

# Graph Coloring Problem

## **Problema:**

Asignar colores a los vértices de modo que vértices adyacentes tengan colores distintos.

## **Intuición:**

Colorear el grafo evitando choques. El objetivo típico es minimizar el número total de colores usados (número cromático).

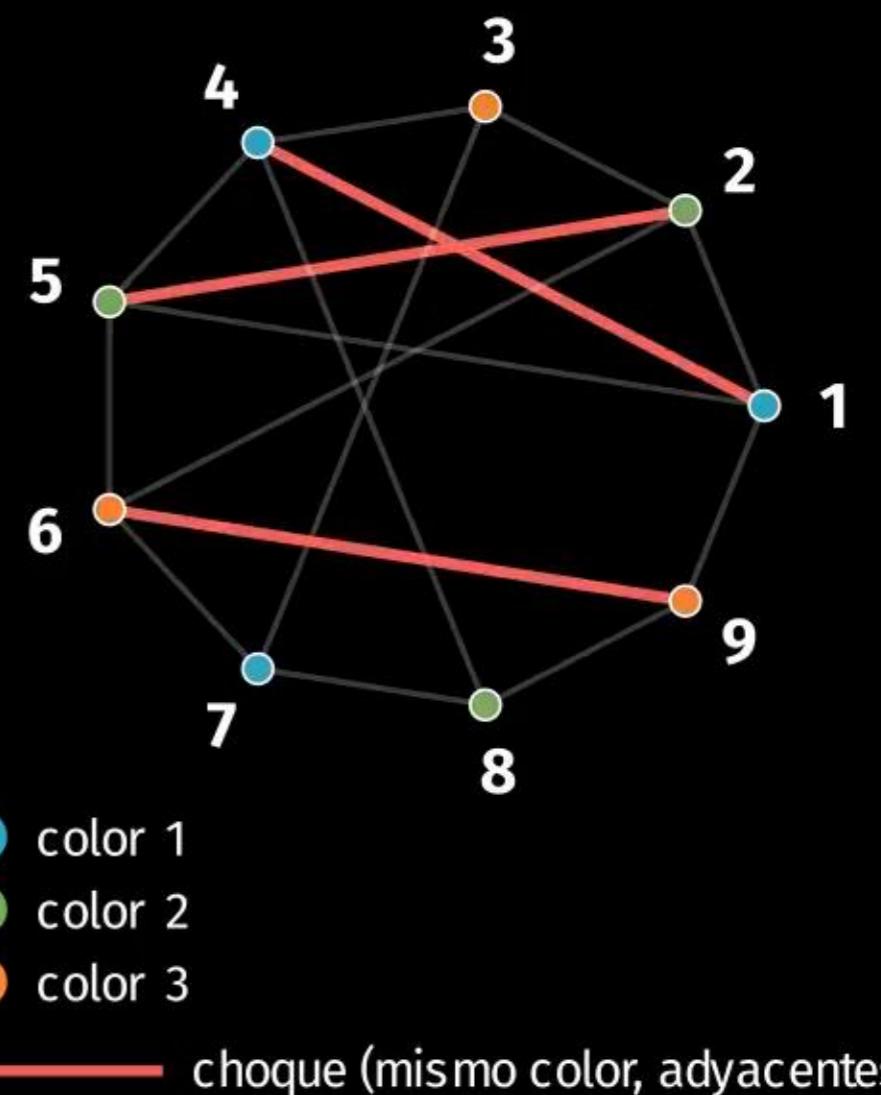
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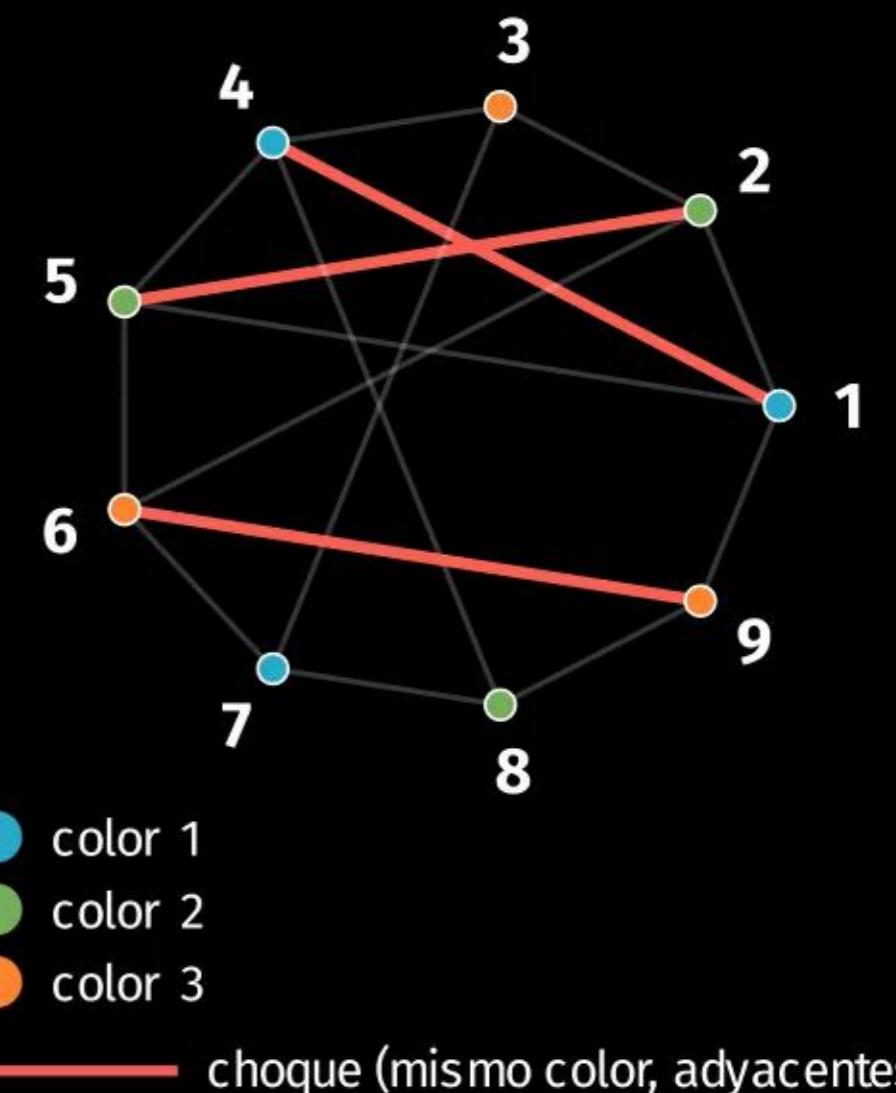
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$$x_{v,c} \in \{0, 1\}; \sum_c x_{v,c} = 1 \quad \forall v$$

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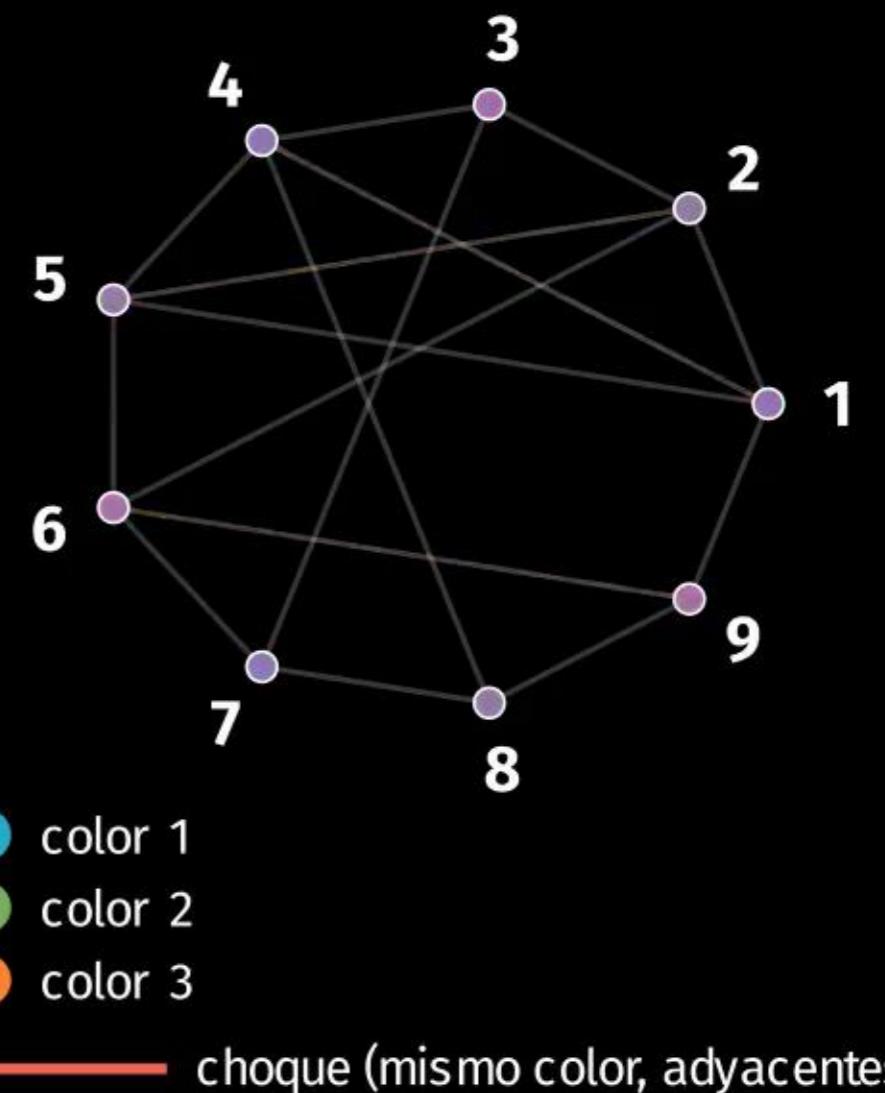
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Colores usados: **0**

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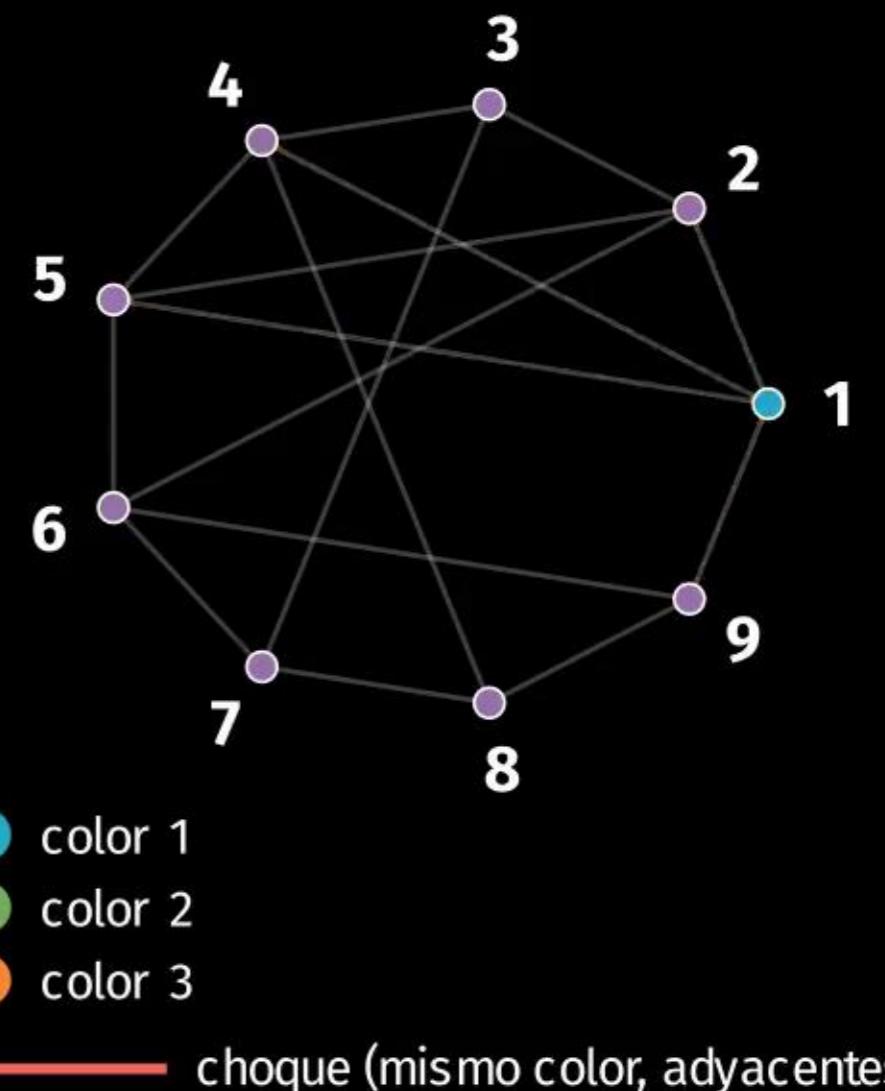
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Colores usados: 1

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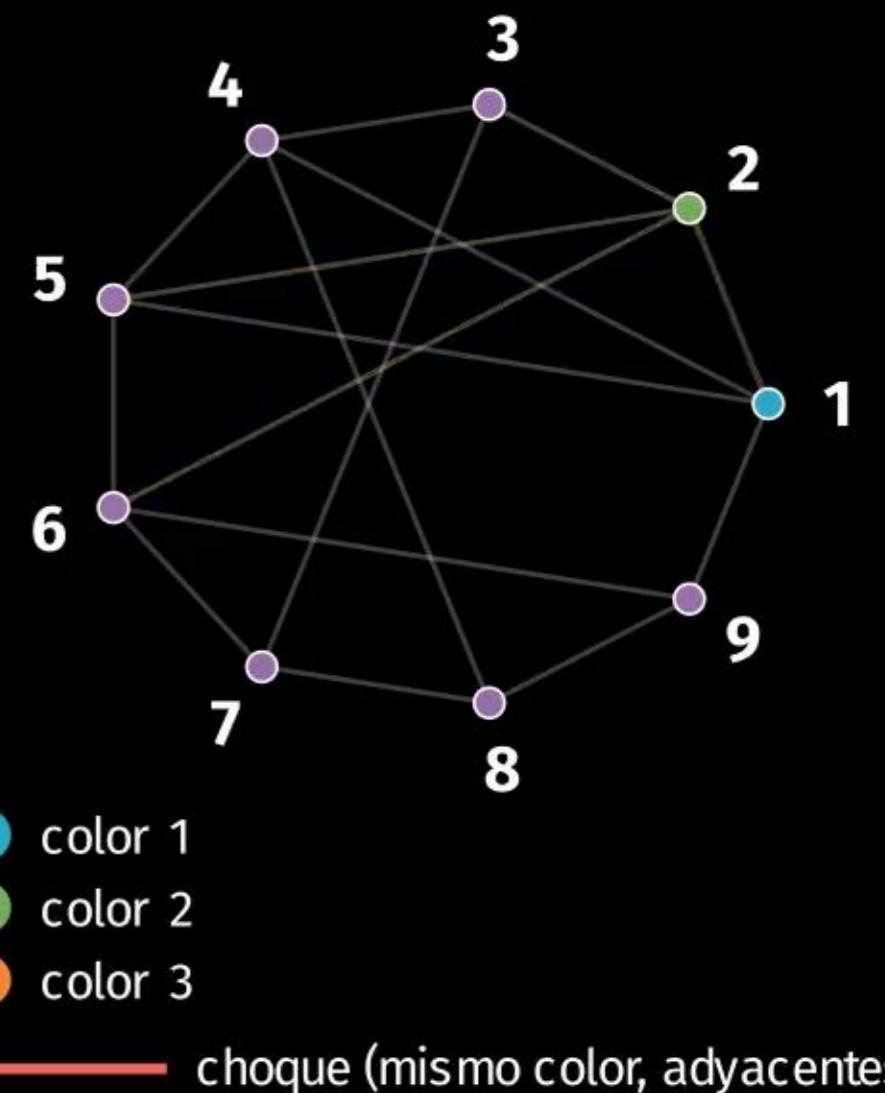
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Colores usados: **2**

# Graph Coloring Problem

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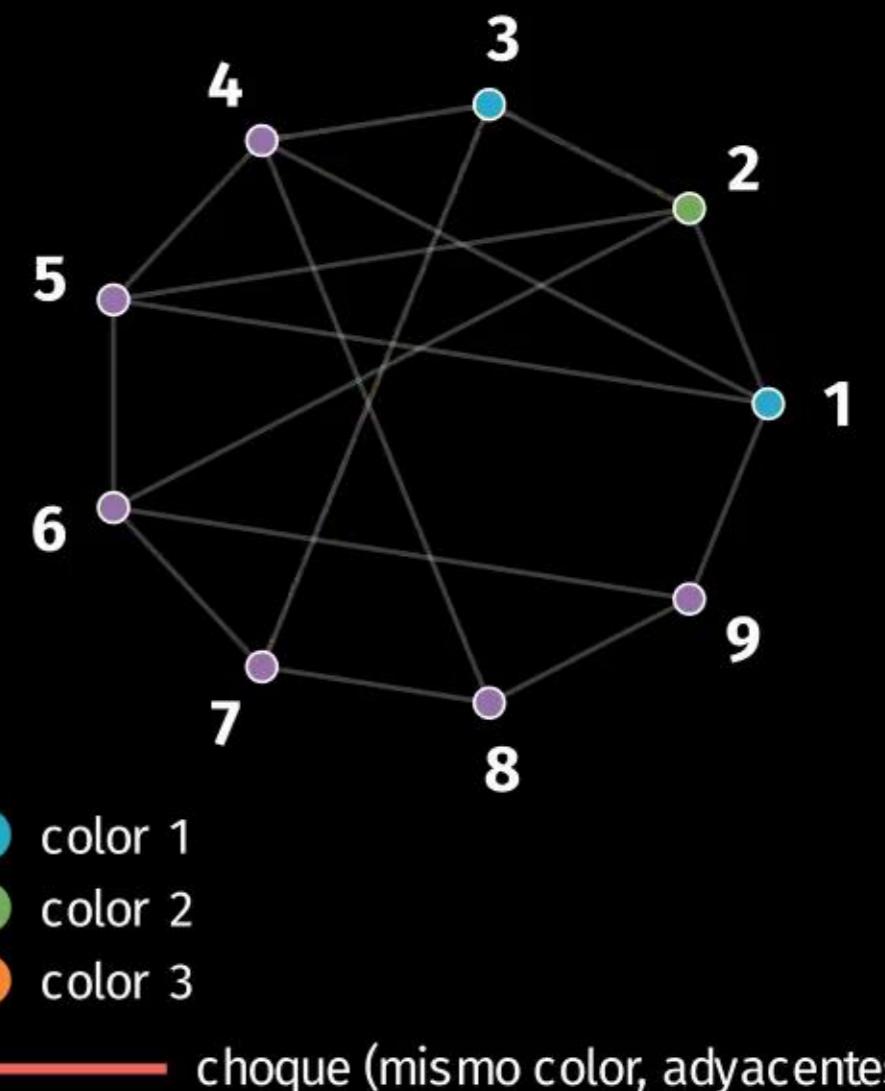
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Colores usados: **2**

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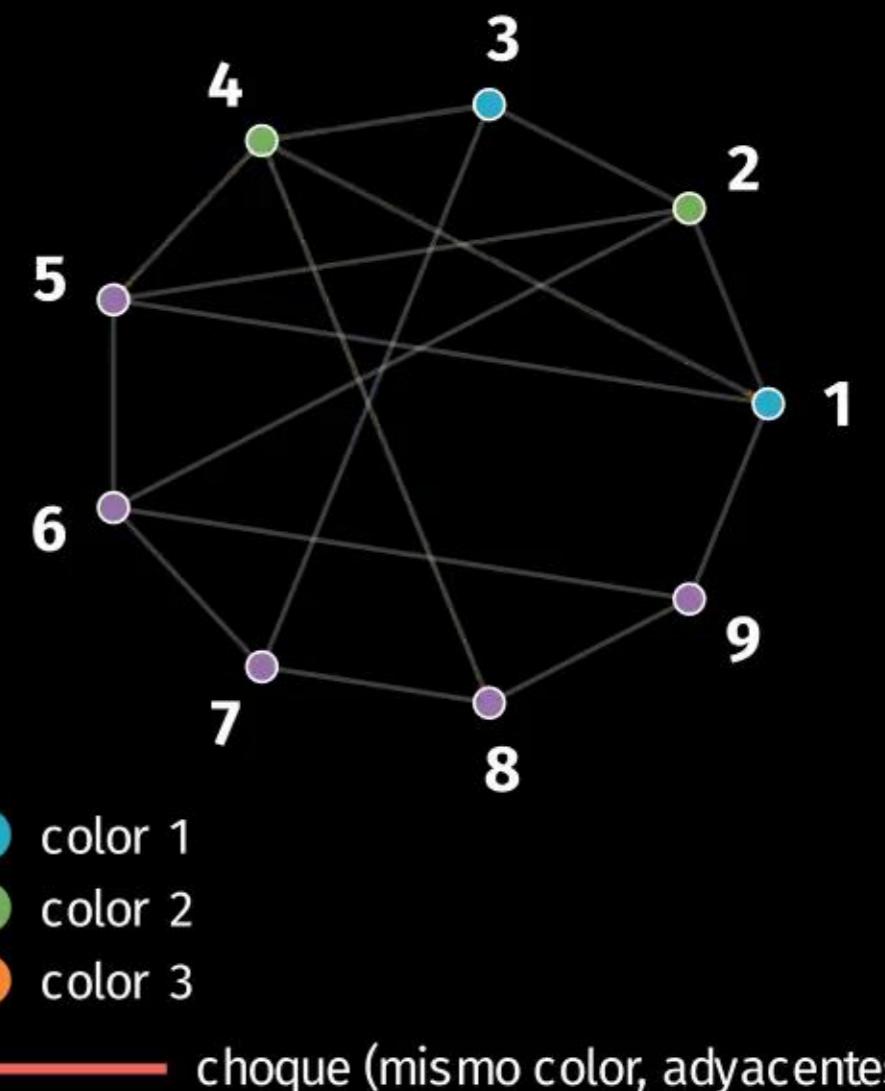
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Colores usados: **2**

# Graph Coloring Problem

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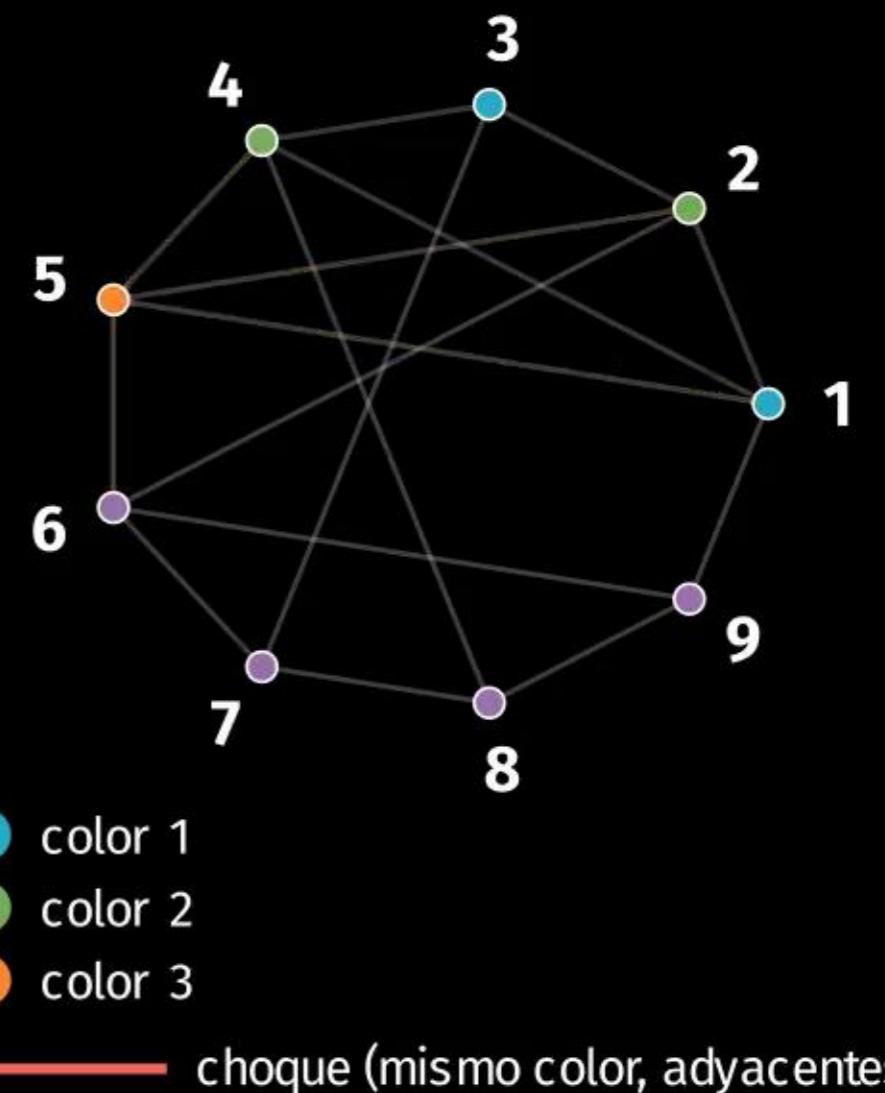
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Colores usados: 3

# Graph Coloring Problem

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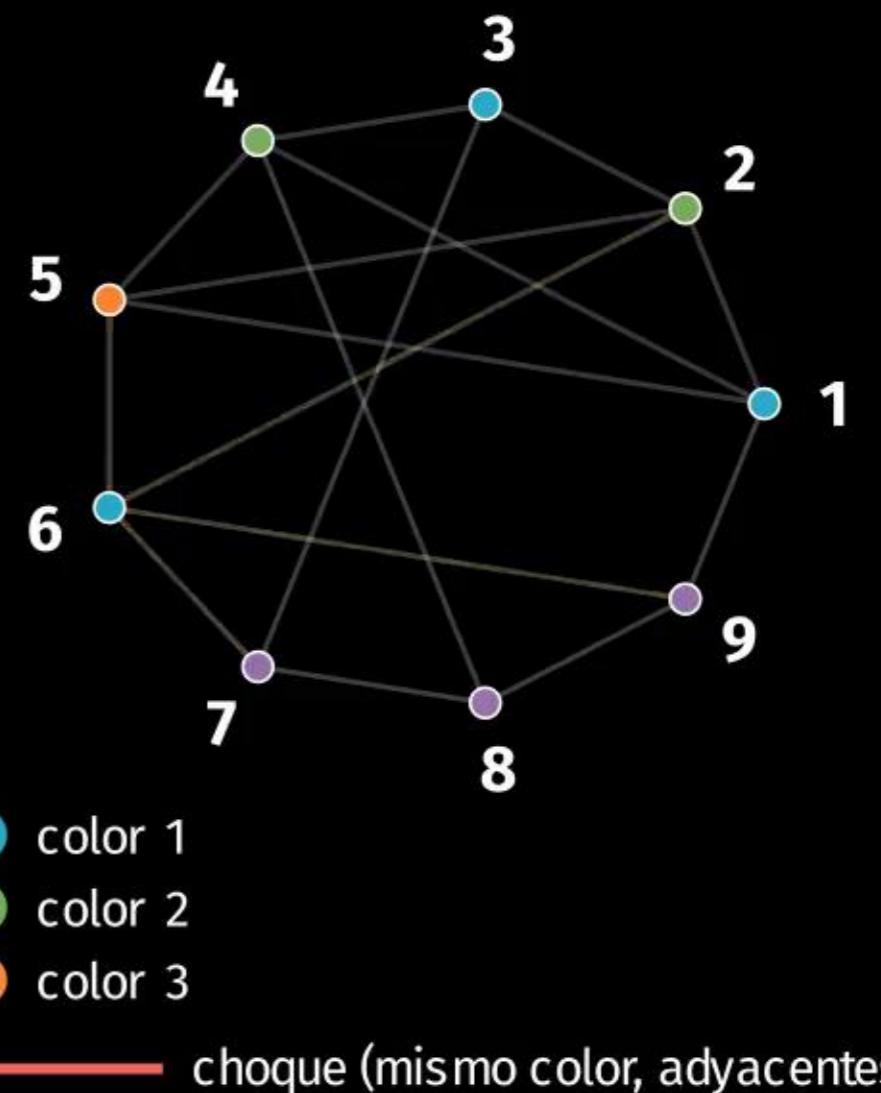
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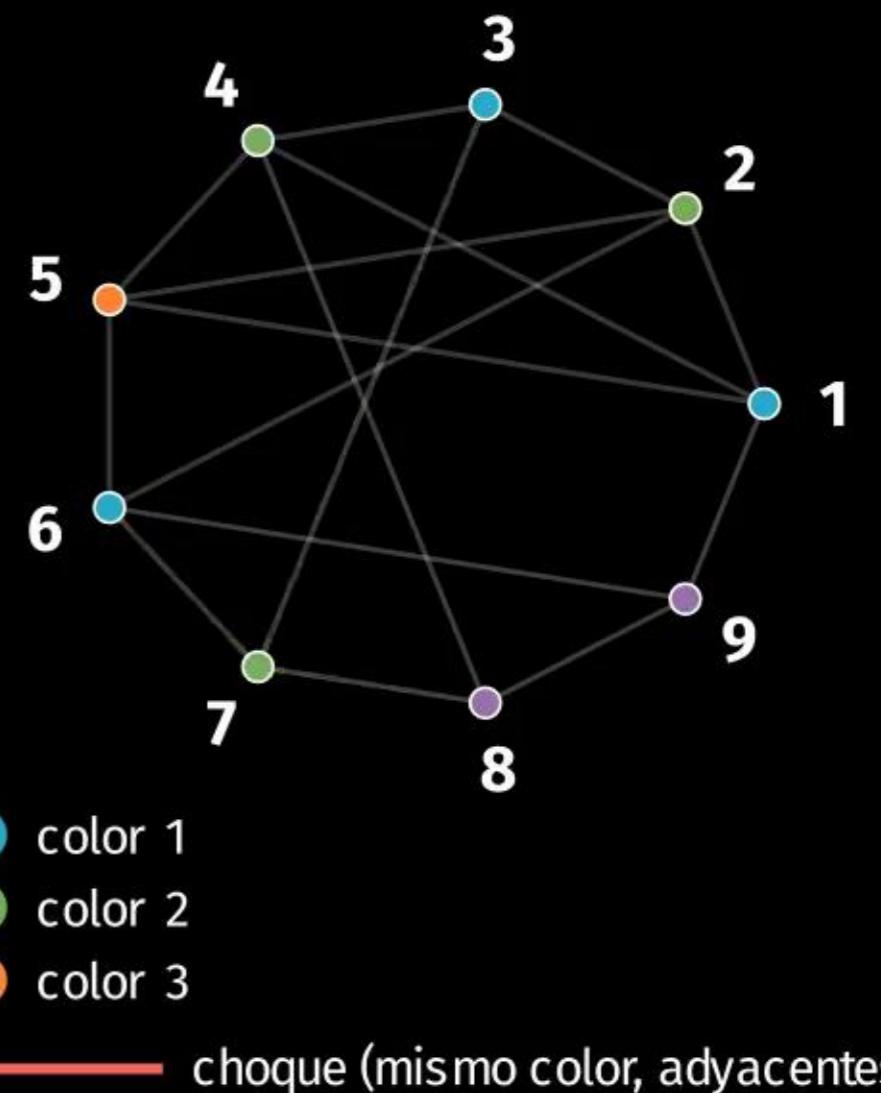
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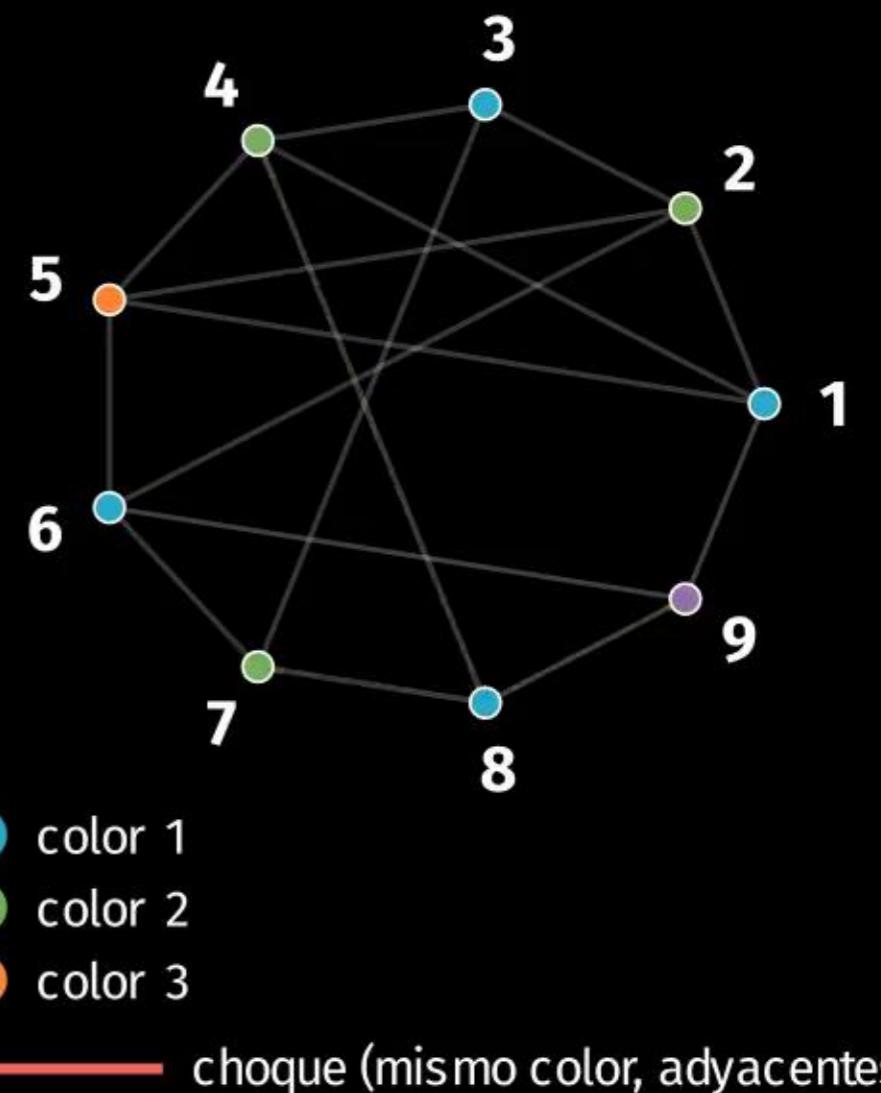
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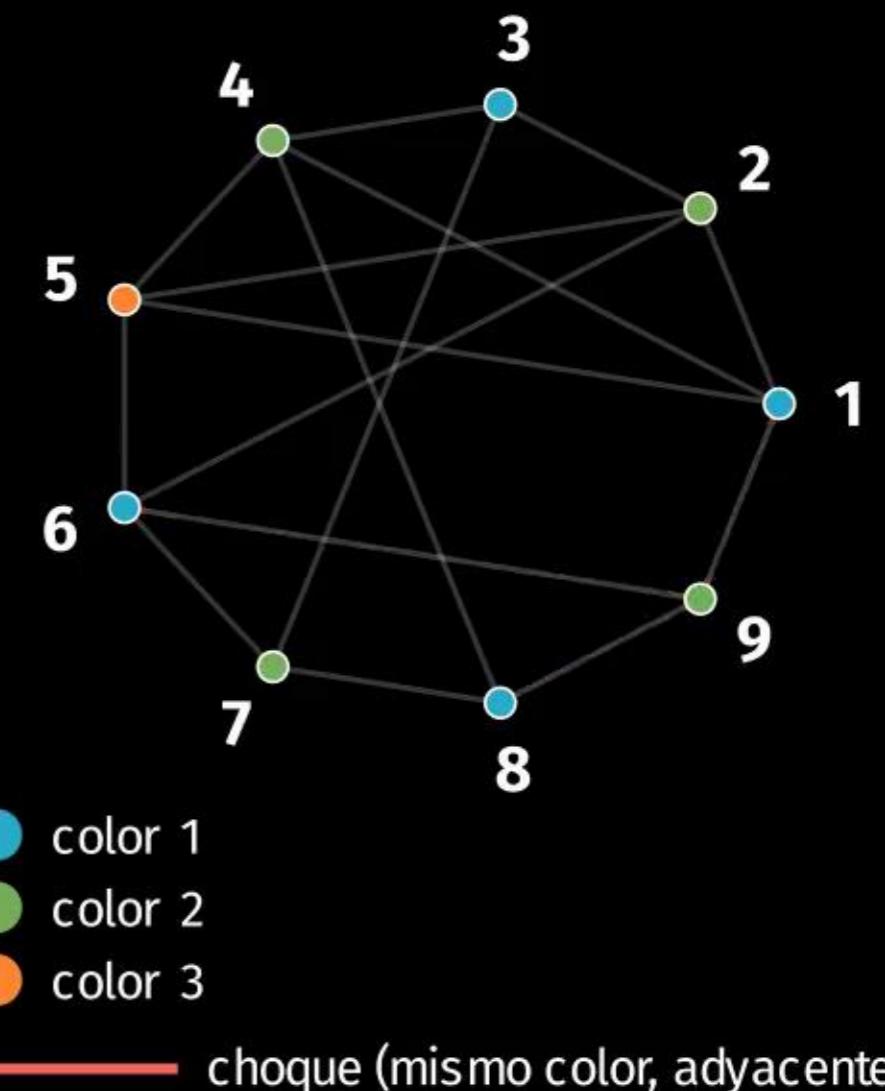
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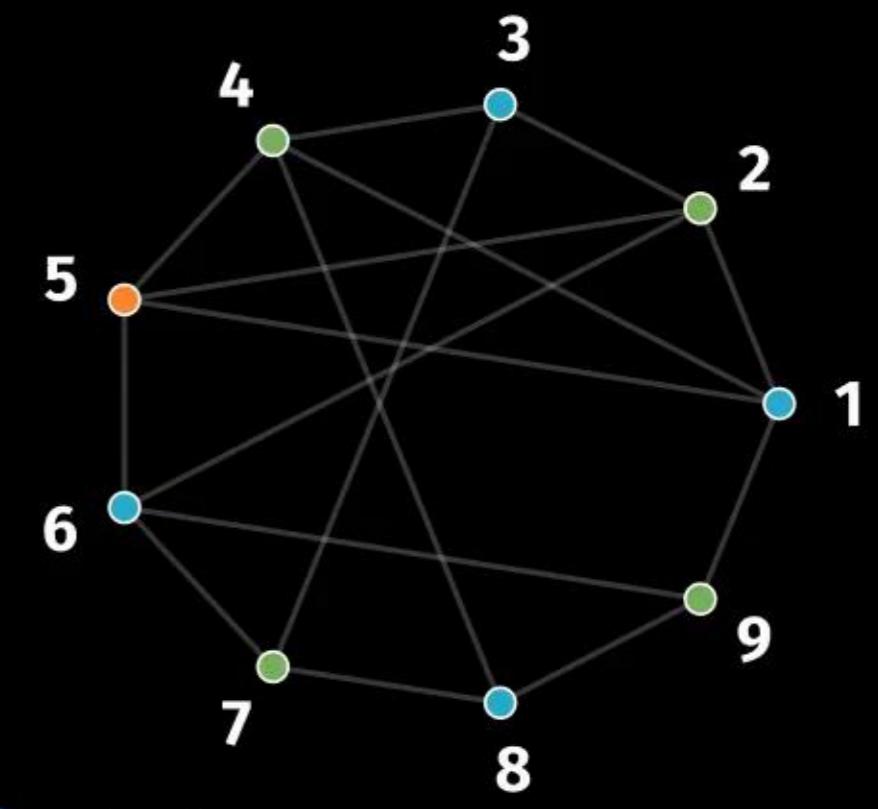
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Colores usados: 3

