Quantitative Macroeconomics

Firms and Business Cycles: Clementi and Palazzo (2016)

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Firms and Business Cycles

Broadly the literature on firm dynamics contribute to the business cycles literature on three dimensions:

- How does entry/exit and firm's cohorts propagate aggregate shocks?
  - Clementi and Palazzo (2016, AEJ: Macro); Sedlacek and Sterk (2017, AER).
- What are the effects of firm-level capital adjustment costs in aggregate investment?
  - Kahn and Thomas (2008, ECMA); Bachmann and Bayer (2013, JME); Winberry (2021, AER)
- What is the effect of financial frictions in boom-bust cycles?
  - Arellano, Bai and Kehoe (2019, JPE); Ottonello and Winberry (2020, ECMA); Khan and Thomas (2013, JPE).

Firms and Business Cycles: Empirical Facts

- **Size** (Moscarini and Postel-Vinay, 2012, AER):
  - Large employers on net destroy proportionally more jobs relative to small employers when in a recession;
  - This pattern holds for continuing firms, as well as for older, established firms.
  - It holds for different countries and industries.

- **Age** (Sedlacek, 2020, JME):
  - Young firms (less than 6 years) account for 38% of all fluctuations in employment...
  - ...but only 16% of employment share!
  - About half of the above contribution of young firms comes from the number of young firms, the other half by changes in size of young firms.
  - Firm entry account for almost all extensive margin variation, change in survival rate do very little.

Firms and Business Cycles: Empirical Facts

- **Cohort** (Sedlacek and & Sterk, 2017, AER):
  - (i) Employment created by startups is volatile and procyclical.
  - (ii) Cyclical variations of startup employment persist into later years.
  - (iii) Cyclical variations of cohort-level employment are mainly driven by fluctuations in firm size, with an increasing importance as cohorts age.
- Cohorts of firms that enter during a recession grow less and that shows up years later in "missing employment".
  - there are strong cohort effects in firm-level data $\Rightarrow$ role of entry (and startup conditions) important for macro;

Cohort Effects

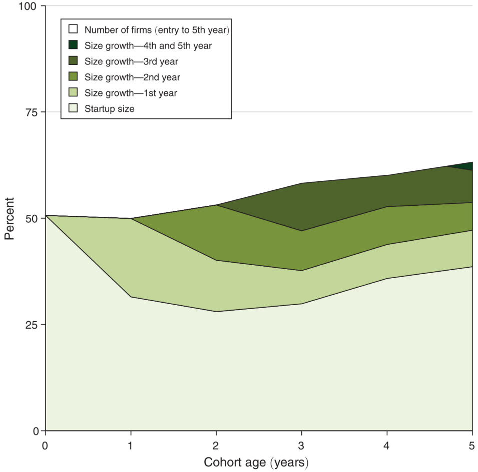

FIGURE 3. CONTRIBUTIONS TO VARIATION IN COHORT-LEVEL EMPLOYMENT

Note: Contributions (in percent) of changes in the number of firms and in average firm size at different ages to the variation in cohort-level employment.

Source: BDS

Clementi and Palazzo (2016)

- **Question:** how does entry/exit propagate aggregate shocks?
- Firm dynamics model a la Hopenhayn over the business cycles.
- Results point out that entry is crucial in shaping the recovery from economic recessions.
- The effect on entry is not so important at impact but grows larger as time passes by.

Model Setup

- Infinite horizon, discrete time. Production function uses capital and labor:

$$y_t = z_t s_t (k_t^{\alpha} l_t^{1-\alpha})^{\theta} \qquad \text{where} \quad 0 < \theta, \alpha < 1.$$

where z_t is an aggregate (common to all firms) shock:

$$\log z_t = \rho_z \log z_{t+1} + \sigma_z \varepsilon_{z,t+1} \qquad \varepsilon_z \sim N(0,1),$$

and s_t is an idiosyncratic (firm-specific) shock:

$$\log s_t = \rho_s \log s_{t+1} + \sigma_s \varepsilon_{s,t+1} \qquad \varepsilon_s \sim N(0,1),$$

- Denote the conditional distribution of s by $H(s_{t+1}|s_t)$.

