

# Advanced Macroeconomics

Financial Frictions and Development: Midrigan & Xu (2014)

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INSPER

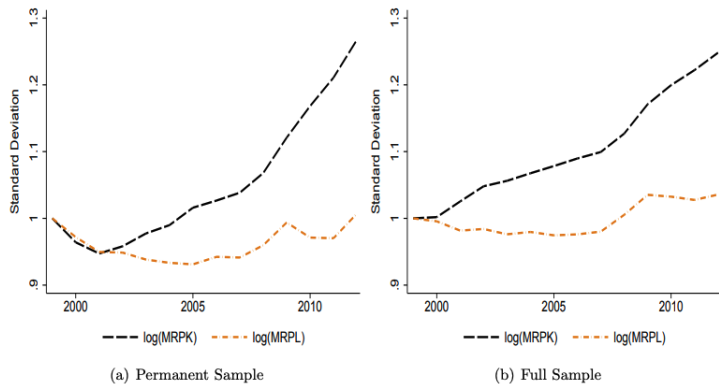
# Introduction

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- Large differences in  $TFP$  across countries.
- Many have shown that dispersion in  $MPK$  can go a long way in explaining these differences..
- Financial markets are much less developed in poor countries  $\Rightarrow$  Potential source of misallocation!
- Financial frictions can distort the economy in two ways:
  - ▶ **Extensive Margin**: Entry or technology adoption;
  - ▶ **Intensive Margin**: Misallocation of resources among operating producers.

# Decline of TFP is related with increase in $\text{std}(\ln(\text{MRPK}))$

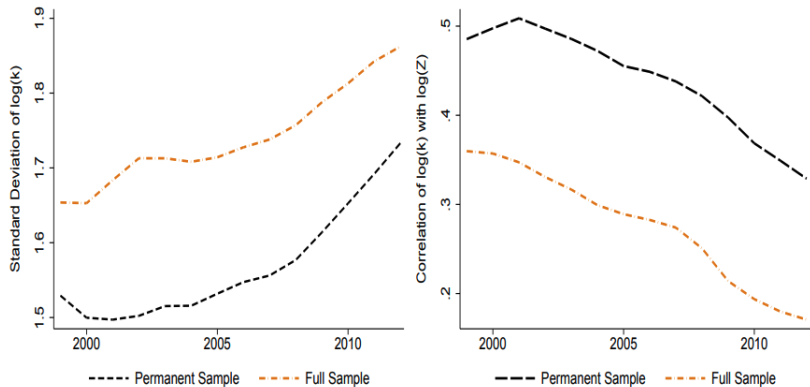
Figure: Dispersion of Capital



Source: Gopinath et al (2017, QJE). Capital Allocation and Productivity in South Europe.

# Decline of TFP is related with decline in $\text{Corr}(\ln(K), \ln(Z))$

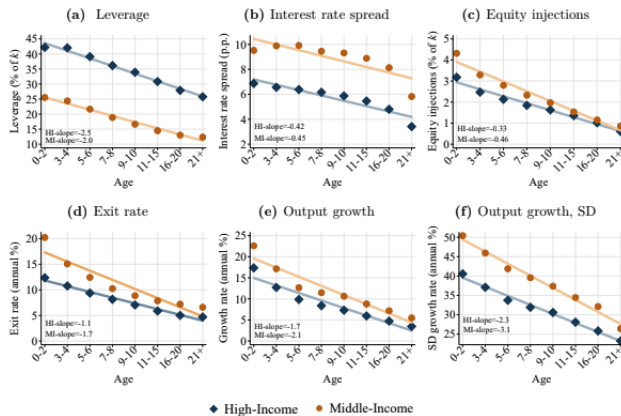
Figure: Dispersion of  $\log(\text{MRPK})$  in Spain



Source: Gopinath et al (2017, QJE). Capital Allocation and Productivity in South Europe.