# Graph Coloring Problem



- color 1
  - color 2
  - color 3
- choque (mismo color, adyacentes)

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Colores usados: 3



# **Simulated Annealing (SA)**

## Idea:

Búsqueda local probabilística que puede aceptar peores movimientos para escapar de óptimos locales; el control lo da la temperatura  $T$ .

## Regla de aceptación (minimización):

$$P[\text{aceptar}] = \begin{cases} 1, & \Delta \leq 0 \\ e^{-\Delta/T}, & \Delta > 0 \end{cases}$$

$$\Delta = E(s') - E(s)$$

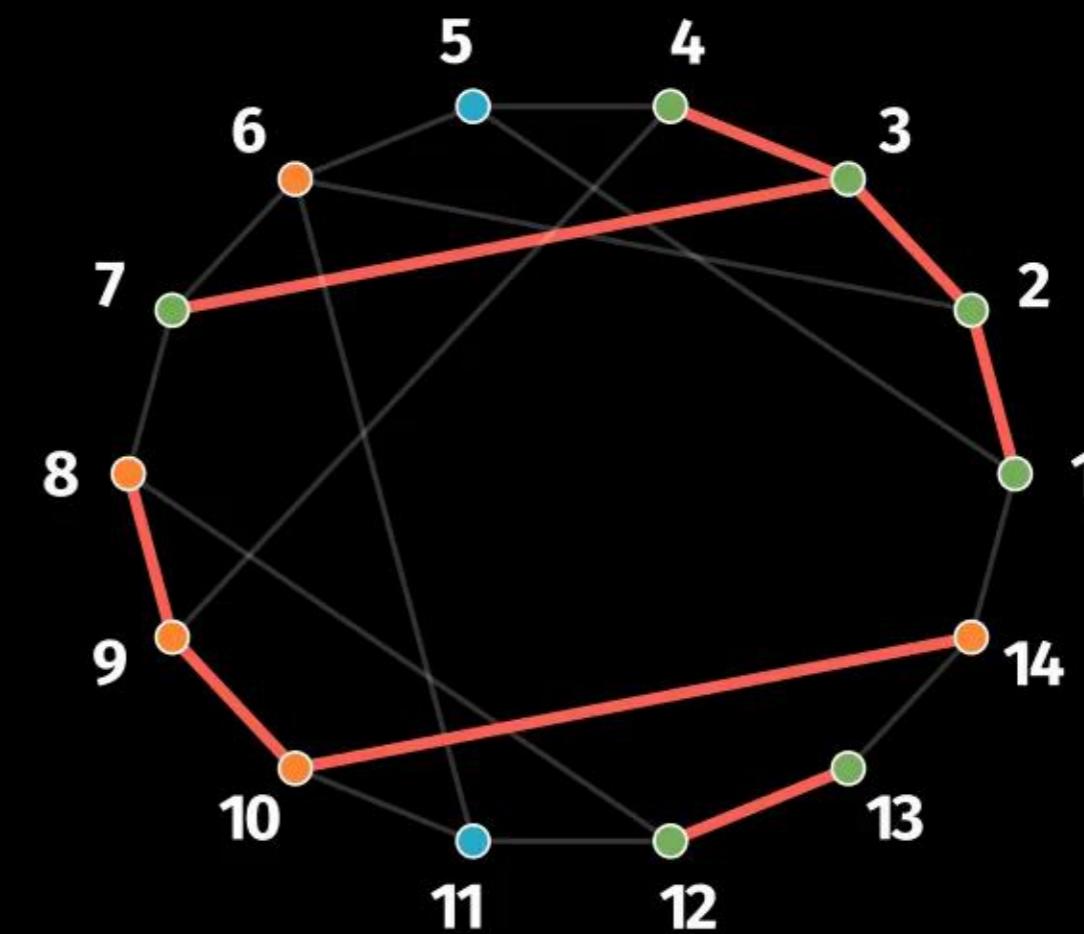
## Schedule y detención:

$$T_{k+1} = \alpha T_k \quad (0 < \alpha < 1) \quad \text{o bien enfriamientos log/linear}$$



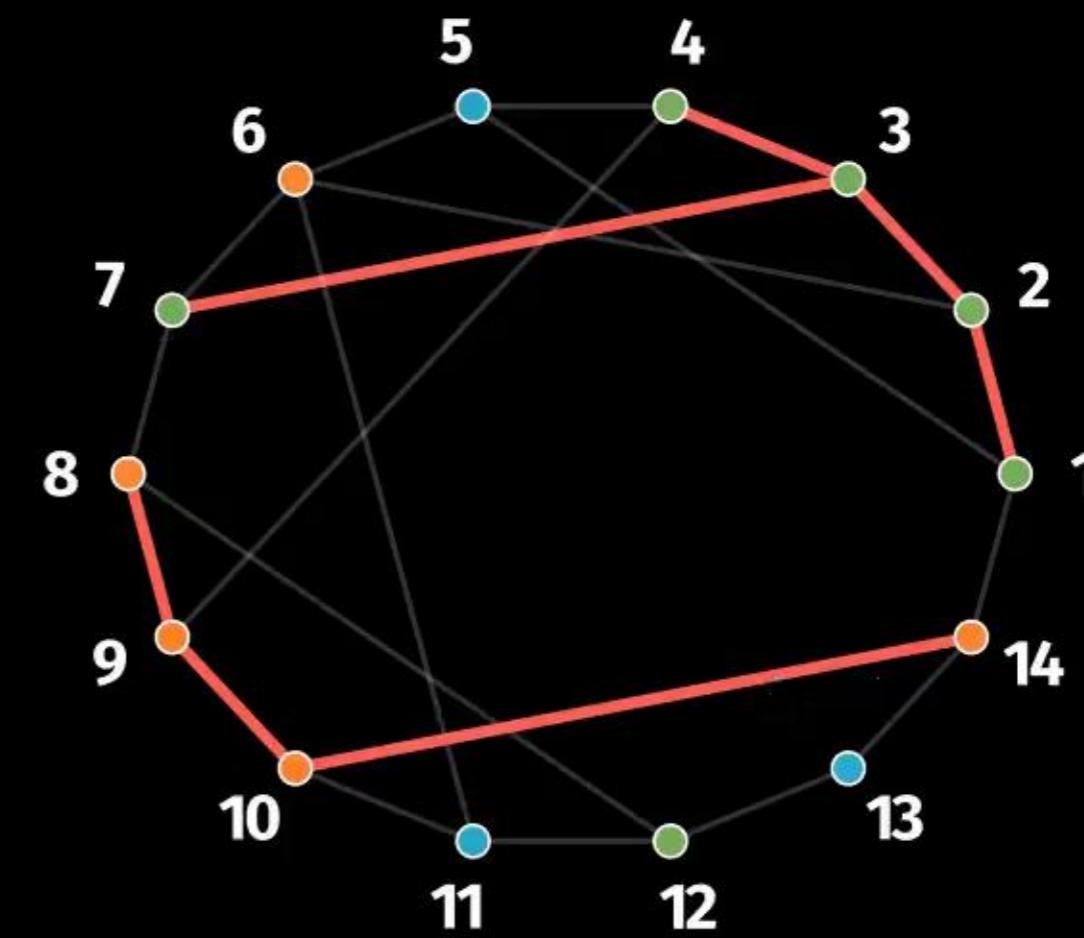
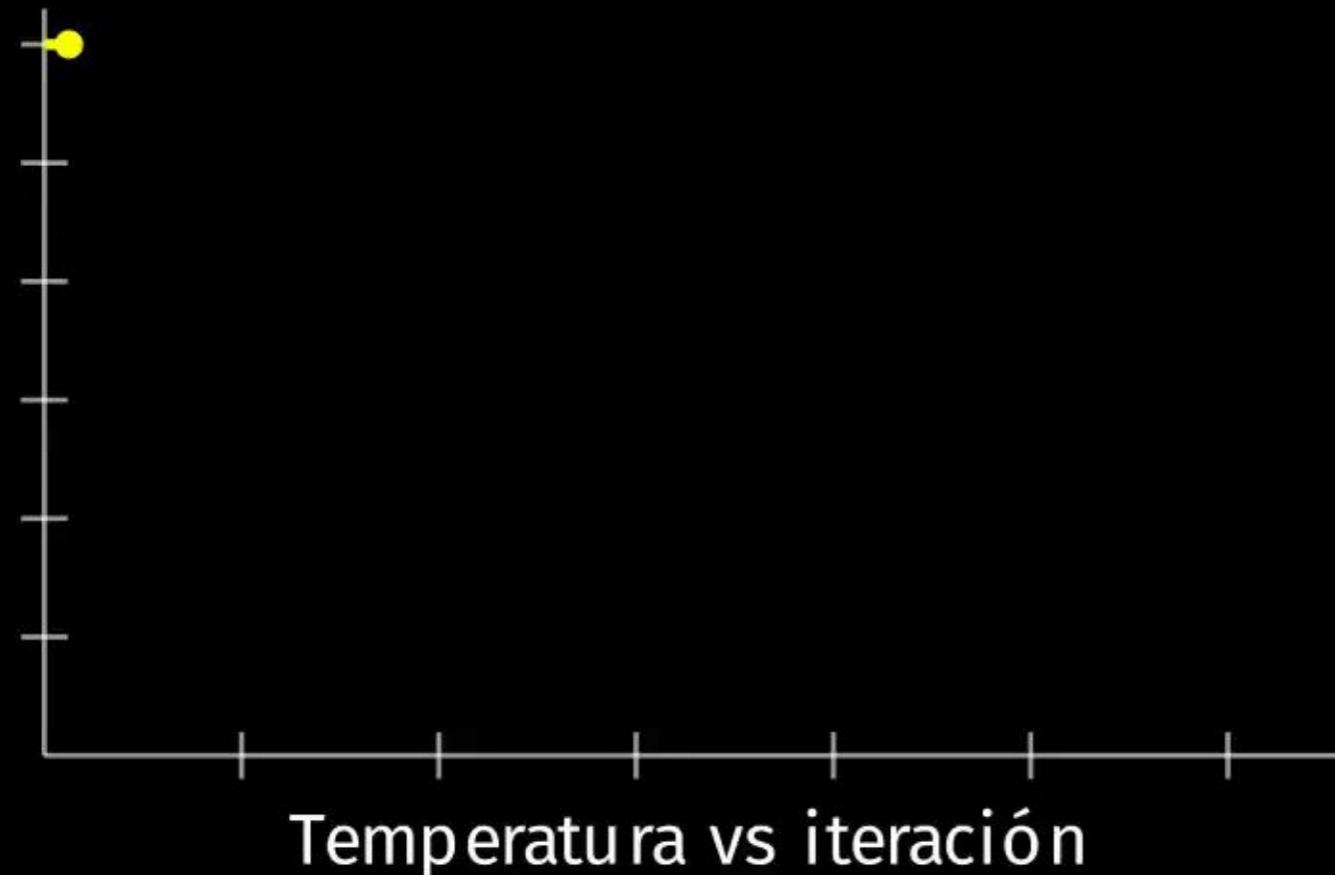
# Simulated Annealing – Graph Coloring

# Simulated Annealing – Graph Coloring



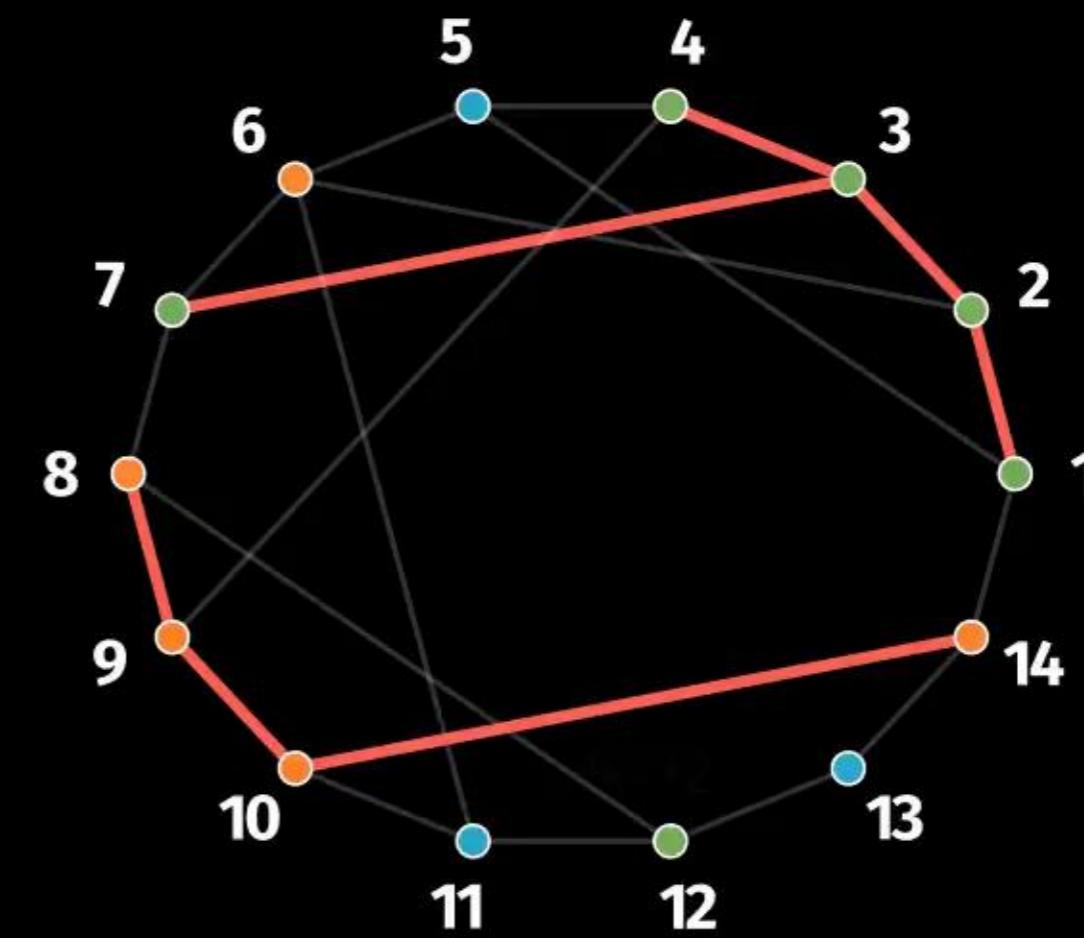
Temperatura: **2.000** Conflictos: **8** Incumbente: **8**  $\Delta$ : **+0**

# Simulated Annealing – Graph Coloring



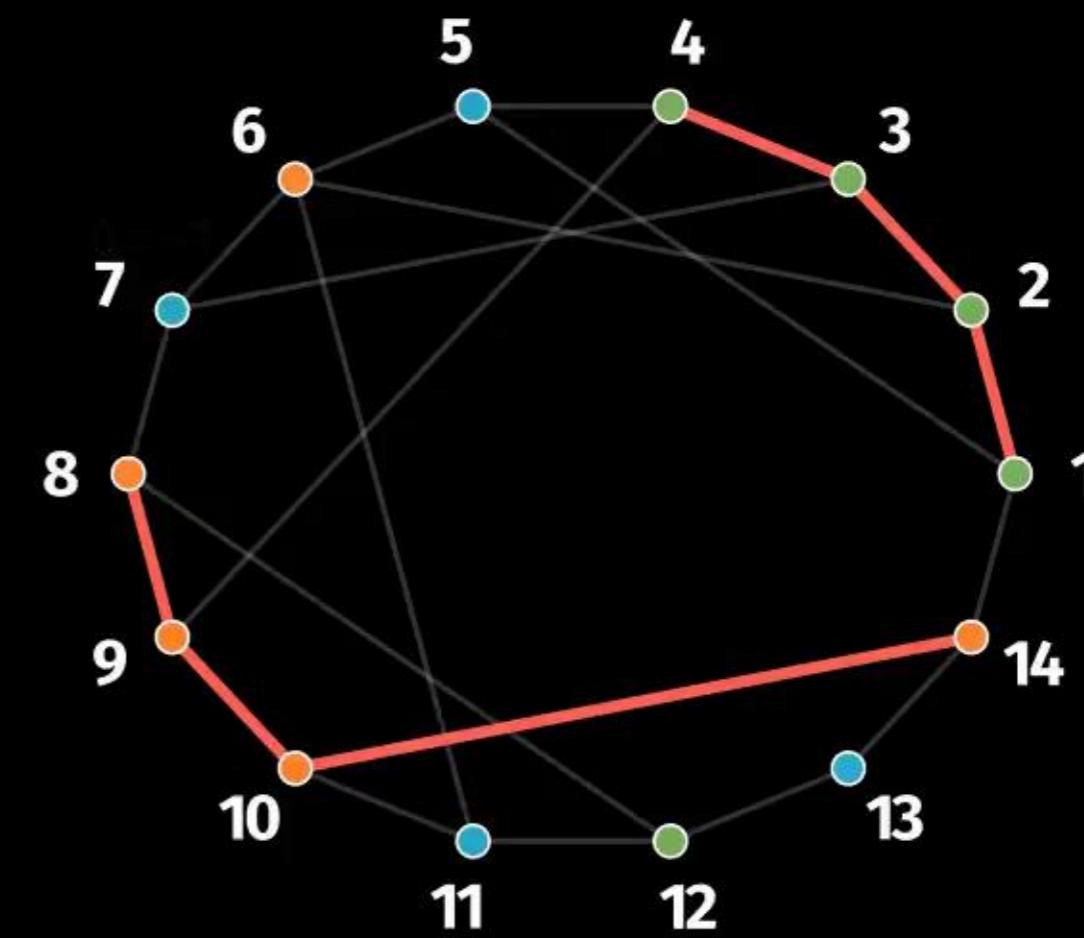
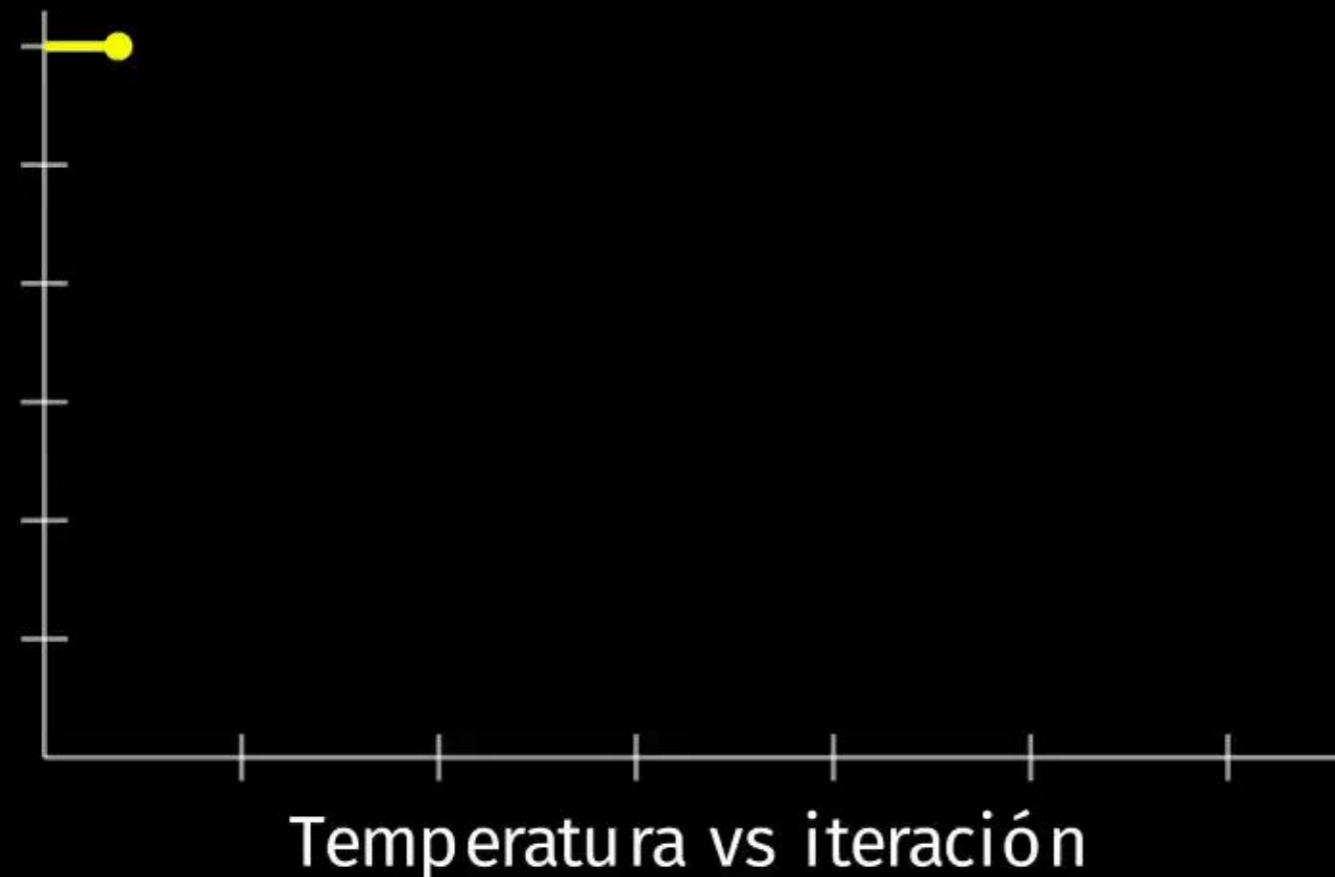
Temperatura: **2.000** Conflictos: **7** Incumbente: **7**  $\Delta$ : **-1**

# Simulated Annealing – Graph Coloring



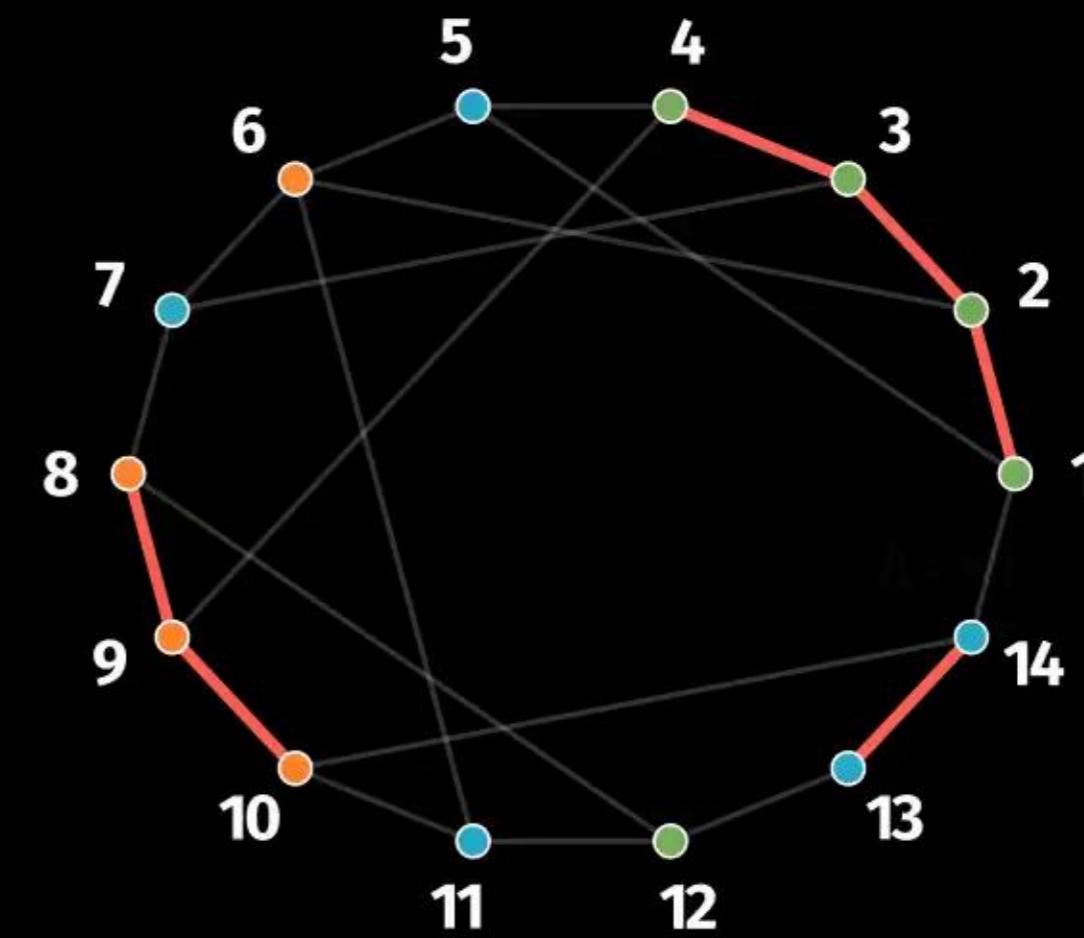
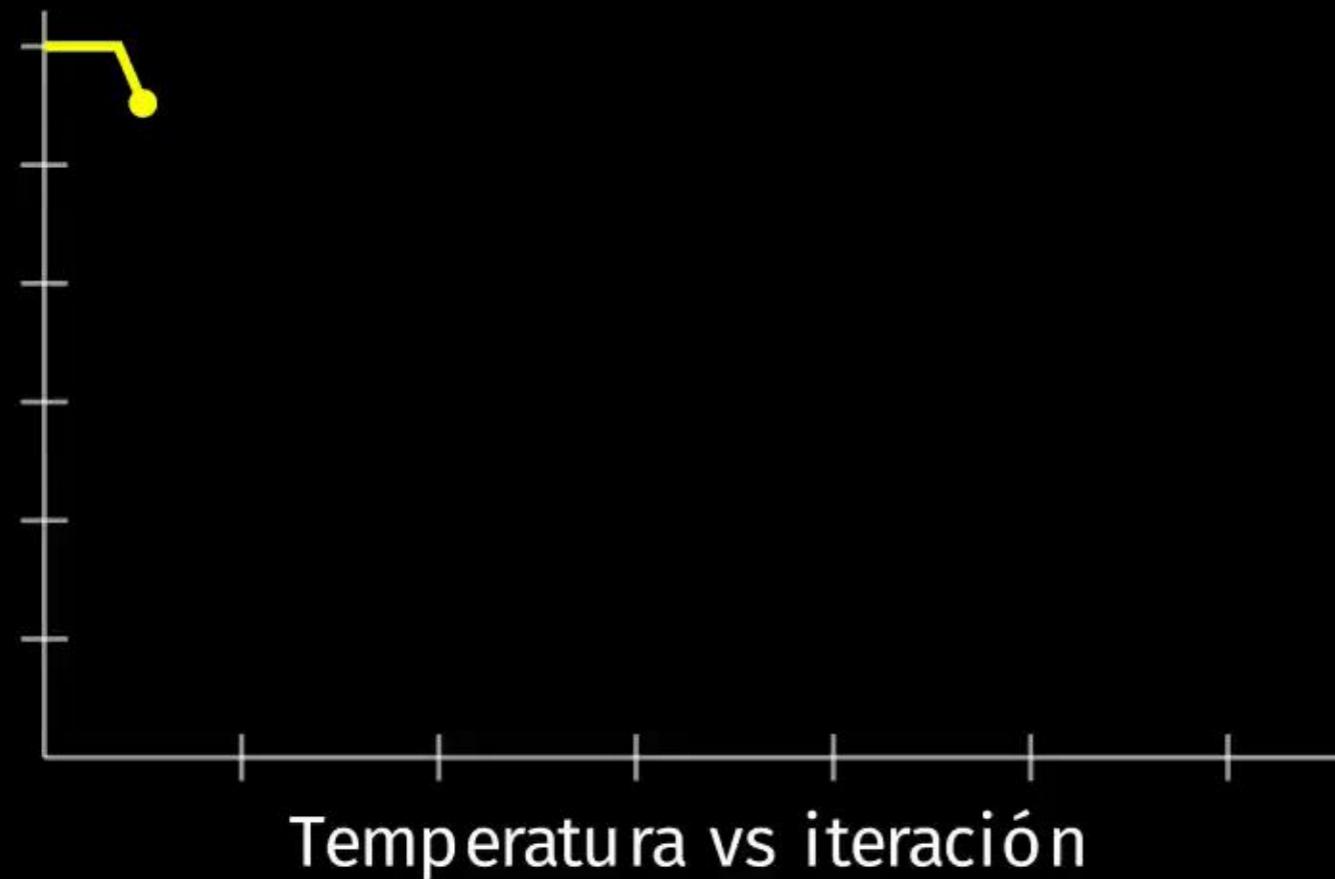
Temperatura: **2.000** Conflictos: **7** Incumbente: **7**  $\Delta$ : **+2**