Model Setup

- Rest of the model is standard. Capital depreciates at rate $\delta \in (0, 1)$.
- Incumbents pay a (stochastic) fixed cost, $c_f \sim G$;
- Entrants observe a signal of productivity and pay an entrant cost c_e.
- Model is solved in **partial equilibrium** along some dimensions:
  - Labor supply is given by $L_s(w) = w^\gamma$...
  - ...but capital is infinitely elastic and gross interest rate $R > 1$ is fixed.
- There will be a distribution of firms $\Gamma_t(k, s)$. This distribution fluctuates over time because of aggregate shocks.
  - Denote the aggregate state by $\lambda_t = (\Gamma_t, z_t)$ and transition operator $J(\lambda_{t+1}|\lambda_t)$.

Adjustment Cost

- Let firm's investment be:

$$x = k' - k(1 - \delta).$$

- The investment adjustment costs are sum of a fixed part and a convex part:

$$g(x, k) = \mathcal{I}_{\{x \neq 0\}} c_0 k + c_1 \left(\frac{x}{k}\right)^2 k,$$

where $c_0, c_1 \geq 0$ and $\mathcal{I}$ is an indicator function.

Timing

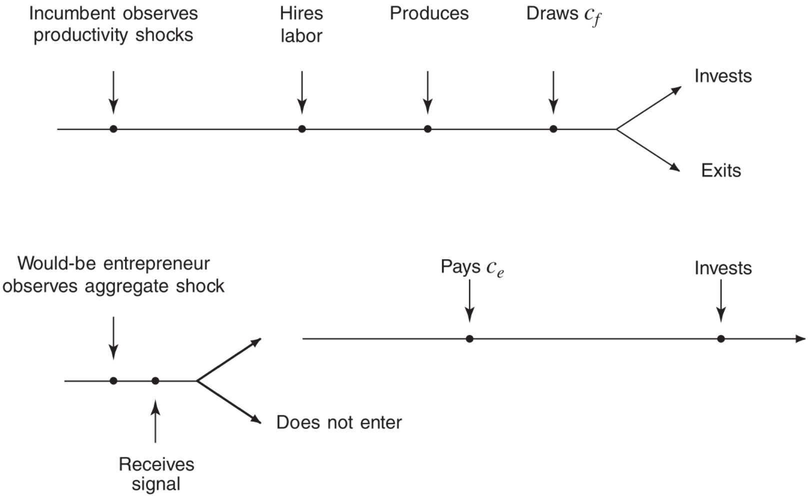

FIGURE 1. TIMING IN PERIOD t

Incumbent's Problem

- If a firm decides to exit, it gets the undepreciated portion of k plus the adjustment cost:

$$V_x(k) = k(1-\delta) - g(-k(1-\delta), k).$$

- Given aggregate state λ, capital and productivity, profits is:

$$\pi(k, s, \lambda) = \max_{l} sz(k^{\alpha} l^{1-\alpha})^{\theta} - wl$$

- The Bellman equation of an incumbent:

$$\begin{aligned}
V(k, s, \lambda) &= \pi(k, s, \lambda) + \int \max\{V_x(k), \tilde{V}(k, s, \lambda) - c_f\} dG(c_f) \\
\tilde{V}(k, s, \lambda) &= \max_{x} -x - g(x, k) + \frac{1}{R} \int \int V(k', s', \lambda') dH(s'|s) dJ(\lambda'|\lambda) \\
\text{s.t.} \quad & x = k' - k(1-\delta).
\end{aligned}$$

Entrants' Problem

- There is a constant mass $M > 0$ of prospective entrants.
- Each receives a signal $q \sim Q(q)$ about her productivity, where $Q(q)$ is Pareto.

$$V_e(q, \lambda) = \max_{k'} -k' + \frac{1}{R} \int \int V(k, s, \lambda) dH(s'|q) dJ(\lambda'|\lambda)$$

