
Panel 1: Change in Macroeconomic Aggregates (%)				
Agg. Output	38.2%	25.3%	22.0%	45.9%
Agg. Capital	43.3%	23.0%	23.5%	53.3%
TFP	25.4%	18.5%	15.3%	30.0%
Panel 2: Occupational Choice				
Informal Ent.	0.014	0.000	0.000	0.084
Formal Ent.	0.095	0.040	0.103	0.016
Paid Workers	0.891	0.960	0.897	0.900
Inf. among Paid Workers	28.2%	0.0%	28.1%	2.0%

Notes: The table displays the effects of eliminating financial frictions ($\lambda_f = 100$) in alternative model economies: i) baseline economy, ii) economy with no informality, iii) economy with no informal business (no ext. margin), iv) economy with no informal paid workers (no intensive margin).

5.3 Institutions and Financing Social Security

Social security is costly because its financing leads to lower output. In this subsection, we show two results about the costs of social security. First, the (steady-state) output costs of social security crucially depend on the economy's degree of financial frictions. Second, the output costs in the baseline economy are lower if social security is financed with sales taxes rather than payroll taxes. However, the opposite is true in the economy with no financial friction.

We find dramatic differences in the cost of financing social security in our baseline model economy relative to an economy with no financial friction. The elimination of the social security system in our baseline economy leads to an increase in output of 19.2% *together* with an increase in government tax revenue of 11% (see Table 9). However, when social security taxation is eliminated in the economy with no financial friction output increases by 10.9% (about half the value in the baseline economy), and the government tax revenue *decreases* by 10%. As we discuss below, the output costs of social security are much larger in the presence of financial frictions because the payroll tax hurts borrowing constrained entrepreneurs, who tend to operate their business with low capital to labor ratio. By reducing profits, social security taxes hinder the accumulation of net worth by entrepreneurs, making credit constraints tighter, and distorting the efficient allocation of productive resources across entrepreneurs. Hence, the elimination of social security (payroll taxes) in our baseline economy relaxes credit constraints, leads to a better allocation of resources, and ultimately to a large output expansion (relative to an economy with no financial friction). The large increase in output, together with the formalization of entrepreneurs, caused by the elimination of payroll taxes boost the tax revenue from sales, leading to an increase in aggregate tax revenue. On the contrary, in an economy with no financial friction, the size of the informal economy is small to start with and the increase in output after the elimination of social security is smaller. As a result, the increase in revenues from sales taxes does not compensate for the lost revenue from the elimination of payroll taxes, and the overall government tax revenue decreases.

Financial frictions also have consequences for the government's decision of whether to use payroll taxes or sales taxes to fund pensions. Table 9 reports, for the baseline economy and the economy with no financial friction, the long-run effects of eliminating payroll taxes and replacing the lost revenue with an increase in sales taxes. We find that financing social security with sales taxes increases output, capital, and TFP in the baseline economy by 3.5%, 2.4%, and 3.1%. However, this policy change leads to *negative* changes in all of these macroeconomic variables in the economy with no financial friction (-10.7% in output, -27% in capital, -2.7% in TFP). Why are the results so different across model economies?

Table 9: The Effects of Financing Social Security

	No Social Security	No Soc. Security (Ec. with No FF)	No Payroll Tax	No Payroll Tax (Ec. with No FF)
Panel 1: Change Macroeconomic Aggregates (%)				
Agg. Output	19.2%	10.9%	3.8%	-10.7%
Agg. Capital	86.8%	52.4%	2.4%	-27.0%
TFP	0.7%	-1.0%	3.1%	-2.7%
Panel 2: Occupational Choice				
Informal Ent.	0.002	0.000	0.103	0.100
Formal Ent.	0.210	0.042	0.019	0.011
Paid Workers	0.788	0.958	0.878	0.888
Inf. among Paid Workers	0.0%	0.0%	3.6%	3.2%
Panel 3: Change in Government Tax Revenue (%)				
S.S. Tax Rev.	-100.0%	-100.0%	-100.0%	-100.0%
Sales Tax Rev.	37.8%	11.5%	22.1%	21.1%
Total Tax Rev.	11.1%	-10.0%	0.0%	0.0%

Notes: The first two columns display the effects of eliminating the social security system ($\tau_{ss} = 0$ and $b = 0$) in the baseline economy and in the economy with no financial frictions ($\lambda_f = 100$). The third and fourth columns report the effects of financing the social security system with sales taxes rather than payroll taxes ($\tau_{ss} = 0$ and increase sales taxes) in the baseline economy ($\tau_y = 0.324$) and in the economy with no financial frictions ($\lambda_f = 100$ and $\tau_y = 0.422$).