# Simulated Annealing – Graph Coloring



Temperatura: **2.000** Conflictos: **6** Incumbente: **6**  $\Delta$ : **-1**

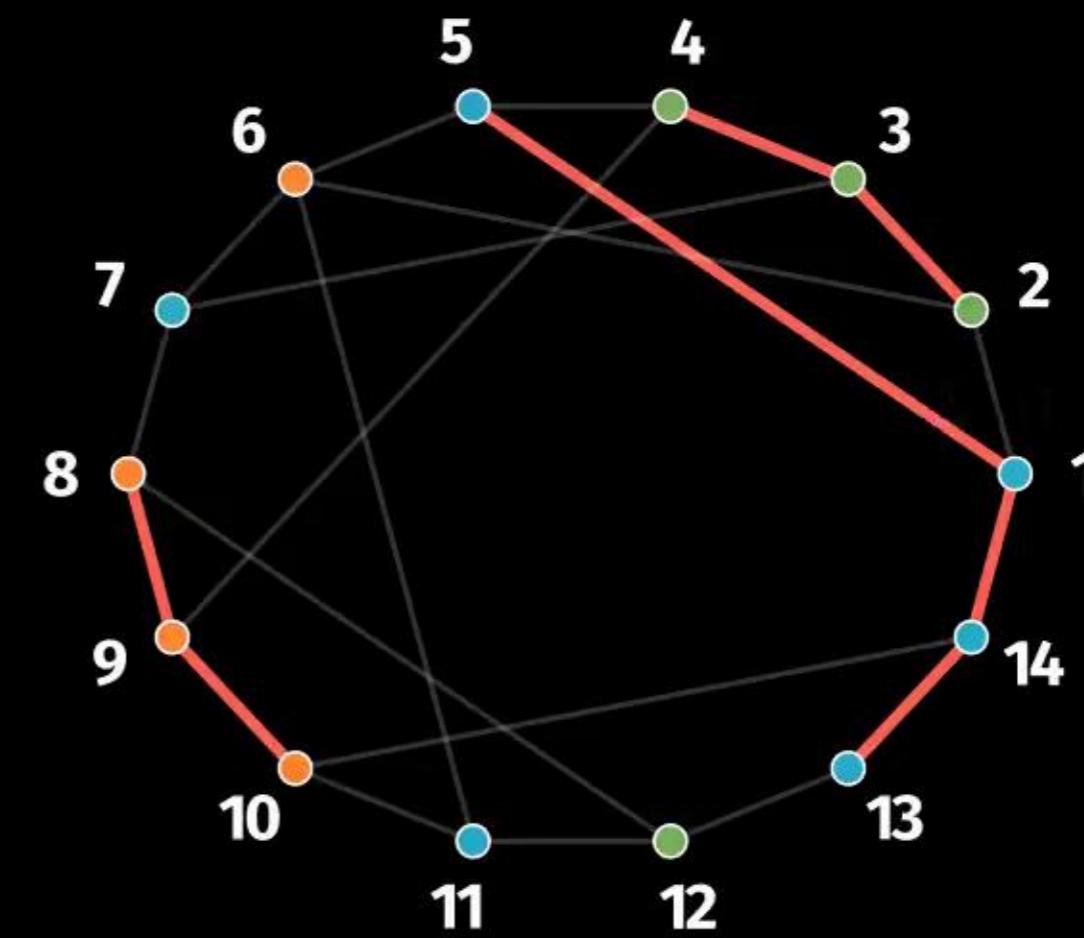
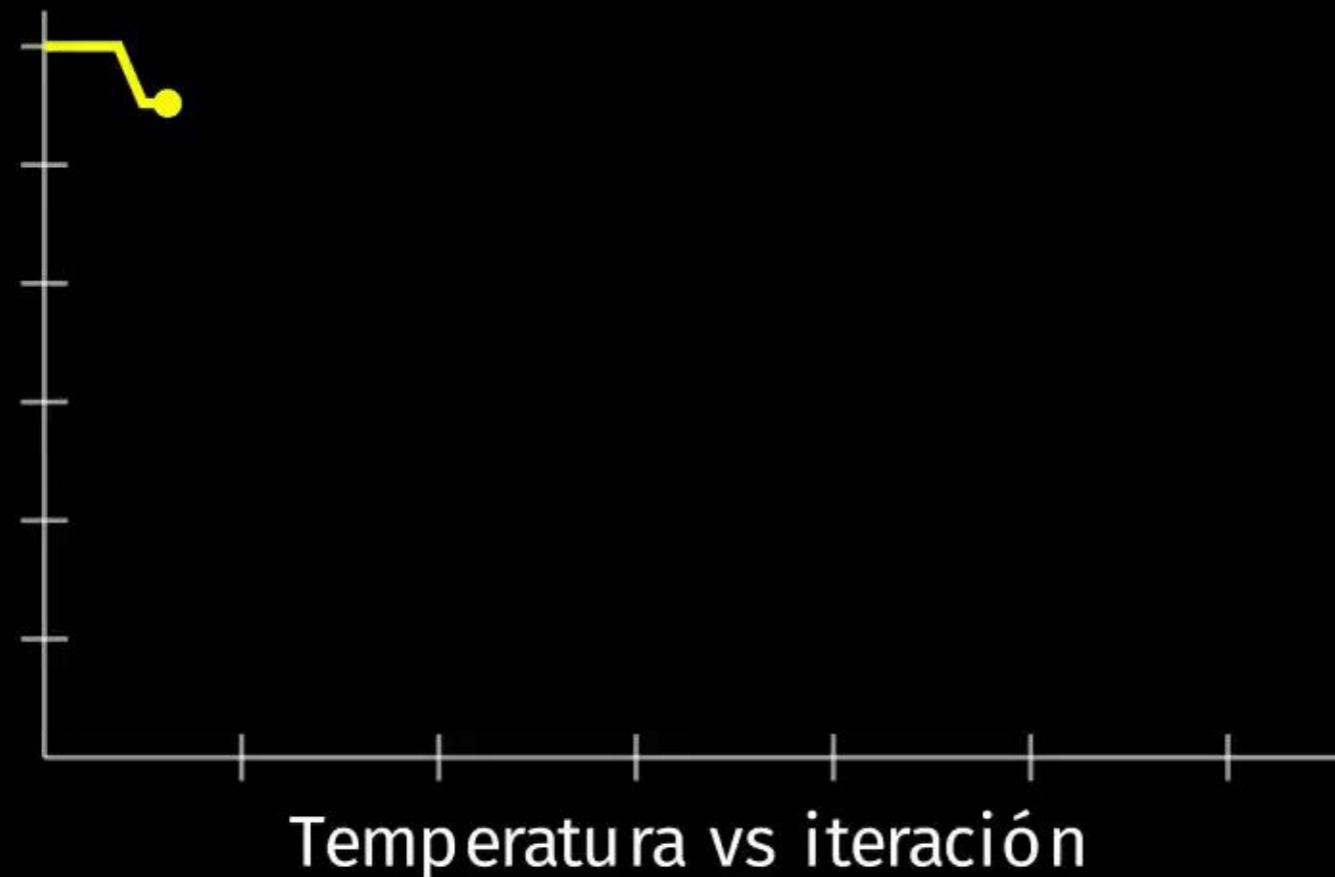
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Movimiento aceptado

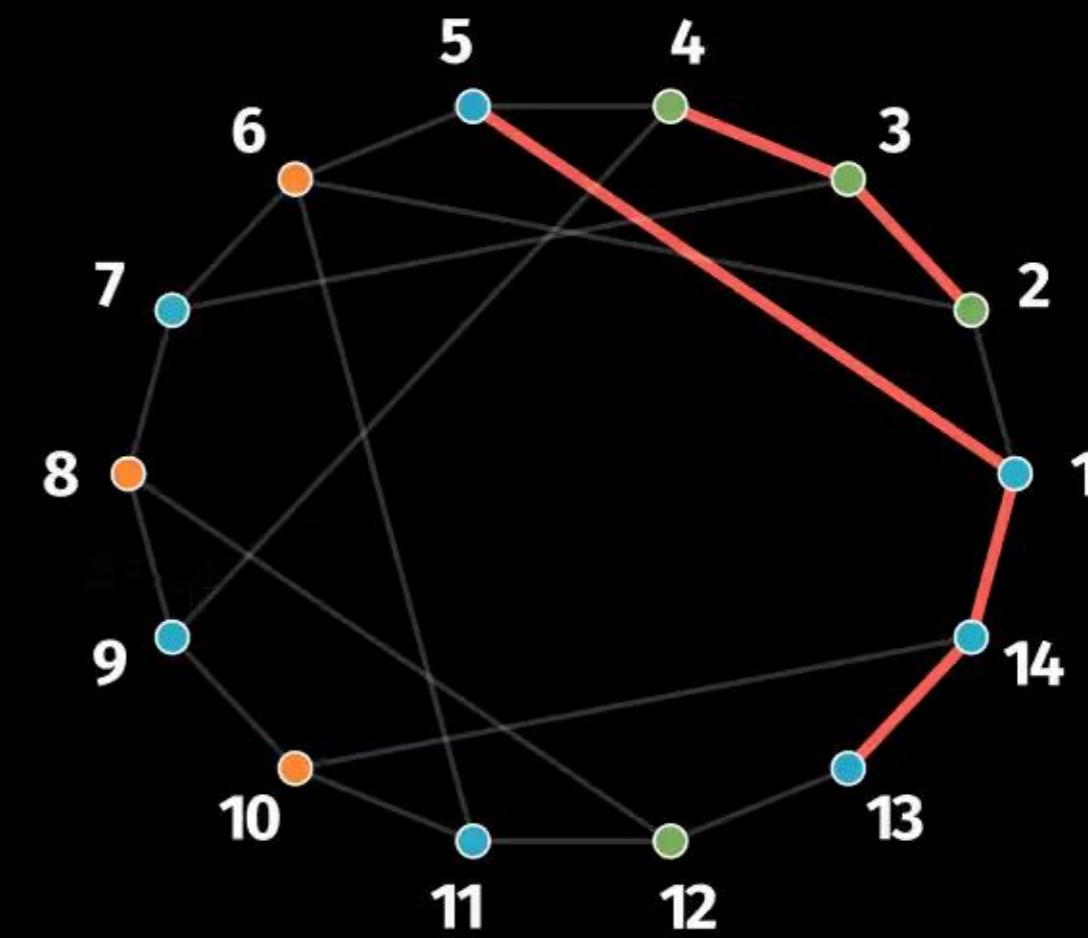
Temperatura: **1.840** Conflictos: **6** Incumbente: **6**  $\Delta$ : **+0**

# Simulated Annealing – Graph Coloring



Temperatura: **1.840** Conflictos: **7** Incumbente: **6**  $\Delta$ : **+1**

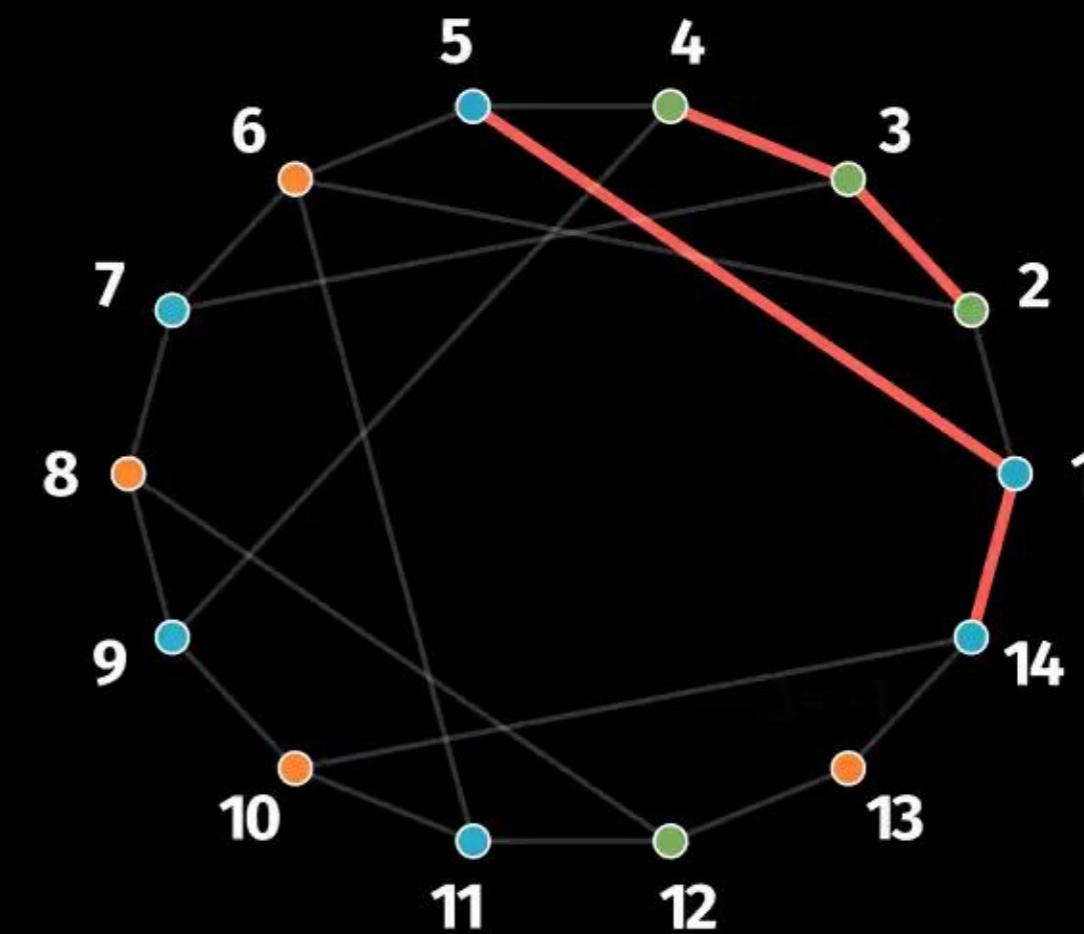
# Simulated Annealing – Graph Coloring



Movimiento aceptado

Temperatura: **1.840** Conflictos: **5** Incumbente: **5**  $\Delta$ : **-2**

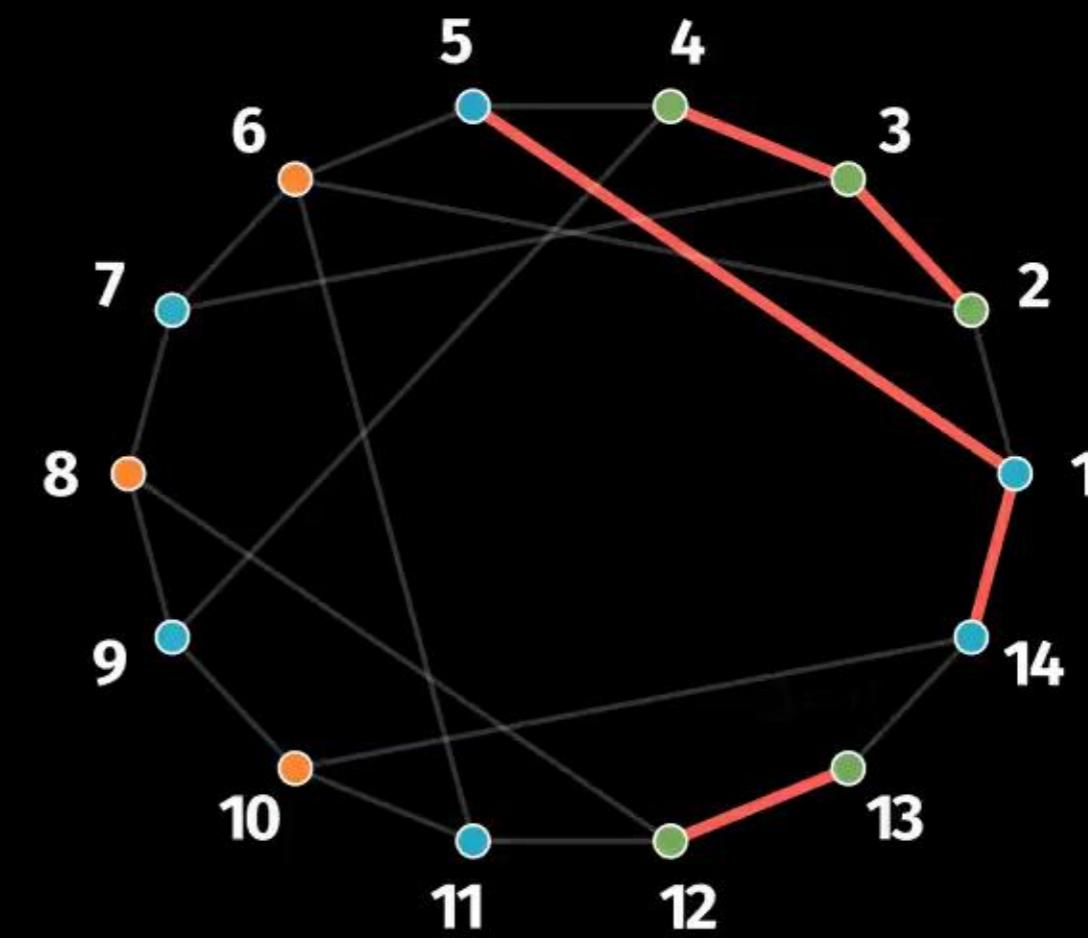
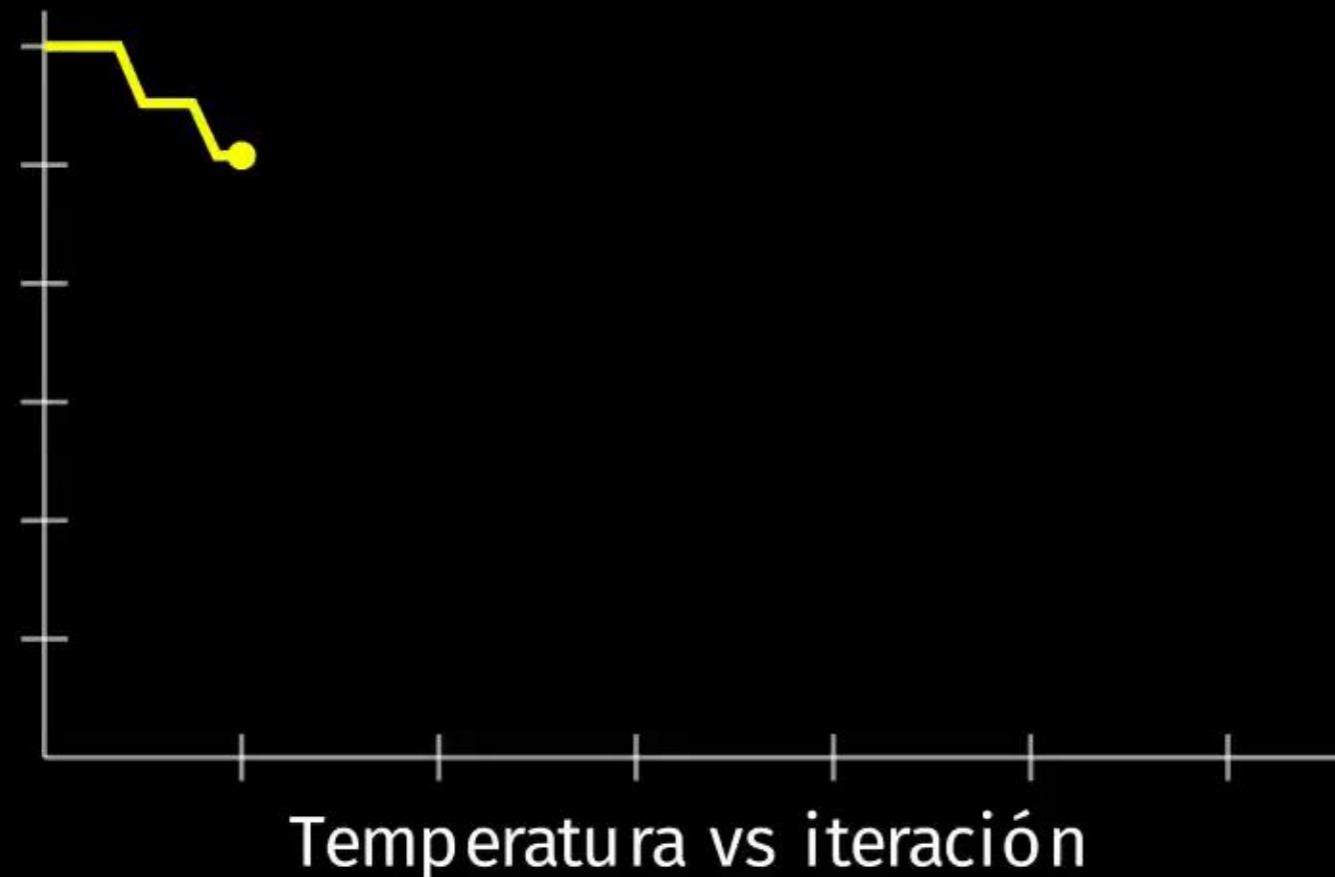
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Movimiento aceptado

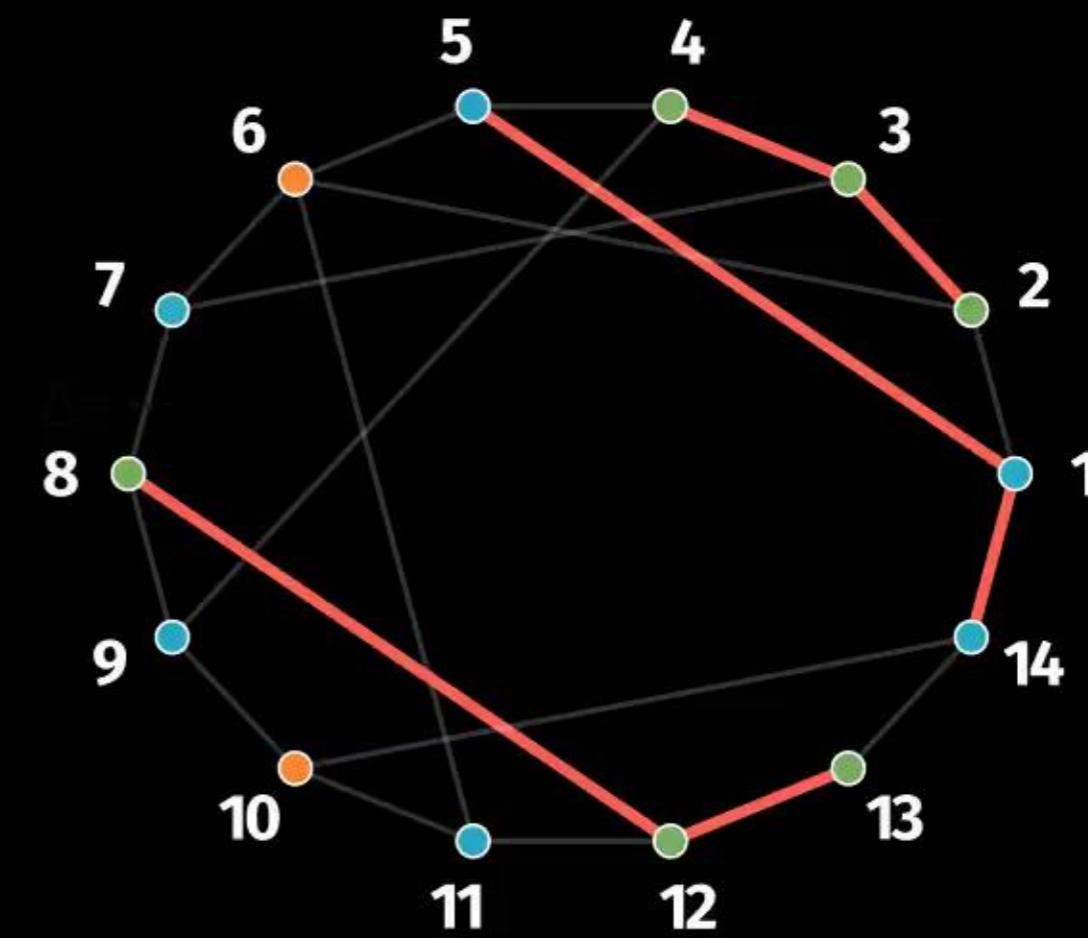
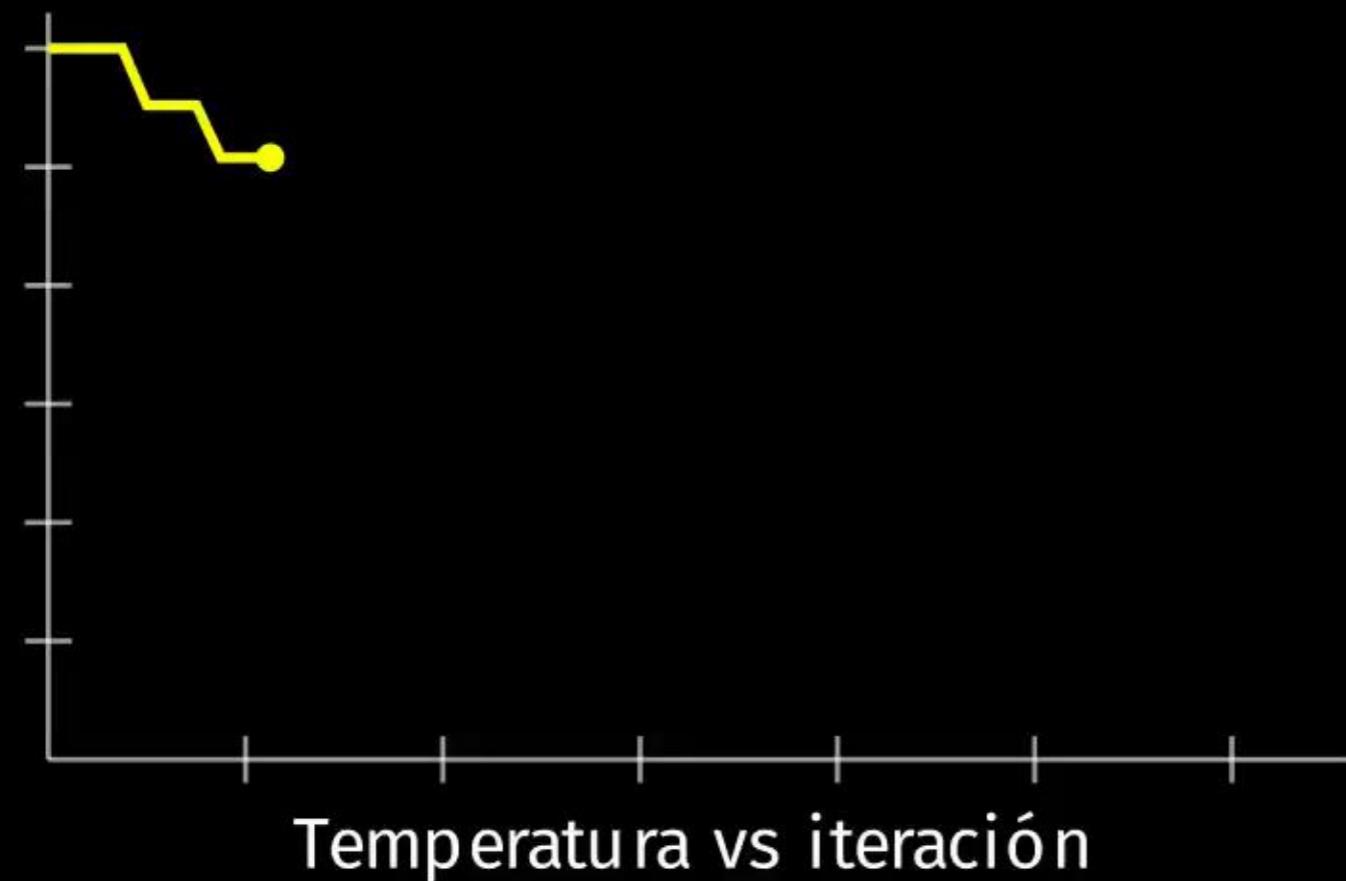
Temperatura: **1.693** Conflictos: **4** Incumbente: **4**  $\Delta$ : **-1**

# Simulated Annealing – Graph Coloring



Temperatura: **1.693** Conflictos: **5** Incumbente: **4**  $\Delta$ : **+1**

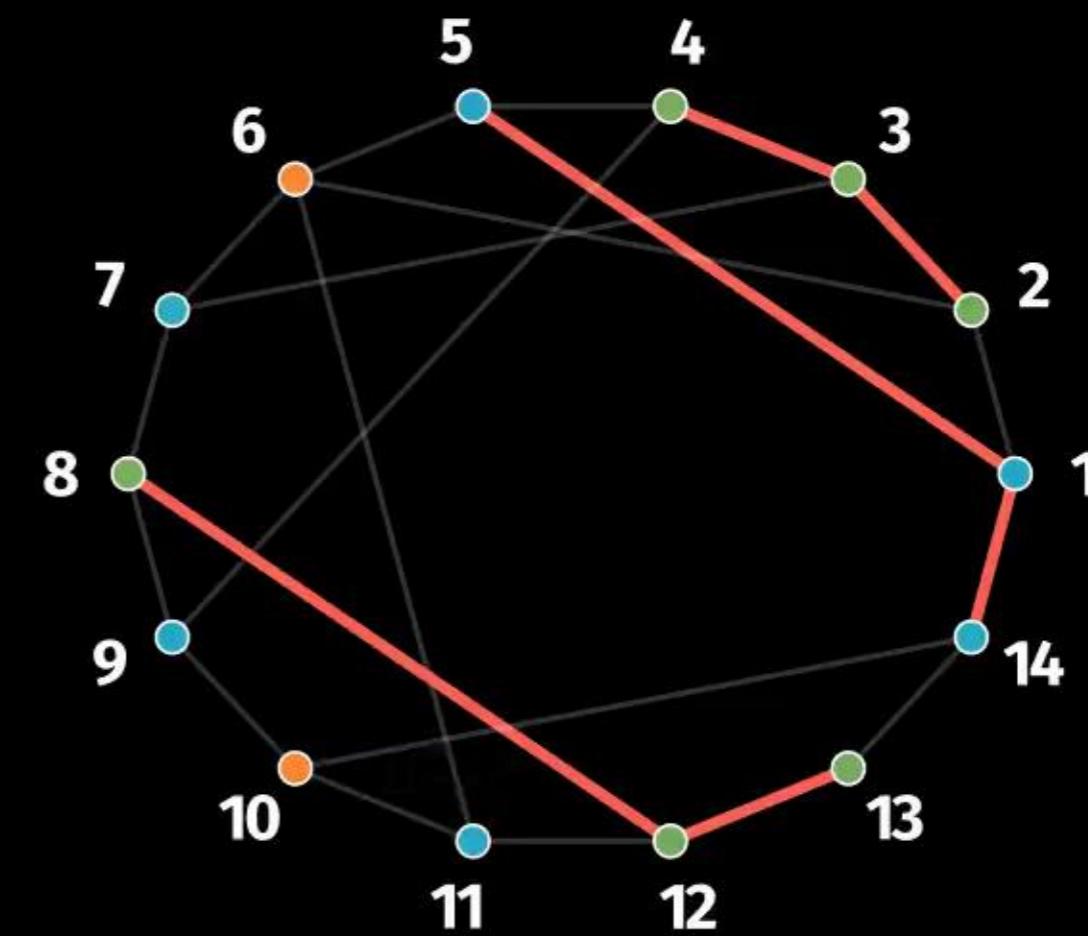
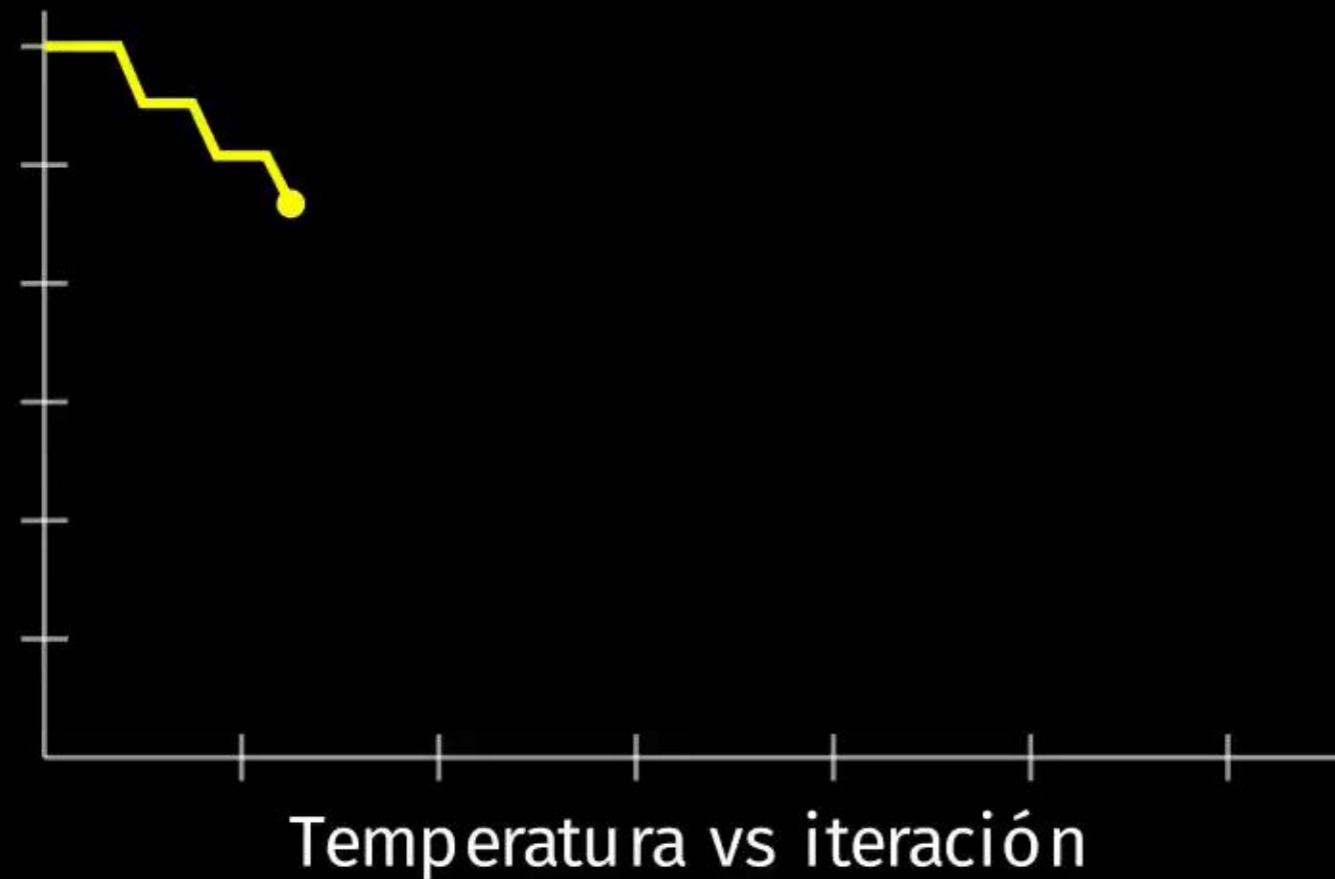
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## Movimiento aceptado