- Firms decide to invest and operate if $V_e(q, \lambda) \geq c_e$.
- There will be a threshold $q^*(\lambda)$ such that all firms with $q \geq q^*$ decide to enter.
- The mass of entrants with productivity less than $\bar{s}$ is:

$$M \int_{q^*}^{\infty} \int_0^{\bar{s}} dH(s|q) dQ(q)$$

Equilibrium

- There is a distribution $\Gamma_t(k, s)$ over the idiosyncratic state of the firm.
- The distribution has to satisfy a law-of-motion defined by the policy functions of incumbents (investment and exit) and entrants (initial capital and entry).
  - The policy functions dependent of the aggregate state of the economy $\lambda_t = (\Gamma_t, z_t)$.
  - See the paper.
- We must solve for the equilibrium wage using the labor market clearing condition:

$$\int l(k, s, \lambda_t) d\Gamma_t(k, s) = L_s(w_t) = w_t^\gamma$$

Calibration and Micro Implications

- Conventional parameters are taken from the data/literature: $R, \delta, \alpha, \theta$.
- Parameters regarding the idiosyncratic shock, adjustment costs, and fixed cost are calibrated using microdata on investment/firm dynamics in the **stationary equilibrium**.
- Parameters related to aggregate fluctuations $(\gamma, \sigma_z, \rho_z)$ are chosen to match time series of aggregate output and employment.
- The model is solved numerically using the Krusell-Smith method.

Calibration

TABLE 1—PARAMETER VALUES

Description	Symbol	Value
Capital share	α	0.3
Span of control	θ	0.8
Depreciation rate	δ	0.1
Interest rate	R	1.04
Labor supply elasticity	γ	2.0
Mass of potential entrants	M	1,766.29
Persistence idiosyncratic shock	ρ_s	0.55
Variance idiosyncratic shock	σ_s	0.22
Operating cost – mean parameter	μ_{c_f}	−5.63872
Operating cost – var parameter	σ_{c_f}	0.90277
Fixed cost of investment	c_0	0.00011
Variable cost of investment	c_1	0.03141
Pareto exponent	ξ	2.69
Entry cost	c_e	0.005347

Calibration

Table 2—Calibration Targets

Statistic	Model	Data
Mean investment rate	0.153	0.122
SD investment rate	0.325	0.337
Investment autocorrelation	0.059	0.058
Inaction rate	0.067	0.081
Entry rate	0.062	0.062
Entrants' relative size	0.58	0.60
Exiters' relative size	0.47	0.49

Micro Implications

- As in the Hopenhayn model, age matters unconditionally: young firms have very low productivity on average, so they are more likely to exit.

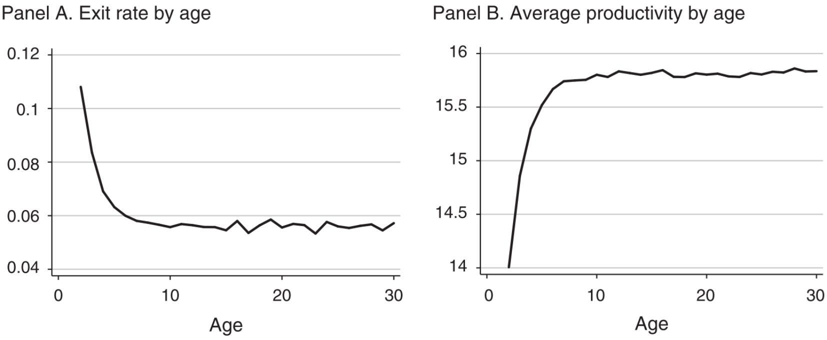

FIGURE 4. THE EXIT HAZARD RATE

- Also, all firms have a chance of exiting since c_f is random.

Micro Implications

- Establishment growth is (unconditionally) negatively correlated with size/age (as in Hopenhayn).