Let us first focus on the results for the economy with no financial friction. The aggregate capital is about 27% lower when social security is financed with sales taxes instead of payroll taxes. The sales tax has such a large negative effect on the capital demanded by entrepreneurs because it decreases the marginal product of *both* labor and capital. Given that capital and labor are complements in production, both of these effects reduce aggregate capital. The payroll tax has a lower negative impact on the demand for capital because, by rising labor costs, it makes entrepreneurs produce with higher capital to labor ratio (see Equation (31)).

In the presence of financial frictions, taxation has subtle and heterogeneous effects on the demand for capital across entrepreneurs. Consider the static problem faced by an entrepreneur with productivity z and assets a:[29]

$$\max_{k,l} z(1-\tau_y)k^{\alpha\theta}l^{(1-\alpha)\theta} - w(1+\tau_{ss})l - r(k-a),$$

$$\text{s.t.} \quad k \leq \lambda\, a.$$

[29]For simplicity, we ignore the possibility of hiring informal workers.

Denoting by μ the Lagrange multiplier on the collateral constraint, the FOC yields:

$$\frac{k}{l} = \frac{\alpha}{(1-\alpha)} \frac{w(1+\tau_{ss})}{[r+\mu(z,a)]}, \tag{31}$$

$$\text{where} \quad \mu(z,a) = \max\{C_\mu \left[\frac{(1-\tau_y)z}{\lambda a}\right]^{\frac{1}{1-\theta(1-\alpha)}} \left[\frac{1}{w(1+\tau_{ss})}\right]^{\frac{(1-\alpha)\theta}{1-\theta(1-\alpha)}} - r, 0\},$$

and C_μ is a constant that depends on the parameters (α, θ).

When the borrowing constraints bind ($\mu(a,z) > 0$), the capital to labor ratio vary across entrepreneurs (see Equation (31)). Intuitively, as the labor input is not directly affected by the collateral constraint, entrepreneurs facing tight borrowing constraints (high μ due to a low a/z ratio) rely relatively more on labor than on capital in their production than entrepreneurs with loose borrowing constraints (high a/z ratio). As a result, a decrease in payroll taxes benefits relatively more entrepreneurs with a tight borrowing constraint. A switch from payroll taxes to sales taxes redistributes tax liabilities from borrowing constrained entrepreneurs to unconstrained entrepreneurs, relaxing credit constraints, and reallocating labor to the more productive entrepreneurs. The rise in the equilibrium diminishes the mass of informal businesses by 0.07, leading to a TFP increase of 3.1%. On the contrary, in the absence of financial frictions, the increase in sales taxes rises the mass of informal businesses rises by 0.10 because low productivity entrepreneurs find it optimal to operate at a small scale to avoid taxes. This response results in a decrease of TFP of 2.7%.

We find it interesting to interpret this finding in terms of the life cycle of a new entrepreneur starting with low assets and with a tight borrowing constraint. During an initial stage, the entrepreneur will produce with low capital to labor ratio and will thus benefit from low payroll taxes. Over time, the entrepreneur will tend to accumulate capital, increase production, and pay more sales taxes. In sum, a switch from payroll taxes to sales taxes redistributes tax liabilities over the life cycle of entrepreneurs from an early borrowing-constrained stage to a later unconstrained stage. In this regard, the finding is consistent with the view of Itskhoki and Moll (2019) who advocate for an initial phase of low labor taxation as an economy moves to its steady-state from an initial situation with low capital.

6 Conclusion

We develop a quantitative theory of entrepreneurship to study how informality in Brazil affects occupational choice, capital accumulation, resource allocation, and government tax revenue. Our results point to the importance of modeling the intensive and extensive margins of informality and their interaction with financial frictions for understanding informality in

Brazil and for assessing the consequences of regulations and institutions. We find that entry costs, financial frictions, and the taxes funding the social security system have distinct effects on the intensive and extensive margin of informality. Moreover, they all interact in non-trivial ways. In the presence of financial frictions, policies that eliminate the intensive margin alone have pervasive effects on output and tax revenue. Hiring workers off-the-books allow small but productive businesses to outgrow borrowing constraints. Without this option, most entrepreneurs operate on a small scale in the informal sector, making the negative impact of financial frictions in macroeconomic variables much more severe. In general, the joint effect of both informality margins is large and different from the sum of each effect on macroeconomic aggregates. This is true for output and TFP, but it is particularly evident for the change in government tax revenue.