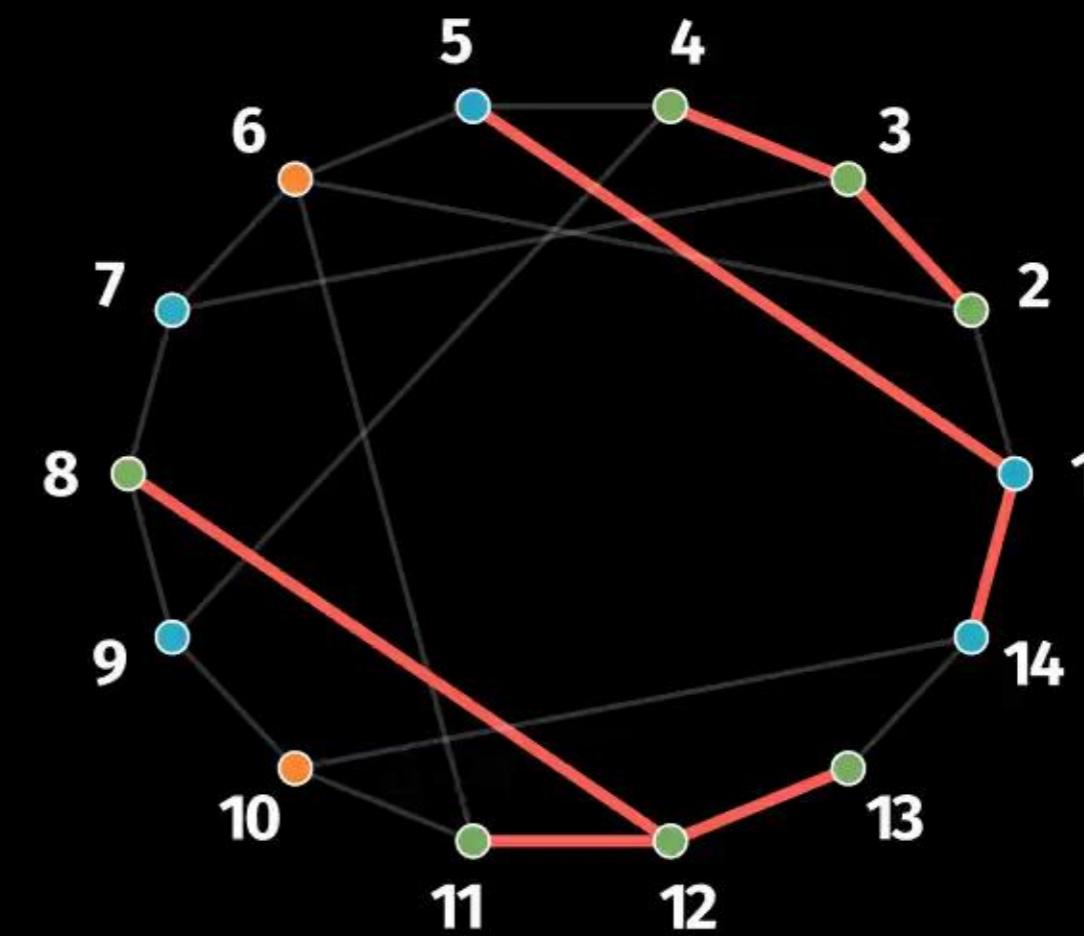
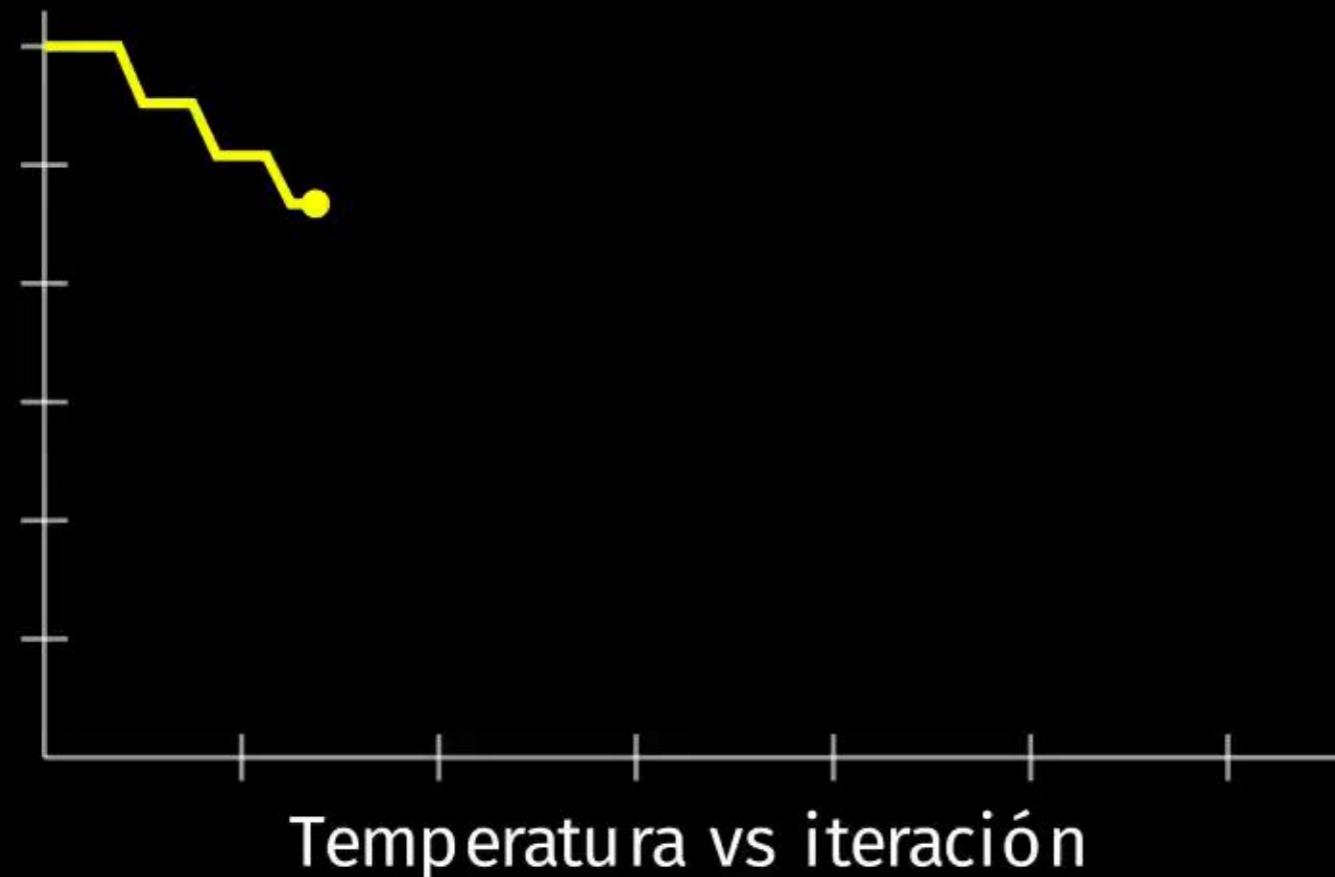
Temperatura: **1.693** Conflictos: **6** Incumbente: **4** Δ: **+1**

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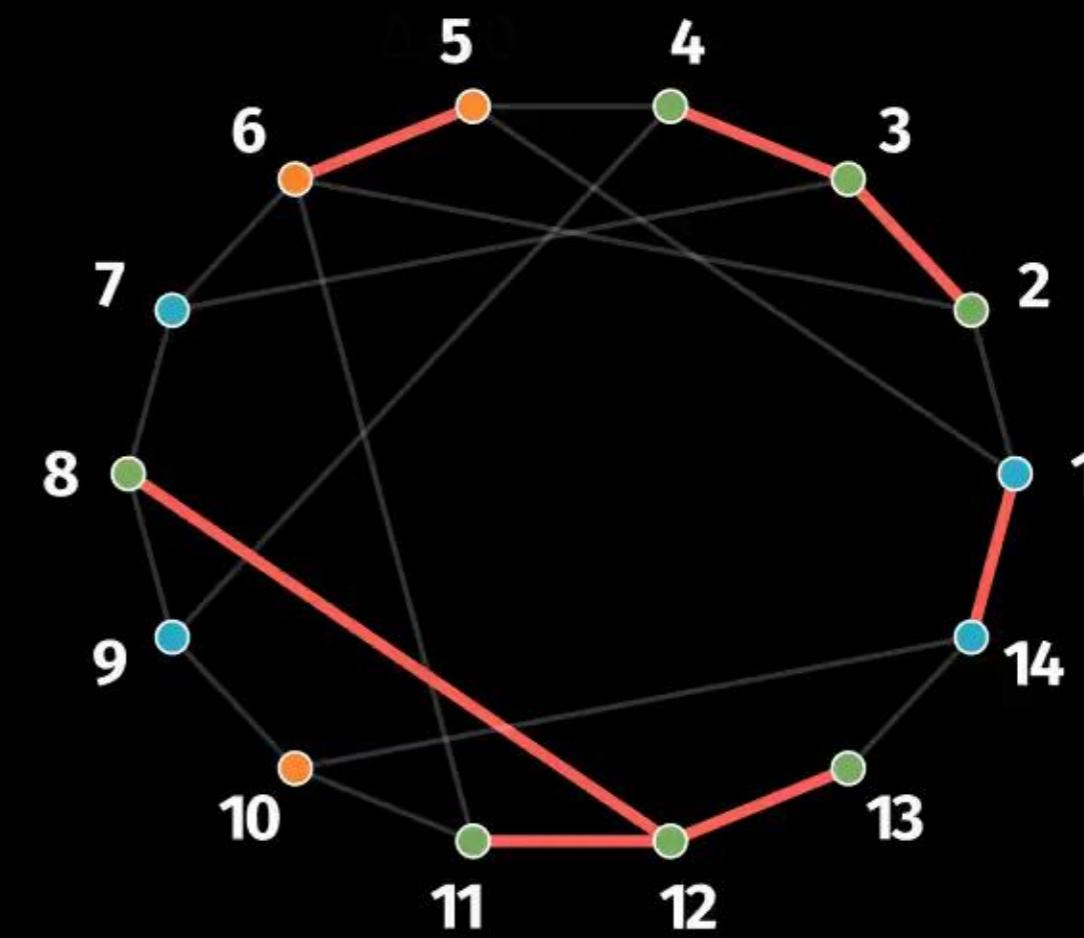
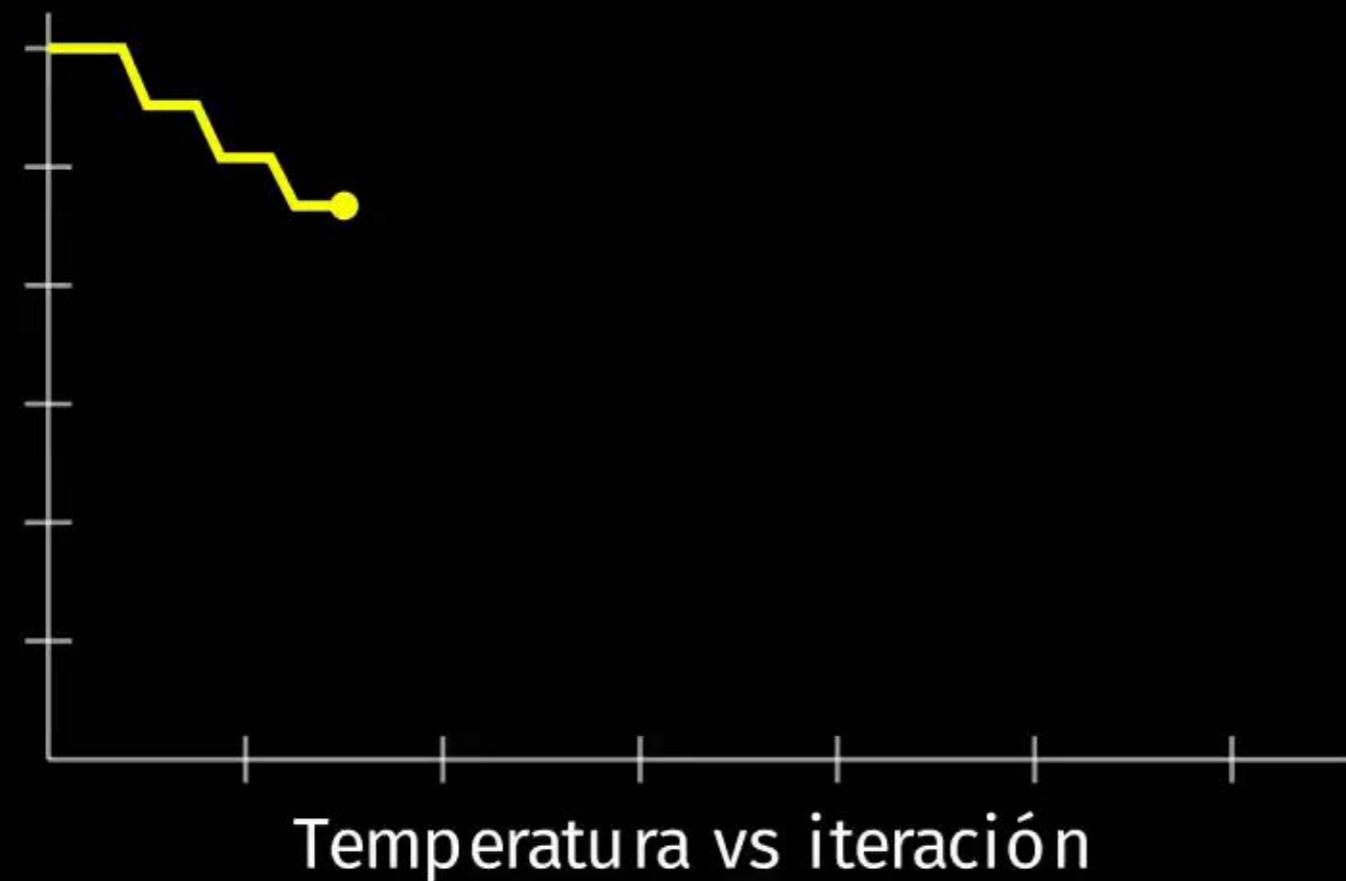
Temperatura: **1.557** Conflictos: **6** Incumbente: **4**  $\Delta$ : **+2**

# Simulated Annealing – Graph Coloring



Temperatura: **1.557** Conflictos: **7** Incumbente: **4**  $\Delta$ : **+1**

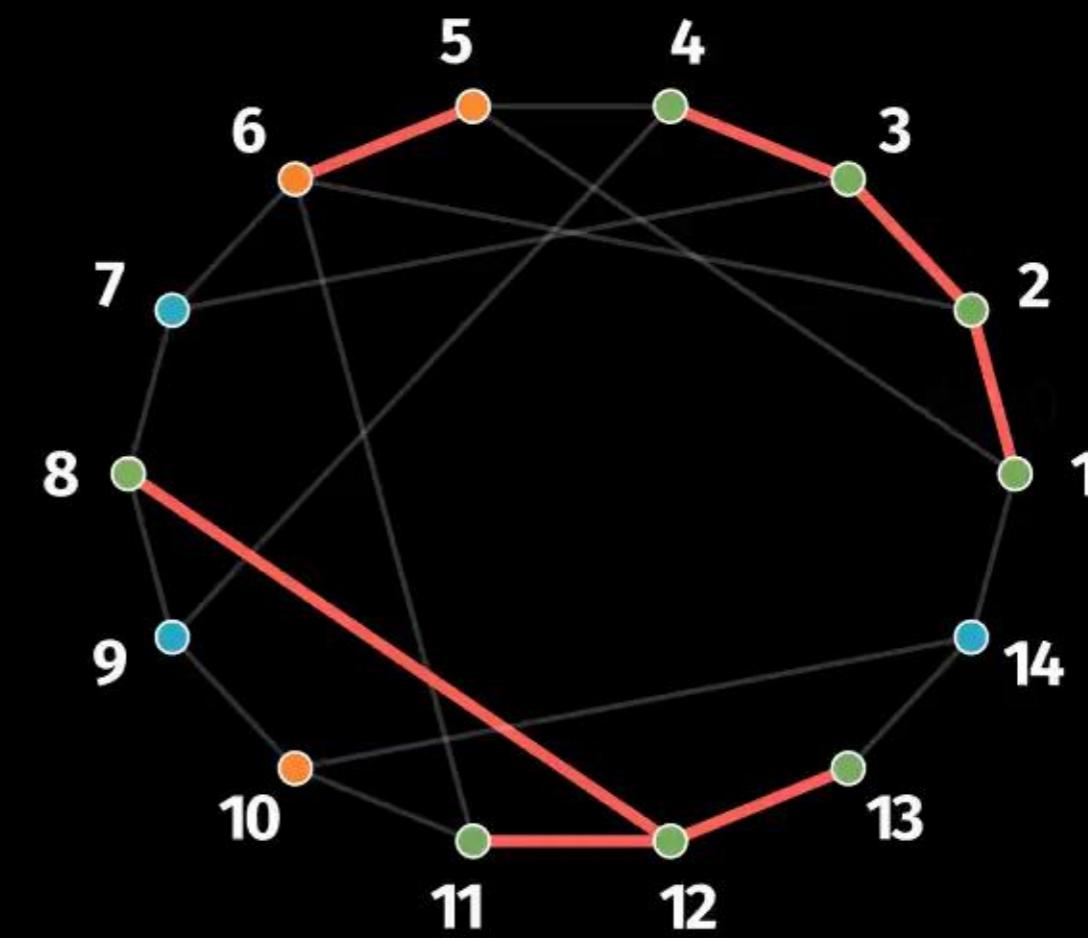
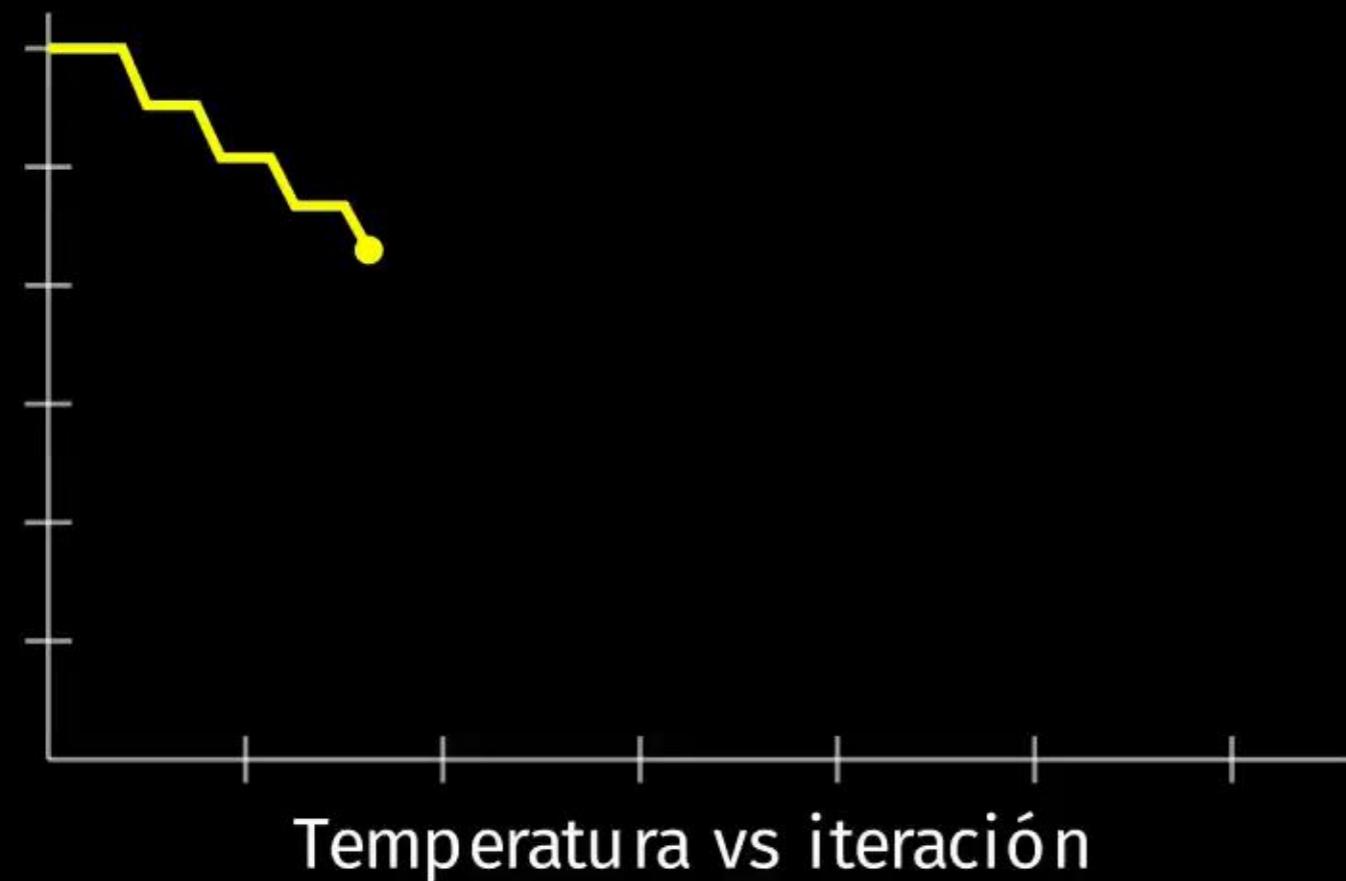
# Simulated Annealing – Graph Coloring



## Movimiento aceptado

Temperatura: **1.557** Conflictos: **7** Incumbente: **4** Δ: **+0**

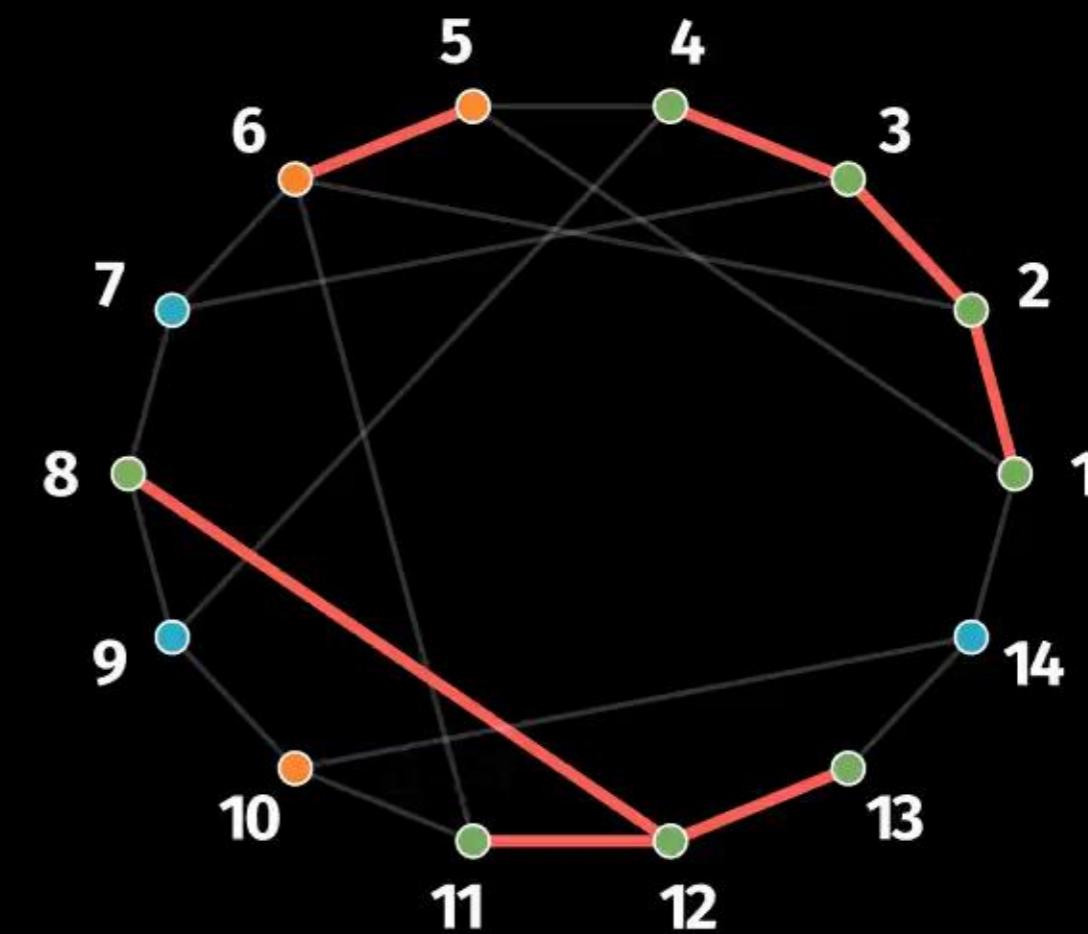
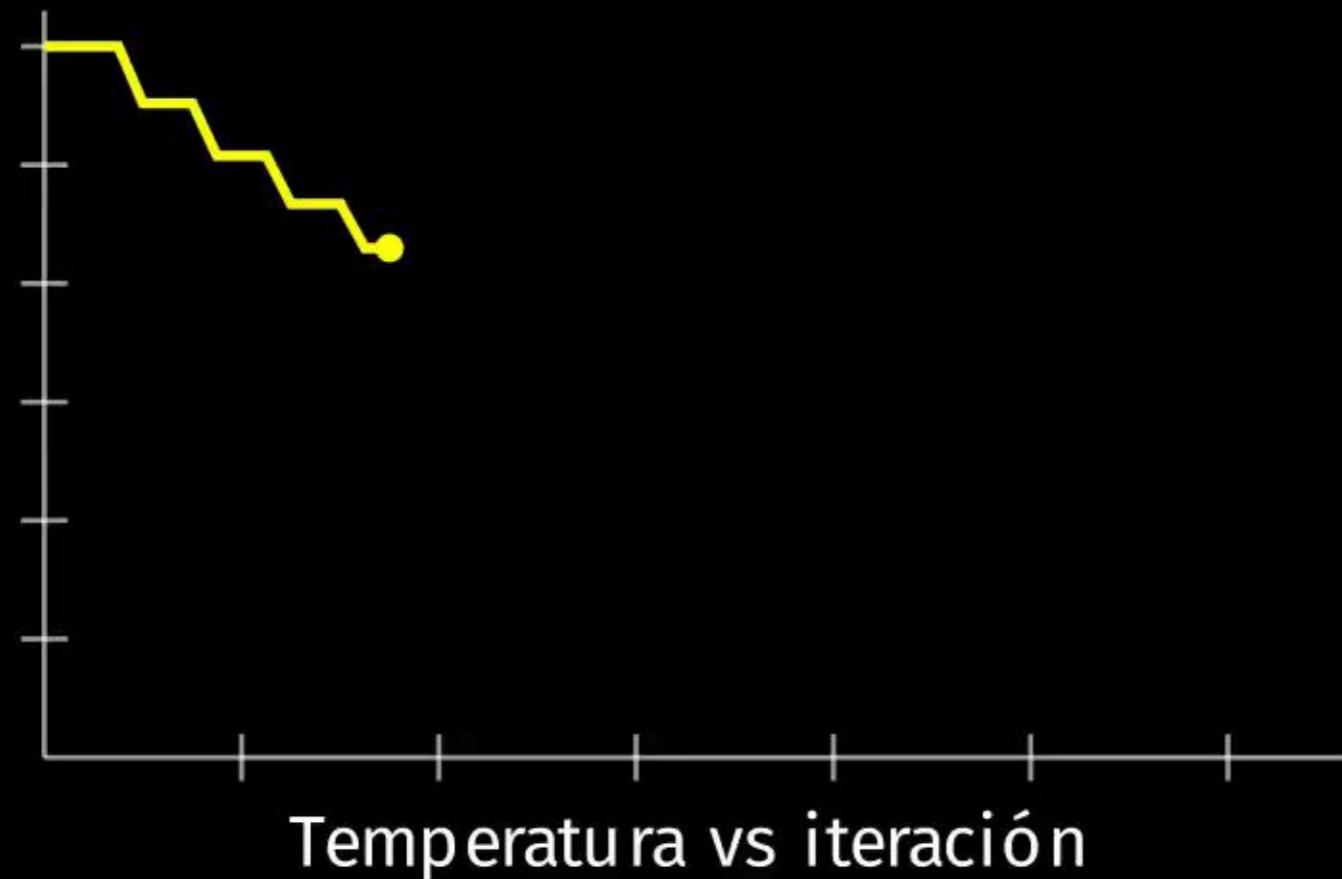
# Simulated Annealing – Graph Coloring