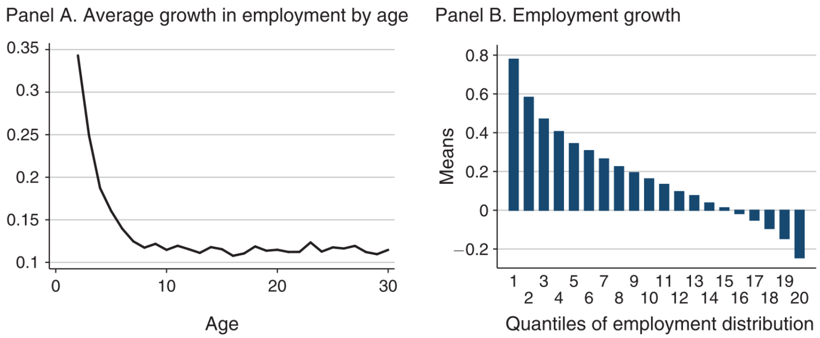

FIGURE 5. UNCONDITIONAL RELATIONSHIP BETWEEN GROWTH, AGE, AND SIZE

- However, after controlling by size there is no role for age in Hopenhayn.

Micro Implications

- As opposed to Hopenhayn, the data shows that firm growth declines with age even controlling by size.
- In C&P there is a role of age **even conditioning for age**.
- Why?
    - Young firms start with lower productivity, which creates scope for young firms to growth (mean reverting of s).
    - Young firms also start with low capital.
    - Because adjustment costs some firms will be low k/high s, while others will be high k/low s.
    - High k/low s will shrink and tend to be older, since to have high k they must have grown in the first place.
- Hence, one way to make age matter is to include adjustment costs. Another way is including financial frictions

Micro Implications

- Distribution displays positive skewness, and it decreases by age.

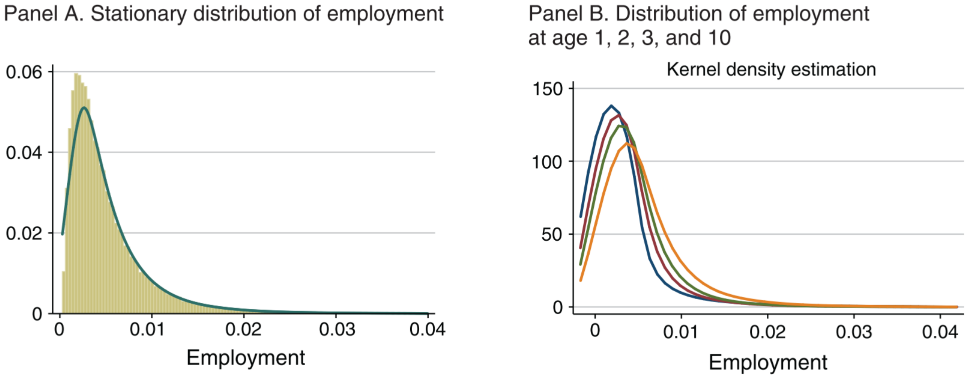

FIGURE 6. EVOLUTION OF A COHORT'S SIZE DISTRIBUTION

Aggregate Fluctuations and Macro Implications

- From labor market clearing, the equilibrium wage satisfies:

$$\log w_t = \frac{\log[(1-\alpha)\theta z_t]}{1+\gamma[1-(1-\alpha)\theta]} + \frac{1-(1-\alpha)\theta}{1+\gamma[1-(1-\alpha)\theta]}\Omega_t$$

  where $\Omega_t = \log\left[\int (sk^{\alpha\theta})^{\frac{1}{1-(1-\alpha)\theta}} d\Gamma_t(k,s)\right]$.

- Thus, wage depends on the realization of the aggregate state z_t, as well as the distribution Γ_t

- In order to forecast w, the firm must form expectations over Γ!

- Hard problem since Γ is a infinite dimension object.

Aggregate Fluctuations and Macro Implications

- As in the **Krusell-Smith algorithm**, suppose the moment Ω_{t+1} is a linear function of Ω_t and $\log z_{t+1}$.

