

# Quantitative Macroeconomics

## Misallocation

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UnB

# References

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- Restuccia and Rogerson\* (2017, JEP) and Hopenhayn\* (2014, Annual Review of Econ.): Survey papers.
- Restuccia and Rogerson (2008, RED): Policy distortions and aggregate productivity with heterogeneous establishments.
- Hsieh and Klenow (2009, QJE): Misallocation and Manufacturing TFP in China and India.
- Also, there are two nice “master” classes with [Richard Rogerson](#) and [Pete Klenow](#) on youtube.

## What accounts for productivity differences across countries?

- Frontier technologies and best practice methods are slow to diffuse to low-income countries.
  - ▶ Poor countries are not using the best technology available.
- Low-income countries are not as effective in allocating their factors of production to their most efficient use.
  - ▶ Production units are not equalizing their marginal products  $\Rightarrow$  Misallocation of resources!

# Sources of Misallocation

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- Suppose a Hopenhayn-type of economy where heterogeneous firms produce a good according a DRS technology:  $y_i = A_i f(k_i, n_i)$ .
- Three channels will affect the amount of output, and hence the overall level of productivity.
  - ▶ **Technology channel**  $\Rightarrow$  reflects the values of the producer-level productivity  $A_i$ ; if all of the  $A_i$  are larger, output will be greater.
  - ▶ **Selection channel**  $\Rightarrow$  reflects the choice of which producers should operate.
  - ▶ **Misallocation channel**  $\Rightarrow$  reflects the choice of how capital and labor are allocated among those producers that operate.
- Conceptually, selection is also a type of misallocation. Hard to measure: we do not observe producers who do not operate.

# Sources of Misallocation

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## Three broad categories of sources

- **Statutory provisions, tax code and regulations.**
  - ▶ Tax code that vary with firm characteristics;
  - ▶ Tariffs applied to narrowly defined categories of goods;
  - ▶ Labor market/product market/land market regulations.
- **Discretionary provisions made by the government or other entities.**
  - ▶ Government corruption and “crony capitalism”;
  - ▶ Subsidies, tax breaks, or low interest rate loans granted to specific firms;
  - ▶ Preferential market access, or selective enforcement of taxes and regulations.
- **Market imperfections.**
  - ▶ Monopoly power;
  - ▶ Market frictions such as financial frictions;
  - ▶ Enforcement of property rights.

- How large is the effect of misallocation on aggregate TFP?
- Restuccia and Rogerson use idiosyncratic producer tax/subsidy rates to broadly reflect factors that generate misallocation.
- Misallocation can cause quantitatively large output and productivity losses, on the order of 30 to 50%.
- Distortions may or may not be correlated with firm-size, generally larger effects if correlated with size.

- Simple version of Hopenhayn model in SS, calibrated to match features of US economy.
- Basic features:
  - ▶ Inelastic labor supply;
  - ▶ Production function  $y = zk^\alpha n^\gamma$ , where  $0 < \alpha + \gamma < 1$ .
  - ▶ Fixed cost,  $c_f$ , and entry cost,  $c_e$ ;
  - ▶ Exogenous exit probability,  $\lambda > 0$ ;
  - ▶ Normalize price of final good to one; solve for  $r$  and  $w$ .
  - ▶ Productivity level of the establishment,  $z$ , remains constant over time.

- Representative consumer maximizes

$$\sum_{t=0}^{\infty} \beta^t u(C_t)$$

subject to the period budget constraint

$$C_t + K_{t+1} = N_t w_t + (1 - \delta + r_t) K_t + \Pi_t + T_t$$

where  $\Pi_t$  and  $T_t$  aggregate profits and net taxes.

