Quantitative Macroeconomics

Life Cycle Economies: Storesletten, Telmer & Yaron (2004, JME)

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UnB

Introduction

- **Goal:** introduce the incomplete markets framework in a OLG economy.
- Study a classic paper as an example: **Storesletten, Telmer & Yaron (2004): Consumption and risk sharing over the life cycle**.
- The life cycle structure is useful to study many questions where age interacts with inequality:
  - **Early age**: Education;
  - **Middle age**: Labor market;
  - **Old age**: Social security, health;

Storesletten, Telmer & Yaron (2004): Motivation

- **Stylized Facts:**
  1. Inequality in consumption and earnings increase substantially during the life cycle;
  2. The increase in inequality of consumption is less than earnings;
  3. The increase is approximately linear.
- Can noninsurable idiosyncratic shocks to labor earnings explain this facts?
- What is the role of initial heterogeneity in comparison to earnings shocks during the life cycle?

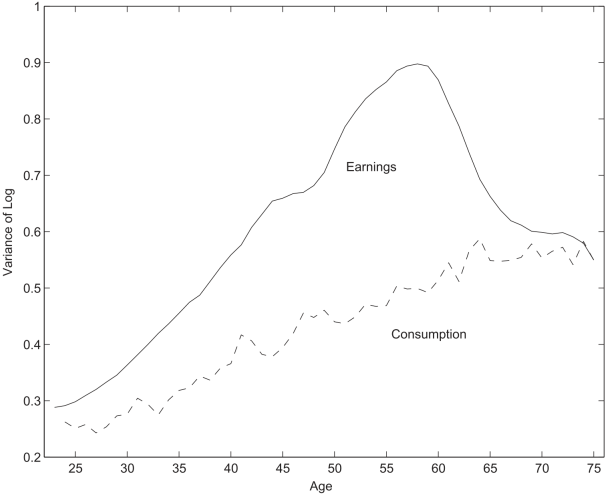
Storesletten, Telmer & Yaron (2004): Method

- Estimate a rich labor earnings process using the PSID:
  - Individual fixed effects;
  - Persistent shocks;
  - Transitory shocks.
- Input the earnings process in an OLG model without consumption risks sharing.
  - General equilibrium pins down the level of wealth.
- Only two sources of insurance:
  - Self-insurance;
  - Pension system financed by labor tax.

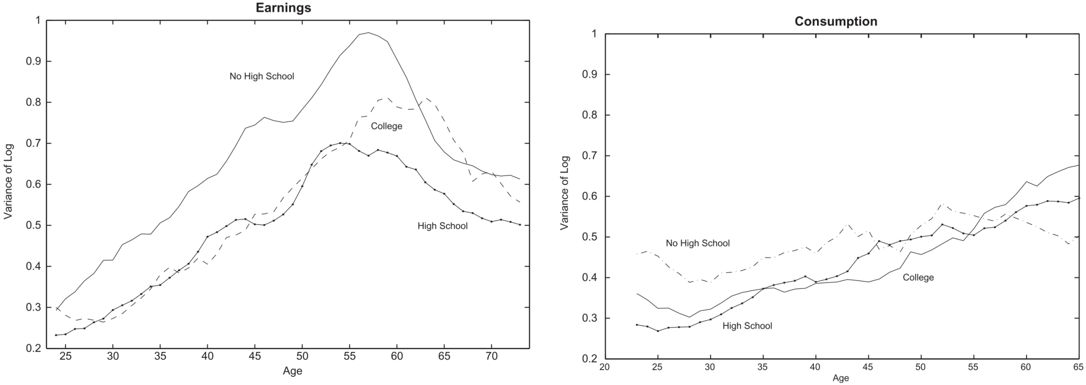
Empirical Evidence

- **Data**
  - **Panel Study of Income Dynamics (PSID, 1969-1992)**: Household survey; Panel data. Earnings: wage income before taxes, plus transfers.
  - **Consumption Expenditure Survey (CEX, 1980-1990)**: Consumption survey; Consumption: nonmedical and nondurable expenditures on goods and services by urban U.S. households.
- Unit of study: household.
- Clean for cohort effects using a linear regression.