- Plugging back in the previous equation, we have the forecasting rule:

$$\log w_{t+1} = \beta_0 + \beta_1 \log w_t + \beta_2 \log z_{t+1} + \beta_3 \log z_t + \varepsilon_{t+1}$$

- The aggregate variables reduce to the pair (w_t, z_t).
  - Also, they are positively correlated.

- Numerically, we must iterate on the β's to solve for the model (just like in the typical KS algorithm).

Aggregate Fluctuations and Macro Implications

- In the model with aggregate fluctuations, changes in the aggregate state (w_t, z_t) affects the **entry** and **exit** of firms.

- An increase in z_t or a decline in w_t:
  - Increases entry, but the average efficiency of entrants will be lower.
  - Decreases exit, but the average efficiency of exiters also decline.

- There is a role for a **cleasing recession**, which is consistent with the data.

TABLE 5—CORRELATIONS WITH OUTPUT

Entry rate	Exit rate	Entrants' size	Exiters' size
0.402	−0.779	−0.725	−0.892

Aggregate Fluctuations and Macro Implications

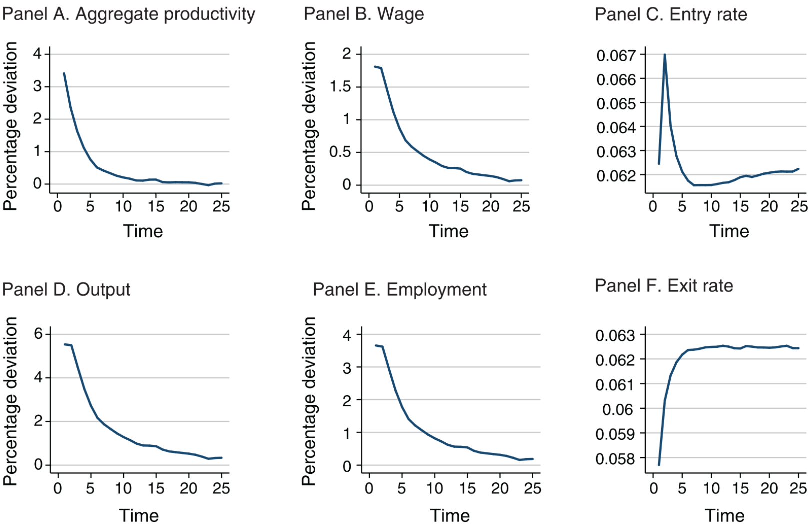

FIGURE 9. RESPONSE TO A POSITIVE PRODUCTIVITY SHOCK

Aggregate Fluctuations and Macro Implications

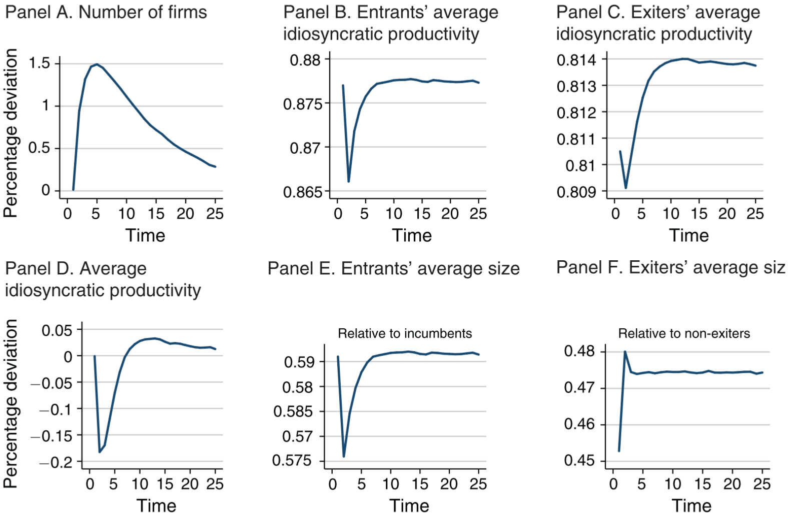

FIGURE 10. RESPONSE TO A POSITIVE PRODUCTIVITY SHOCK

The Role of Entry and Exit

- Allowing for entry and exit enhances the model internal **amplification** and **propagation**.