# Spreads and leverage are important for the firm life cycle

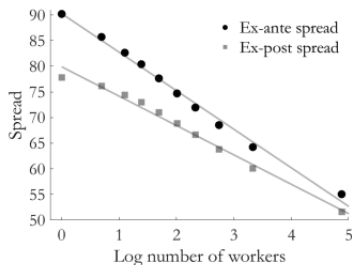
**Figure 2:** Finance, Survival, and Growth Over the Life Cycle of Firms  
High-Income and Middle-Income Countries



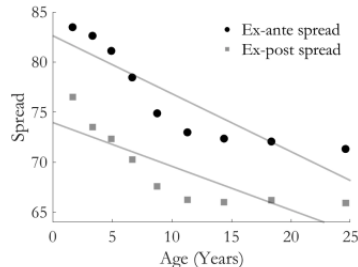
Source: Finance Over the Life Cycle of Firm.s Kochen (2025, WP).

# Spreads in Brazil

Figure 1: Ex Ante and Ex Post Spreads



(a) Spreads by firm size.



(b) Spreads by firm age.

**Source:** Financing Costs and Development. Cavalcanti et al (2025, WP).

Cavalcanti et al  $\Rightarrow$  dispersion of interest rate is huge in Brazil and not explained by observables.

# Introduction: Midrigan and Xu

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- M&XU: Financial frictions can have large effects on TFP, but these effects are mostly from the **extensive margin**.
- Why FF cannot generate strong misallocation among existent producers?
  - ▶ They accumulate internal funds and *un-do* the effects of financial frictions.
- Financial frictions might also be useful to generate micro behavior consistent with the data:
  - ▶ Larger growth from young firms.

# Model Setup

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- Model is populated by a measure one of workers and a measure  $N_t$  of producers.
- Efficiency of labor grows at rate  $\gamma > 1$  (i.e., it is a growth model);
- **Two sectors:** traditional and modern sector.
- **Traditional:** only labor and an unproductive technology.
- **Modern:** capital and labor, productive technology, requires upfront investment (entry cost).
- A measure of  $(\gamma - 1)N_t$  producers enter every period. All producers enter in the traditional sector.



# Model Setup

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- Workers and Producers are heterogeneous in their net worth and productivities.
- They have log preferences over consumption:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \log(C_t)$$

- Producers productivity has two dimensions:
  - ▶  $z$ : permanent (fixed) productivity;
  - ▶  $e$ : transitory productivity that follows a Markov process with probabilities  $f_{ij}$ ;
- Entrants start with zero net worth and draw productivities from their invariant distribution.
- Workers' labor productivity  $\nu_t$  follows a finite state Markov.

# Traditional Sector

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- Production function uses only labor:  $Y_t = \exp(z + e)^{1-\eta} L_t^\eta$ , where  $\eta < 1$ .
- Producers in the traditional sector choose consumption-savings and whether to enter the modern sector.
- If they stay in the traditional sector, their budget constraint is:

$$C_t = Y_t - W L_t - (1 + r)D_t + D_{t+1}$$

where  $D_{t+1} \leq 0$  is their debt position. They cannot borrow.

## Traditional Sector

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- If they enter the modern sector, they must finance expenditure on physical capital  $K_{t+1}$  and intangibles  $\exp(z)\kappa$ . To finance it, they can use their own funds, borrow or issue equity.
- The producer who enters can borrow up to a certain limit of its capital (collateral):

$$D_{t+1} \leq \theta(K_{t+1} + \exp(z)\kappa).$$

where  $\theta \in [0, 1]$  governs the strength of financial frictions.

- Producers can issue claims to a fraction  $\theta\chi$  of their future profits (equity), where  $\chi \in [0, 1]$ .
- The budget constraint of a entrepreneur entering in the modern sector is:

$$C_t + K_{t+1} + \exp(z)\kappa = Y_t - WL_t - (1+r)D_t + D_{t+1} + \theta\chi P_t$$

where  $P_t$  is the price of a share.