## Movimiento aceptado

Temperatura: **1.433** Conflictos: **7** Incumbente: **4** Δ: **+0**

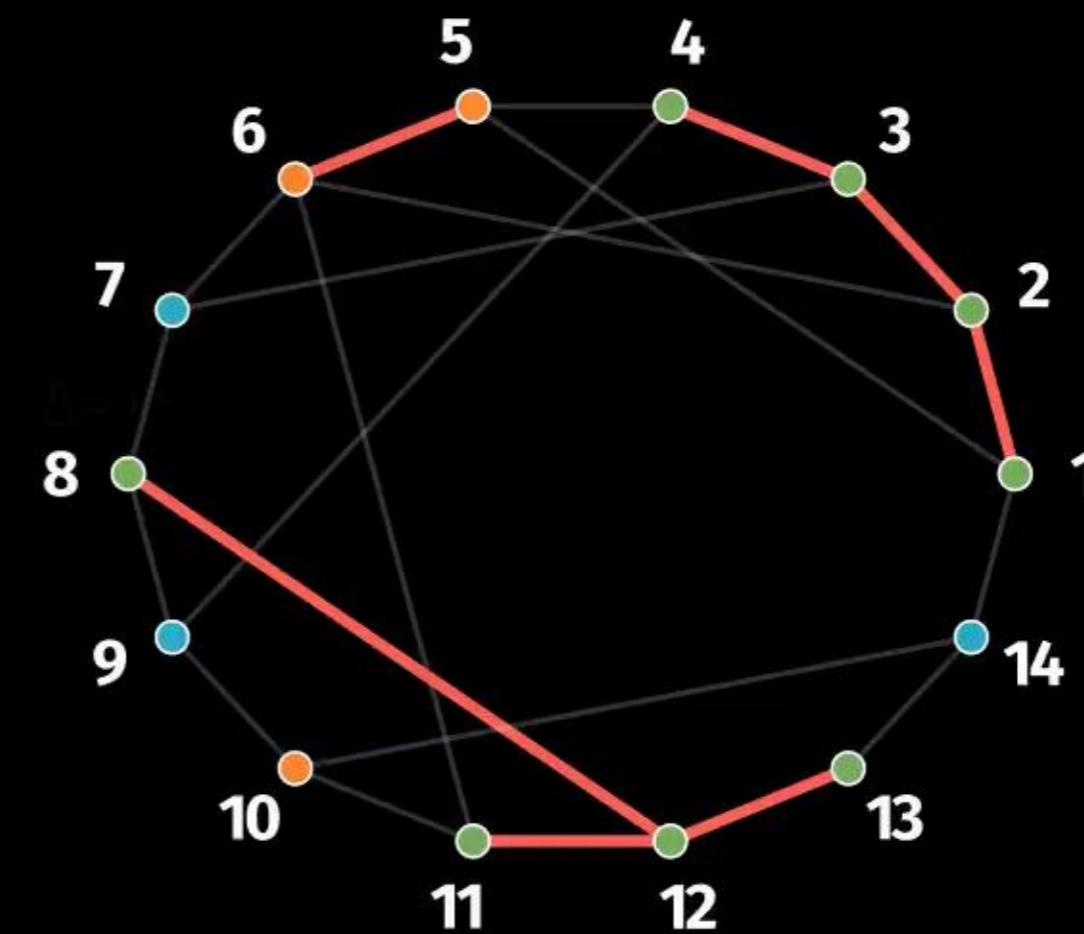
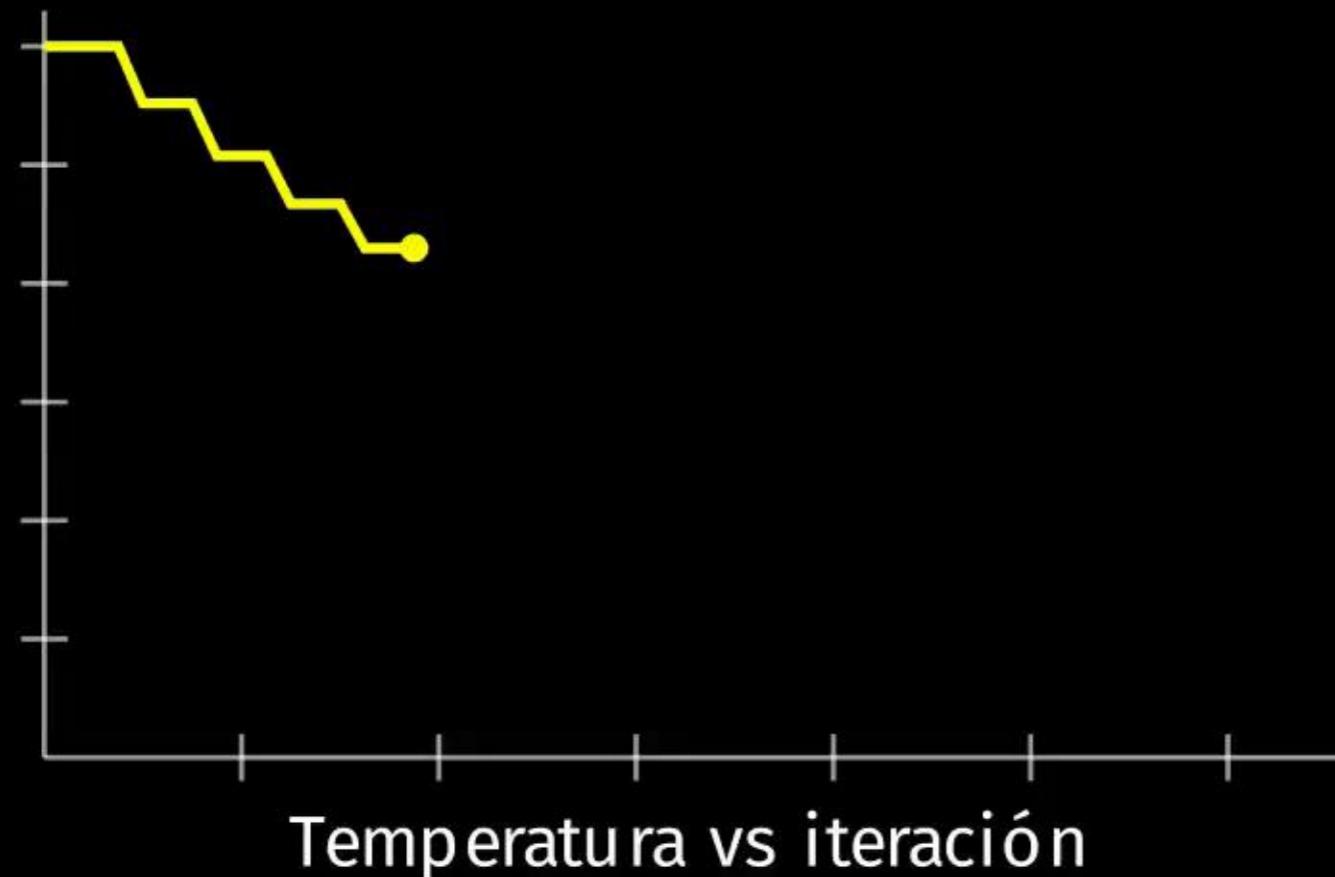
# Simulated Annealing – Graph Coloring



Movimiento rechazado

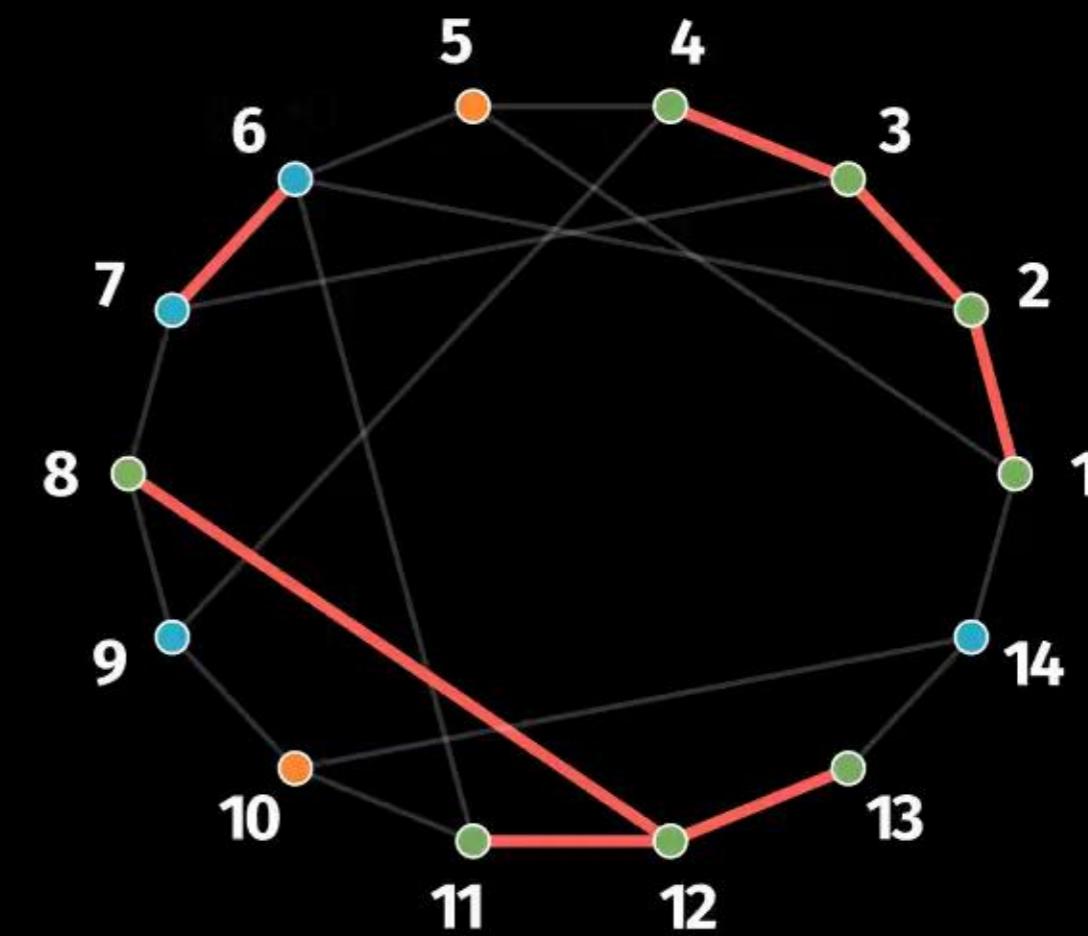
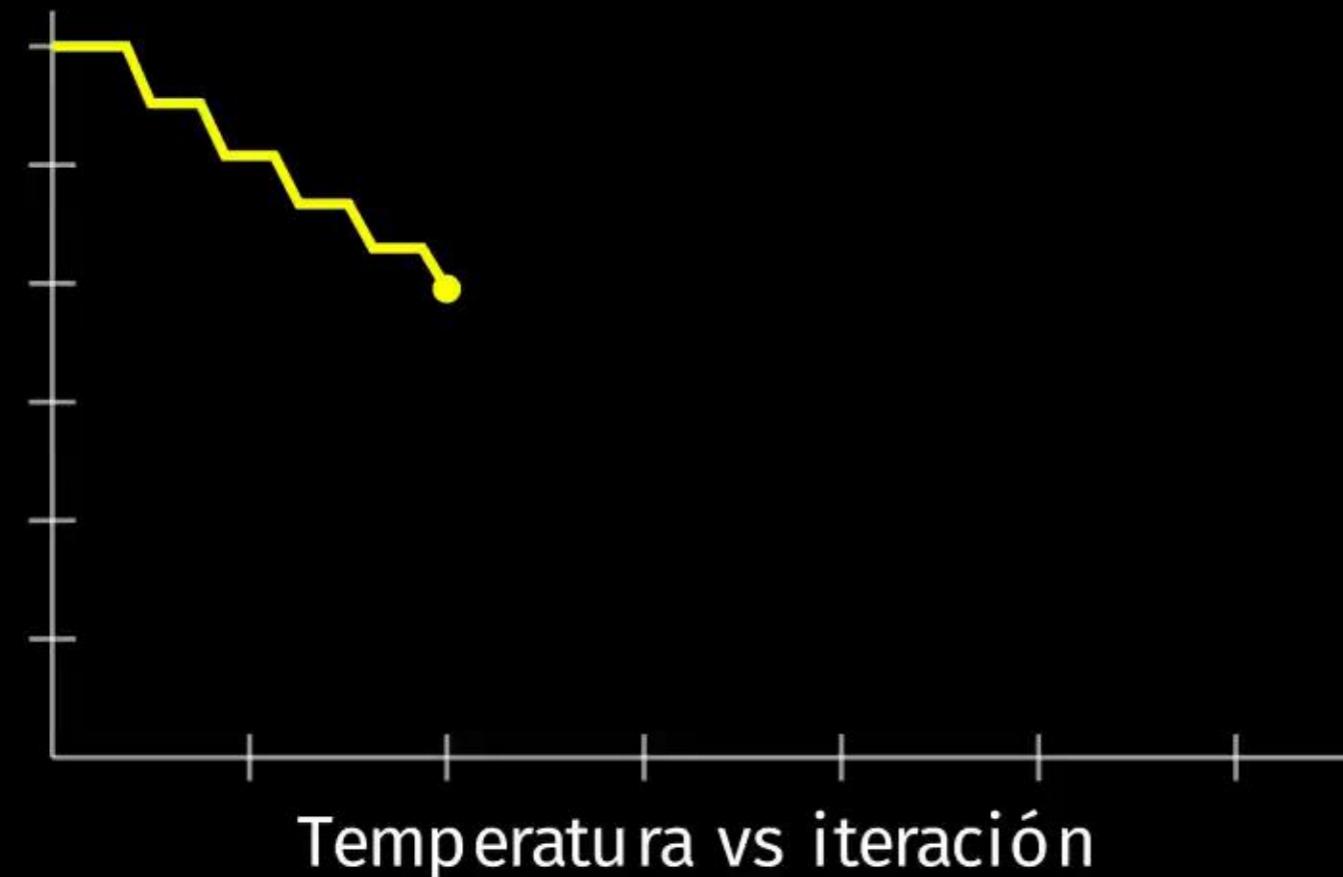
Temperatura: **1.433** Conflictos: **7** Incumbente: **4**  $\Delta$ : **+1**

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Temperatura: **1.433** Conflictos: **7** Incumbente: **4**  $\Delta$ : **+1**

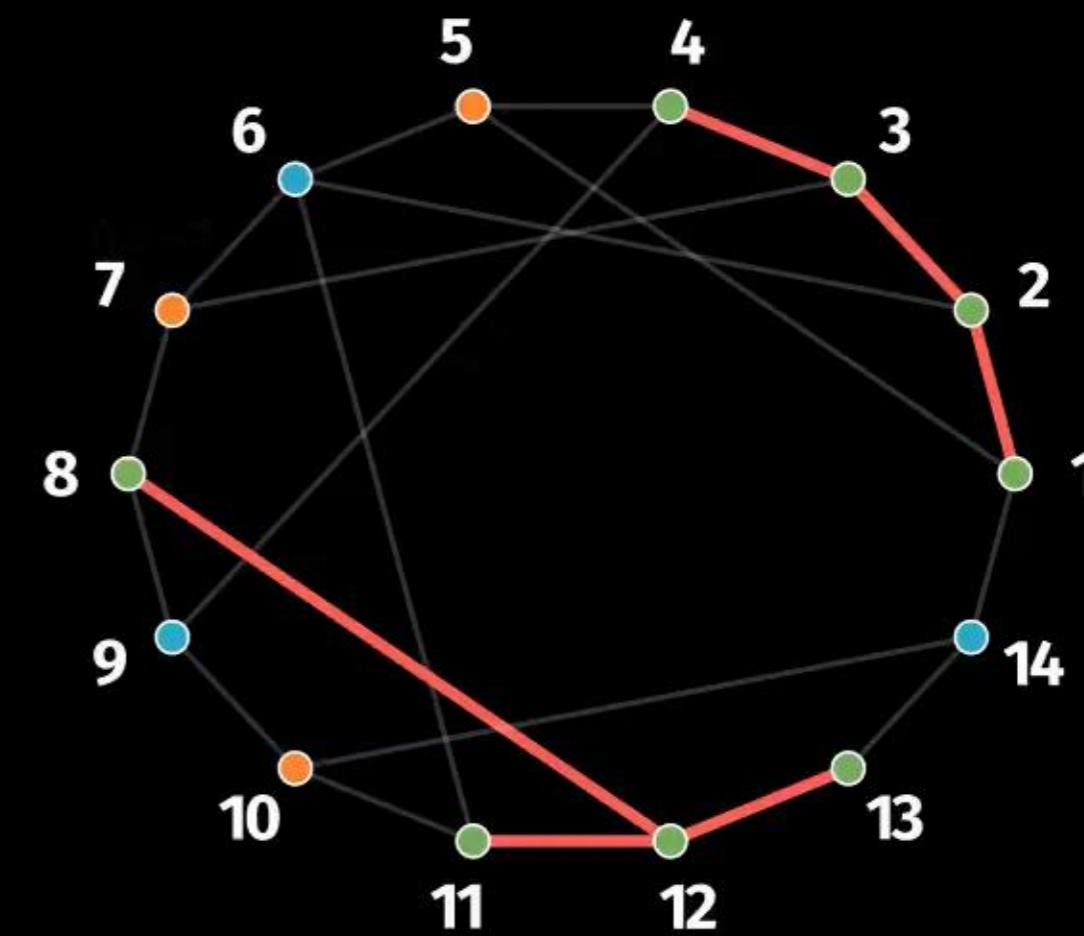
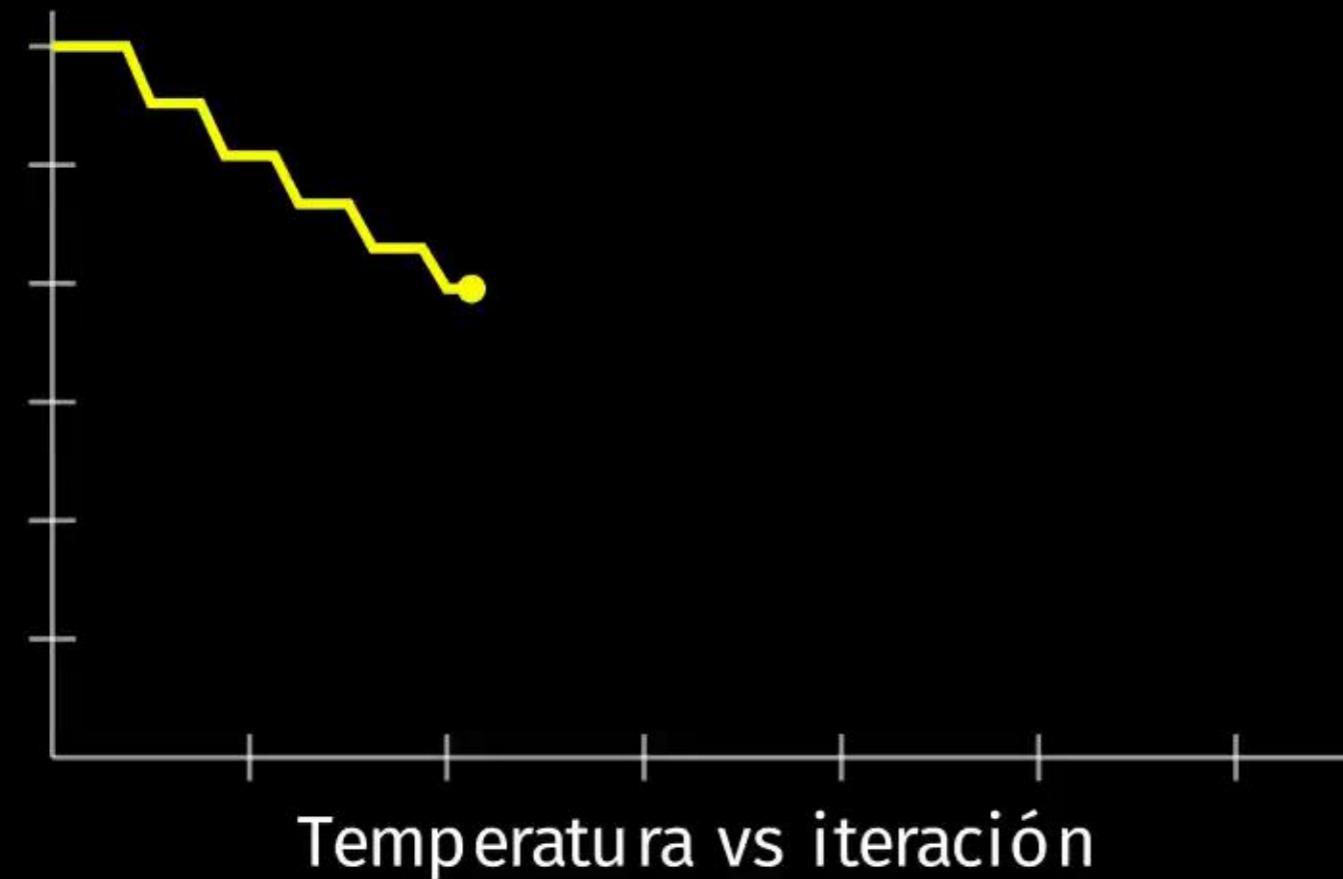
# Simulated Annealing – Graph Coloring



Movimiento aceptado

Temperatura: **1.318** Conflictos: **7** Incumbente: **4**  $\Delta$ : **+0**

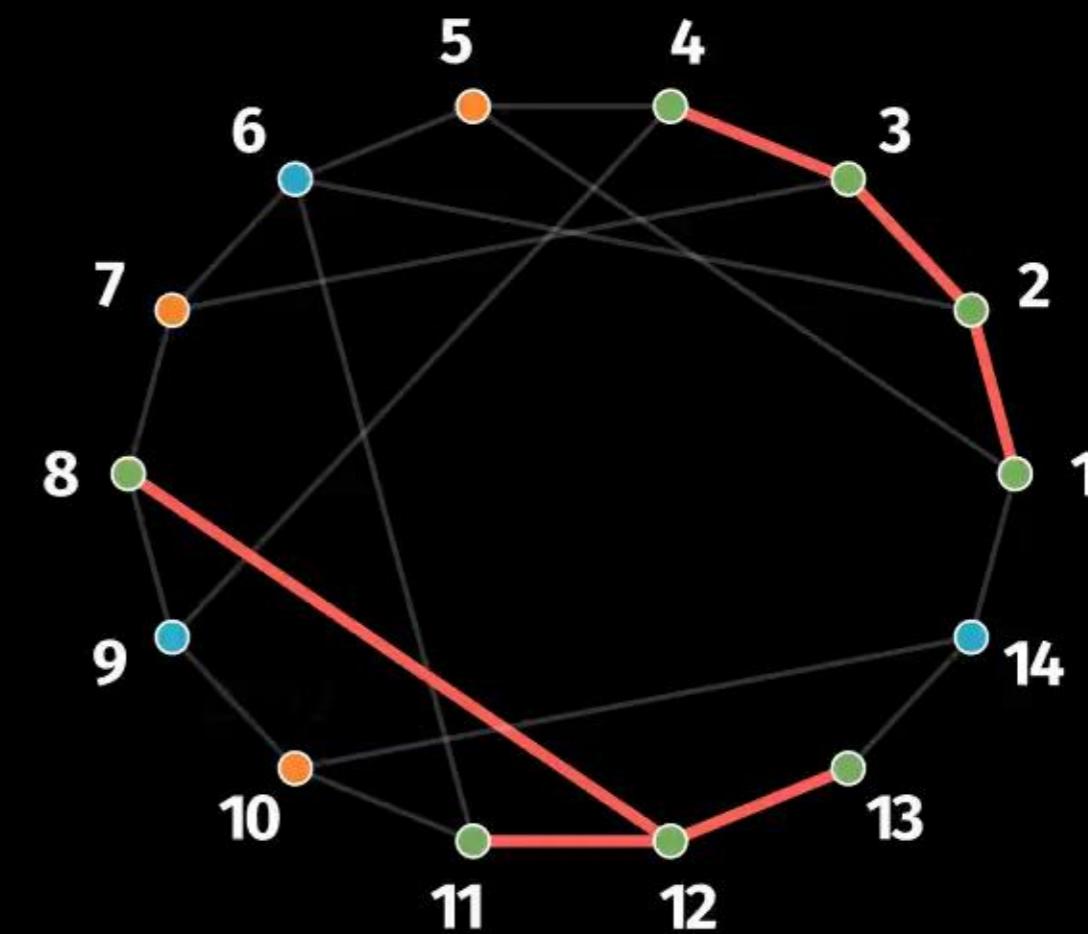
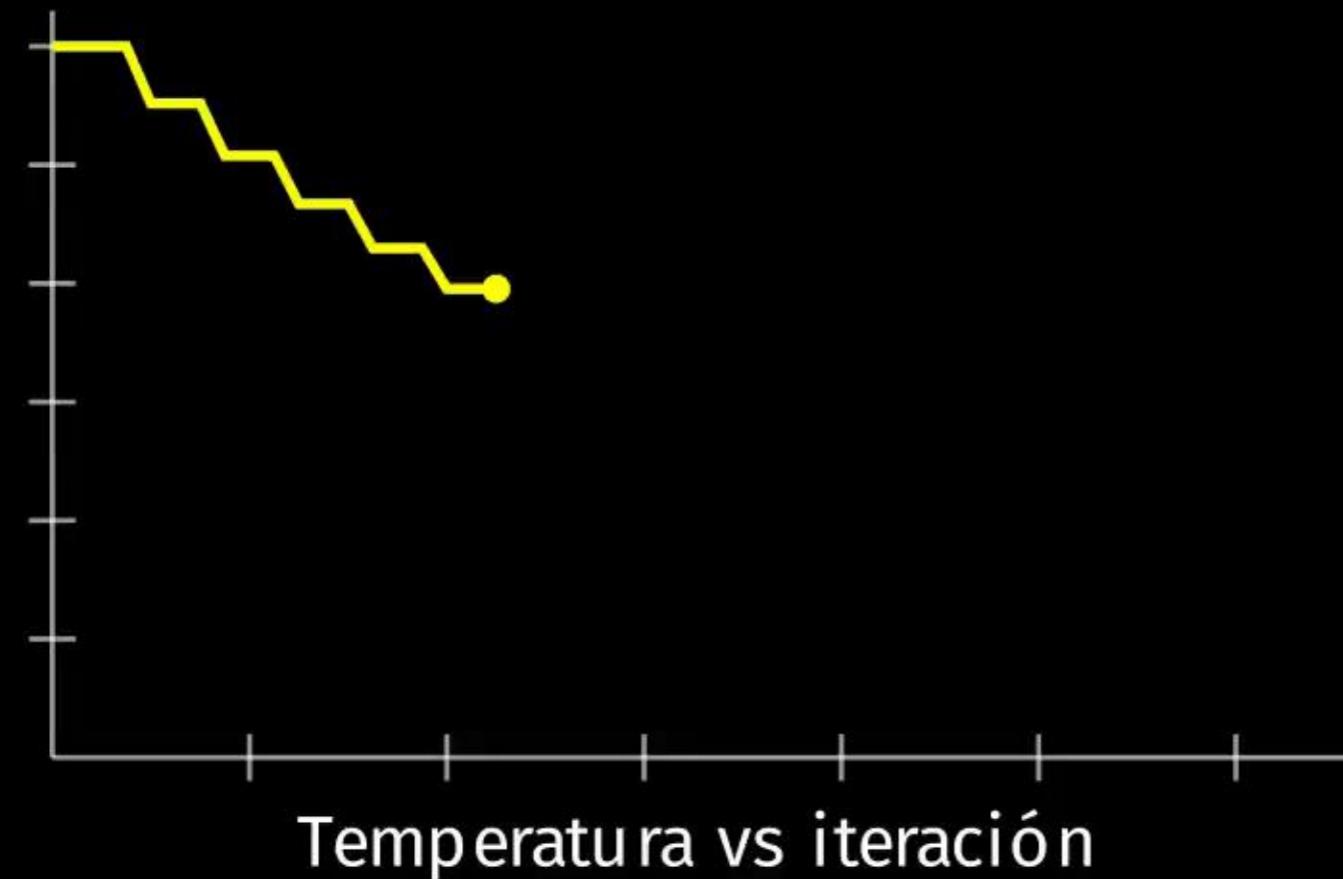
# Simulated Annealing – Graph Coloring



Movimiento aceptado

Temperatura: **1.318** Conflictos: **6** Incumbente: **4**  $\Delta$ : **-1**

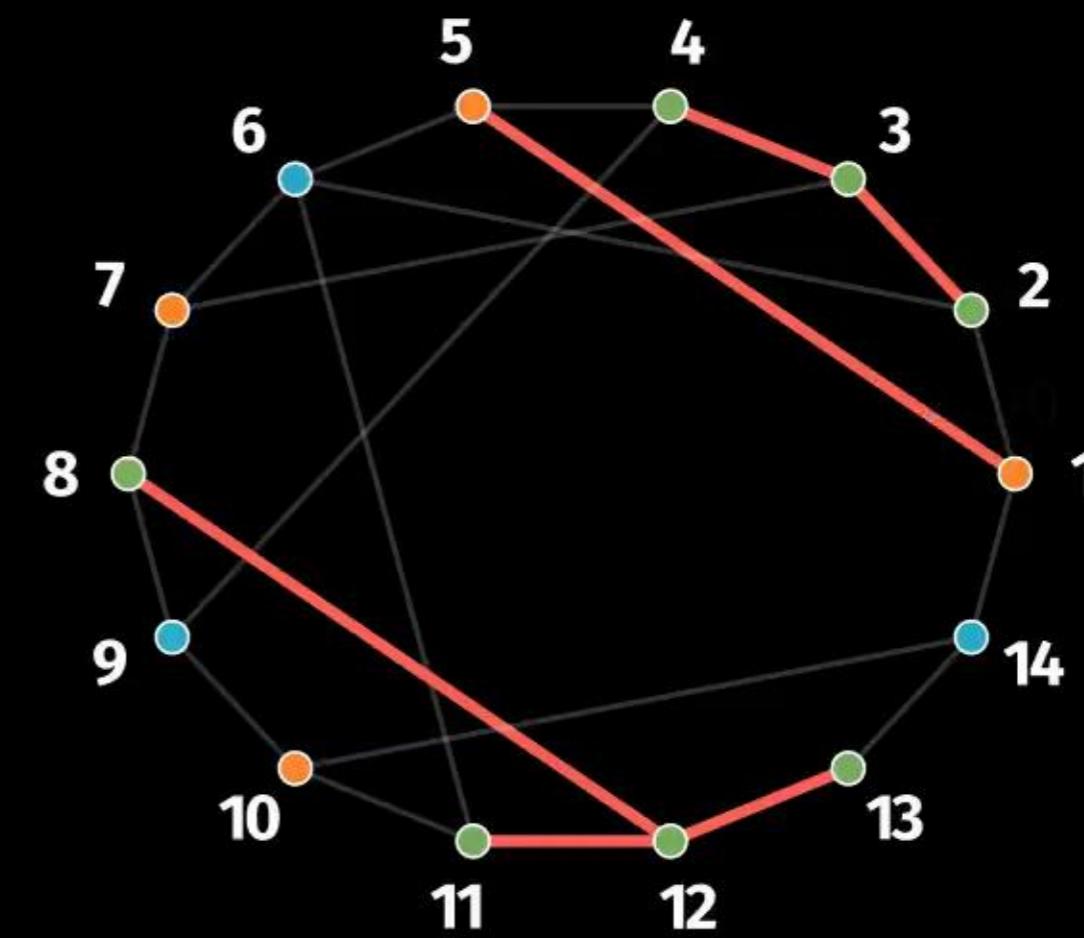
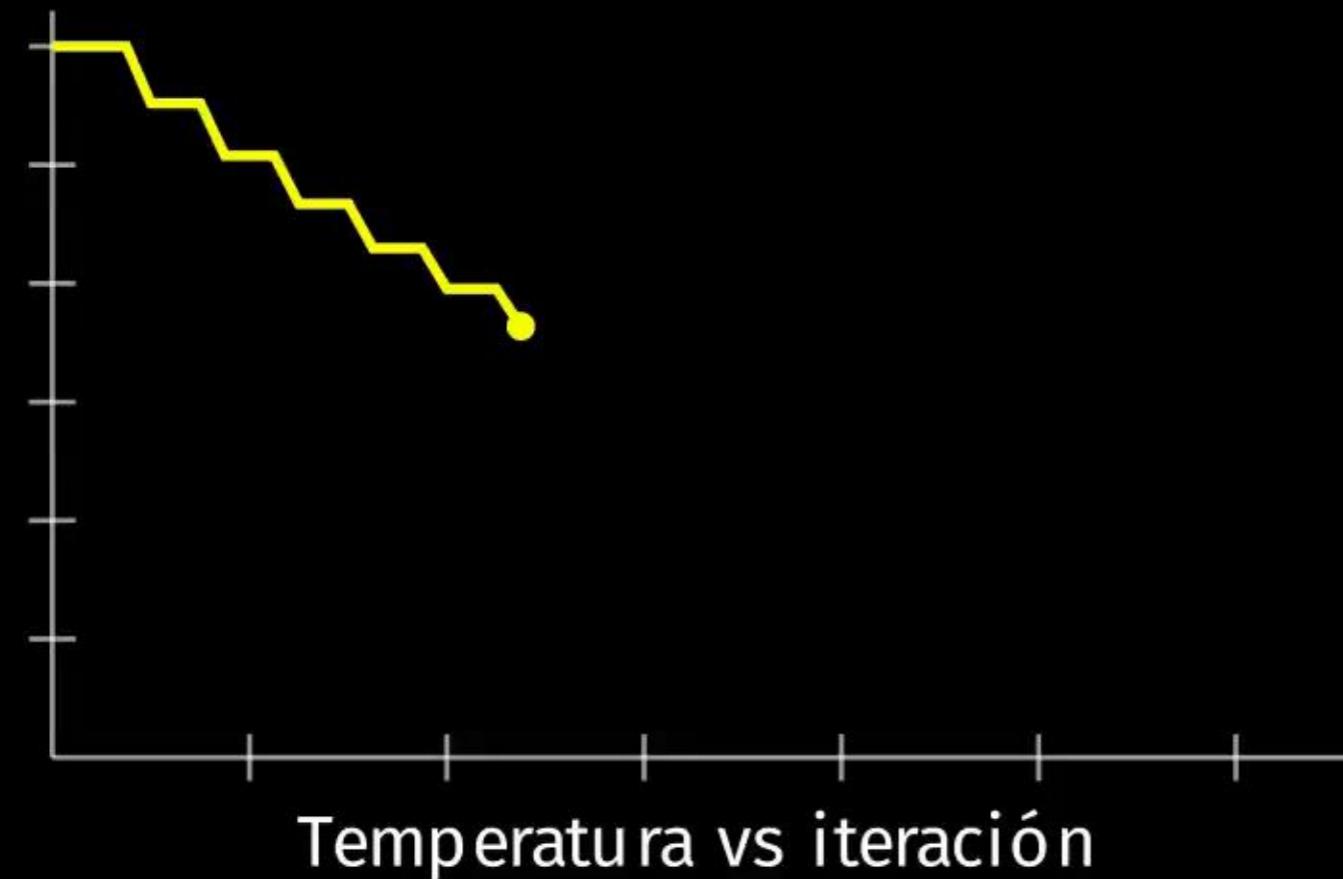
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Movimiento rechazado