- In the steady state (use the euler equation):

$$r = 1/\beta + 1 - \delta.$$



- Face output distortions  $\tau$ . It can be positive (a tax,  $\tau^+$ ) or negative (a subsidy,  $\tau^-$ ).
- Static profits are:

$$\pi(z, \tau) = \max_{n, k} \{ (1 - \tau) z k^\alpha n^\gamma - wn - rk - c_f \}.$$

- Optimal factor demands of this establishment are thus given by:

$$k(z, \tau) = \left( \frac{\alpha}{r} \right)^{\frac{1-\gamma}{1-\gamma-\alpha}} \left( \frac{\gamma}{w} \right)^{\frac{\gamma}{1-\gamma-\alpha}} (z(1-\tau))^{\frac{1}{1-\gamma-\alpha}}$$
$$n(z, \tau) = \left( \frac{z(1-\tau)\gamma}{w} \right)^{\frac{1}{1-\gamma}} k^{\frac{\alpha}{1-\gamma}}$$

- Because both  $\tau$  and  $z$  are constant over time, the value for incumbents is

$$W(z, \tau) = \pi(z, \tau) + \frac{1 - \lambda}{1 + r} W(z, \tau) \quad \Rightarrow \quad W(z, \tau) = \frac{\pi(z, \tau)}{1 - \rho},$$

where  $\rho = \frac{1 - \lambda}{1 + r}$  is the discount rate of the establishment.

- Entrants pay  $c_e$  and draw a pair  $(z, \tau)$  from a distribution  $G(z, \tau) = H(z)P(\tau|z)$ .
  - ▶ The draw of  $\tau$  may be correlated with  $z$ .
- Denote  $\chi(z, \tau) = 1$ , the decision to enter after observing the draw. Free entry implies

$$W_e = \int \max_{\chi(z, \tau) \in \{0, 1\}} \{ \chi(z, \tau) W(z, \tau) - c_e \} dG(z, \tau) = 0$$

# Invariant Distribution

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- Let  $\mu(z, \tau)$  denote the joint distribution of  $(z, \tau)$  and  $E$  the mass of entrants, the law of motion is:

$$\mu'(z, \tau) = \mu(z, \tau)(1 - \lambda) + \chi(z, \tau)G(z, \tau)E$$

where  $\rho = \frac{1 - \lambda}{1 + r}$  is the discount rate of the establishment.

- In a stationary equilibrium this simplifies to

$$\mu(z, \tau) = E \frac{\chi(z, \tau)}{\lambda} G(z, \tau)$$

# Market Clearing and Aggregation

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- Capital and labor Market clearing (assume mass one of HH):

$$\int n(z, \tau) d\mu(z, \tau) = 1 \quad \int k(z, \tau) d\mu(z, \tau) = K$$

- Goods market clearing:

$$C + \delta K + c_e E = \int [zk(z, \tau)^\alpha n(z, \tau)^\gamma - c_f] d\mu(z, \tau)$$

- Government balance budget:

$$\int \tau z k(z, \tau)^\alpha n(z, \tau)^\gamma d\mu(z, \tau) = T$$

**Table 1**  
Benchmark calibration to US data

Parameter	Value	Target
$\alpha$	0.283	Capital income share
$\gamma$	0.567	Labor income share
$\beta$	0.96	Real rate of return
$\delta$	0.08	Investment to output ratio
$c_e$	1.0	Normalization
$c_f$	0.0	Benchmark case
$\lambda$	0.1	Annual exit rate
$s$ range	[1, 3.98]	Relative establishment sizes
$h(s)$	see Fig. 1	Size distribution of establishments

**Table 2**

Distribution statistics of benchmark economy

	Establishment size (number of employees)		
	< 5	5 to 49	$\geq 50$
Share of establishments	0.56	0.39	0.05
Share of output	0.08	0.34	0.58
Share of labor	0.08	0.34	0.58
Share of capital	0.08	0.34	0.58
Average employment	2.4	15.5	183.0

Since capital/labor ratios are equalized across producers, the distribution of labor and capital is the same as the distribution of output across producers.

# Quantitative analysis

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- Introduce random idiosyncratic tax rates,  $\tau_i$  on value added of each producer  $i$ .
- Tax rate is only realized post-entry, may be conditioned on realized productivity, and remains constant thereafter.
- Consider four cases:  $\tau^+ = \{0.1, 0.2, 0.3, 0.4\}$ ,  $\tau^-$  is chosen so SS  $K$  is unchanged.
- Two cases: **uncorrelated distortions** and **correlated distortions**.
  - ▶ Uncorrelated distortions:  $\tau$  is independent of  $z$ .
  - ▶ Correlated:  $\tau$  is correlated with  $z$ .