# Traditional Sector

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- Let net worth be:  $A = K - D$ . Divide everything by  $\exp(z)$ , i.e.  $a \equiv A / \exp(z)$ .
- The Bellman equation of the producers in the traditional sector is:

$$V^{\tau}(a, e) = \max_{a', c} \log(c) + \beta \max \left\{ \mathbb{E}[V^{\tau}(a', e')|e], \mathbb{E}[V^m(a', e')|e] \right\}$$

s.t.  $c + x = \pi^{\tau}(e) + (1 + r)a$

$$x = \begin{cases} a' & \text{if stays in the traditional sector} \\ a' + \kappa - \theta \chi p(a', e) & \text{if enter in the modern sector.} \end{cases}$$

where  $\pi^{\tau}(e)$  is the profit of a producer in the traditional sector, and  $p(a', e)$  is the price of a share.

# Modern Sector

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- Production function:  $Y_t = \exp(z + e + \phi)^{1-\eta} (K_t^\alpha L_t^{1-\alpha})^\eta$ , where  $\phi \geq 0$ .
- Producers in the modern sector are subject to the same collateral constraint of entrants.
- Bellman equation:

$$\begin{aligned} V^m(a, e) &= \max_{a', c} \log(c) + \beta \mathbb{E}[V^m(a', e') | e] \\ \text{s.t.} \quad c + a' &= (1 - \theta\chi)\pi^m(a, e) + (1 + r)a \end{aligned}$$

where profits  $\pi^m(a, e)$  are given by:

$$\pi^m(a, e) = \max_{k, l} \exp(e + \phi)^{1-\eta} (k^\alpha l^{1-\alpha})^\eta - Wl - (r + \delta)k \quad \text{s.t.} \quad k \leq \frac{1}{1 - \theta}a + \frac{\theta}{1 - \theta}\kappa$$

- Workers choose consumption-savings. Savings can be in risk-free assets or shares from modern firms.
- Let  $\omega_i$  be the number of shares from producer  $i$ , the budget constraint is:

$$c_t + a_{t+1} + \int P_t^i \omega_{t+1}^i di = W \gamma^t \nu_t + (1 + r)a_t + \int (P_t^i + \Pi_{t,}^{m,i}) \omega_t^i di$$

which implies a savings function and the no-arbitrage condition:

$$P_t = \frac{\mathbb{E}_t[P_t + \Pi_{t+1}^{m,i}]}{1 + r} \quad \Rightarrow \quad p(a, e) = \frac{\mathbb{E}[p(a', e') + \pi^m(a', e')|e]}{1 + r}.$$

# Distribution

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- Let  $n_t^m(a, e)$  and  $n_t^\tau(a, e)$  be the measure of modern and traditional producers.
- The evolution of the measure of modern producers follows:

$$n_{t+1}^m(A, e_j) = \int_A \sum_i f_{i,j} \mathbf{1}_{\{a^m(a, e_i) \in A\}} dn_t^m(a, e_i) + \int_A \sum_i f_{i,j} \mathbf{1}_{\{\xi(a, e_i)=1, a^{\tau,s}(a, e_i) \in A\}} dn_t^{\tau,s}(a, e_i)$$

where  $\xi(a, e_i) = 1$  represents the decision of switching sector.

- The law-of-motion for the traditional sector:

$$n_{t+1}^\tau(A, e_j) = \int_A \sum_i f_{i,j} \mathbf{1}_{\{\xi(a, e_i)=0, a^\tau(a, e_i) \in A\}} dn_t^\tau(a, e_i) + (\gamma - 1) N_t \mathbf{1}_{\{0 \in A\}} \bar{f}_j$$

where  $\bar{f}_j$  is the invariant distribution associated to the Markov process of  $e$ .

# Equilibrium

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- Must solve for  $r$  and  $W$  that clear labor and assets markets:
- Let  $l^m(a, e)$  and  $l^\tau(a, e)$  the labor demand of producers, labor market clearing is:

$$L_t = \int_{A \times E} l^\tau(a, e) dn_t^\tau(a, e) + \int_{A \times E} l^m(a, e) dn_t^m(a, e)$$

where  $L_t$  is the amount of labor efficiency units supplied by workers.