Temperatura: **1.318** Conflictos: **6** Incumbente: **4**  $\Delta$ : **+2**

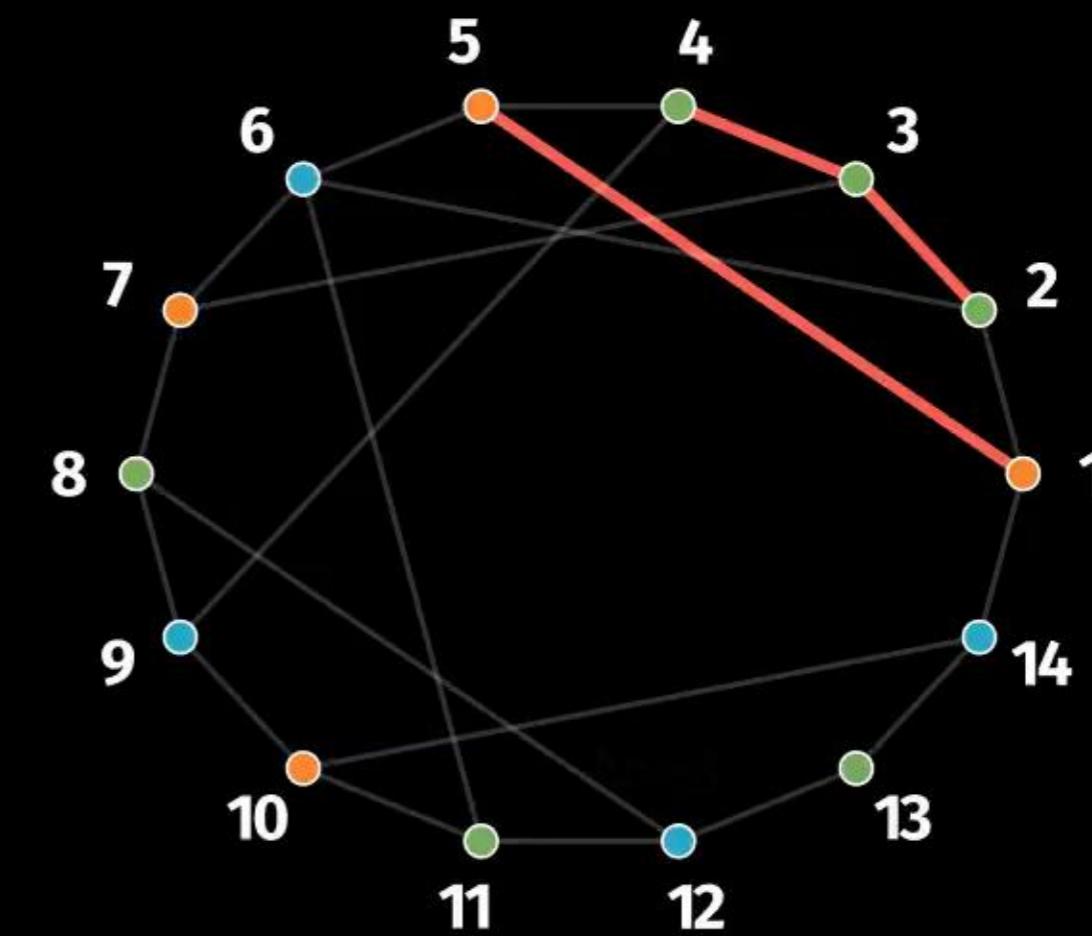
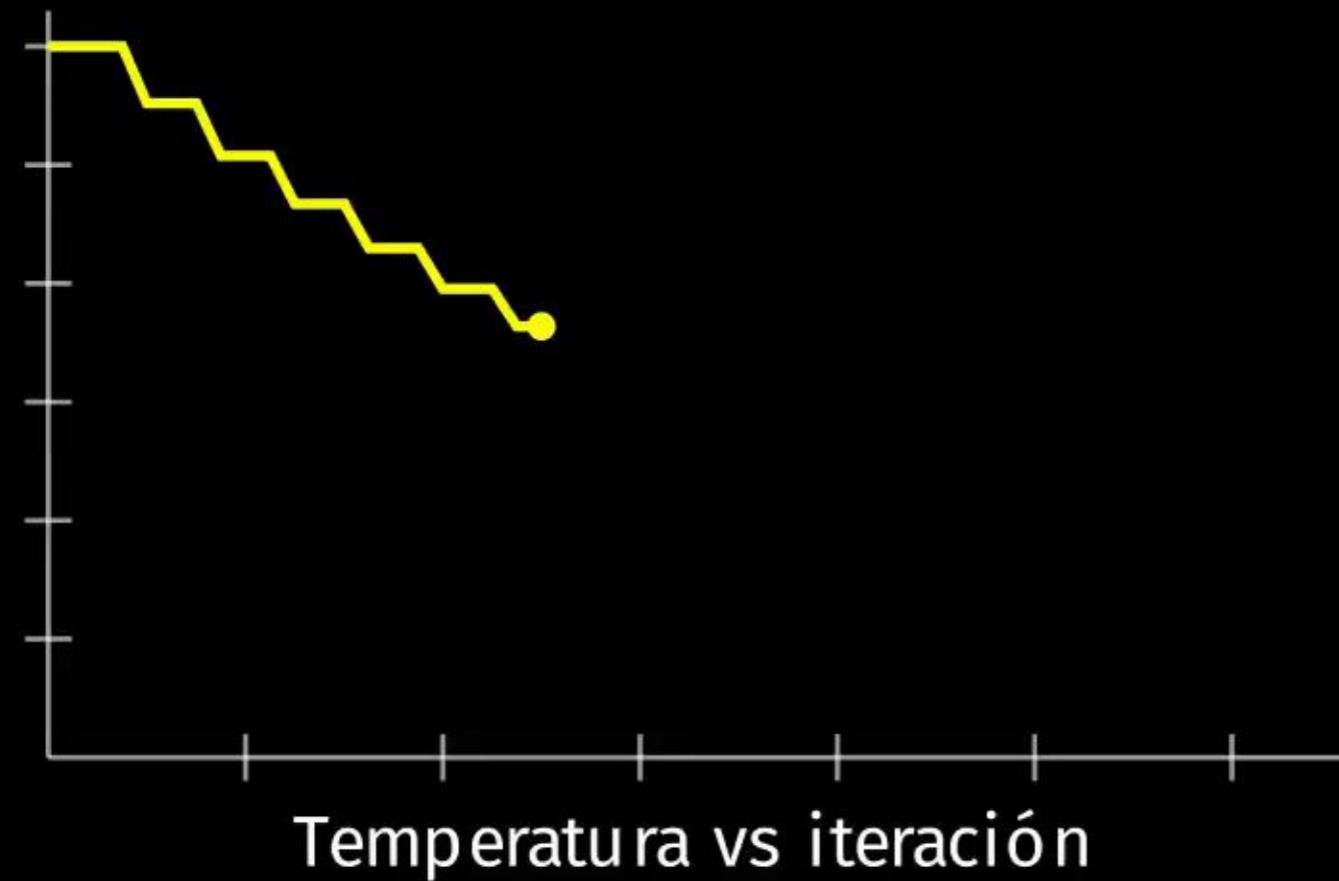
# Simulated Annealing – Graph Coloring



Movimiento aceptado

Temperatura: **1.213** Conflictos: **6** Incumbente: **4**  $\Delta$ : **+0**

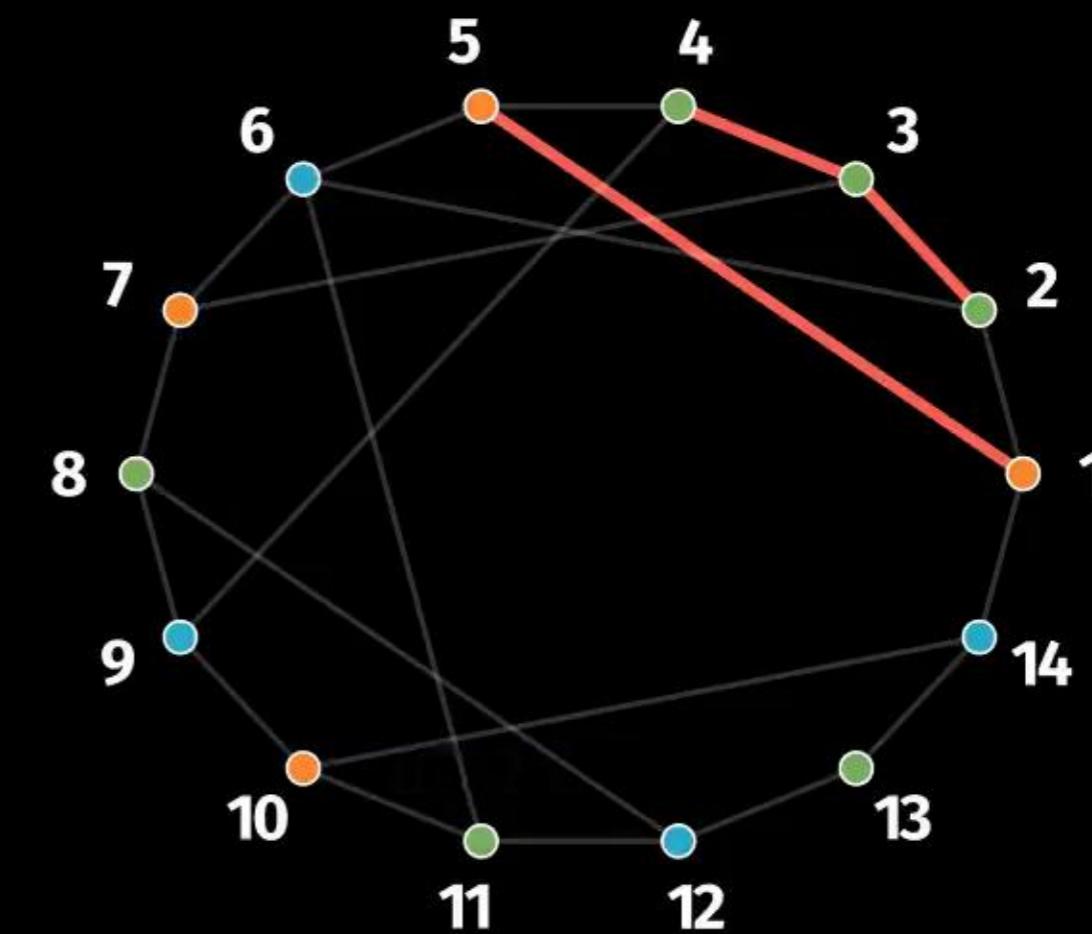
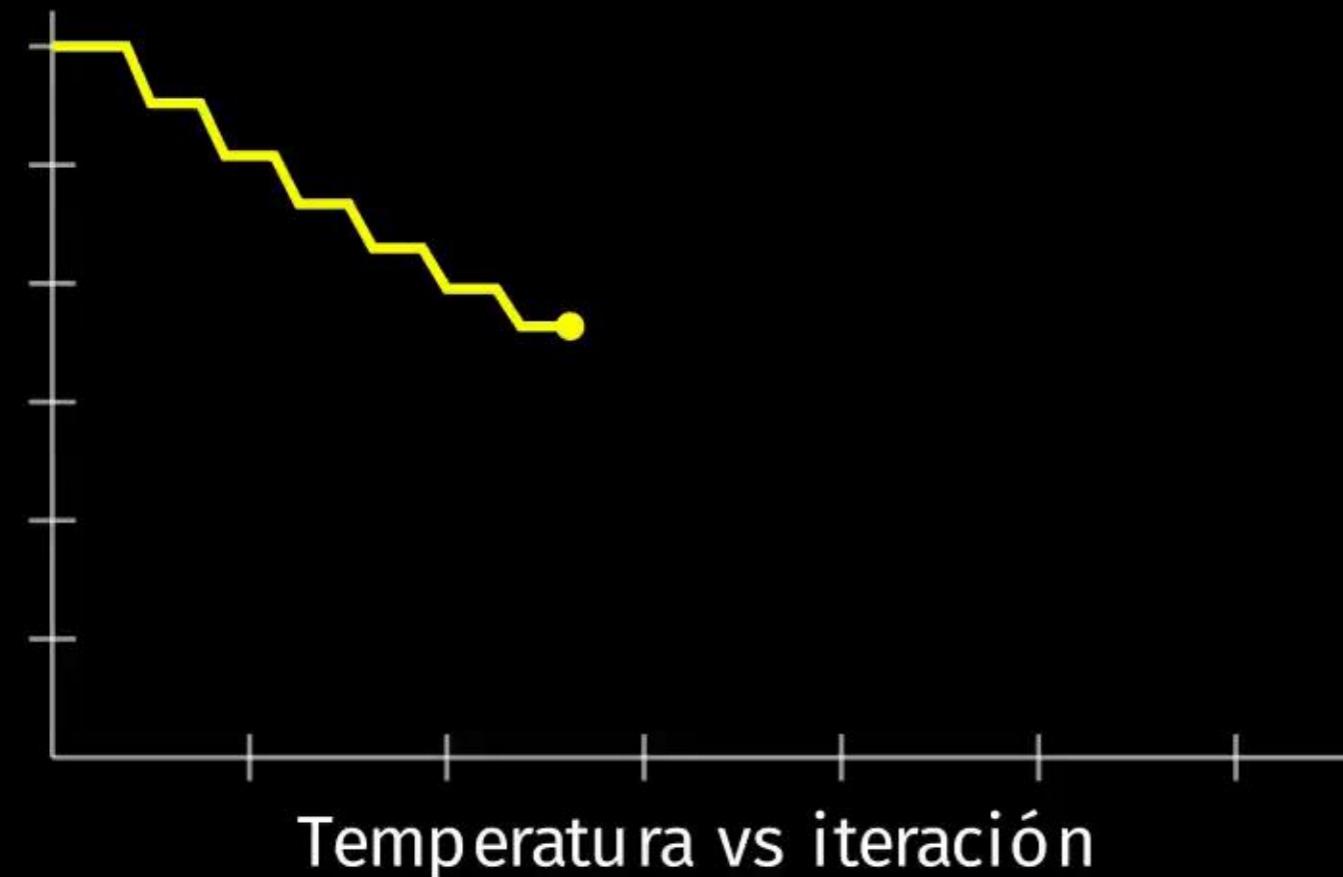
# Simulated Annealing – Graph Coloring



Movimiento aceptado

Temperatura: **1.213** Conflictos: **3** Incumbente: **3**  $\Delta$ : **-3**

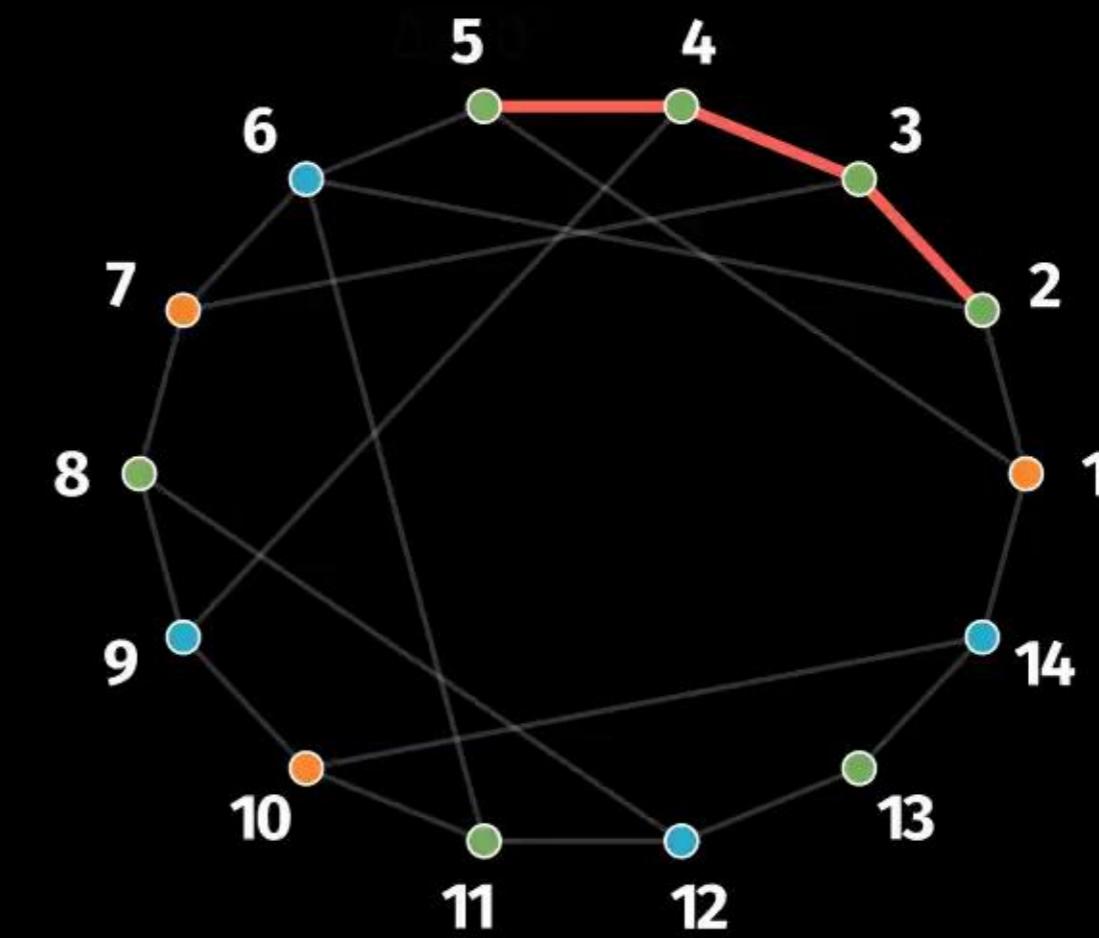
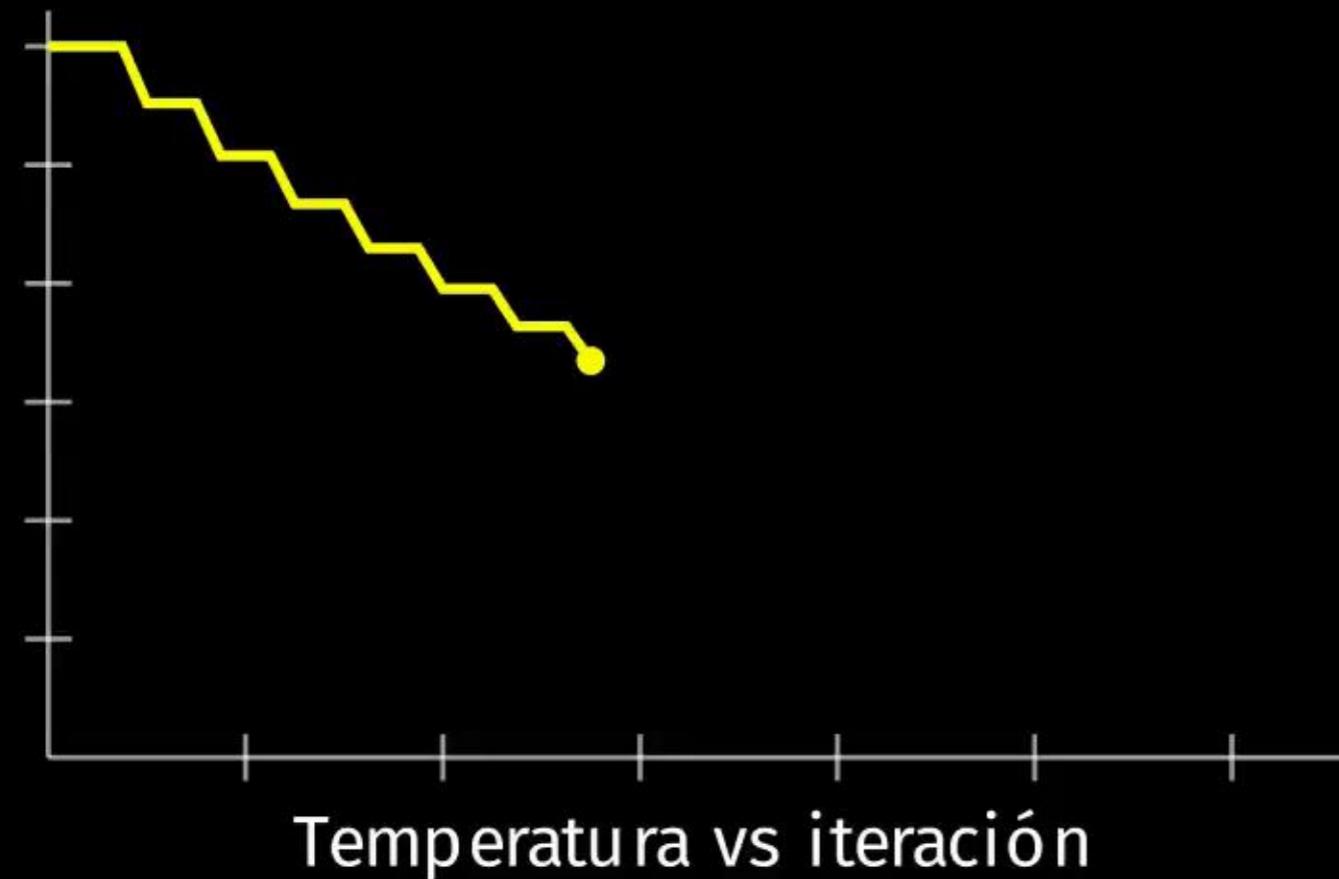
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Movimiento rechazado

Temperatura: **1.213** Conflictos: **3** Incumbente: **3**  $\Delta$ : **+2**

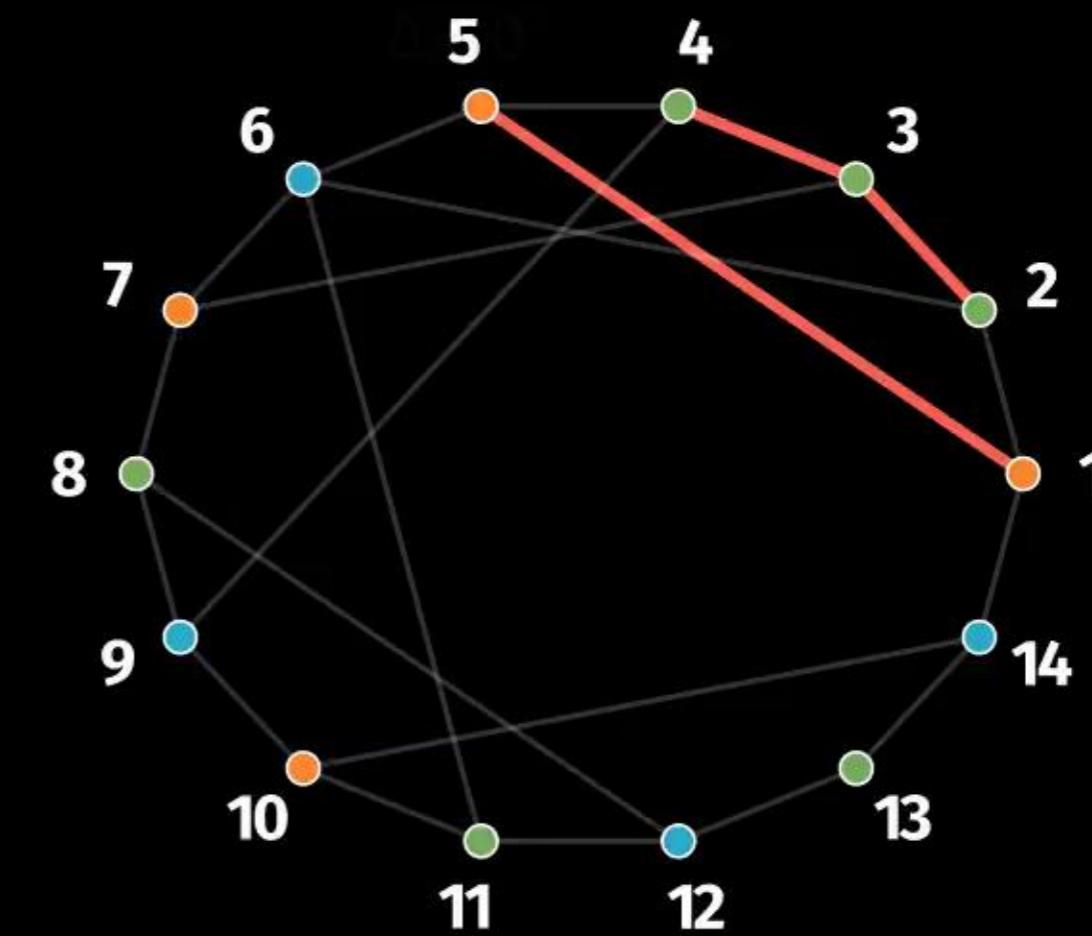
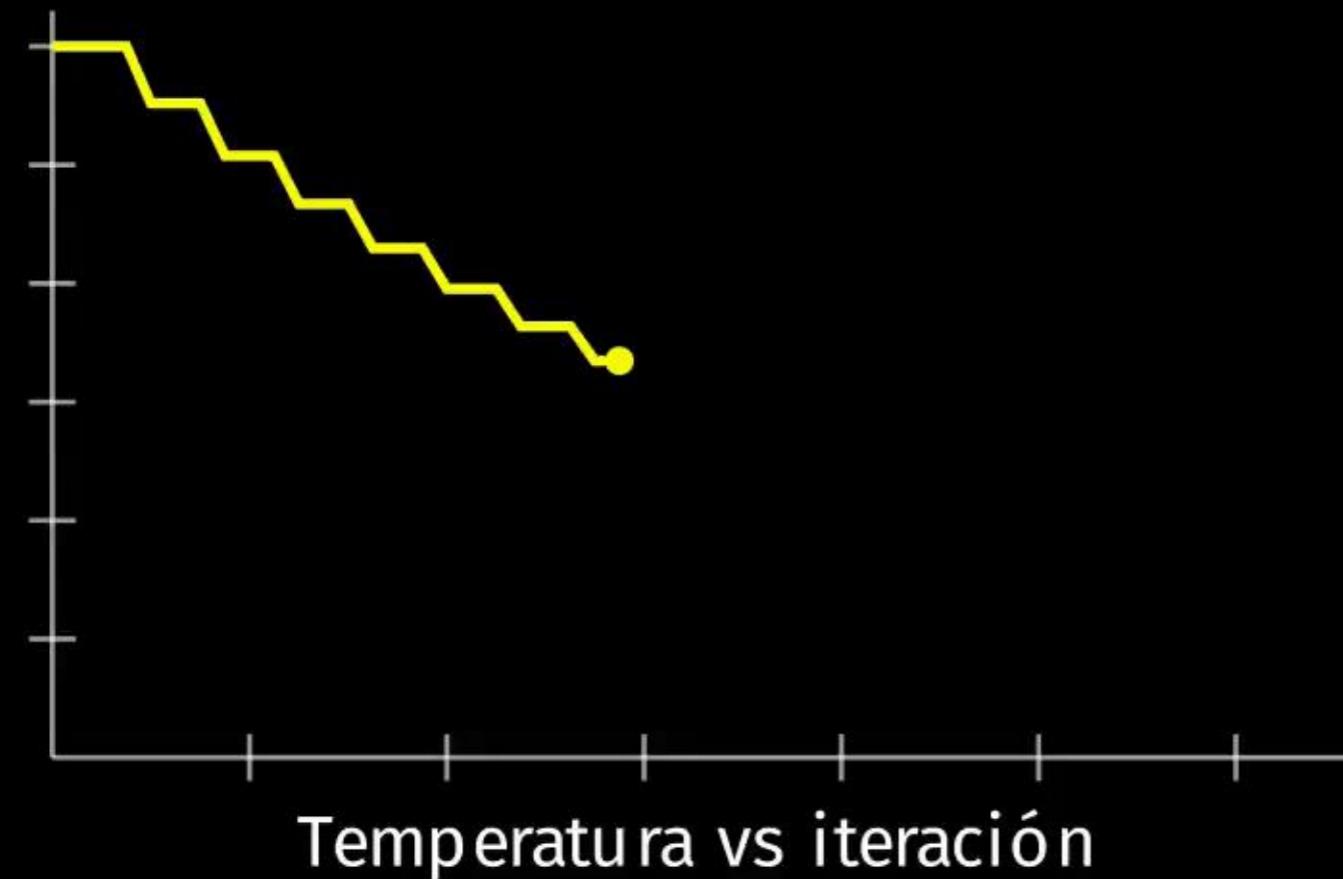
# Simulated Annealing – Graph Coloring



Movimiento aceptado

Temperatura: **1.116** Conflictos: **3** Incumbente: **3**  $\Delta$ : **+0**

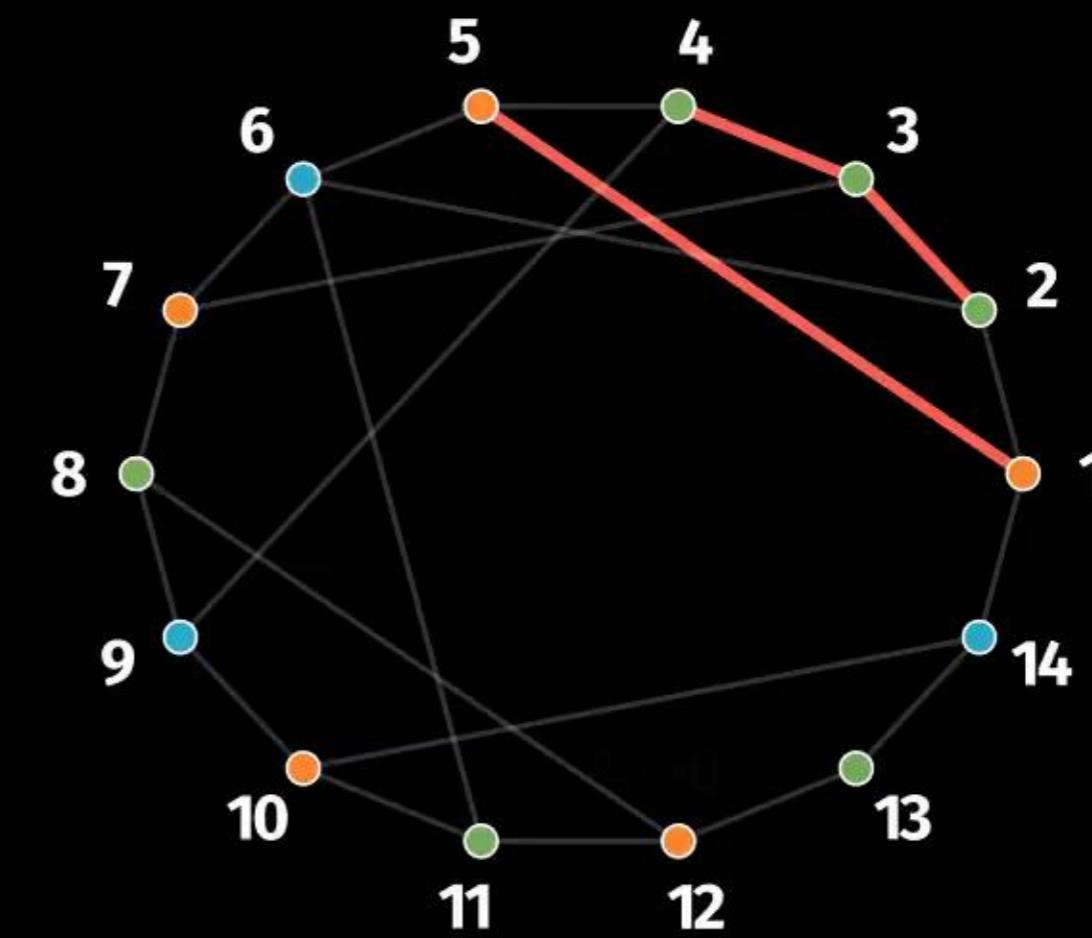
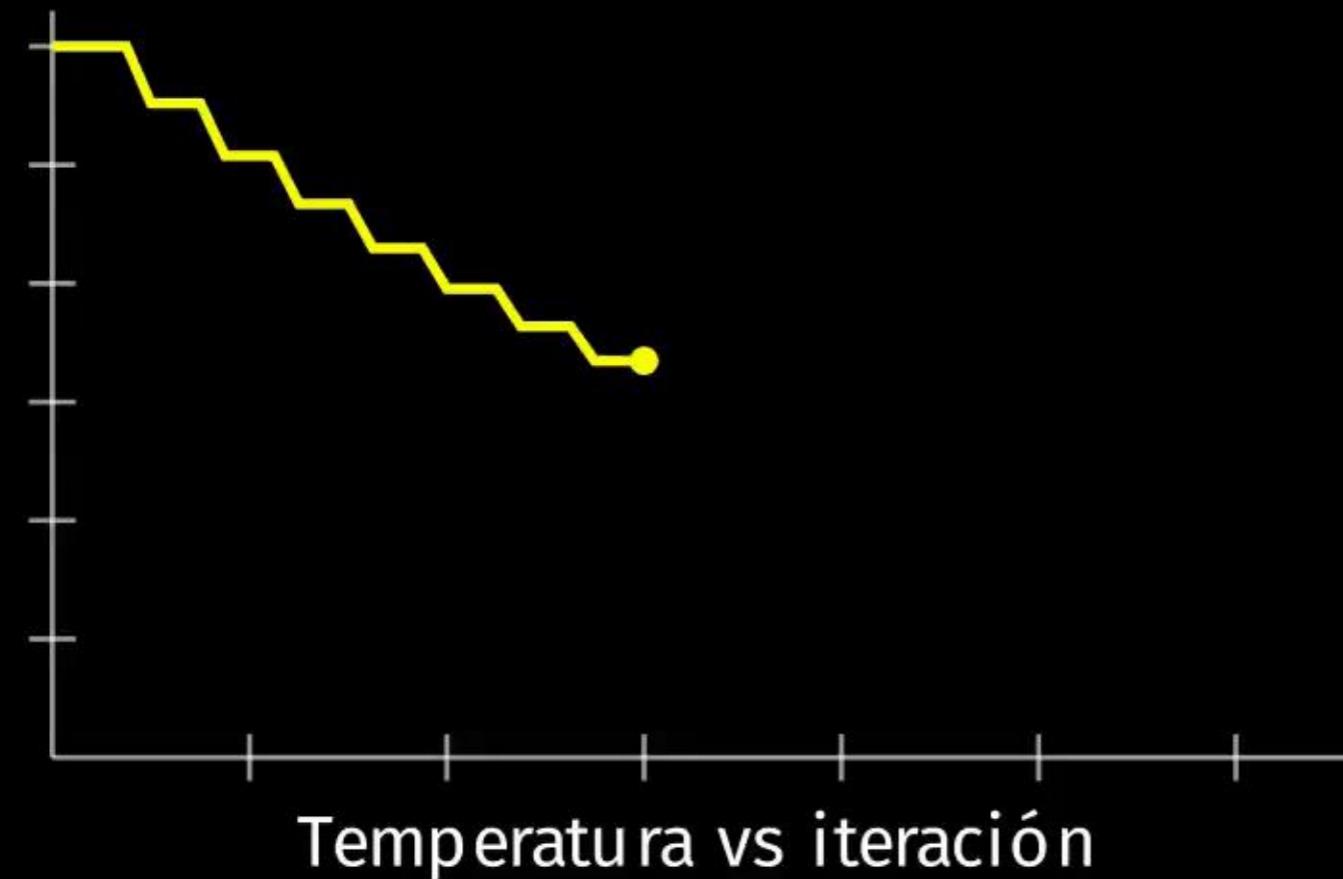
# Simulated Annealing – Graph Coloring