# Uncorrelated Distortions

**Table 3**

Effects of idiosyncratic distortions—uncorrelated case

Variable	$\tau_t$			
	0.1	0.2	0.3	0.4
Relative Y	0.98	0.96	0.93	0.92
Relative TFP	0.98	0.96	0.93	0.92
Relative E	1.00	1.00	1.00	1.00
$Y_S/Y$	0.72	0.85	0.93	0.97
$S/Y$	0.05	0.08	0.09	0.10
$\tau_S$	0.06	0.09	0.10	0.11

**Table 4**

Relative TFP—uncorrelated distortions

Fraction of establishments taxed (%):	$\tau_t$			
	0.1	0.2	0.3	0.4
90	0.92	0.84	0.78	0.74
80	0.95	0.89	0.84	0.81
60	0.98	0.94	0.91	0.89
50	0.98	0.96	0.93	0.92
40	0.99	0.97	0.95	0.94
20	1.00	0.99	0.98	0.97
10	1.00	0.99	0.99	0.99

$Y_S/Y$  denotes output share of subsidized firms,  $S/Y$  aggregate subsidy as share of output.



# Correlated Distortions

**Table 5**

Effects of idiosyncratic distortions—correlated case

Variable	$\tau_t$			
	0.1	0.2	0.3	0.4
Relative Y	0.90	0.80	0.73	0.69
Relative TFP	0.90	0.80	0.73	0.69
Relative E	1.00	1.00	1.00	1.00
$Y_s/Y$	0.42	0.67	0.83	0.92
$S/Y$	0.17	0.32	0.43	0.49
$\tau_s$	0.40	0.48	0.52	0.53

**Table 6**

Relative TFP—correlated distortions

Fraction of establishments taxed (%):	$\tau_t$			
	0.1	0.2	0.3	0.4
90	0.81	0.66	0.56	0.51
80	0.84	0.70	0.62	0.57
60	0.88	0.77	0.69	0.65
50	0.90	0.80	0.73	0.69
40	0.92	0.82	0.76	0.72
20	0.95	0.89	0.84	0.81
10	0.97	0.92	0.88	0.86

Low productivity is subsidized, high productivity is taxed.

# Summary

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- Purely random misallocation likely not that costly.
  - ▶ Reallocation occurs within productivity class.
- Correlated distortions have larger impacts on TFP.
  - ▶ Systematic reallocation across productivity classes.
  - ▶ Also more costly to finance the subsidy.
- Amount of misallocation is larger when most taxed and few subsidized.
- Subsidizing most productive and taxing least productive is also TFP-reducing, but effects are much smaller because less scope for misallocation.

# Measuring Misallocation

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- The R&R exercise does not attempt to measure misallocation.
- Instead it assesses the **potential** of resource misallocation across productive units to generate large aggregate income per capita differences.
- Note that in their benchmark they abstract from entry issues such as selection. This can increase the negative impact of the wedge.
- How to measure misallocation?

# Measuring Misallocation

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- **Direct Approach:** assess one specific source of misallocation.
  - ▶ Usually requires a full structural model;
  - ▶ Use the model to compute a counterfactual (efficient) economy;
  - ▶ Requires a quantitative measure of the underlying source of misallocation.
- **Indirect Approach:** identify the extent of misallocation without identifying the underlying source of the misallocation.
  - ▶ Requires little structure (usually a production function);
  - ▶ Requires good data to compute the MPN and MPK given a production function;
  - ▶ How to deal with measurement error and other sources of dispersion of MPN/MPK?

# Measuring Misallocation: Indirect Approach

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- The indirect approach identifies misallocation from its symptoms.
- Key symptoms? Efficient requires of marginal products:  $MPN_j = MPN_i$ .
- For usually assumed production function (i.e.,  $y = zn^\alpha$ ), this also implies equalization of the average product:

$$\frac{y_i}{n_i} = \frac{y_j}{n_j}$$

- Given data on  $y$  and  $n$ , we can identify the size of misallocation.