- Let  $a_{t+1}(a, e)$  the savings decision of entrepreneurs, and  $A_{t+1}^w$  aggregate worker savings, the asset market clearing:

$$A_{t+1}^w + \sum_{i=m, \tau} \int_{A \times E} a_{t+1}^i(a, e) dn_t^i(a, e) = \int_{A \times E} k_{t+1}^m(a, e) dn_{t+1}^m(a, e)$$



# Financial Frictions

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- What is the effect of financial frictions (i.e., low  $\theta$ )?
- Distortions come from two channels:
- **Entry:**
  - ▶ Entrepreneur must pay entry cost  $\kappa$  in order to operate in the modern sector;
  - ▶ Need financing to pay these costs or use their own funding.
- **Allocations in the Modern Sector:**
  - ▶ Collateral constraint: producers with little collateral operates with small scale.

# Financial Frictions: Misallocation in the Modern Sector

- The collateral constraint distorts allocations in the modern sector. Recall the static profit maximization:

$$\pi^m(a, e) = \max_{k, l} \exp(e + \phi)^{1-\eta} (k^\alpha l^{1-\alpha})^\eta - Wl - (r + \delta)k \quad \text{s.t.} \quad k \leq \frac{1}{1-\theta}a + \frac{\theta}{1-\theta}\kappa$$

- Let  $\mu(a, e)$  be the multiplier of the collateral constraint. F.O.C imply:

$$MPK = \alpha \eta \frac{y(a, e)}{k(a, e)} = r + \delta + \mu(a, e)$$

$$MPL = (1 - \alpha) \eta \frac{y(a, e)}{l(a, e)} = W$$

- $\downarrow a$  or  $\uparrow e \Rightarrow \uparrow \mu$ : Low-collateral entrepreneurs operate with low  $k(a, e)/l(a, e)$  than optimal  $\Rightarrow$  lower profits.

# Financial Frictions: Misallocation in the Modern Sector

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- The distortion also changes the returns of savings. Suppose a simplified two-period version:

$$\begin{aligned} & \max_{c, c', a'} \log(c) + \beta \mathbb{E}[\log(c') | e] \\ \text{s.t. } & c = \pi(a, e) + (1 + r)a - a' \quad \text{and} \quad c' = \pi(a', e') + (1 + r)a' \end{aligned}$$

- F.O.C implies in the modified Euler Equation:

$$\frac{1}{c(a, e)} = \beta \mathbb{E} \left[ \left( 1 + r + \frac{\partial \pi(a', e')}{\partial a'} \right) \frac{1}{c(a', e')} | e \right]$$

where  $\partial \pi(a', e') / \partial a' > 0$  iff  $\mu(a', e') > 0$ .

- Constrained entrepreneurs have higher returns to savings.

# Financial Frictions: Misallocation in the Modern Sector

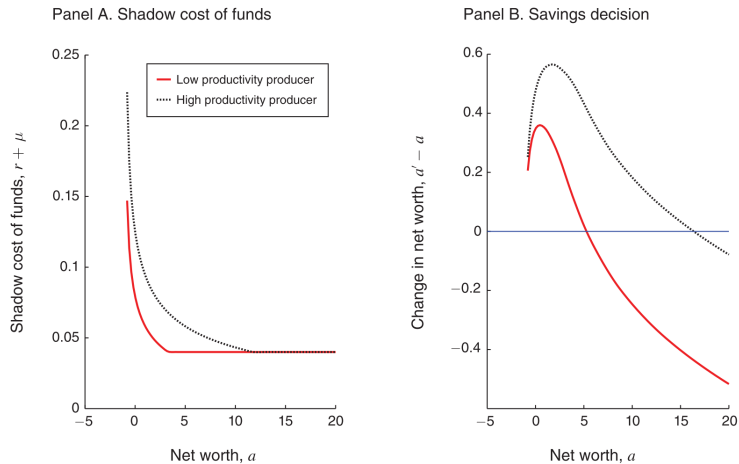


FIGURE 1. DECISION RULES: MODERN SECTOR

# Financial Frictions: Entry

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- Recall that to enter the modern sector a producer must pay the entry cost  $\kappa$ , and receives an injection  $\theta\chi p(a', e)$ .
- In the absence of financial frictions, a productive entrepreneur decides to enter no matter what.
  - ▶ She can always borrow to finance a large fixed cost;
- If the financial frictions are high:
  - ▶ Producer cannot borrow to pay the fixed cost;
  - ▶ Money from equity is lower because future profits are lower;
- The entrepreneur must first accumulate funds and then decide to enter. A productive but poor producer will not operate a business.