We find that the output costs of financing social security in Brazil are about twice as large as the ones in an economy with no financial friction. Moreover, while our model implies that the output costs would be lower if social security in Brazil were financed with sales taxes rather than payroll taxes, the opposite is true in the economy with no financial friction. A switch from payroll taxes to sales taxes in Brazil redistributes tax liabilities from borrowing constrained entrepreneurs to unconstrained entrepreneurs, relaxing credit constraints, and leading to higher output, capital, and TFP. In sum, our results highlight the importance of jointly modeling financial frictions and the informal economy along both margins for understanding the impact of taxes and institutions in Brazil.

Our framework features many elements used in macro-development: financial frictions, size-dependent policies, technological differences across sectors, and entry barriers. Although these micro-distortions have been studied in different contexts, few studies have highlighted the impact of their interaction on public finance. We provide the first step in this direction, yet the impact of a thorough tax reform with these distortions remains to be done. We believe this is an important avenue for future research.

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Online Appendix

Public Financing with Financial Frictions and Underground Economy

Andrés Erosa, Luisa Fuster, Tomás R. Martinez

A Data Appendix

A.1 Additional Tables

Table A.1: Cumulative mass of formal firms and workers by size of formal firms

Size	Mass of Formal Firms	Mass of Formal Workers
≤ 5	0.698	0.142
≤ 10	0.839	0.242
≤ 20	0.922	0.355
≤ 50	0.972	0.498
≤ 250	0.995	0.723
≤ 1000	0.999	0.890

Notes: Size is defined by the number of paid workers (does not include the entrepreneur). Source: RAIS 2003.

Table A.2: Share of Informal Workers and Entrepreneurs

Variable	2003	2012
Share of Informal Workers (out of total paid workers)	0.350	0.220
Share of Formal Workers (out of total paid workers)	0.650	0.780
Share of Entrepreneurs (out of employed population)	0.322	0.319
Share of Workers (out of employed population)	0.678	0.681

Notes: Urban workers and entrepreneurs in private firms. Source: PNAD 2003 and PNAD-C 2012.

Table A.3: Loan Source by Formal and Informal Firms

Loan Source (%)	Formal	Informal
Friends and Family	9.71	18.19
Banks (Public or Private)	73.66	53.05
Own Supplier	10.67	17.7
Other People or Companies	5.97	11.06
N	977	2054
Share who got a loan (%)	15.37	4.86

Notes: Entrepreneurs who got a loan, credit or financing from Aug/03 to Oct/03. Source: ECINF 2003

Table A.4: Conditional Summary Statistics

Variable	Formal (≤ 5)	Informal	Ratio (F/I)
Size	2.00	1.18	1.698
Business Tenure (months)	119.17	112.28	1.061
Business Tenure (months $\geq$12)	127.92	122.98	1.040
Value Added p/ workers	1,589.60	686.08	2.317
Aggregate Debt/Y	0.431	0.311	1.386
Aggregate K/Y	1.377	1.043	1.320

Notes: Summary statistics conditional on sector, state, gender, education and experience of entrepreneur. Size includes paid workers (formal and informal) and business owners. Aggregate K/Y excludes housing and vehicles. Values in 2003 Brazilian Reals. Firms with 5 or less employees. Source: ECINF 2003.

Table A.5: Tax Revenue by Percent of GDP

	All Governments
Tax Revenue	
Income Tax	6.86
Payroll Tax	0.30
Property Tax	1.15
Sales Tax	16.77
Other Taxes	0.53
Social Sec. Contribution	6.24
Other Revenue Sources	4.76
Total	36.61

Notes: Government Revenue by Percent of GDP (2006). All governments include federal, state and local administration. Other taxes include revenue from tariffs and other transfers. Other revenue sources include property income, fines, sales of goods and services and other. Source: IMF Government Finance Statistics.

A.2 Share of Informal Workers in Formal Firms in 2003

A key moment used in the calibration of the model is the share of informal workers in formal firms. Unfortunately, such statistic is not available in 2003, the year in which the model is calibrated. To determine an estimate of this moment, we proceed by finding a lower and a upper bound of it. At a first step, we use the updated version of PNAD to calculate the lower bound of the share of informal workers in formal firms. The PNAD-C (*Pesquisa Nacional por Amostra de Domiclios Contnua*) begun to roll in 2012, and asks the worker both the formality status of the firm she is working and her own formality status. Using the first year available of PNAD-C, Table 2 shows that, out of all workers employed informally, 62% were located in formal firms. Given that the overall share of informal workers decreased from 35% in 2003 to 22% in 2012 (see Table A.2), the share of informal workers in formal firms in formal firms is likely higher in 2003 as well. Therefore, we take 62% as the lower bound of the share of informal workers in formal firms in 2003.

