

# Quantitative Macroeconomics

## Problem Set 2

Deadline: 26/04

1. **(The Hopenhayn & Rogerson Model).** Consider the Hopenhayn & Rogerson model. Every period the representative household decides how much to consume and how much labor to supply to the market. The problem is summarized as:

$$\max_{C,N} \ln C - AN \quad \text{s.t.} \quad pc = N + \Pi + T. \quad (1)$$

The solution is given by:  $C = 1/Ap$  and  $N = 1/A - \Pi - T$ , where  $\Pi$  is the aggregate profits and  $T$  is the aggregate tax revenue.

Incumbent firms produce the final good  $y$  according to the following production function:  $y = zn^\alpha$ , where  $n$  denotes the labor hired by the firm. They are subject to a per-period fixed cost  $c_f$  and a firing tax  $g(n_t, n_{t-1}) = \tau \max\{0, n_{t-1} - n_t\}$ . The productivity,  $z$ , follows an AR(1):

$$\log z_t = \mu(1 - \rho) + \rho \log z_{t-1} + \sigma \varepsilon_t \quad \varepsilon \sim N(0, 1).$$

At the beginning of the period, before the realization of  $z$ , incumbents decide whether to continue operations or exit. In case the firm decides to exit, it receives  $-g(0, n_{t-1})$  this period and 0 in all the others. The value function of incumbent reads:

$$V(z, n; p) = \max_{n'} \{pz(n')^\alpha - n' - pc_f - g(n', n) + \beta \max[-g(0, n'), \mathbb{E}_{z'|z} V(z', n'; p)]\},$$

where  $p$  is the price of the final good, and the wage,  $w$ , is normalized to one. Denote the firm's policy function as  $n' = n^d(z, n)$  and  $\chi = \{0, 1\}$ , where  $\chi = 1$  denotes the exit decision.

Potential entrants are ex-ante identical. They must pay an entry cost,  $c_e$ , and start producing in the next period. They draw their productivity from the distribution  $G(z)$ . Let  $M \geq 0$  be the mass of entrants, the free entry condition implies that in equilibrium:

$$V^e(p) = \beta \int V(z, 0; p) dG(z) - c_e \leq 0 \quad (2)$$

with equality whenever the mass of entrants is positive,  $M > 0$ .

We solve for the stationary equilibrium, the equilibrium where prices are constant and there is an invariant distribution  $\mu(z, n)$  over the employment-productivity space. The market clearing conditions of the goods and labor market are:

$$Y = \int z(n'(z, n))^\alpha - c_f d\mu(z, n) = C \quad (3)$$

$$N = L^d + Mc_e. \quad (4)$$

- (a) Outline a computational algorithm to find a stationary competitive equilibrium of the model.
- (b) Solve the model in the computer using the following parameters:  $\alpha = 2/3$ ,  $\beta = 0.8$ ,  $A = 0.01$ ,  $c_e = 20.0$ ,  $c_f = 20.0$ ,  $\rho = 0.9$ ,  $\sigma = 0.2$  and  $\mu = 1.0$ . Discretize the productivity process using the Tauchen algorithm. Make sure to have enough nodes (i.e.,  $n_z \geq 45$ ). The employment grid is discretized between  $n_{n,1} = 0$  and  $n_{n,max} = 10000$ . An easy way to discretize the employment space is to assume a vector of the type:

$$g_N = [0(1)20; 22(2)100; 105(5)500; 550(50)1000; 1100(100)5000; 5500(500)10000]',$$

where the number inside the () denotes the step size between grid points.<sup>1</sup>

Solve the model for  $\tau = 0.1$  and show: (i) the average firm size; (ii) the average firm productivity; (iii) exit/entry rate; and (iv) the average misallocation of the economy, where misallocation of firm  $i$  is defined as

$$\text{misalloc}_i = \frac{|MPN_i - 1/p|}{1/p},$$

and  $MPN_i$  is the marginal product of labor.

- (c) Solve the model again for no tax  $\tau = 0.0$  and a high firing tax  $\tau = 0.5$  how misallocation has changed in the economy?
- (d) Solve the model for a low persistence productivity process  $\rho = 0.5$ . Compute the statistics for the three cases:  $\tau = (0.0; 0.1; 0.5)$ . How does the persistence of the stochastic process matter for the impact of the firing tax?
2. **(Labor Tax in the Hopenhayn Model).** Consider the standard Hopenhayn & Rogerson model presented in the previous question. The parameter's values are the same as the previous question.

Suppose there is a government deciding between two types of taxes: a firing tax as modeled in the previous question, or a traditional labor tax of  $\tau_n$  of the equilibrium wage. The value function of the incumbents read:

$$V(z, n; p) = \max_{n'} \{pz(n')^\alpha - n'(1 + \tau_n) - pc_f - g(n', n) + \beta \max[-g(0, n'), \mathbb{E}_{z'|z} V(z', n'; p)]\}.$$

The government already has a firing tax in place and it is thinking about whether to move to the other tax scheme. The reform should be revenue-neutral, i.e, it should yield the same tax revenue as before.

- (a) Solve the model without the labor tax,  $\tau_n = 0.0$ , but with a firing tax of  $\tau = 0.1$ . Report the aggregate tax revenue, the aggregate output, average misallocation, average productivity, and the equilibrium price.

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<sup>1</sup>For instance,  $[0(1)3; 4(2)10] = [0, 1, 2, 3, 4, 6, 8, 10]$ .

- (b) Shut down the firing tax  $\tau = 0.0$ . Find the labor tax  $\tau_n > 0$  that yields the same aggregate tax revenue (if it exists).<sup>2</sup> Report the aggregate output, average misallocation, average productivity, and the equilibrium price. What is the tax scheme that maximizes aggregate output?

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<sup>2</sup>You can find it by trying different taxes rates or can write a root-finding function. Note that an exact tax rate may be hard to find because of the grid approximation. In this case, feel free to report an approximation.