Movimiento aceptado

Temperatura: **1.116** Conflictos: **3** Incumbente: **3**  $\Delta$ : **+0**

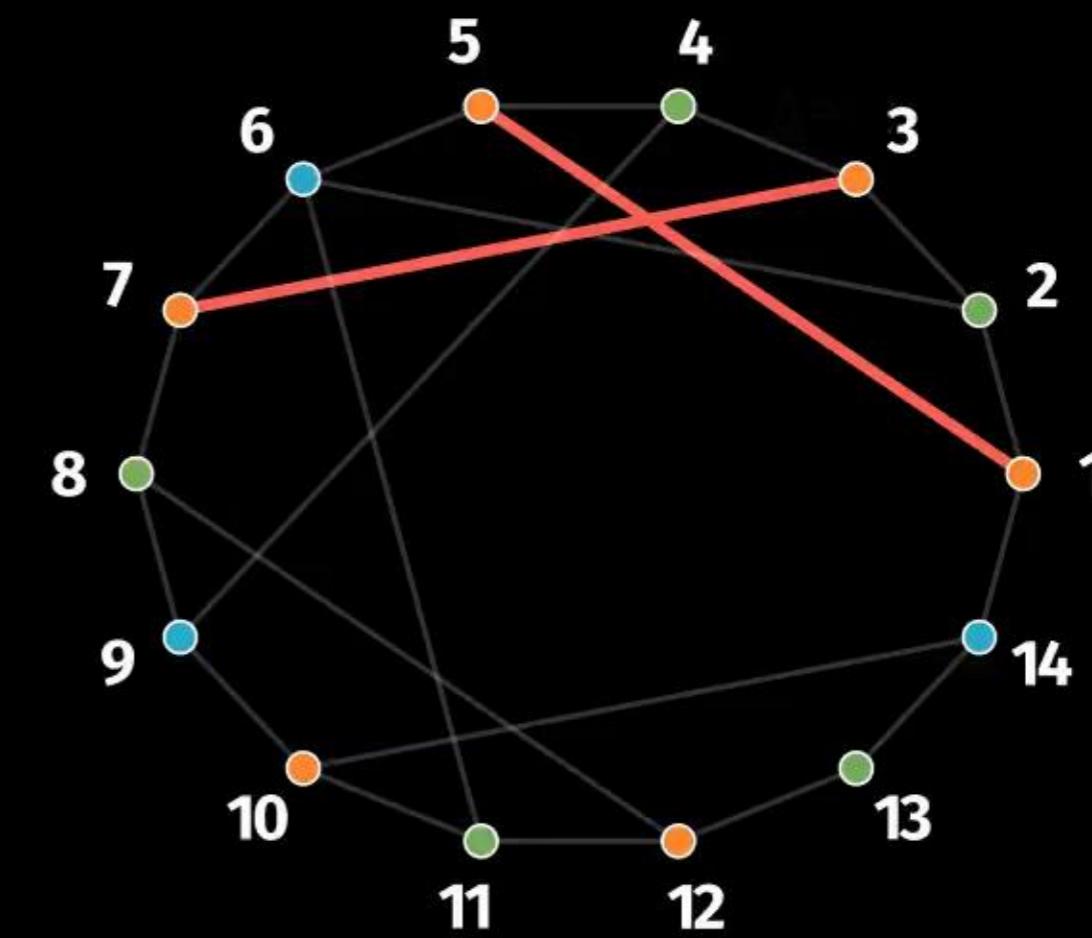
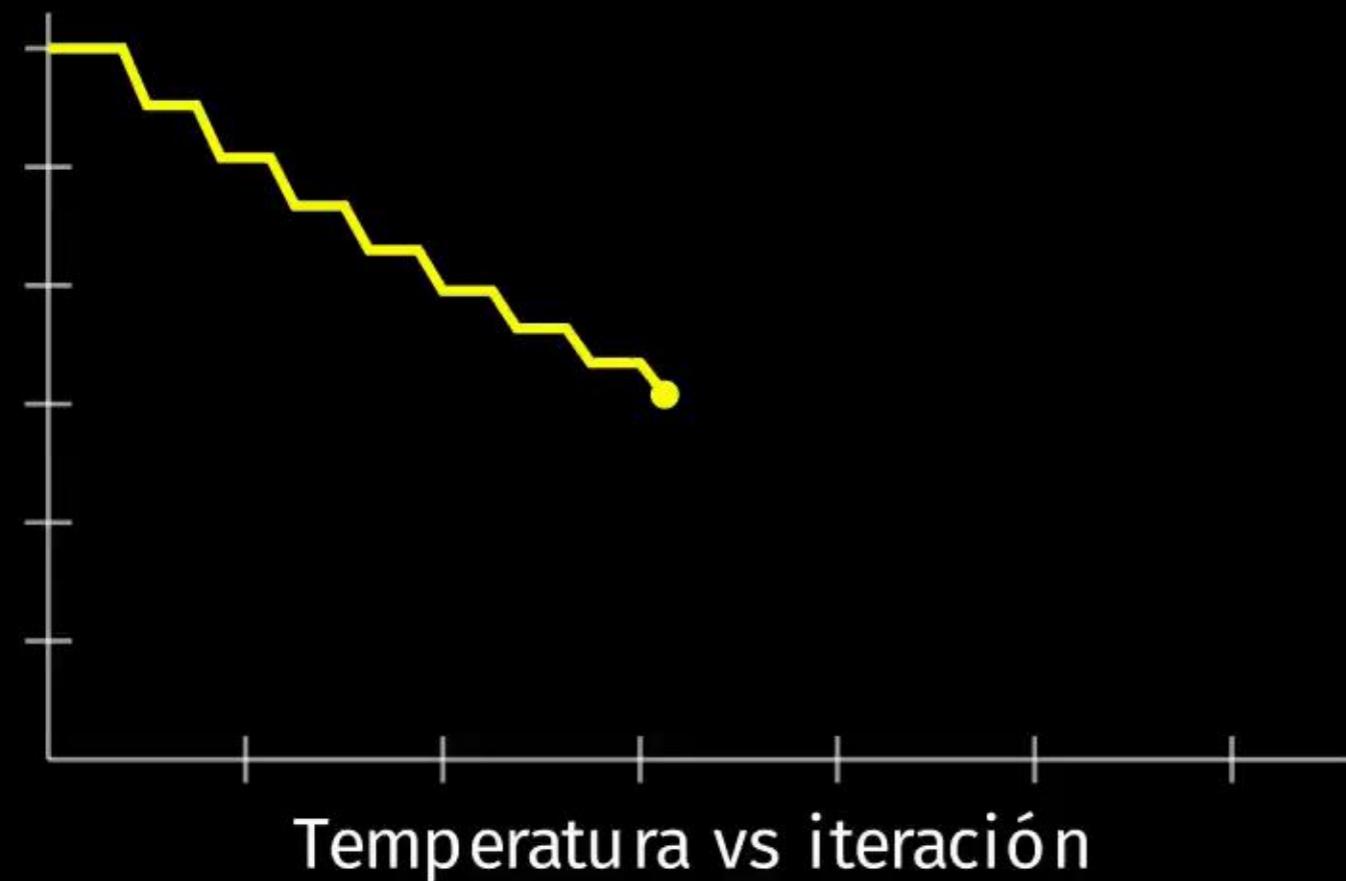
# Simulated Annealing – Graph Coloring



Movimiento aceptado

Temperatura: **1.116** Conflictos: **3** Incumbente: **3**  $\Delta$ : **+0**

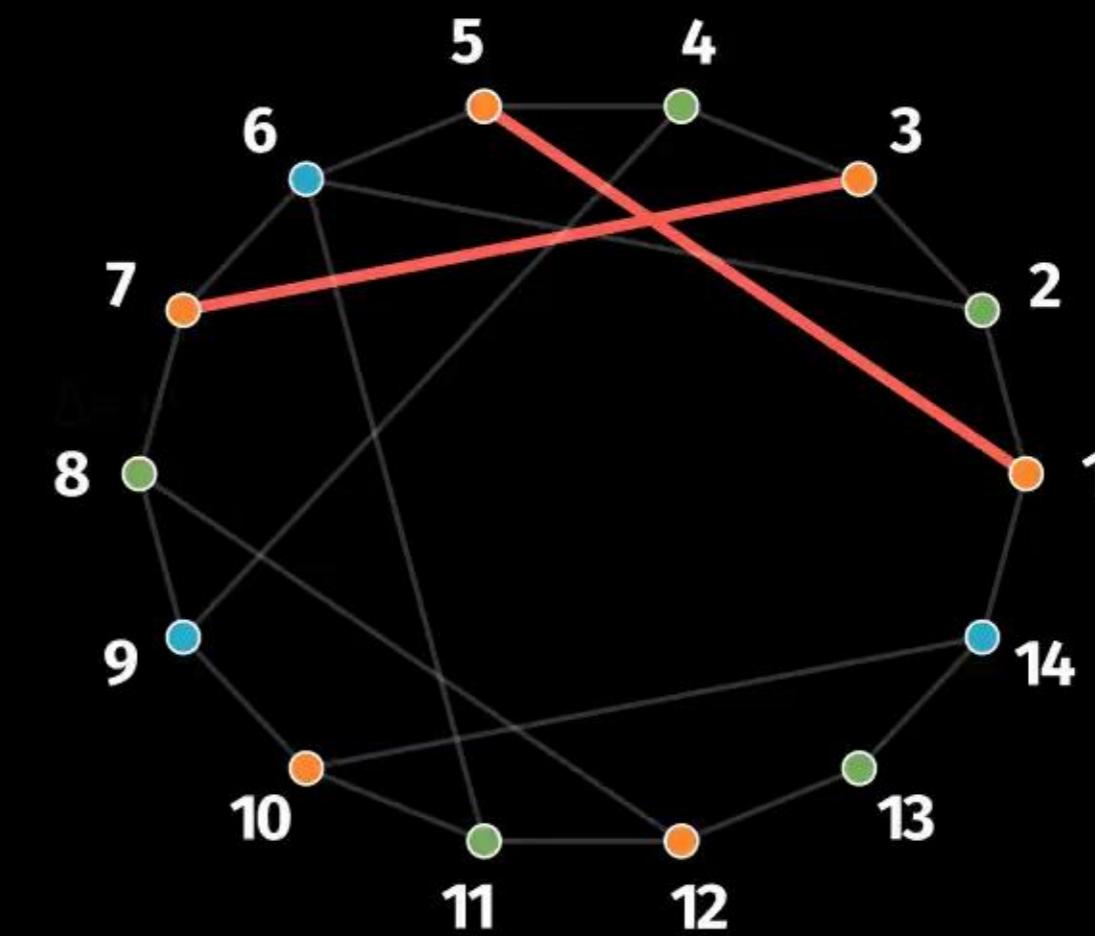
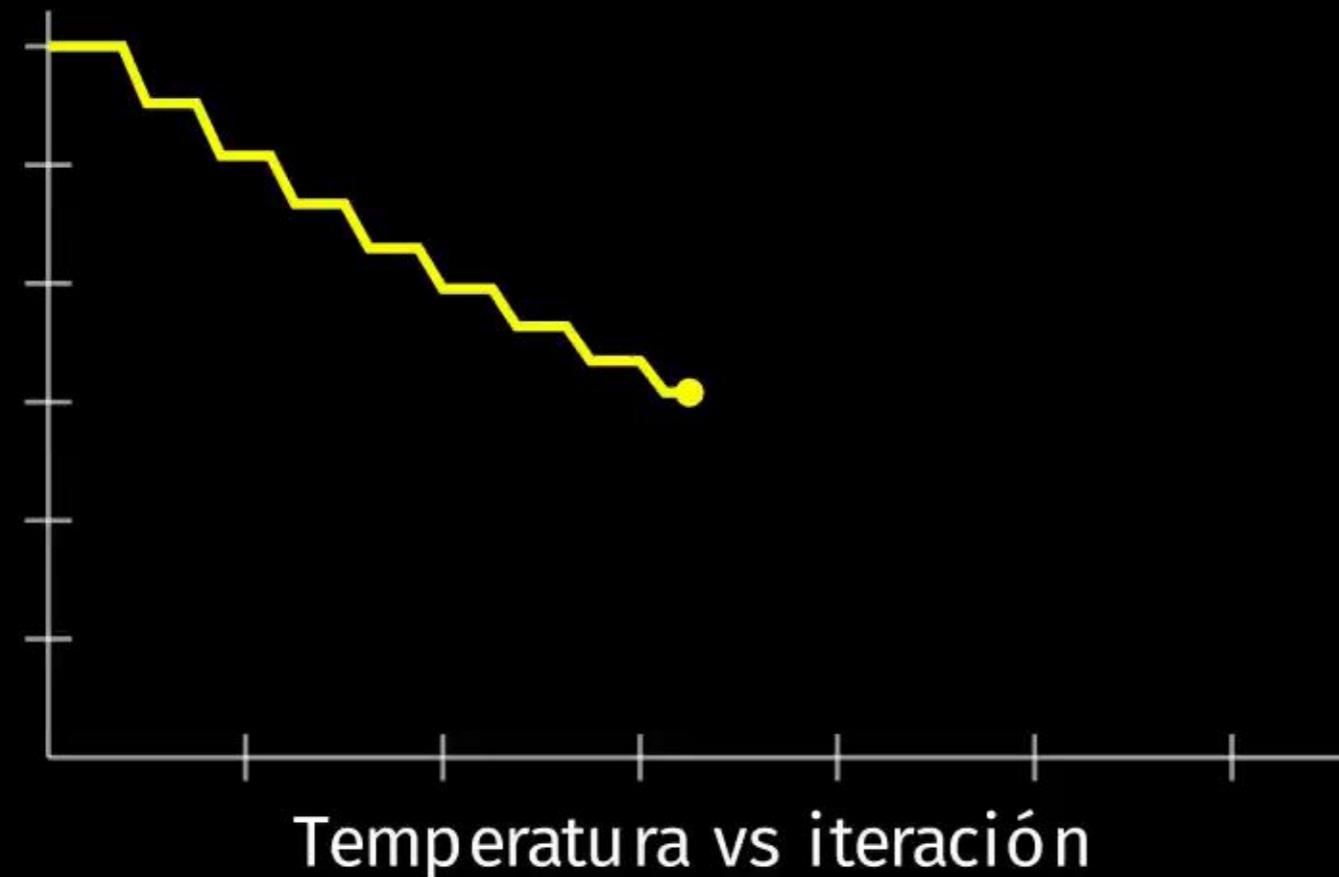
# Simulated Annealing – Graph Coloring



## Movimiento aceptado

Temperatura: **1.026** Conflictos: **2** Incumbente: **2** Δ: **-1**

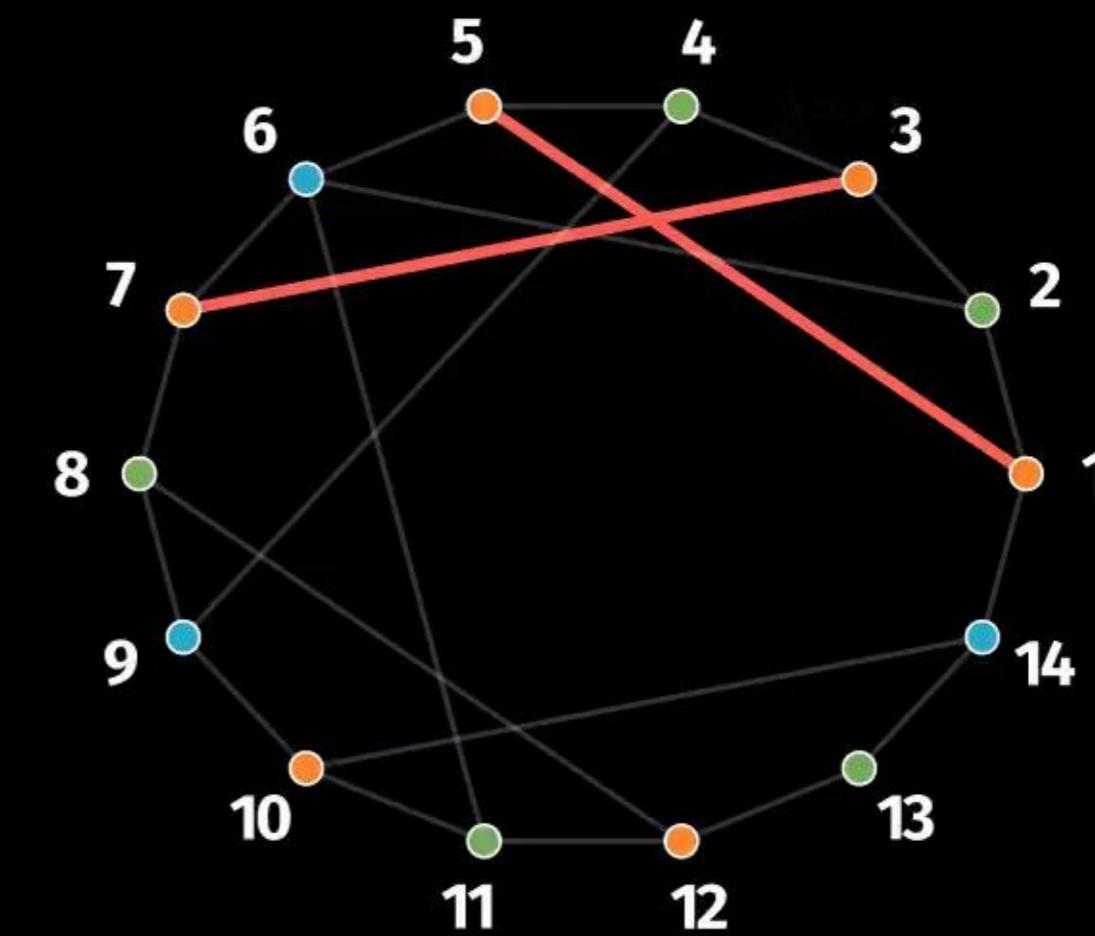
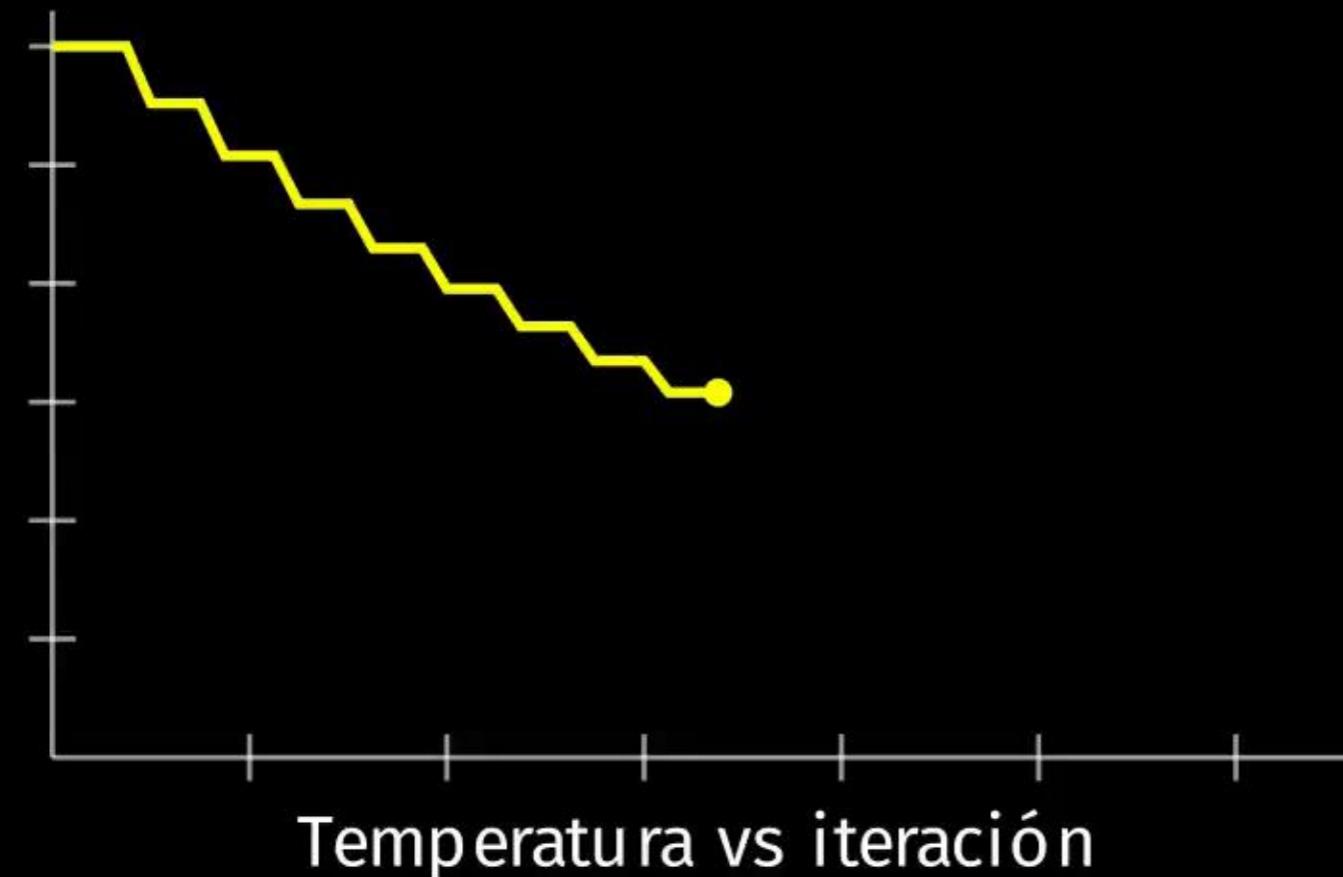
# Simulated Annealing – Graph Coloring



Movimiento rechazado

Temperatura: **1.026** Conflictos: **2** Incumbente: **2**  $\Delta$ : **+1**

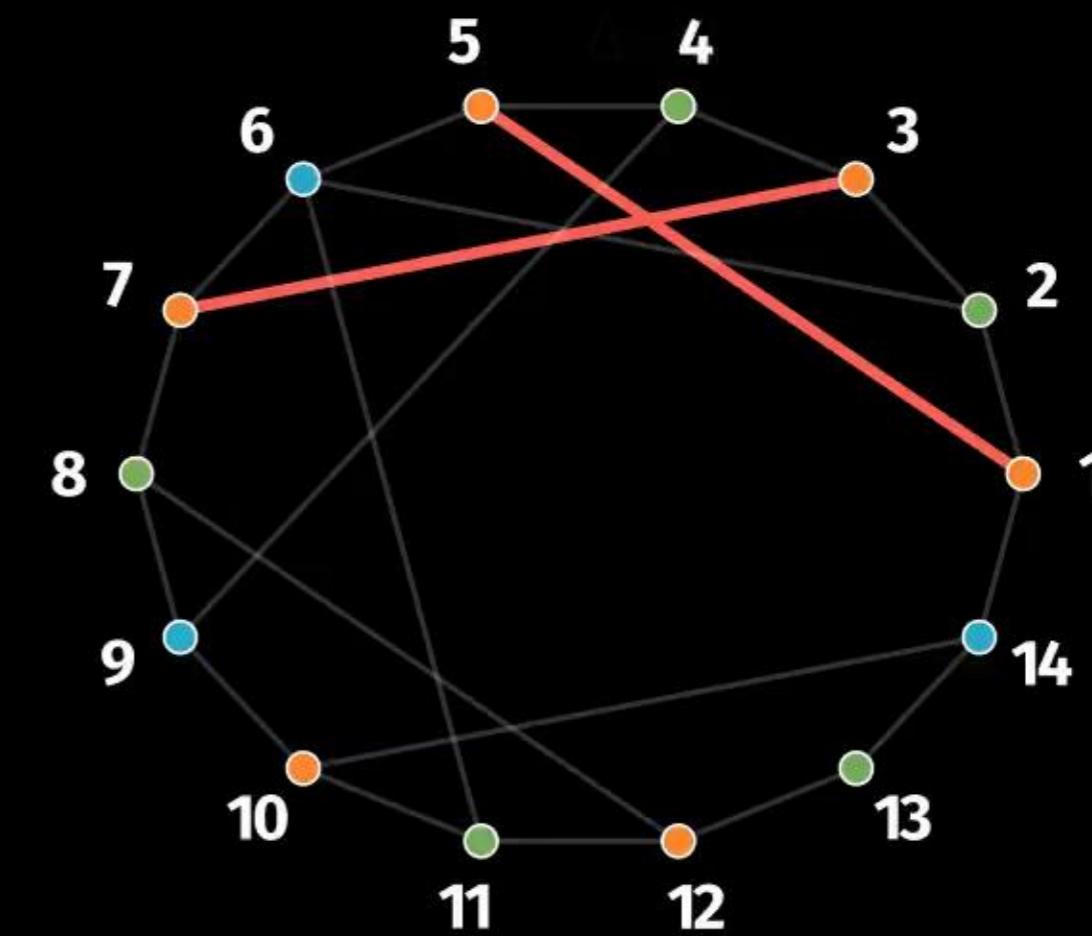
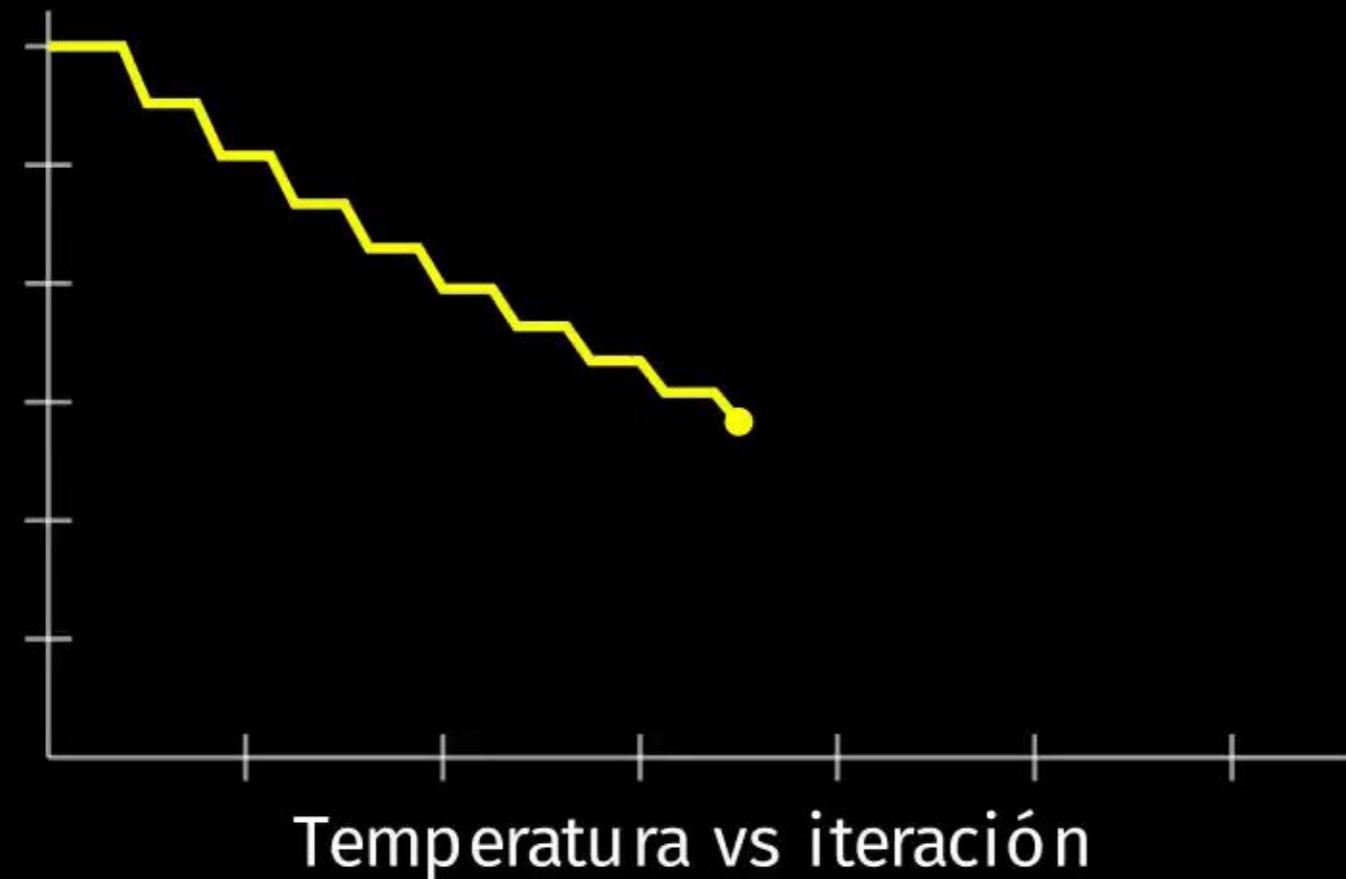
# Simulated Annealing – Graph Coloring



Movimiento rechazado

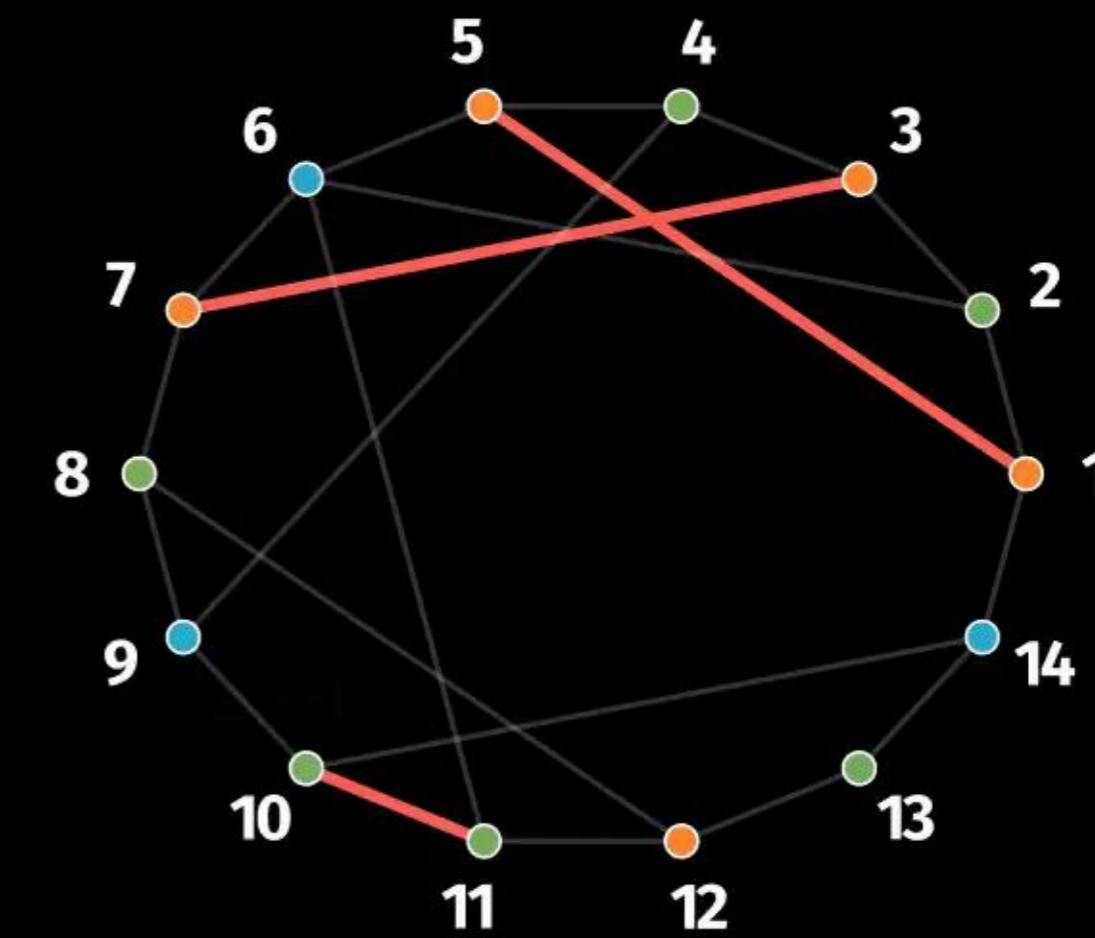
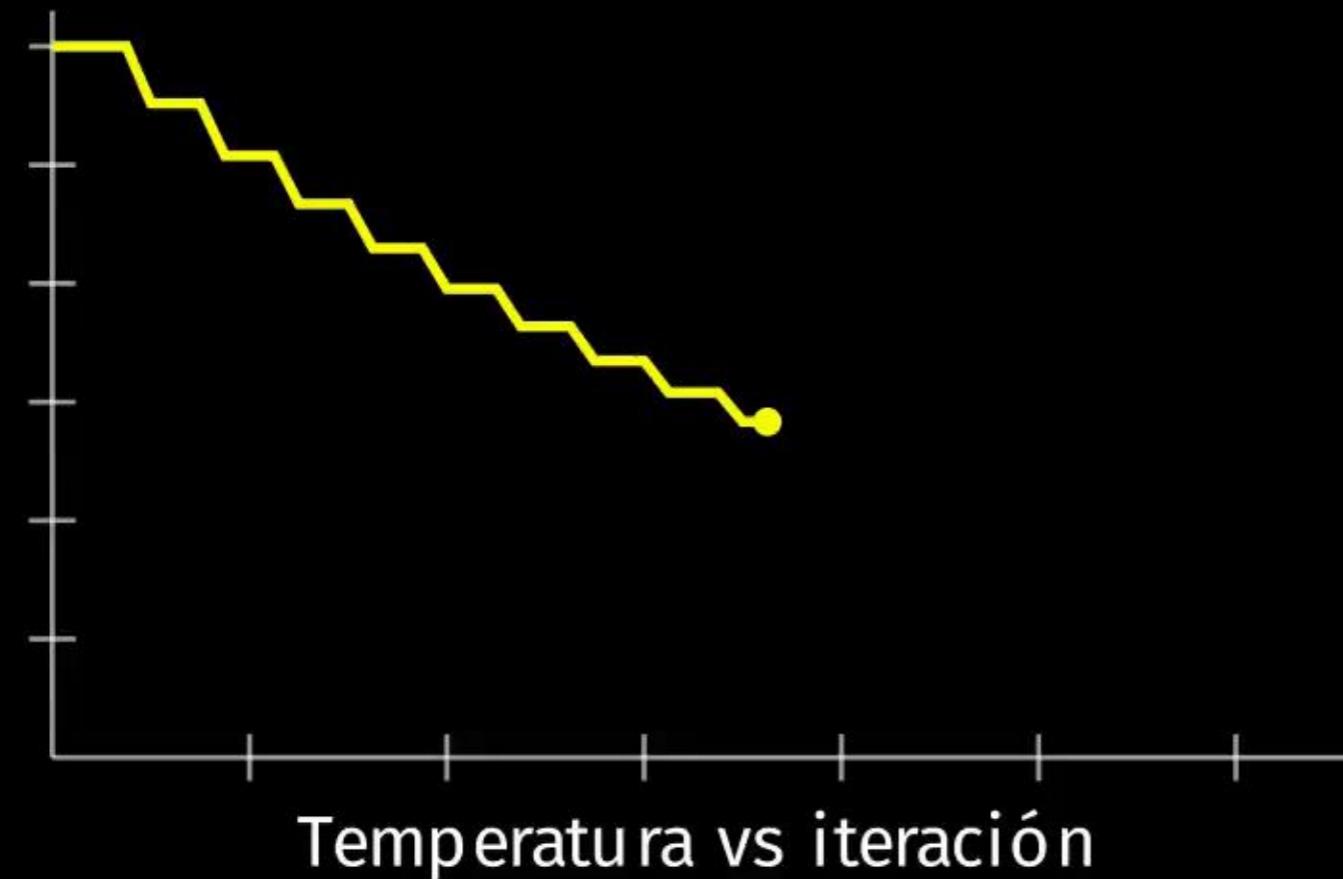
Temperatura: **1.026** Conflictos: **2** Incumbente: **2**  $\Delta$ : **+1**