To calculate the upper bound, we make use of both ECINF and PME. The initial step involves determining the share of informal workers by business size. Table A.6 indicates that 35.5% of the informal workers are located in micro firms (with less or equal five employees), while 64.5% are located in larger firms. As shown in Table 1, the likelihood of a firm with more than five employees be informal is negligible. Hence, by assuming that all firms with more than five employees are formal, we have that at least 64.5% of all informal workers are in formal firms. The second step requires finding out the share of informal workers employed in formal business conditional that the firm has five employees or less. Using the ECINF, we found that 32.2% of informal workers in micro firms are employed in formal firms. Therefore, the total share of informal workers in formal firms is equal to the share of informal workers in firms larger than five employees (64.5%) plus the share of informal workers in formal micro firms ($32.2\% \times 35.5\% = 11.4\%$).

Table A.6: Informality by Business Size Distribution in 2003

Variable	Micro (≤ 5)	Small (≥ 6 and ≤ 10)	Large (> 10)
Mass of Informal Workers (by size)	0.355	0.115	0.530
Mass of Formal Workers (by size)	0.066	0.074	0.859
Mass of Workers (by size)	0.148	0.086	0.766
Fraction of Informal Workers	0.678	0.379	0.195

Notes: Urban paid employees in private firms only. Source: PME 2003.

Hence, by combining both ECINF and PME, we infer that the share of informal employees in formal business in 2003 is equal to 75.9%. Yet, because PME samples only workers from

the six largest metropolitan regions, it overstates the number of large business with respect all the other data sets. For comparison, in RAIS (in Table A.1), firms with more than 10 employees accounts for 75.8% of all the workers, while in PME this number is equal to 85.9%. We decide to interpret the 75.9% as an upper bound. To find a good compromise between the lower bound (62.9%) and the upper bound (75.9%), we decide to calibrate the share of informal workers in formal business in 2003 to 70%.

B Theory Appendix

B.1 Cost of Hiring Informal Workers

We parametrize the cost of hiring a informal worker by a formal firm to depend of the share of informal workers hired by the firm. The functional form is given by:

$$\tau_f(l_i, l_f) = \tau_{1,f} l_i^{\tau_{2,f}} \left(\frac{l_i}{l_i + l_f} \right)^{\omega} = \tau_{1,f} l_i^{\tau_{2,f}+\omega} (l_i + l_f)^{-\omega} \tag{A.1}$$

The marginal cost of hiring a formal and informal worker:

$$MC_f = w \left(1 + \tau_{ss} - \omega \tau_{1,f} l_i^{(\tau_{2,f}+\omega)} (l_i + l_f)^{(-\omega-1)} \right) \tag{A.2}$$

$$MC_i = w \left(1 + \tau_{1,f} [(\tau_{2,f} + \omega) l_i^{(\tau_{2,f}-1+\omega)} (l_i + l_f)^{-\omega} - \omega l_i^{(\tau_{2,f}+\omega)} (l_i + l_f)^{(-\omega-1)}] \right) \tag{A.3}$$

The entrepreneur hires informal workers until $MC_i = MC_f$.

$$\tau_{1,f}[(\tau_{2,f} + \omega) l_i^{(\tau_{2,f}-1+\omega)} (l_i + l_f)^{-\omega} - \omega l_i^{(\tau_{2,f}+\omega)} (l_i + l_f)^{(-\omega-1)}] = \tau_{ss} - \omega \tau_{1,f} l_i^{(\tau_{2,f}+\omega)} (l_i + l_f)^{(-\omega-1)}$$

$$\tau_{ss} = \tau_{1,f} l_i^{(\tau_{2,f}-1+\omega)} (l_i + l_f)^{-\omega} (\tau_{2,f} + \omega)$$

$$l_i = \left(\frac{\tau_{ss}}{\tau_{1,f}(\tau_{2,f} + \omega)} \right)^{1/(\tau_{2,f}-1+\omega)} (l_i + l_f)^{\omega/(\tau_{2,f}-1+\omega)} \tag{A.4}$$

Hence, if $\omega > 0$, the number of informal workers increase with the size. If $\omega = 0$, all firms have the same cutoff. By setting $\tau_{2,f} = 2$ and taking logs, equation (21) follows.