# Financial Frictions: Entry

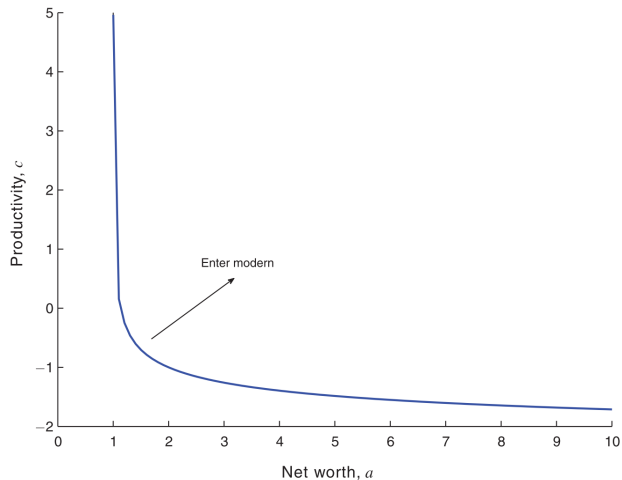


FIGURE 2. DECISION TO ENTER MODERN SECTOR

- TFP losses in the Modern sector can be found by aggregating producers:

$$Y = \exp(\phi)^{1-\eta} \frac{\left( \int_{i \in M} \exp(e_i) (r + \delta + \mu_i)^{-\frac{(1-\alpha)\eta}{1-\eta}} di \right)^{1-\alpha\eta}}{\underbrace{\left( \int_{i \in M} \exp(e_i) (r + \delta + \mu_i)^{\frac{\alpha\eta-1}{1-\eta}} di \right)^{(1-\alpha)\eta}}_{=TFP}} (K^\alpha L^{1-\alpha})^\eta$$

where  $M$  is the set of producers in the modern sector.

- The efficient TFP:

$$Y = \exp(\phi)^{1-\eta} \left( \int_{i \in M} \exp(e_i) di \right)^{1-\eta} (K^\alpha L^{1-\alpha})^\eta.$$

# Efficient Allocations

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- TFP losses from financial frictions in the modern sector arise from *dispersion in the shadow cost of funds*  $\mu(a, e)$ .
- The previous equations kept fixed the measure  $M$ .
- To compute the fully *efficient (first-best) allocations* (i.e., allocation across modern and traditional sectors), one should solve and specify a planners' problem.
- The planner chooses the measure of producers in the two sectors  $n^\tau$  and  $n^m$ , and the stock of capital, subject to the resources available.
  - ▶ Check the paper for the full problem.



# Data and Calibration

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## Data

- Korean manufacturing sector between 1991-1999;
- Information about revenue, wage bill, intermediate inputs, investment and capital stock for plants with more than five workers. Panel data.

## Calibration

- Aggregate technology parameters from other studies and standard values  $(\alpha, \eta, \delta, \beta, \gamma, \phi, \nu)$ .
- Microdata on value-added to estimate productivity process; data on credit to calibrate  $\theta$  and  $\chi$ .
- Data on capital and labor is used to validate the model.

