Análise Matemática II

2017/18

Lista de Exercícios 2 Algumas Respostas

1. Determine,utilizando a definição, as derivadas parciais de 1^a ordem das seguintes funções, nos pontos indicados:

a) $\dfrac{\partial f}{\partial x}(1,1) = -8$ e $\dfrac{\partial f}{\partial y}(1,1) = 17$.

b) $\dfrac{\partial g}{\partial s}(1,1) = \dfrac{1}{2}$ e $\dfrac{\partial g}{\partial t}(1,1) = \dfrac{1}{2}$.

2.
$\dfrac{\partial f}{\partial x}(x,y,z) = 2xyz^2$; $\quad \dfrac{\partial f}{\partial y}(x,y,z) = x^2z^2$; $\quad \dfrac{\partial f}{\partial z}(x,y,z) = 2x^2yz$

4. $\dfrac{\partial f}{\partial x}(1,0) = 0$ e $\dfrac{\partial f}{\partial y}(0,0) = 0$.

5.
a) 2.
b) $1 + 10\sqrt{2}$.

6. Determine a derivada direccional $\dfrac{\partial f}{\partial u}$ nos pontos P indicados.

a) $f(x,y) = e^{5xy}$, $u = (1,1)$ e $P = \left(\dfrac{2}{3}, -\dfrac{1}{3}\right)$.

b) $g(s,t) = \log\left(2 + s + y^2\right)$, $u = (1,0)$ e $P = \left(-\dfrac{1}{2}, \dfrac{1}{\sqrt{2}}\right)$.

c) $h(x,y) = \sqrt{x^2 + y^2,}\ $ $u = (1,2)$ e $P = (1,0)$.

8.
a) $Df = \mathbb{R}^2 \backslash \{(0,0)\}$.
b) Não existe o $\lim\limits_{(x,y)\to(0,0)} f(x,y)$.
c) Para $(x,y) \neq (0,0)$: $\dfrac{\partial g}{\partial x}(x,y) = \dfrac{x^4 + 4x^2y^2 - y^4}{\left(x^2+y^2\right)^2}$;
$\dfrac{\partial g}{\partial y}(x,y) = \dfrac{-4x^3y}{\left(x^2+y^2\right)^2}$,
$\dfrac{\partial g}{\partial x}(0,0) = 1$ e $\dfrac{\partial g}{\partial y}(0,0) = 0$.

10.

$a)\ \dfrac{\partial^2 f}{\partial x^2} = 0,$ $\quad b)\ \dfrac{\partial^2 f}{\partial y^2} = -xz^2 \operatorname{sen}(yz),$ $\quad c)\ \dfrac{\partial^2 f}{\partial x \partial y} = z\cos(yz),$

$d)\ \dfrac{\partial^2 f}{\partial y \partial z} = y\cos(yz),$ $\quad e)\ \dfrac{\partial^3 f}{\partial x^3} = 0,$ $\quad f)\ \dfrac{\partial^3 f}{\partial x \partial y^2} = -z^2 \operatorname{sen}(yz).$

11.

$a)\ \dfrac{\partial f}{\partial x} = 2xy^3 \cos x - x^2y^3 \operatorname{sen} x,$ $\quad b)\ \dfrac{\partial f}{\partial y} = 3x^2y^2 \operatorname{sen} x,$

$c)\ \dfrac{\partial^2 f}{\partial x \partial y} = 6xy^2 \cos x - 3x^2y^2 \operatorname{sen} x,$ $\quad d)\ \dfrac{\partial^2 f}{\partial y^2} = 6x^2 y \cos x,$

$e)\ \dfrac{\partial g}{\partial y} = -\dfrac{2x^3}{\left(y\ x^2 + z^2\right)^2},$ $\quad f)\ \dfrac{\partial g}{\partial z} = \dfrac{4xz}{\left(y\ x^2 + z^2\right)^2},$

$g)\ \dfrac{\partial^2 g}{\partial z^2} = \dfrac{16xz^2}{\left(y\ x^2 + z^2\right)^3} - \dfrac{4x}{\left(y\ x^2 + z^2\right)^2},$ $\quad h)\ \dfrac{\partial^2 h}{\partial y \partial z} = -\dfrac{2x^3y^2z^2}{\left(1 + x^2y^2z^2\right)^2} + \dfrac{x}{1 + x^2y^2z^2},$

$i)\ \dfrac{\partial^2 h}{\partial y^2} = -\dfrac{2x^3yz^3}{\left(1 + x^2y^2z^2\right)^2},$ $\quad j)\dfrac{\partial p}{\partial x} = 18\left(3x + 2y\right)^5,$

$l)\ \dfrac{\partial^3 p}{\partial y \partial x^2} = 2160\left(3x + 2y\right)^3,$ $\quad m)\ \dfrac{\partial^4 p}{\partial y^2 \partial x^2} = 12960\left(3x + 2y\right)^2.$

13.
a) Descendo.
b) Subindo.

14.
a) $\dfrac{\partial f}{\partial x}(0,0) = 0$ e $\dfrac{\partial g}{\partial y}(0,0) = 0$

b) $z = x^2 \log\left(\dfrac{x}{y}\right)$ no ponto $(1,1)$ para os acréscimos $dx = -0,2$ e $dy = 0,2$.

17. $df = \dfrac{x_1}{\|(x_1,...,x_n)\|^2}dx_1 + ... + \dfrac{x_n}{\|(x_1,...,x_n)\|^2}dx_n.$

18.
a) $f(1.02, 0.96) \simeq 8.06$.