Macroeconomics I
Problem Set 6

1. **(OLG with Government).** Consider the standard OLG model where the agent works when young and consumes savings when old. The utility of an agent born in t is:

$$\ln(c_t^1) + \beta \ln(c_{t+1}^2).$$

The government taxes the young via lump-sum taxation, τ_t. Suppose $\delta = 0$. The budget constraints of the agent when young and old for all generations $t \geq 1$ are:

$$\begin{aligned} c_t^1 + s_t &\leq w_t - \tau_t \\ c_{t+1}^2 &\leq (1 + r_{t+1}) s_t \end{aligned}$$

The government uses taxation to finance its fiscal policy. Suppose the budget is balanced every period, i.e., the government's budget constraint for all $t \geq 1$ is:

$$\tau_t L_t = G_t \equiv g_t L_t,$$

where $G_t > 0$ is the aggregate spending (and is exogenous), and g_t is spending per effective unit of labor. The production function is: $Y_t = K_t^\alpha L_t^{1-\alpha}$, with $\alpha \in (0, 1)$ and the population grows $L_t = (1 + n)^t L_0$ where $L_0 = 1$. The initial old generation has $K_0 > 0$ given.

(a) Solve the households' problem. Find s_t in terms of the parameters, prices, and tax τ_t.

(b) Write down the equilibrium conditions in the goods market and in the asset market.

(c) Suppose the prices are given by $r_t = \alpha k_t^{\alpha-1}$ and $w_t = (1-\alpha) k_t^\alpha$ for all t, where $k_t \equiv K_t / L_t$. Find an equation that describes the evolution of the equilibrium capital in the model (k_{t+1} in terms of k_t and parameters). How does an increase in government spending per worker alter the capital accumulation dynamics?

(d) Suppose $n = 0$ (for simplicity). In the discrete-time neoclassical growth model with infinitely-lived dynasties and fiscal policy financed by lump-sum taxation, the Euler equation is given by:

$$c_{t+1} = c_t[\beta(1 + r_{t+1})] \qquad \forall t.$$

Find the k_t in the steady state in the neoclassical growth model. How does the level of government spending per effective unit of labor, g, affect capital in the steady state in the neoclassical growth model? Compare with the OLG model.

2. **(Capital Utilization a la Burnside and Eichenbaum (1996)).** Consider the RBC model with variable capital utilization. The utility of the representative family is standard and given by:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \theta \frac{N_t^{1+\phi} - 1}{1+\phi} \right)$$

where $0 < \beta < 1$, C_t is consumption, N_t is the labor time. We will solve for the optimal equilibrium allocations.

The social planner is subject to the economy's resource constraints and respects the capital motion law. He chooses consumption, investment, household labor, and capital utilization, u_t, to maximize the utility of the representative family subject in all periods to:

$$\begin{aligned} Y_t &= C_t + I_t \\ Y_t &= Z_t (u_t K_t)^{\alpha} N_t^{1-\alpha} \\ K_{t+1} &= (1 - \delta_f(u_t)) K_t + I_t \end{aligned}$$

where $\delta_f(u_t)$ is a function:

$$\delta_f(u_t) = \frac{\delta}{\psi} u_t^{\psi}, \qquad \psi > \alpha.$$

Finally, Z_t follows a stationary AR(1) process.

(a) Solve the central planner's problem. Which conditions are *intratemporal* and which conditions are *intertemporal*?

(b) Use the FOC of u_t and show that the production function can be written as:

$$Y_t = \left(\frac{\alpha}{\delta}\right)^{\frac{\alpha}{\psi-\alpha}} Z_t^{\frac{\psi}{\psi-\alpha}} K_t^{\frac{\alpha(\psi-1)}{\psi-\alpha}} N_t^{\frac{\psi(1-\alpha)}{\psi-\alpha}}.$$

(c) Explain intuitively how the introduction of capital utilization u_t alters the amplification and persistence of the model. In particular, answer how a positive shock in Z_t changes N_t and Y_t. How does this depend on the elasticity of capital utilization ψ?[1]

3. **(Money in Utility Function).**[2] Consider the following RBC model with money in the utility function. Households value money in their utility (you can interpret this as a need for money for transactions or liquidity preference). The utility is given by:

$$\mathbb{E}_t \sum_{t=0}^{\infty} \beta^t \left(\log C_t - \theta \frac{N_t^{1+\phi} - 1}{1+\phi} + \frac{(M_t/P_t)^{1-\nu} - 1}{1-\nu} \right)$$

[1] Hint: observe the production function above. Compare the elasticity of production with respect to the quantity of hours worked in the case with variable utilization and without variable utilization. How do wages respond to a shock Z_t?

[2] Based on Walsh (2010, chap. 2)

where $0 < \beta < 1$, C_t is consumption, N_t is the labor time, P_t is the price of the final good (in units of money), and M_t/P_t is the quantity of real money holdings of the household (or real balances). Households choose how much to consume, C_t, how much to work, N_t, how much money to hold, M_t, how much capital to invest K_{t+1}, and how many bonds to buy B_{t+1}:

The household's budget constraint is:

$$P_t C_t + P_t(K_{t+1} - (1-\delta)K_t) + B_{t+1} + M_t = P_t w_t N_t + P_t \hat{r}_t K_t + (1 + i_{t-1})B_t + M_{t-1} + P_t T_t$$

Where i_t is the nominal interest rate paid on bonds and T_t is a lump-sum transfer paid by the government. Government transfer is financed via money printing:

$$P_t T_t = M_t - M_{t-1}.$$

Production is standard and follows a Cobb-Douglas function: $Y_t = K_t^{\alpha} N_t^{1-\alpha}$, where $\alpha \in (0, 1)$. The household is subject to a no-Ponzi scheme constraint. Define $P_{t+1}/P_t \equiv 1 + \pi_t$ where π_t is the inflation rate of the economy.

(a) Describe and solve the household's problem. Write the demand for real balances, M_t/P_t, as a function of C_t and i_t. How does it depend on i_t and C_t?

(b) Find the Fisher equation $i_t = \mathbb{E}_t r_{t+1} + \pi_{t+1}$, where $r_t \equiv \hat{r}_t - \delta$.

(c) Suppose that in the steady state the money growth rate is $M_{t+1}/M_t = 1 + \mu$ and that real balances are constant in the steady state $M_t/P_t = M_{t+1}/P_{t+1} = m$. Find a system of (eight) equations that solves the problem in the steady state for the variables: (r, w, K, N, C, i, m, T).

(d) Suppose an increase in the money growth rate: μ. How does this affect the variables in the steady state? How do changes in M_t affect the price level and other endogenous variables? What would happen if agents expected an increase in the money supply in $t + 1$?