TABLE 1—MOMENTS

	Data Korea	Benchmark	Adoption	Exit
<i>Panel A. Used to calibrate model</i>				
SD output growth	0.59	0.58	0.58	0.59
SD output	1.31	1.30	1.37	1.30
1-year autocorrelation	0.90	0.90	0.89	0.91
3-year autocorrelation	0.87	0.87	0.83	0.87
5-year autocorrelation	0.85	0.86	0.80	0.86
Intangibles invest. to output, percent	4.6	4.6	4.6	4.6
Output growth rate, percent	8.0	8.0	8.0	8.0
Debt to output	1.2	1.2	1.2	1.2
Equity to output	0.3	0.3	0.3	0.3

TABLE 2—PARAMETER VALUES

		Benchmark	Adoption	Exit
<i>Assigned parameters</i>				
Labor elasticity	$\alpha$	0.67	0.67	0.67
Span of control	$\eta$	0.85	0.85	0.85
Capital depreciation	$\delta$	0.06	0.06	0.06
Discount factor	$\beta(1 + \mu)^{-1}$	0.92	0.92	0.92
Growth rate	$\gamma$	1.08	1.08	1.08
Persistence unit worker state	$\lambda_1$	0.79	0.79	0.79
Persistence zero worker state	$\lambda_0$	0.50	0.50	0.50
Relative efficiency in modern sector	$(1 - \eta)\phi$	0.20	0.20	0.20
<i>Calibrated parameters</i>				
Collateral constraint	$\theta$	0.86	0.78	0.68
Equity issuance constraint	$\chi$	0.10	0.08	0
SD transitory shocks	$\sigma_\varepsilon$	0.50	0.50	0.96
Persistence transitory shocks	$\rho$	0.25	0.11	0.40
Cost of entering modern sector	$\kappa$	1.19	0.30	2.66
Variance exogenous permanent component	$\text{var}(z)$	1.47	1.43	1.44
Relative efficiency of productive technology	$(1 - \eta)\phi_p$	—	0.27	—
Cost of adopting productive technology	$\kappa_p$	—	1.83	—
Fixed cost of operating in modern sector	$\bar{F}$	—	—	0.27

# Experiments

TABLE 3—AGGREGATE IMPLICATIONS OF FINANCE FRICTIONS: OPEN ECONOMY

	Efficient	“Korea”	$\theta = 1$	$\theta = 0.75$	$\theta = 0.50$	$\theta = 0.25$	$\theta = 0$
<i>Panel A. With equity issuance (<math>\chi = 0.10</math>)</i>							
Debt to output (modern)		1.18	1.30	0.92	0.35	−0.14	−0.60
Equity to output (modern)		0.29	0.32	0.30	0.26	0.14	0
Interest rate		0.047	0.047	0.047	0.047	0.047	0.047
Fraction constrained		0.17	0	0.44	0.69	0.78	0.83
Capital to output (modern)	1.93	2.59	2.65	2.46	2.25	2.14	2.05
TFP (modern)	1.003	1.000	1.003	0.989	0.926	0.869	0.827
Loss misallocation, percent	0	0.3	0.0	1.4	3.9	4.4	4.7
Fraction producers modern	0.93	0.93	0.93	0.93	0.70	0.48	0.35
Fraction output modern	0.99	0.99	0.99	0.99	0.90	0.73	0.58
Consumption	1.02	1.00	1.01	0.98	0.90	0.85	0.82
Output	1.50	1.68	1.70	1.62	1.41	1.24	1.13
<i>Panel B. Without equity issuance (<math>\chi = 0</math>)</i>							
Debt to output (modern)			1.59	0.85	0.34	−0.13	−0.60
Interest rate			0.047	0.047	0.047	0.047	0.047
Fraction constrained			0	0.50	0.68	0.77	0.83
Capital to output (modern)			2.65	2.42	2.27	2.15	2.05
TFP (modern)			1.003	0.918	0.878	0.849	0.827
Loss misallocation, percent			0.0	2.7	3.8	4.4	4.7
Fraction producers modern			0.93	0.61	0.49	0.41	0.35
Fraction output modern			0.99	0.88	0.78	0.67	0.58
Consumption			1.01	0.91	0.87	0.84	0.82
Output			1.70	1.42	1.29	1.20	1.13