# Measuring Misallocation: Indirect Approach

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## Conceptual Issues:

- Must assume a production function. We are looking to the data through the lens of the model.
- All the differences in  $MPN$  are interpreted as misallocation. Can the model specified wrong? Could the differences in  $MPN$  be model misspecification instead (i.e., different production functions)?

## Practical Issues:

- Microdata typically reports revenue  $p_i y_i$  instead of quantities  $y_i$ .
- If prices vary, we cannot identify  $z_i$  without separating  $p_i$  from  $y_i$ .
- Need a framework that permits price heterogeneity  $\rightarrow$  Melitz.

- Hsieh & Klenow use the indirect approach to infer misallocation in the US, China and India from measured gaps in marginal product.
- They find a large dispersion of marginal products across manufacturing firms in the data.
- **Counterfactual experiment:** Compute the hypothetical gains from reallocating capital and labor:
  - ▶ eliminating distortions entirely: TFP gains 40% in US vs. 130% in India.
  - ▶ reducing distortions to US level: of 30%–50% in China and 40%–60% in India

- Final output  $Y$  is a Cobb-Douglas aggregate of all industries  $s$  output:

$$Y = \prod_{s=1}^S Y_s^{\theta_s}, \quad \sum_{s=1}^S \theta_s = 1.$$

- An industry output is a CES aggregate of  $M_s$  firms indexed by  $i$  that produce differentiated products:

$$Y_s = \left( \sum_{i=1}^{M_s} Y_{is}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 1.$$

- Individual firms produce with a Cobb-Douglas aggregate of capital and labor:

$$Y_{is} = A_{is} K_{is}^{\alpha_s} L_{is}^{1-\alpha_s}, \quad 0 < \alpha_s < 1$$



# Firms and Distortions

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- Monopolistic competition implies that a firm is subject to a demand function:

$$Y_{is} = \left( \frac{P_{is}}{P_s} \right)^{-\sigma} Y_s.$$

- Let the firm-specific (idiosyncratic) distortions be:
  - ▶  $\tau_{Y,is}$ : distortions to marginal product of capital and labor.
  - ▶  $\tau_{K,is}$ : distortions to marginal product of capital relative to labor.
- Profit maximization problem of a firm  $i$  in industry  $s$ :

$$\begin{aligned} \pi_{is} = & \max_{P_{is}, Y_{is}, L_{is}, K_{is}} (1 - \tau_{Y,is}) P_{is} Y_{is} - w L_{is} - (1 + \tau_{K,is}) r K_{is} \\ \text{s.t. } & Y_{is} = \left( \frac{P_{is}}{P_s} \right)^{-\sigma} Y_s, \quad Y_{is} = A_{is} K_{is}^{\alpha_s} L_{is}^{1-\alpha_s} \end{aligned}$$

# Firms and Distortions

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- Marginal Revenue Product of  $K$  and  $L$  equalizes the marginal cost:

$$MRPK_{is} \equiv P_{is}MPK_{is} \left( \frac{\sigma - 1}{\sigma} \right) = \alpha_s \left( \frac{\sigma - 1}{\sigma} \right) \frac{P_{is}Y_{is}}{K_{is}} = \frac{(1 + \tau_{K,is})}{(1 - \tau_{Y,is})} r$$

$$MRPL_{is} \equiv P_{is}MPL_{is} \left( \frac{\sigma - 1}{\sigma} \right) = (1 - \alpha_s) \left( \frac{\sigma - 1}{\sigma} \right) \frac{P_{is}Y_{is}}{L_{is}} = \frac{1}{(1 - \tau_{Y,is})} w$$

- Compute firm-level “wedges”  $\tau_K$  and  $\tau_Y$  as a residual:
  - ▶ Calibrate  $\alpha_s$ ,  $\sigma$  and  $r$ .
  - ▶ Use firm-level data on revenue  $P_{is}Y_{is}$ , capital  $K_{is}$  and wage-bill  $wL_{is}$ .
- $\tau_K$  and  $\tau_Y \Rightarrow$  how much firms deviate from their marginal products!