B.2 Profit Maximization Problem

Informal Entrepreneurs. Profit maximization of an informal entrepreneur is given by:

$$\pi_i(a, z; w, r) = \max_{k, l_i} z q_i(k, l) - (r + \delta)k - w l_i + (1 + r)a - c_i, \tag{A.5}$$

$$\text{s.t.} \quad k \leq \lambda_i a, \tag{A.6}$$

$$l = l_i + \bar{l} \qquad \text{and} \qquad l_i \geq 0, \tag{A.7}$$

That leads to the following first order condition:

$$\frac{\partial \pi_i(\cdot)}{\partial l_i} = MPL(k, l) - w + \mu_l = 0$$

$$\frac{\partial \pi_i(\cdot)}{\partial k} = MPK(k, l) - (r + \delta) - \mu_k = 0$$

where μ_l and μ_k are the associated multipliers of the non-negativity constraint (A.7) and the collateral constraint (A.6), respectively.

Assuming that the collateral constraint does not bind ($\mu_k = 0$) leads to the following unconstrained demand for capital:

$$k_u(z) = \left(\frac{\theta_i \alpha_i z}{r+\delta}\right)^{\frac{1}{1-\alpha_i\theta_i}} l(z)^{\frac{(1-\alpha_i)\theta_i}{1-\alpha_i\theta_i}}, \tag{A.8}$$

and the associated labor demand:

$$l(z) = \max\left\{\left(z\theta_i\left(\frac{(1-\alpha_i)}{w}\right)^{1-\alpha_i\theta_i}\left(\frac{\alpha_i}{r+\delta}\right)^{\alpha_i\theta_i}\right)^{\frac{1}{1-\theta_i}}, \bar{l}\right\}. \tag{A.9}$$

In the case collateral constraint binds, the capital demand is equal to $k_c(a) = \lambda_i a$, and labor demand is:

$$l(a,z) = \max\left\{\left(\frac{\theta_i(1-\alpha_i)z}{w}\right)^{\frac{1}{1-(1-\alpha_i)\theta_i}} k_c(a)^{\frac{\alpha_i\theta_i}{1-(1-\alpha_i)\theta_i}}, \bar{l}\right\}. \tag{A.10}$$

Finally, the capital demand that solves the problem is:

$$k(a,z) = \min\left\{k_u(z), k_c(a)\right\}, \tag{A.11}$$

together with its associated labor demand function.

Formal Entrepreneurs. Profit maximization of a formal entrepreneur is given by:

$$\begin{aligned}
\pi_f(a,z;w,r) = \max_{k,l_f,l_i\geq 0} &(1-\tau_y)zq_f(k,l) - (r+\delta)k - w(l-\bar{l}) - \tau_{ss}wl_f \qquad (A.12)\\
&- w\tau(l_i,l_f) + (1+r)a - c_f,\\
\text{s.t.} \quad & k \leq \lambda_f a, \qquad (A.13)\\
& l = l_i + l_f \geq \bar{l}, \qquad (A.14)\\
& l_f \geq 0. \qquad (A.15)
\end{aligned}$$

The associated first order condition of the problem are:

$$\frac{\partial \pi_f(\cdot)}{\partial l_i} =(1-\tau_y)MPL(k,l) - w\left(1+\tau_{1,f} l_i^{\tau_{2,f}+\omega-1}(l_i+l_f)^{-\omega}\left(\tau_{2,f}+\omega-\omega\frac{l_i}{l_i+l_f}\right)\right)+\mu_l = 0, \tag{A.16}$$

$$\frac{\partial \pi_f(\cdot)}{\partial l_f} =(1-\tau_y)MPL(k,l) - w\left(1+\tau_{ss}-\omega\tau_{1,f} l_i^{\tau_{2,f}+\omega}(l_i+l_f)^{-\omega-1}\right)+\mu_l+\mu_f = 0, \tag{A.17}$$

$$\frac{\partial \pi_f(\cdot)}{\partial k} =(1-\tau_y)MPK(k,l) - (r+\delta)-\mu_k = 0, \tag{A.18}$$

where μ_k, μ_l, and μ_f are the associated multipliers to the constraints (A.13), (A.14), and (A.15), respectively.[1]

When the collateral constraint does not bind ($\mu_k = 0$), the capital demand is

$$k_u(z,l(z)) = \left(\frac{(1-\tau_y)\theta_f\alpha_f z}{r+\delta}\right)^{\frac{1}{1-\alpha_f\theta_f}} l(z)^{\frac{(1-\alpha_f)\theta_f}{1-\alpha_f\theta_f}}. \tag{A.19}$$

Optimal capital demand is given by

$$k(a,z) = \min\left\{k_u(z,l^*(z)), \lambda_f a\right\}, \tag{A.20}$$

where $l^*(z)$ is the optimal labor demand when the collateral constraint does not bind.