Empirical Evidence



Empirical Evidence: by Education



Earnings Stochastic Process

- Let household i of age h. Denote the **residual** log of annual earnings as u_{ih} (i.e., log income with mean zero and net of cohort effects).
- The stochastic process of u_{ih} is defined as:

$$
\begin{aligned}
u_{ih} &= \alpha_i + \epsilon_{ih} + z_{ih} \\
z_{ih} &= \rho z_{i,h-1} + \eta_{ih}
\end{aligned}
$$

  where $\alpha_i \sim N(0, \sigma_\alpha^2)$, $\epsilon_{ih} \sim N(0, \sigma_\epsilon^2)$, $\eta_{ih} \sim N(0, \sigma_\eta^2)$, and $z_{i0} = 0$.

- Interpretation of each idiosyncratic shock:
  - **Fixed effect**, α_i: Innate ability, early education investments, etc.
  - **Transitory shock**, ϵ_{ih}: Earnings bonus, transitory health problems, etc.
  - **Persistent shock**, η_{ih}: Unemployment shocks with scarring effects, promotions, etc.

Earnings Stochastic Process

- At $h = 0$ (age 22), the variance of u_{i0} is $V(u_{i0}) = \sigma_\alpha^2 + \sigma_\epsilon^2 + \sigma_\eta^2$.
- At $h = 1$ (age 23), the variance of u_{i1} is $V(u_{i1}) = \sigma_\alpha^2 + \sigma_\epsilon^2 + \sigma_\eta^2 + \rho^2\sigma_\eta^2$.
- The variance of u_{ih} for age h:

$$V(u_{ih}) = \sigma_\alpha^2 + \sigma_\epsilon^2 + \sigma_\eta^2 \sum_{j=0}^{h-1} \rho^{2j}.$$

- The variance of earnings increases during the life-cycle as persistent shocks accumulate!
- The rate of the increase depends on how persistent are the shocks: ρ.
  - If $\rho = 1$, shocks are permanent and their effects never fade out.

Earnings Stochastic Process

- The goal is to estimate the parameters: ρ, σ^2_η, σ^2_α, and σ^2_ϵ.
- Use GMM to estimate the parameters. Identification intuition (case $\rho = 1$). Take the difference

$$\Delta u_{ih} = \Delta\epsilon_{ih-1} + \eta_{ih}$$

  and use the moments:

$$V(\Delta u_{ih}) = 2\sigma^2_\epsilon + \sigma^2_\eta$$
$$COV(\Delta u_{ih}, \Delta u_{ih+1}) = -\sigma^2_\epsilon$$

  - To recover σ^2_α, use σ^2_η, σ^2_ϵ and the variance of levels, $V(u_i h)$.
  - To estimate ρ, an extra time period is required so we need a panel of at least 4 time periods.
  - STY actually use all moments in levels. The broad idea is similar.

Model

- The economy is populated by H overlapping generations. Denote ϕ_h as the unconditional probability of surviving up to age h, preferences are:

$$\mathbb{E}\sum_{t=1}^{H}\beta^h\phi_h\frac{c_h^{1-\gamma}}{1-\gamma}, \qquad \text{where } \beta\in(0,1).$$

- Agents begin to work at 22 and, conditional on surviving, retire at 65. At 100 die with certainty.

- Technology: $Y = ZK^{\theta}N^{1-\theta}$.
    - Firms hire labor and rent capital at prices W and R.
    - Law of motion: $K' = Y - C + (1-\delta)K$.
    - The economy has SS growth rates of g, so some variables must be normalized.

Budget Constraint

- Budget constraint of a working agent:

$$c_h + (1+g)a'_h \leq a_h R/\xi_h + n_h(1-\tau)W$$

where τ is a labor tax, and $\xi_h = \phi_h/\phi_{h-1}$ is the survivor's premium.

- The labor endowment process is given by:

$$\log n_h = \kappa_h + u_h$$

where κ_h are the age-profile earnings common to all agents, while u_h is the individual-specific stochastic process as defined before.

Budget Constraint

- Budget constraint of a retired agent:

$$c_h + (1+g)a'_h \leq a_h R/\xi_h + B(\overline{n}_h)W$$

where $B(\overline{n}_h)$ is the pension replacement rate that is a function of the average labor endowments over the life cycle, $\overline{n}_h$.