# Firms and Distortions

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- Intuitively, variation in capital/labor ratio across firms reveals misallocation of capital:

$$(1 + \tau_{K,is}) = \frac{\alpha_s}{(1 - \alpha_s)} \frac{wL_{is}}{rK_{is}}$$

- Variation in labor share reveals overall misallocation to both capital and labor:

$$(1 - \tau_{Y,is}) = \frac{1}{(1 - \alpha_s)} \left( \frac{\sigma}{\sigma - 1} \right) \frac{wL_{is}}{P_{is}Y_{is}}$$

- Using the production function, we can also compute the optimal price given distortions:

$$P_{is} = \underbrace{\frac{\sigma}{\sigma - 1}}_{\text{markup}} \underbrace{\left( \frac{r}{\alpha_s} \right)^{\alpha_s} \left( \frac{w}{1 - \alpha_s} \right)^{1 - \alpha_s} \frac{1}{A_{is}}}_{\text{marginal cost}} \underbrace{\frac{(1 + \tau_{K,is})^{\alpha_s}}{1 - \tau_{Y,is}}}_{\text{wedges}}$$

# Firms and Distortions

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- We are also interested to recover firm-level productivity.
- Let  $TFPQ_{is}$  denote physical productivity (quantity based measure of TFP):

$$TFPQ_{is} \equiv A_{is} = \frac{Y_{is}}{K_{is}^{\alpha_s} L_{is}^{1-\alpha}}$$

- **Problem:** we typically do not observe firm-level output but just revenue. It is useful to define revenue productivity  $TFPR_{is}$  (revenue based measure of TFP):

$$TFPR_{is} \equiv P_{is} A_{is} = \frac{P_{is} Y_{is}}{K_{is}^{\alpha_s} L_{is}^{1-\alpha}}$$

# Firms and Distortions

- In the efficient benchmark  $TFPQ_{is}$  will vary across firms, but  $TFPR_{is}$  will be constant across firms (prices scale with  $A_{is}$ ).
- Wedges show up in  $TFPR_{is}$ . Recall prices:

$$P_{is} = \underbrace{\frac{\sigma}{\sigma-1}}_{\text{markup}} \underbrace{\left(\frac{r}{\alpha_s}\right)^{\alpha_s} \left(\frac{w}{1-\alpha_s}\right)^{1-\alpha_s}}_{\text{marginal cost}} \underbrace{\frac{1}{A_{is}} \frac{(1+\tau_{K,is})^{\alpha_s}}{1-\tau_{Y,is}}}_{\text{wedges}}$$

- Using the equations for  $MRPK$  and  $MRPL$  and the equation above, easy to see that

$$TFPR_{is} \equiv P_{is} A_{is} \propto MRPK_{is}^{\alpha_s} MRPL_{is}^{1-\alpha_s} \propto \frac{(1+\tau_{K,is})^{\alpha_s}}{1-\tau_{Y,is}}$$

# Firms and Distortions

- We can also recover  $TFPQ$  (up to a scale factor) by inverting the demand:

$$P_{is} = \left( \frac{Y_{is}}{Y_s} \right)^{-\frac{1}{\sigma}} P_s \quad \Rightarrow \quad P_{is} Y_{is} = Y_{is}^{\frac{\sigma-1}{\sigma}} P_s Y_s^{\frac{1}{\sigma}}$$
$$Y_{is} = (P_{is} Y_{is})^{\frac{\sigma}{\sigma-1}} \kappa_s \quad \text{where} \quad \kappa_s = (P_s Y_s^{\frac{1}{\sigma}})^{-\frac{\sigma}{\sigma-1}}$$

- Thus  $TFPQ_{is}$  can be retrieve as:

$$A_{is} = \frac{Y_{is}}{K_{is}^{\alpha_s} L_{is}^{1-\alpha}} = \kappa_s \frac{(P_{is} Y_{is})^{\frac{\sigma}{\sigma-1}}}{K_{is}^{\alpha_s} L_{is}^{1-\alpha}}$$

where  $\kappa_s$  absorbs the industry terms.