To characterize a solution we will proceed by first assuming that the collateral does not bind, and then by solving for all possible cases. In all the cases, we verify that the Kuhn-Tucker conditions are satisfied. Whenever $k_u(z,l^*(z)) > \lambda_f a$, we set $k^* = \lambda_f a$ and solve for the associated $l^*(z,a)$.

The first case is when no formal labor is hired (i.e., (A.15) is binding) and the constraint (A.14) is slack. In this case, we have $l_f = 0$, $\mu_f \geq 0$ and $\mu_l = 0$, and the first order condition (A.16) collapses to:

$$(1-\tau_y)\theta_f(1-\alpha_f)zk^{\alpha_f\theta_f}l_i^{(1-\alpha_f)\theta_f-1} = w(1+\tau_{1,f}\tau_{2,f}l_i^{\tau_{2,f}-1}). \tag{A.21}$$

By using (A.19), we are able to solve for l_i. The condition $\mu_f \geq 0$ can be checked using the first order condition (A.17). After we recover $k_u(z,l_i(z))$, we check for the collateral constraint. In case it is binding, we set $k = \lambda_f a$, and solve again for $l_i(z)$.

The second case is when both (A.15) and (A.14) are binding. In this case, we have $l_f = 0$, $\mu_f \geq 0$, $l_i = \bar{l}$ and $\mu_l \geq 0$. Capital is given by: $k(a,z) = \min\left\{k_u(z,\bar{l}), \lambda_f a\right\}$. The conditions $\mu_f \geq 0$ and $\mu_l \geq 0$ are checked using the first order conditions.

[1] Note that we are ignoring the constraint $l_i \geq 0$. This case is only relevant if $\tau_{ss} = 0$.

The third case is when the entrepreneur hires both formal and informal labor, and the constraint (A.14) is slack. In this case, we have $\mu_f = 0$ and $\mu_l = 0$ and the solution is characterized by $l^*(z)$ and $k_u(z, l^*(z))$. After wise, we check for the collateral constraint. In case it is binding, we set $k = \lambda_f a$, and solve again for $l^*(z)$. The optimal number of informal workers is given by equation (A.4). Under the assumption that the marginal worker is formal, we can combine equations (A.19), (A.17) and (A.4) and solve for $l^*(z)$, $k_u(z, l^*(z))$.[2] The fourth case is when the entrepreneur hires both formal and informal labor, and the constraint (A.14) is biding. In this case, we have $l = l_f + l_i = \bar{l}$, $\mu_l \geq 0$ and $\mu_f = 0$. Capital demand is given by $k_u(a, z) = \min\left\{k_u(z, \bar{l}), \lambda_f a\right\}$. and the optimal number of informal workers is given by equation (A.4). Once we have l_i, we check whether $l_f = \bar{l} - l_i > 0$ and $\mu_l \geq 0$ holds by using the first order condition (A.17).

[2] Alternatively, we could assume that the marginal worker is informal and use the informal FOC to solve for $l^*(z)$. With both solutions in hand, we could check whether the marginal cost of the formal worker is indeed lower. Under reasonable calibration values the marginal worker is always formal.

C Computational Appendix

C.1 Discretization and Value Function Approximation

To bring the model to the computer, we discretize state space of the value and policy functions. Specifically, we discretize the asset space in 1200 grid points equidistant over the log space. The entrepreneurial idea, z is discretized in the same 60 points. Again, the grid points are equidistant over the log space, where the initial is point is given by z_0 and final grid point to the value associated to the 0.9999 percentile of the Γ_{z_s}. The distribution of productivity in the corporate sector, z_c, is uniformly discretized over 10000 grid points.