- The avg. labor endowment, $\overline{n}_h$, summarizes the social security contribution and evolves as following:

$$\overline{n}_{h+1} = \begin{cases} \overline{n}_h + n_h/I & \text{if working,} \\ \overline{n}_h & \text{if retired,} \end{cases}$$

where I is the number of years before retirement.

Value Function

- Let V_h denote the value function of an h years old agent. The value function of the agent is:

$$V_h(\alpha, z_h, \epsilon_h, a_h, \overline{n}_h) = \max_{a'_{h+1} \geq \underline{a}(\alpha,z,h)} \left\{ \frac{c_h^{1-\gamma}}{1-\gamma} + \hat{\beta}\xi_{h+1}\mathbb{E}_h[V(\alpha, z'_{h+1}, \epsilon'_{h+1}, a'_{h+1}, \overline{n}'_{h+1})] \right\}$$

  s.t.

$$c_h + (1+g)a'_h = \begin{cases} a_h R/\xi_h + n_h(1-\tau)W & \text{if working,} \\ a_h R/\xi_h + B(\overline{n}_h)W & \text{if retired,} \end{cases}$$

  where $\hat{\beta} = \beta(1+g)^{1-\gamma}$ and $\underline{a}(\alpha, z, h)$ is an age-dependent borrowing constraint.

- You can solve the value function using backward induction, as $V_{H+1} = 0$ and $a'_{H+1} = 0$.

Equilibrium

- Let $S = \{\alpha, z, \epsilon, a, \overline{n}, h\}$ be the state space.
- A stationary equilibrium is defined as prices, R and W; a set of functions, $\{V_h, a'_{h+1}\}_{h=1}^{H}$; aggregate capital stock K and labor supply N; and a cross-sectional distribution μ of agents across S, such that:
  (a) Prices are given by the firm's marginal productivity of labor and capital;
  (b) Functions $\{V_h, a'_{h+1}\}_{h=1}^{H}$ solve the individual's problem;
  (c) Given individual decisions, the distribution μ is stationary;
  (d) Pension tax satisfies the social security budget constraint: $\int_S B(\overline{n}) d\mu = N(1-\tau)$.
  (e) Capital and labor market clears: $K = \int_S a_h d\mu$ and $N = \int_S n_h d\mu$.

Calibration

- **Standard parameters**: $\theta = 0.4$, $\gamma = 2$, $\delta = 0.109$.
- **Stochastic process parameters**: $(\rho, \sigma_\eta^2, \sigma_\epsilon^2, \sigma_\alpha^2, \kappa_h)$ estimated using PSID.
- $B(\overline{n}_h)$ replicates the pension system in the US.
- $\beta = 0.961$ matches wealth-to-income ratio of 3.1 in the US.

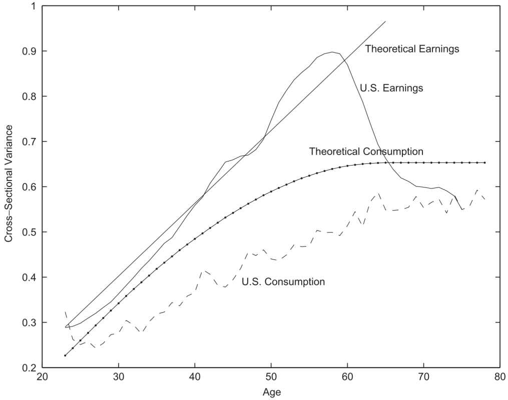
Results

Qualitatively Successful

- Consumption inequality is lower than earnings inequality;
- Earnings inequality increase faster than consumption inequality.

Quantitative: Consumption still a bit off.

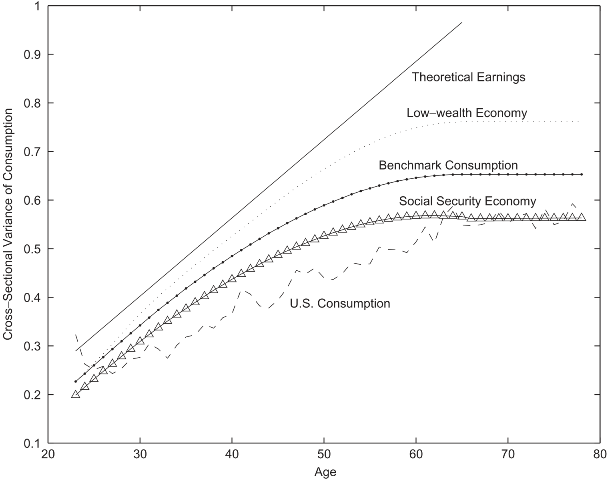
Figure: Model without Social Security ($B(\overline{n}_h) = 0$)