- What matters is relative productivity (and hence reallocation between firms) so assume  $\kappa_s = 1$ . In the paper H&K uses  $wL$  instead of  $L$  so  $w$  goes to  $\kappa_s$  as well.

- Data:

- ▶ Manufacturing plants of U.S., China (firms) and India;
- ▶ Roughly 400 industries (4-digit).
- ▶ Census for large plants, sample for small plants.
- ▶ Labor compensation, value-added, book value capital stock.

- Calibration:

- ▶  $r = 0.1$  (depreciation = 5% and interest rate 5%);
- ▶  $\sigma = 3$  elasticity of substitution across producers within industry;
- ▶  $1 - \alpha_s$  labor share in corresponding US industry;
- ▶  $\theta_s$  industry shares are country specific and follow from the data:  $\theta_s = P_s Y_s / Y$ .

# Distribution of TFPQ

- Small private plants are underrepresented in the Chinese survey (so not very comparable).
- Left tail is thicker in India.
- Consistent with policies favoring the survival of inefficient plants in India relative to the US.

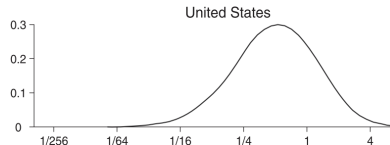
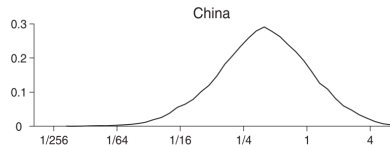
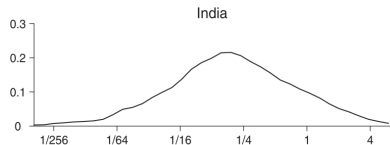


FIGURE I  
Distribution of TFPQ



# Distribution of TFPR

- High dispersion of TFPR is consistent with greater distortions in China and India than the United States.
- Relative to average industry  $\overline{TFPR}_s$ .

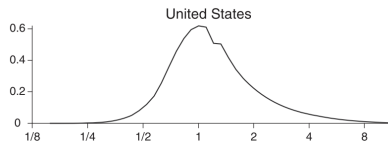
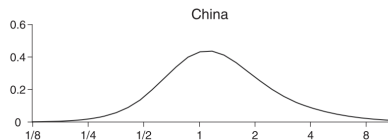
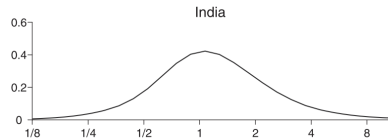


FIGURE II  
Distribution of TFPR

# What Explains Variation of TFPR

TABLE III  
PERCENT SOURCES OF TFPR VARIATION WITHIN INDUSTRIES

	Ownership	Age	Size	Region
India	0.58	1.33	3.85	4.71
China	5.25	6.23	8.44	10.01

*Notes.* Entries are the cumulative percent of within-industry TFPR variance explained by dummies for ownership (state ownership categories), age (quartiles), size (quartiles), and region (provinces or states). The results are cumulative in that “age” includes dummies for both ownership and age, and so on.

- Observables account for roughly 10% of the variation of TFPR in China and 5% in India.
- State ownership seems to be a big driver in China.