C.2 Computation with Taste Shock

To facilitate the numerical solution of the model and improve convergence to an equilibrium, we smooth out the discrete occupational choice by adding a taste shock. The taste shock simply adds noise to the entrepreneurial decisions of indifferent households, but the calibration of the model and all the results are robust to the inclusion of it and are left mostly unchanged. Here we outline the extended model, and for more details we refer to Iskhakov et al. (2017). The major modification and key assumption is that every period individuals receive a vector of additive-separable taste shocks $\epsilon = (\epsilon_w, \epsilon_f, \epsilon_i)$ to the value of being a worker, a formal entrepreneur and a informal entrepreneur. These shocks are $i.i.d$ according to an Extreme Value type I distribution (Gumbel) with scale parameter σ_ϵ. We calibrate the variance to $\sigma_\epsilon = 0.01$.

The modified value function of a worker in state (a, z_s) is given by

$$W(a, z, \epsilon) = \max\{W_w(a, z) + \sigma_\epsilon \epsilon_w, W_f(a, z) + \sigma_\epsilon \epsilon_f, W_i(a, z) + \sigma_\epsilon \epsilon_i\} \tag{A.22}$$

where:

$$\begin{aligned} W_w(a, z) = & \max_{c, a'} u(c) + \beta \rho_R V_{ret}(a') \\ & + \beta(1 - \rho_R) \left(\pi_z E_\epsilon W(a', z, \epsilon) + (1 - \pi_z) \int E_\epsilon W(a', z', \epsilon) d\Gamma_z \right), \\ \text{s.t.} \quad & c + a' = w + (1 + r)a, \end{aligned} \tag{A.23}$$

$$W_f(a,z) = \max_{c,a'} u(c) + \beta\rho_R V_{ret}(a') + \beta(1-\rho_R)E_\epsilon V_f(a',z,\epsilon), \tag{A.24}$$
$$\text{s.t.} \quad c + a' + c_e^f = w + (1+r)a,$$

$$W_i(a,z) = \max_{c,a'} u(c) + \beta\rho_R V_{ret}(a') + \beta(1-\rho_R)E_\epsilon V_i(a',z,\epsilon) \tag{A.25}$$
$$\text{s.t.} \quad c + a' + c_e^i = w + (1+r)a,$$

where $W_j(a,z,\epsilon)$ for $j \in \{w,i,f\}$ represents the value function when worker chooses occupational choice j for next period. The E_ϵ denotes the expectation over future taste shocks.

Note that the introduction of the extreme value taste shock smooth out the kink in the entrepreneurial decision. In fact, the binary choice, which in the absence of the shock can described as an indicator function, is now probabilistic function over the relative values of each choice. Denote, $P_w^j(a,z_s)$, the probability that a worker decides occupational choice $j \in \{w,i,f\}$ in state a,z as

$$P_j^w(a,z) = \frac{\exp\{W_j(a,z)/\sigma_\epsilon\}}{\exp\{W_w(a,z)/\sigma_\epsilon\} + \exp\{W_f(a,z)/\sigma_\epsilon\} + \exp\{W_w(a,z)/\sigma_\epsilon\}} \tag{A.26}$$

Intuitively, the taste shock introduces "noise" in the value function such that an individual may decide stays a worker even so the value of being a entrepreneur surpass the value of being a worker as long the preference shock is large enough. Notice the role of σ_ϵ: a large variance generates too much noise, effectively making the values $W_j(a,z)$ unimportant for the entrepreneurial decision. On the other hand, if $\sigma_\epsilon = 0$, the policy function collapses to the binary case without taste shocks.

The taste shock not only smooth out the primary kink given by the discrete choice in the contemporary value function, but also secondary kinks given by the next period value function. Following Iskhakov et al. (2017), we write the expectation with respect to taste shocks using the *log-sum* formula:

$$E_\epsilon W(a',z,\epsilon) = \sigma_\epsilon \log\left(\exp\left\{\frac{W_w(a',z)}{\sigma_\epsilon}\right\} + \exp\left\{\frac{W_i(a',z)}{\sigma_\epsilon}\right\} + \exp\left\{\frac{W_f(a',z)}{\sigma_\epsilon}\right\}\right). \tag{A.27}$$