- More entry:
    - more output initially at impact $\Rightarrow$ Amplification;
    - young firms are high-growth (cohort effect) $\Rightarrow$ Propagation.

- Recall that in a simple version without capital, aggregate production is given by:

$$Y_t = z_t \left[\int \hat{\Gamma}(s) s^{\frac{1}{1-\alpha}} \right]^{1-\alpha} N_t^{1-\alpha} L_t^{\alpha}$$

  where $\hat{\Gamma}(s)s = \Gamma_t / N_t$

- Two effects that are new relative to a model without entry-exit:
    - Time-varying TFP from the distribution of s_t (in addition to z_t);
    - Number of firms N_t.

Aggregate Fluctuations and Macro Implications

- On impact, the IRF of output with and without entry/exit are similar, but in the longer term entry/exit are critical for generating a persistence.

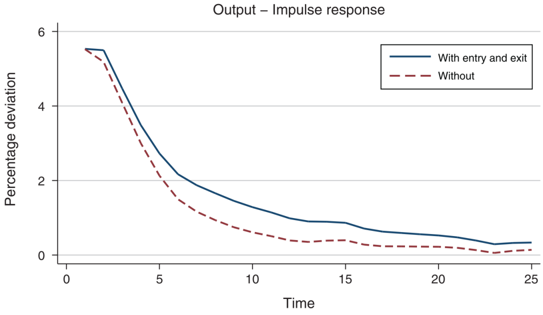

FIGURE 11. THE EFFECT OF ENTRY AND EXIT ON OUTPUT DYNAMICS

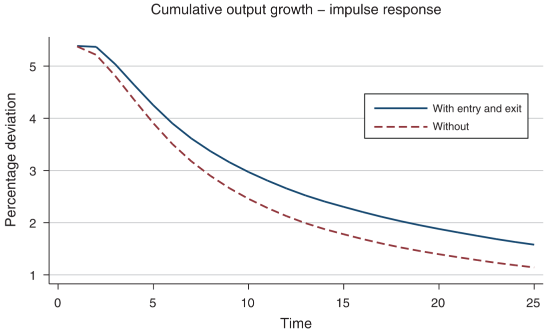

FIGURE 12. CUMULATIVE OUTPUT GROWTH

Accounting for the Great Recession (2007-2009)

- Deep recession characterized by a **slow employment recovery**.
- Unusually large and protracted **drop in the number of establishments**.
  - Conditional on survival, the average plant size relative to incumbents were not that different in the great recession.
- The model is consistent with the evidence on the slow recovery in the Great Recession coming from a **missing cohort of firms**.
  - Slow adjustment comes mainly from the extensive margin (number of firms) and not intensive margin (selection).

Computational Methods

- Methodologically, solving a firm dynamics with aggregate shock is similar to the models with HH heterogeneity.
  - **Usual approaches work**: Krusell-Smith method, Reiter, and Impulse-Response (BKM) method.

- Watch out for some details that sometimes appear in these models: non-convexities (discrete choices such exit/entry), lack of an euler equation, etc.
  - Value function iteration often is solved using **collocation/projection methods**, which are usually faster than brute force the problem.
  - See Nakajima and Simon Mongey's notes on that.

- Good references on how to solve these models are:
  - Terry (2017, JMCB) and Winberry (2018, QE).

Conclusion

- Opening the black box of the aggregate production function matters for the business cycles.

- Many more interesting applications:
  - **Uncertainty and Risk Shocks**: Bloom et al (2018, ECTA), Salgado, Guvenen and Bloom (2020, WP).
  - **Monetary Policy**: Ottonello and Winberry (2020, ECMA); Gonzalez, Nuño, Thaler, Abrazio (2021, WP); Jeenas (2020, WP).
  - **Unemployment Fluctuations**: Sedlacek (2020, JME); Kaas and Kircher (2015, AER).
  - **Granularity**: Carvalho and Grassi (2019, AER); Burstein, Carvalho and Grassi (2021, WP).