# Discussion

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- Benchmark Korea not very constrained (debt-to-GDP 120%  $\Rightarrow \theta = 0.86$ ). Small effects on TFP and output:
  - ▶ Only 17% of the entrepreneurs are constrained.
  - ▶ Loss from misallocation is negligible (0.3%).
  - ▶ Entry is not distorted: fraction in the modern sector is the same in both efficient and “Korea”.
- Increase in financial frictions (i.e.,  $\downarrow \theta$ ):
  - ▶ Large impact on TFP and consumption (at most 17%).
  - ▶ Misallocation within the modern sector accounts for “only” 4.7%.
  - ▶ Most is due to **lack of entry** in the modern sector.
  - ▶ Potentially larger without the possibility of equity issuance ( $\chi = 0$ ) and in a close economy (interest rate adjusts).
- Why misallocation from FF is low? Financially constrained producers *self-finance* and grow out the collateral constraint.

# Microeconomics Implications

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- There are several implications of financial frictions for some micro statistics.
- Financial frictions act like an adjustment cost by preventing constrained firms to adjust capital in response to changes in productivity.
  - ▶ Tighter friction reduces std. dev. of output growth.
- Financial frictions also disproportionately affect young producers who have not yet accumulated internal funds.
  - ▶ With FF, young firms tend to grow faster than old firms since they are accumulating capital.
- The last point may change if the model has technology adoption or endogenous exit.

# Microeconomics Implications

TABLE 6—EXTENSIONS OF THE BENCHMARK MODEL

	“Korea”	No finance	“Korea”	No finance	“Korea”	No finance
Debt to output (modern)	1.2	−0.6	1.2	−0.2	1.2	−0.6
Equity to output (modern)	0.3	0	0.3	0	0.3	0
Consumption	1.00	0.82	1.41	0.97	0.86	0.79
Output	1.68	1.13	2.41	1.47	1.30	1.08
TFP modern sector	1	0.83	1.29	0.99	0.91	0.79
Loss misallocation, percent	0.3	4.7	1.2	6.3	2.1	4.1
Fraction producers modern	0.93	0.35	0.93	0.39	0.36	0.25
Fraction productive producers modern	—	—	0.86	0.18	—	—
Fraction modern producers operating	1	1	1	1	0.67	0.44
SD output growth	0.58	0.32	0.58	0.26	0.59	0.40
Average product of capital, 1–5 versus 11+	0.08	0.73	0.27	0.22	0.13	0.19
Relative output growth, 1–5 versus 11+	0.09	0.12	0.28	0.08	0.10	0.08

**Note:** First column is benchmark; second is model with technology adoption; third is with endogenous exit.

# Conclusion

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- Model of establishment dynamics in which financial frictions may distort aggregate productivity through two channels:
  - ▶ **Entry** (extensive margin);
  - ▶ **Misallocation within sector** (intensive margin).
- Financial frictions potentially generate large losses from inefficiently low levels of entry and technology adoption; but small losses from misallocation within operating producers.
- Why? Productive producers accumulate internal funds over time and quickly grow out of their borrowing constraints.



# Where to Go Now?

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- **Occupational Choice and Development:** Buera, Kaboski and Shin (many papers); Moll (2014, AER);
- **Micro-finance, Optimal Policy and Development:** Itshoki and Moll (2019, ECTA); Buera, Kaboski and Shin (2021, ReStud).
- **Informality and Inequality:** Erosa, Fuster and Martinez (2023, JME); D'Erasmus and Moscoso (2012, JME);
- **Innovation and Government Procurement:** Caggese (2019, AEJ: Macro); Di Giovanni et al (2022, WP).
- **International Trade and Open Economy:** Gopinath et al (2017, QJE); Leibovici (2021, JPE); Manova (2013, ReStud).
- **Inequality and Self-Employment:** Allub and Erosa (2019, JME); Herreño and Ocampo (2022, WP).
- **Firm Financing and Taxes:** Gourio and Miao (2010, AEJ:Macro); Riddick and Whited (2009, J. Finance); Arrelano et al (2012, JME); Kochen (2022, WP).