Similarly, the modified value of an entrepreneur of type $j = \{i,f\}$ is

$$Vj(a,z,\epsilon) = \max\{V_j^j(a,z) + \sigma_\epsilon\epsilon_j, V_j^w(a,z) + \sigma_\epsilon\epsilon_w\}, \tag{A.28}$$

where the value V_j^j is the value function of an entrepreneur that stays operating and V_j^w of an entrepreneur that decides to exit and become a worker in the next period. In recursive form these value functions are given by

$$V_j^j(a,z) = \max_{c,a'} u(c) + \beta(1-\rho_R)\left[(1-\pi_z)E_\epsilon V_j(a',z,\epsilon) + \pi_z \int E_\epsilon W(a',z',\epsilon)d\Gamma_{z'}\right] + \beta\rho_R V_{ret}(a') \tag{A.29}$$

$$V_j^w(a,z) = \max_{c,a'} u(c) + \beta(1-\rho_R)\int E_\epsilon W(a',z',\epsilon)d\Gamma_{z'} + \beta\rho_R V_{ret}(a') \tag{A.30}$$

$$c + a' = \pi_j(a,z). \tag{A.31}$$

The policy function will be, again, given by a logit function:

$$P_j(a,z) = \frac{\exp\{V_j^j(a,z)/\sigma_\epsilon\}}{\exp\{V_j^j(a,z)/\sigma_\epsilon\} + \exp\{V_j^w(a,z)/\sigma_\epsilon\}}. \tag{A.32}$$

where $P^j(a,z)$ is the probability that the entrepreneur in sector j decides to not (endogenously) exit. Finally, the partial expectation (over the taste shock) of a entrepreneur can be written as

$$E_\epsilon V_j(a',z,\epsilon) = \sigma_\epsilon \log\left(\exp\left\{\frac{V_j^j(a',z)}{\sigma_\epsilon}\right\} + \exp\left\{\frac{V_j^w(a',z)}{\sigma_\epsilon}\right\}\right). \tag{A.33}$$

C.3 Numerical Algorithm

1. Guess factor prices (w,r). Compute $V_{ret}(a)$ with an analytical formula.
2. Set the initial guess $E_\epsilon W^n(a,\cdot,\epsilon) = E_\epsilon V_i^n(a,\cdot,\epsilon) = E_\epsilon V_f^n(a,\cdot,\epsilon) = V_{ret}(a)$.
3. Given $E_\epsilon W^n(a,z,\epsilon)$, $E_\epsilon V_i^n(a,z,\epsilon)$ and $E_\epsilon V_f^i(a,z,\epsilon)$, compute $E_\epsilon W^{n+1}(a,z,\epsilon)$:
   (a) Compute $W_j(a,z)$, for $j = \{w,i,f\}$. For that, compute the expectations over Γ_z when applicable, and use the pre-computed $V_{ret}(a)$. The maximization step is carried on using the divide and conquer algorithm of Gordon and Qiu (2018).
   (b) Then, uses the log-sum formula from (A.27) to compute $E_\epsilon W^{n+1}(a,z,\epsilon)$.
   (c) Note that the previous steps define three saving policies, $g_j^w(a,z)$ (one for each of the three occupational choices), and three probability functions, $P_j^w(a,z)$, that describes the probability that a worker choose one of the three occupational choices.

4. Proceed similarly to compute $V_j^{n+1}(a,z)$, for $j = \{i, f\}$. Compute both $V_j^j(a,z)$ and $V_j^w(a,z)$ by taking the expectations over guesses and applying the divide and conquer algorithm in the maximization. Then, apply the associated *log-sum* formula to get $E_\epsilon V_{n+1}^j(a,z,\epsilon)$ and the logit function to calculate $P^j(a,z)$. Again, there will be four associated saving policies (two for formal and two for informal), $(g_f^f(a,z), g_f^w(a,z), g_i^i(a,z), g_i^w(a,z))$.

5. Once $\max\{||E_\epsilon W^{n+1}(a,z,\epsilon) - E_\epsilon W^n(a,z,\epsilon)||, ||E_\epsilon V_f^{n+1}(a,z,\epsilon) - E_\epsilon V_f^n(a,z,\epsilon)||, ||E_\epsilon V_i^{n+1}(a,z,\epsilon) - E_\epsilon V_i^n(a,z,\epsilon)||\} < tol$, stop the value function iteration. Otherwise, update the guess using the values of $n+1$ and go back to step 3. The tolerance specified is equal to 10^{-9}.

6. Discretize the invariant distribution $F(a,z,j)$ and use the savings and occupational choice rules to iterate forward using a non-stochastic simulation method. Stop when $||F^{n+1}(a,z,j) - F^n(a,z,j)|| < tol$. The tolerance specified is equal to 10^{-11}.

7. Compute the excess labor demand (16) and excess capital demand (17) by integrating the decisions rules using the invariant distribution. Compute the total loss function over the square of percentage deviations of the excess demands. If the loss function is below a 10^{-4} stop, otherwise guess new factor prices (w,r) and return to step 1. The minimization of the excess demand functions follows a Nelder-Mead simplex routine.