# Aggregates

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- How large would the aggregate gains be if the cross-sectional allocation was more efficient?
- Must define aggregate output and  $TFP_s$ . We define productivity for industry  $s$  as:

$$TFP_s \equiv \frac{Y_s}{K_s^{\alpha_s} L_s^{1-\alpha_s}}$$

- From the aggregate production function, aggregate output is:

$$Y = \prod_{s=1}^S Y_s^{\theta_s} = \prod_{s=1}^S (TFP_s K_s^{\alpha_s} L_s^{1-\alpha_s})^{\theta_s}$$

- Must get industry aggregates from individual firms. Again, work with the revenue based measure:

$$\overline{TFPR}_s \equiv \frac{P_s Y_s}{K_s^{\alpha_s} L_s^{1-\alpha_s}}$$

# Aggregates

- Industry aggregate demand is the sum of individual firms' demand.

$$K_s = \sum_{i=1}^{M_s} K_{is} = \sum_{i=1}^{M_s} \frac{\alpha_s}{r} \left( \frac{\sigma - 1}{\sigma} \right) P_{is} Y_{is} \frac{(1 - \tau_{Y,is})}{(1 + \tau_{K,is})}$$

$$L_s = \sum_{i=1}^{M_s} L_{is} = \sum_{i=1}^{M_s} \frac{(1 - \alpha_s)}{w} \left( \frac{\sigma - 1}{\sigma} \right) P_{is} Y_{is} (1 - \tau_{Y,is})$$

- Substitute:

$$\overline{TFPR}_s = \frac{\sigma}{\sigma - 1} \left[ \frac{r}{\alpha_s} / \left( \sum_{i=1}^{M_s} \frac{1 + \tau_{K,is}}{1 - \tau_{Y,is}} \frac{P_{is} Y_{is}}{P_s Y_s} \right) \right]^{\alpha_s} \left[ \frac{w}{(1 - \alpha_s)} / \left( \sum_{i=1}^{M_s} \frac{1}{1 - \tau_{Y,is}} \frac{P_{is} Y_{is}}{P_s Y_s} \right) \right]^{1 - \alpha_s}$$

- We can measure  $\overline{TFPR}_s$  using the firm-level revenue and wedges, and industry revenue.

- The optimal price index (recall monopolistically competition):

$$P_s = \left( \sum_{i=1}^{M_s} P_{is}^{1-\sigma} \right)^{1/(1-\sigma)}$$

$$P_s = \left( \sum_{i=1}^{M_s} (TFPR_{is}/A_{is})^{1-\sigma} \right)^{1/(1-\sigma)} \quad \text{where we use } TFPR_{is} = P_{is}A_{is}.$$

- Since  $P_s \times TFP_s = \overline{TFPR}_s$ , we can re-arrange:

$$TFP_s = \left( \sum_{i=1}^{M_s} \left( A_{is} \frac{\overline{TFPR}_s}{TFPR_{is}} \right)^{\sigma-1} \right)^{1/(\sigma-1)}$$

- Without distortions,  $TFPR_{is}$  is the same for all firms and is equal to  $\overline{TFPR}_s$ , thus

$$TFP_s = \left( \sum_{i=1}^{M_s} A_{is}^{\sigma-1} \right)^{1/(\sigma-1)}$$

is a composite of all individual productivities.

- Idiosyncratic distortions increase the dispersion of  $TFPR_{is}$ . Since  $\sigma > 1$ , higher dispersion implies  $\downarrow TFP_s$ .
- In the case  $A_{is}$  and  $TFPR_{is}$  are jointly log-normal, it is possible to show that:

$$\log TFP_s = \frac{1}{1-\sigma} \log \left( \sum_{i=1}^{M_s} A_{is}^{\sigma-1} \right) - \frac{\sigma}{2} \text{Var}(\log TFPR_{is}) + \text{constant}$$

# Equalizing TFPR within Industries

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- Recall:  $Y = \prod_{s=1}^S (TFP_s K_s^{\alpha_s} L_s^{1-\alpha_s})^{\theta_s}$ .
- We can compute how far actual output are from the efficient output:

$$\frac{Y}{Y_{\text{efficient}}} = \prod_{s=1}^S \left[ \sum_{i=1}^{M_s} \left( \frac{A_{is}}{\bar{A}_s} \frac{\overline{TFPR}_s}{TFPR_{is}} \right)^{\sigma-1} \right]^{\theta_s/(\sigma-1)}$$

where  $\bar{A}_s \equiv \left( \sum_{i=1}^{M_s} A_{is}^{\sigma-1} \right)^{1/(\sigma-1)}$ .