Results

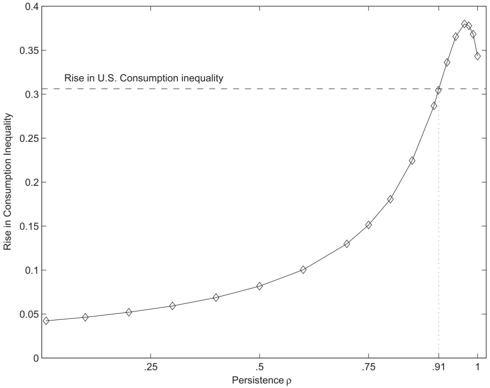
Social Security: it decreases consumption inequality, matches the data better;

Importance of Wealth: ↓ wealth-to-income ratio, ↓ the self-insurance and ↑ consumption inequality.



What matters for Consumption Inequality?

- To generate enough consumption inequality, we need shocks to have enough persistence.
- Borrowing constraints and initial wealth inequality: matters for inequality between 23-29, but it is not very important later.



Life-cycle shocks versus fixed effects

What type of inequality costs more for the agent?

- **Utilitarian measure**: how much consumption the agent is willing to forgo to live in a world without shocks?

- Let ψ the percentage consumption loss. Rewriting the utility function:

$$\mathbb{E}\sum_{t=1}^{H}\beta^h\phi_h\frac{[c_h(1-\psi)]^{1-\gamma}}{1-\gamma}=(1-\psi)^{1-\gamma}\mathbb{E}\sum_{t=1}^{H}\beta^h\phi_h\frac{c_h^{1-\gamma}}{1-\gamma}=(1-\psi)^{1-\gamma}\mathbb{E}V_1(\alpha,z,\epsilon,0),$$

  where $\mathbb{E}V_1(\alpha,z,\epsilon,0)$ is the average lifetime utility of a unborn agent (under the veil of ignorance).

- We can do the same thing for a model without risk, social security, etc.

Life-cycle shocks versus fixed effects

- Solve the model without risk and compute the expected VF at age 1: $\mathbb{E}\hat{V}_1(\alpha, 0|\text{no risk})$.
- What is ψ that equalizes expected utility in both worlds?

$$(1-\psi)^{1-\gamma}\mathbb{E}\hat{V}_1(\alpha, 0|\text{no risk}) = \mathbb{E}V_1(\alpha, z, \epsilon, 0) \iff \psi = 1 - \left(\frac{\mathbb{E}V_1(\alpha, z, \epsilon, 0)}{\mathbb{E}\hat{V}_1(\alpha, 0|\text{no risk})}\right)^{1/(1-\gamma)}$$

- The **consumption equivalent variation** of each type of shock:
  - $\psi_{z,\epsilon} = 27.4\%$.
  - $\psi_{\alpha} = 20.2\%$.
- Shocks are costlier than ex-ante heterogeneity!

Conclusion

- Inequality in earnings and consumption increase during the life cycle.
- Persistent shocks are key to account for this regularity.
- Social security reduces welfare inequality.
- What other policies can achieve less welfare inequality?

Where to go now?

- **Pension System**: Conesa and Krueger (1999), Fuster et al (2007), McKiernan (2021).
- **Inequality over the Life cycle**: Huggett, Ventura and Yaron (2011),
- **Human Capital**: Lochner and Monge-Naranjo (2011), Daruich (2020).
- **Earnings Process**: De Nardi et al (2020), Guvenen et al (2021).
- **Welfare Policy**: Low, Meghir and Pistaferri (2010), Wellschmied (2021).
- **Consumption Insurance**: Kaplan and Violante (2010), Blundell, Pistaferri and Preston (2008).
- **Marriage and Female Labor Supply**: Voena (2015), Attanasio, Low and Sanchez-Marcos (2008).
- **Old Age and Health Shocks**: many papers by Mariacristina Denardi and Eric French.