- Note that we are reallocating labor/capital within industries and not across industries.

# Equalizing TFPR within Industries

TABLE IV  
TFP GAINS FROM EQUALIZING TFPR WITHIN INDUSTRIES

China	1998	2001	2005
%	115.1	95.8	86.6
India	1987	1991	1994
%	100.4	102.1	127.5
United States	1977	1987	1997
%	36.1	30.7	42.9

Notes. Entries are  $100(Y_{\text{efficient}}/Y - 1)$  where  $Y/Y_{\text{efficient}} = \prod_{s=1}^S [\sum_{i=1}^{M_s} (\frac{A_{si}}{A_s} \frac{\overline{\text{TFPR}}_s}{\text{TFPR}_{si}})^{\sigma-1}]^{\theta_s/(\sigma-1)}$  and  $\text{TFPR}_{si} \equiv \frac{P_{si} Y_{si}}{K_{si}^{\alpha_s} (w_{si} L_{si})^{1-\alpha_s}}$ .

Gains have been falling in China, suggesting actual distribution has been improving over time.



# Efficient Distribution

- Plant size is value-added.
- Efficient distribution has more dispersed plant size, fewer middle but more large and more small plants.

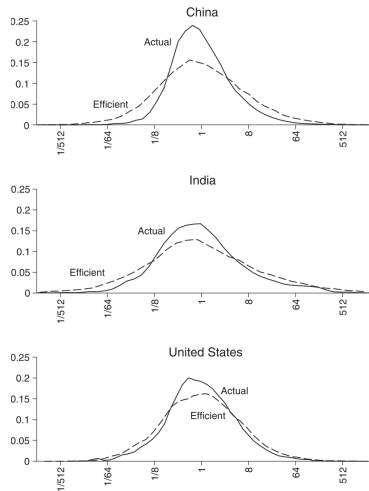


FIGURE III  
Distribution of Plant Size

# Gains from Moving to the U.S. Efficiency

TABLE VI  
TFP GAINS FROM EQUALIZING TFPR RELATIVE TO 1997 U.S. GAINS

China	1998	2001	2005
%	50.5	37.0	30.5
India	1987	1991	1994
%	40.2	41.4	59.2

*Notes.* For each country-year, we calculated  $Y_{\text{efficient}}/Y$  using  $Y/Y_{\text{efficient}} = \prod_{s=1}^S [\sum_{i=1}^{M_s} (\frac{A_{si}}{A_s} \frac{\text{TFPR}_s}{\text{TFPR}_{si}})^{\sigma-1}]^{\theta_s/(\sigma-1)}$  and  $\text{TFPR}_{si} \equiv \frac{P_{si} Y_{si}}{K_{si}^{\alpha_s} (w_{si} L_{si})^{1-\alpha_s}}$ .

We then took the ratio of  $Y_{\text{efficient}}/Y$  to the U.S. ratio in 1997, subtracted 1, and multiplied by 100 to yield the entries above.

These are just “static” gains. They could be magnified by dynamic responses such as endogenous capital accumulation.

# Equalizing TFPR within Industries

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- Probably a lot of the dispersion of TFPR is **NOT** misallocation:
- TFPR dispersion due to...
  - ▶ Measurement error in revenue or inputs;
  - ▶ Unavoidable adjustment costs and transportation costs;
  - ▶ Compensating differentials for labor (amenities and disamenities);
  - ▶ Differential riskiness of investments in capital, R&D, etc.
  - ▶ Misspecification of the production function: within-industry variation in technology/markup.
- Many robustness checks in the paper...

# Conclusion

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- Misallocation can go a long way in explaining cross-country differences in income per capita.
- **Indirect Approach:**
  - ▶ Compute ALL the misallocation directly from the data on mg. products.
  - ▶ Cannot give an exact solution because it does not identify the direct source of wedges.
- Need the **Direct Approach** for that. Possible explanations:
  - ▶ Size-dependent policies/taxes?
  - ▶ Financial frictions?
  - ▶ Entry and land use restrictions?
  - ▶ Discrimination?
- Must specify full model.