# Simulated Annealing – Graph Coloring



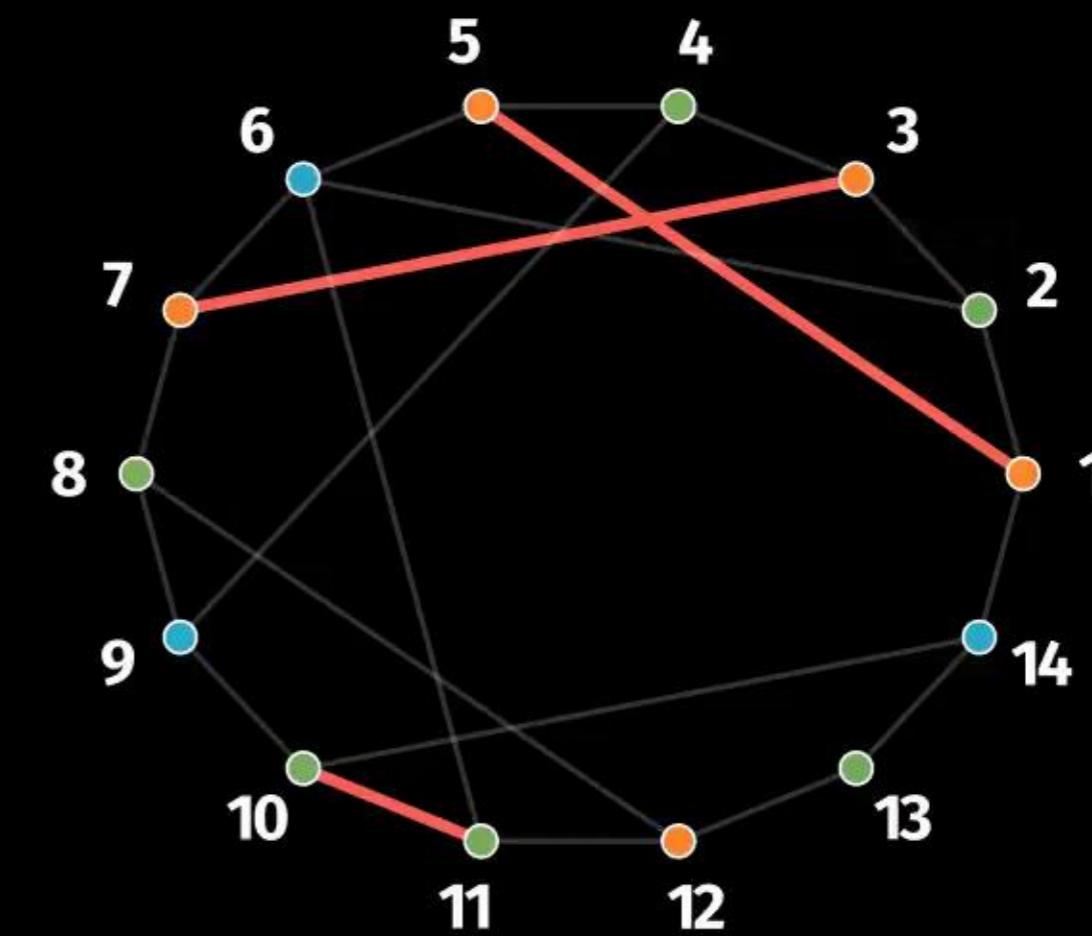
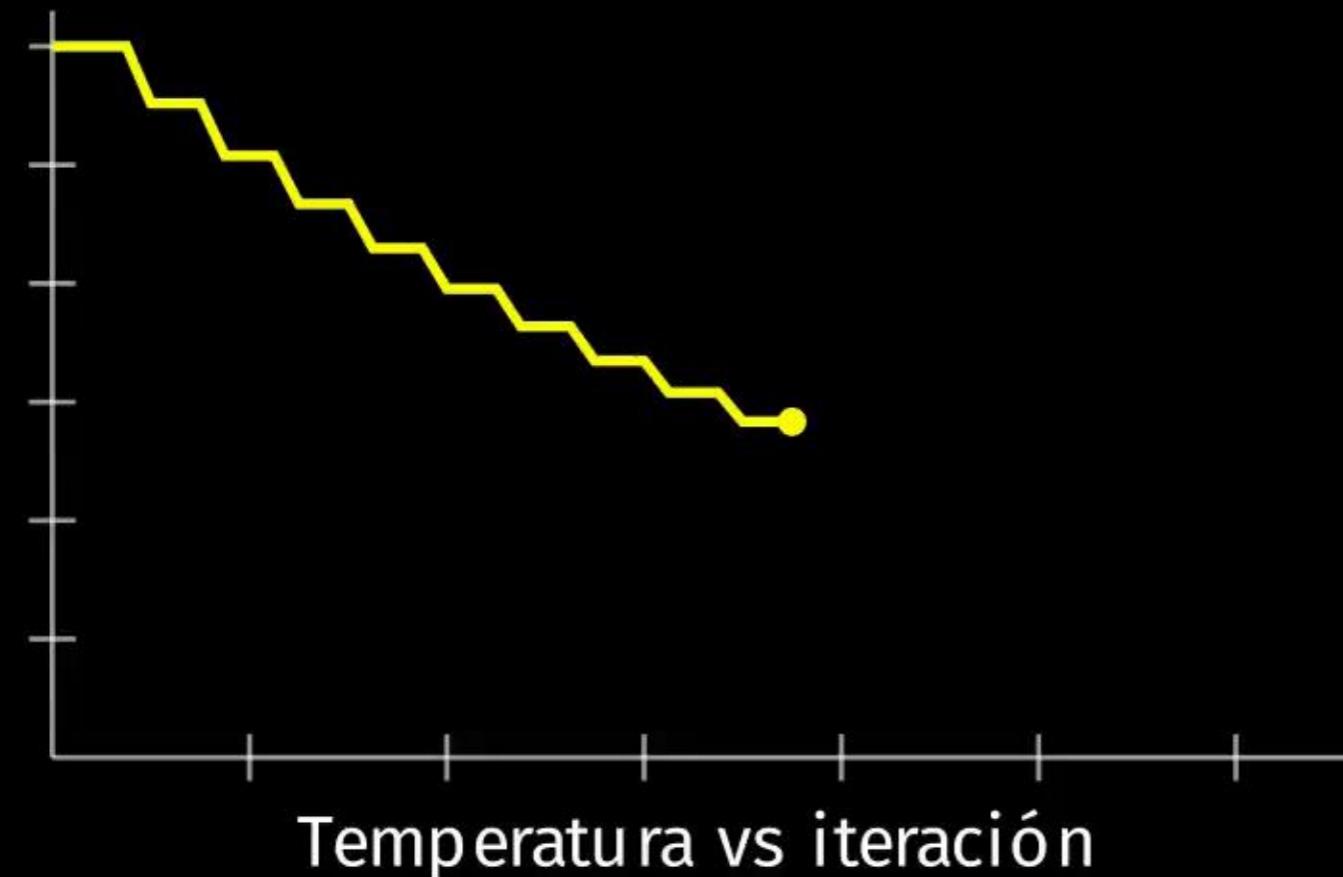
Temperatura: **0.944** Conflictos: **2** Incumbente: **2**  $\Delta$ : **+2**

# Simulated Annealing – Graph Coloring



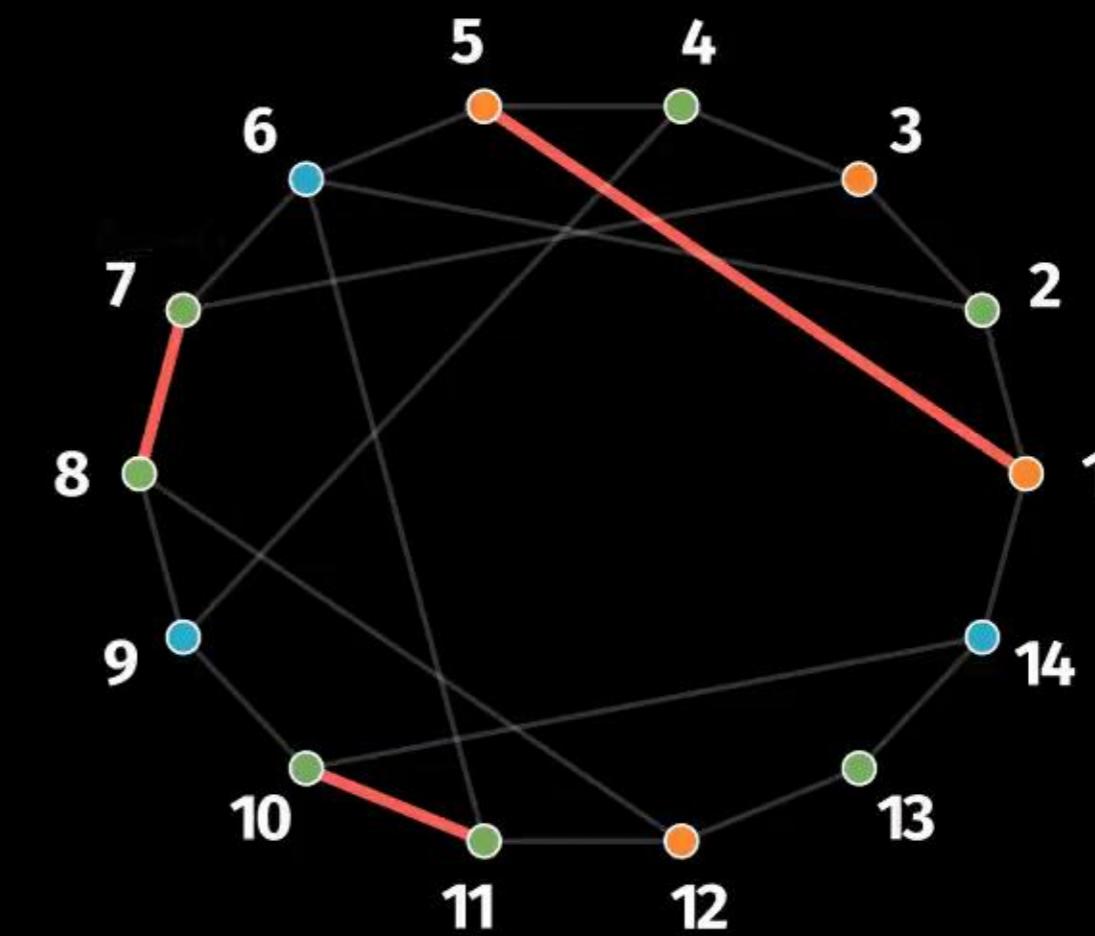
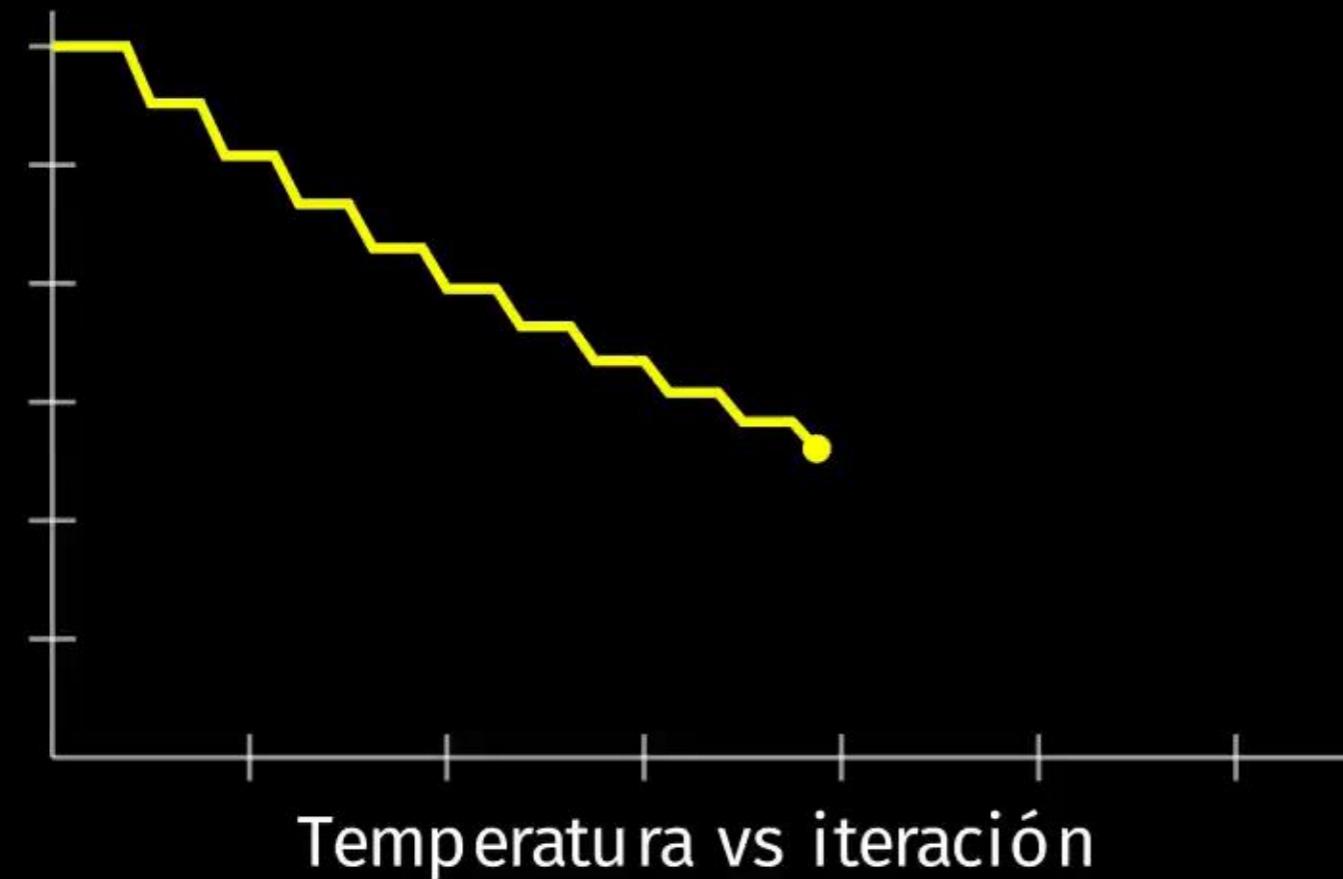
Temperatura: **0.944** Conflictos: **3** Incumbente: **2**  $\Delta$ : **+1**

# Simulated Annealing – Graph Coloring



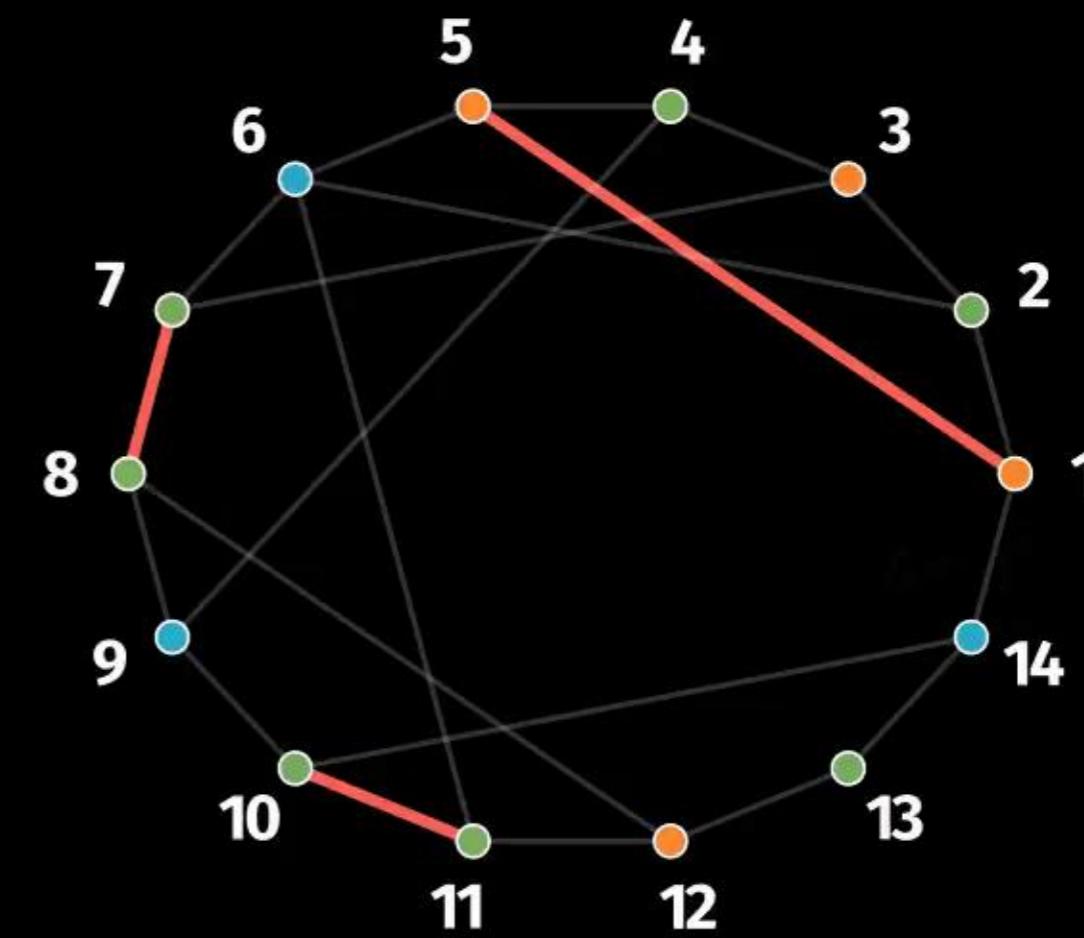
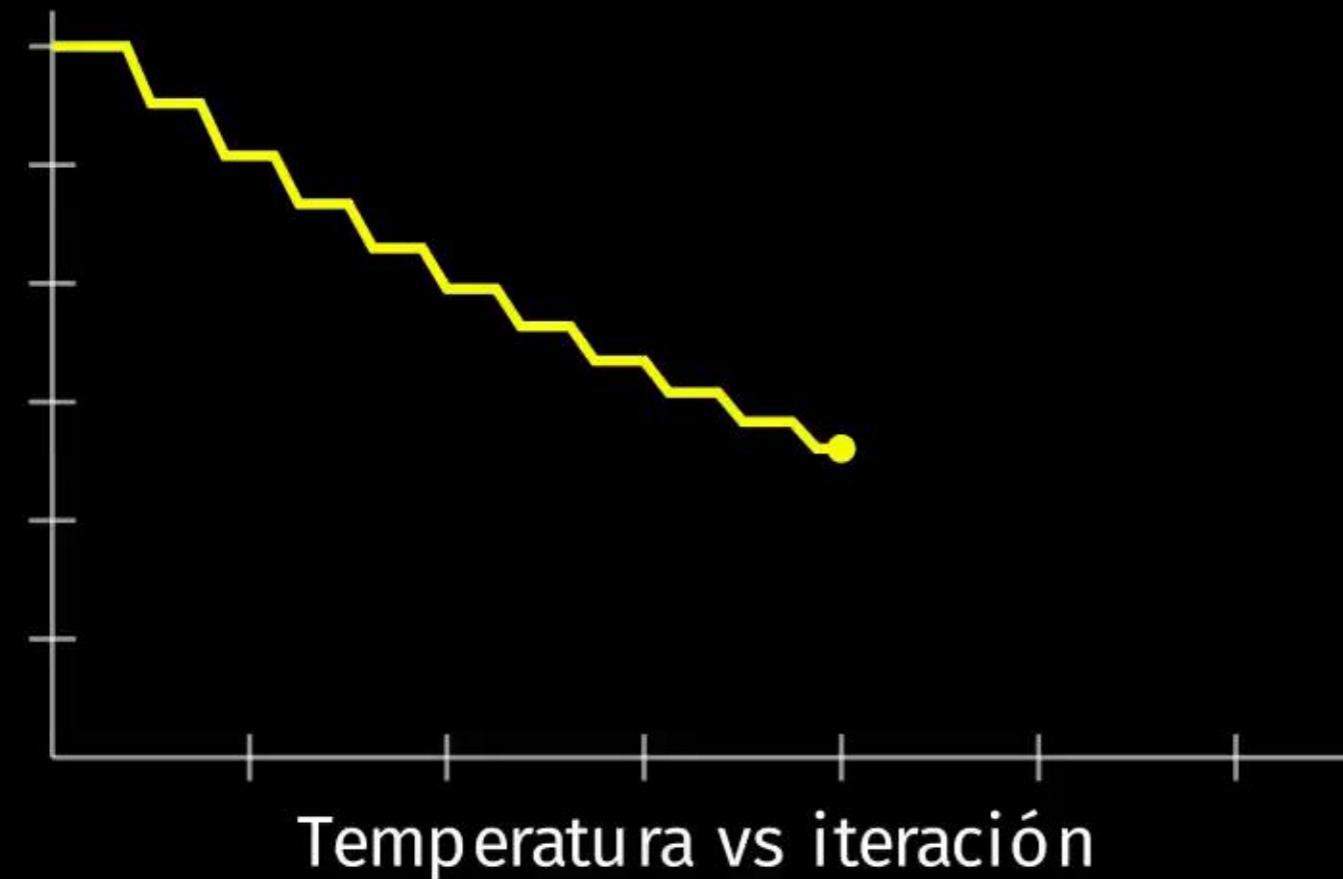
Temperatura: **0.944** Conflictos: **3** Incumbente: **2**  $\Delta$ : **+1**

# Simulated Annealing – Graph Coloring



Temperatura: **0.869** Conflictos: **3** Incumbente: **2**  $\Delta$ : **+0**

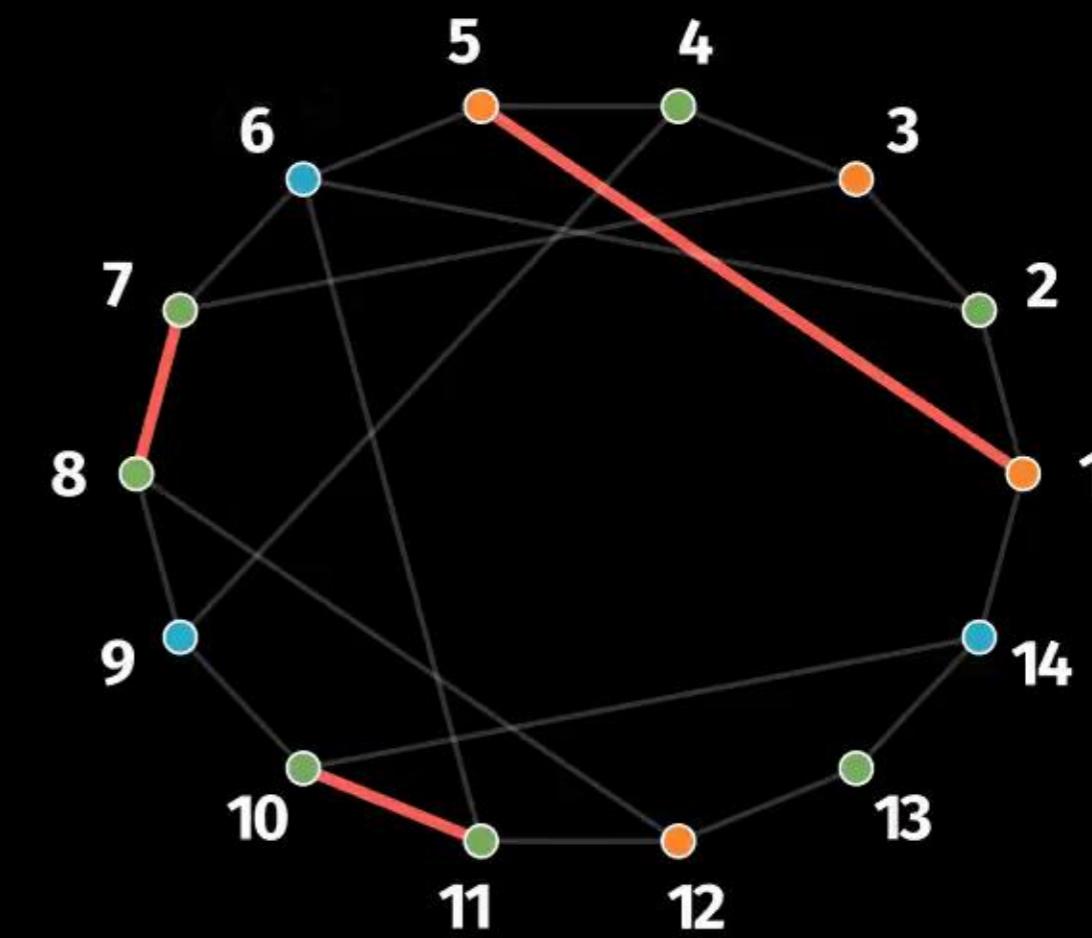
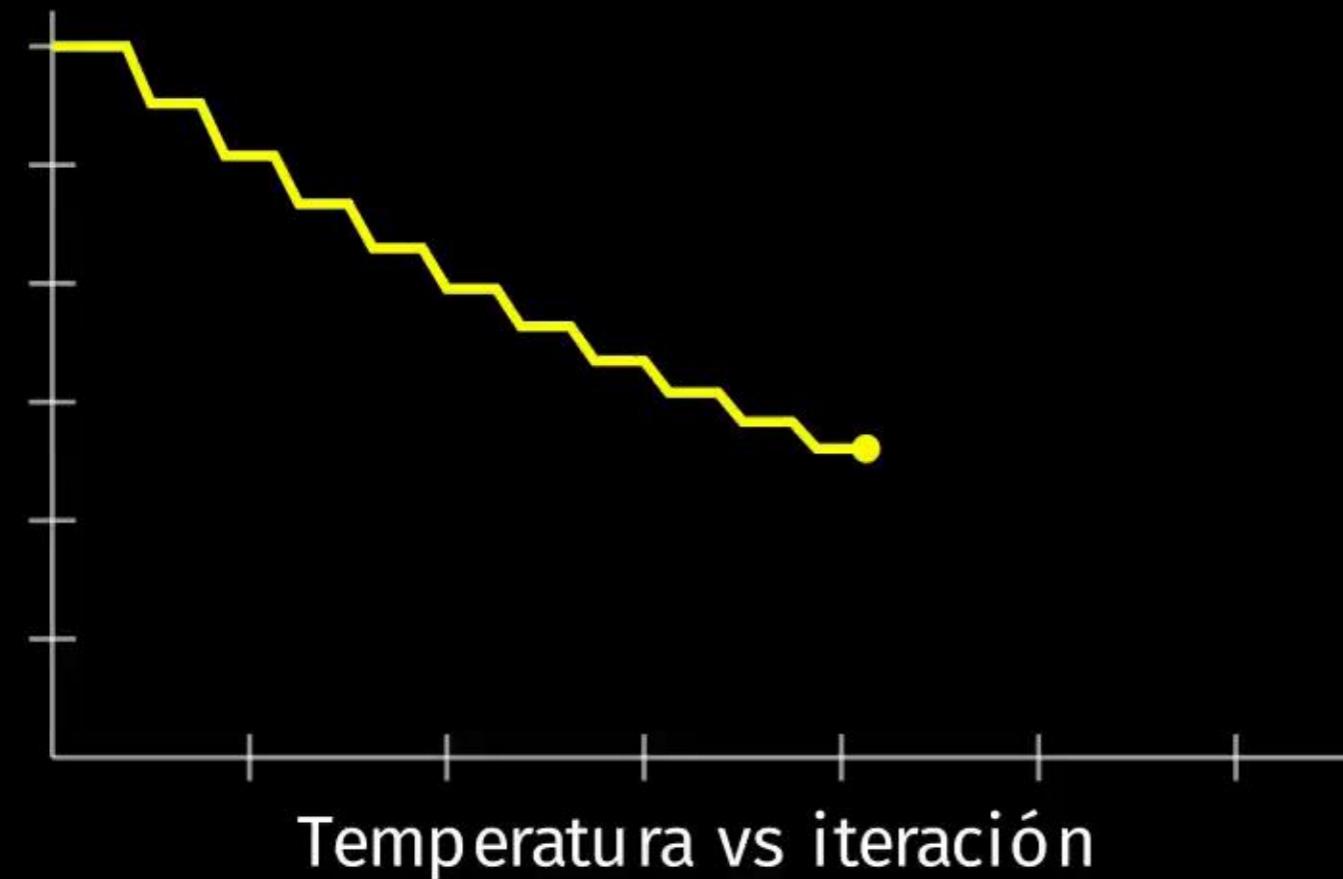
# Simulated Annealing – Graph Coloring



Movimiento rechazado

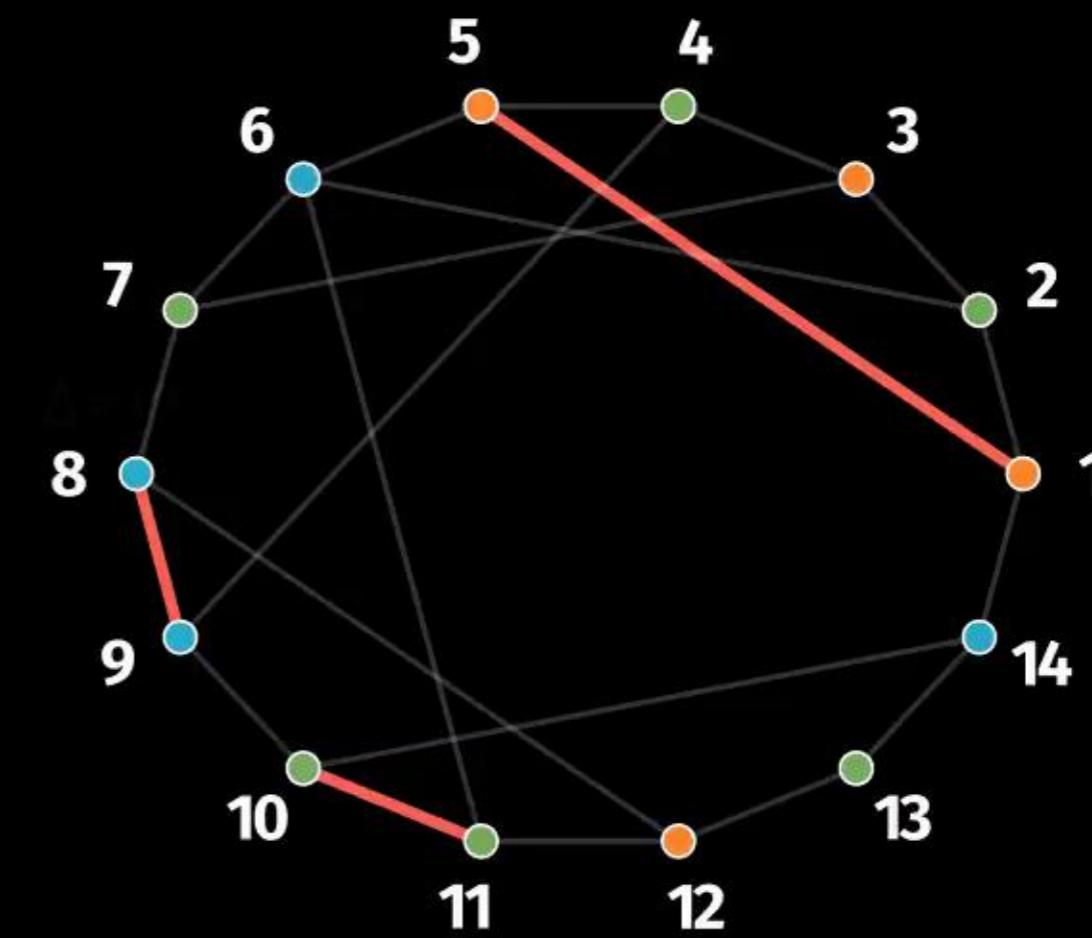
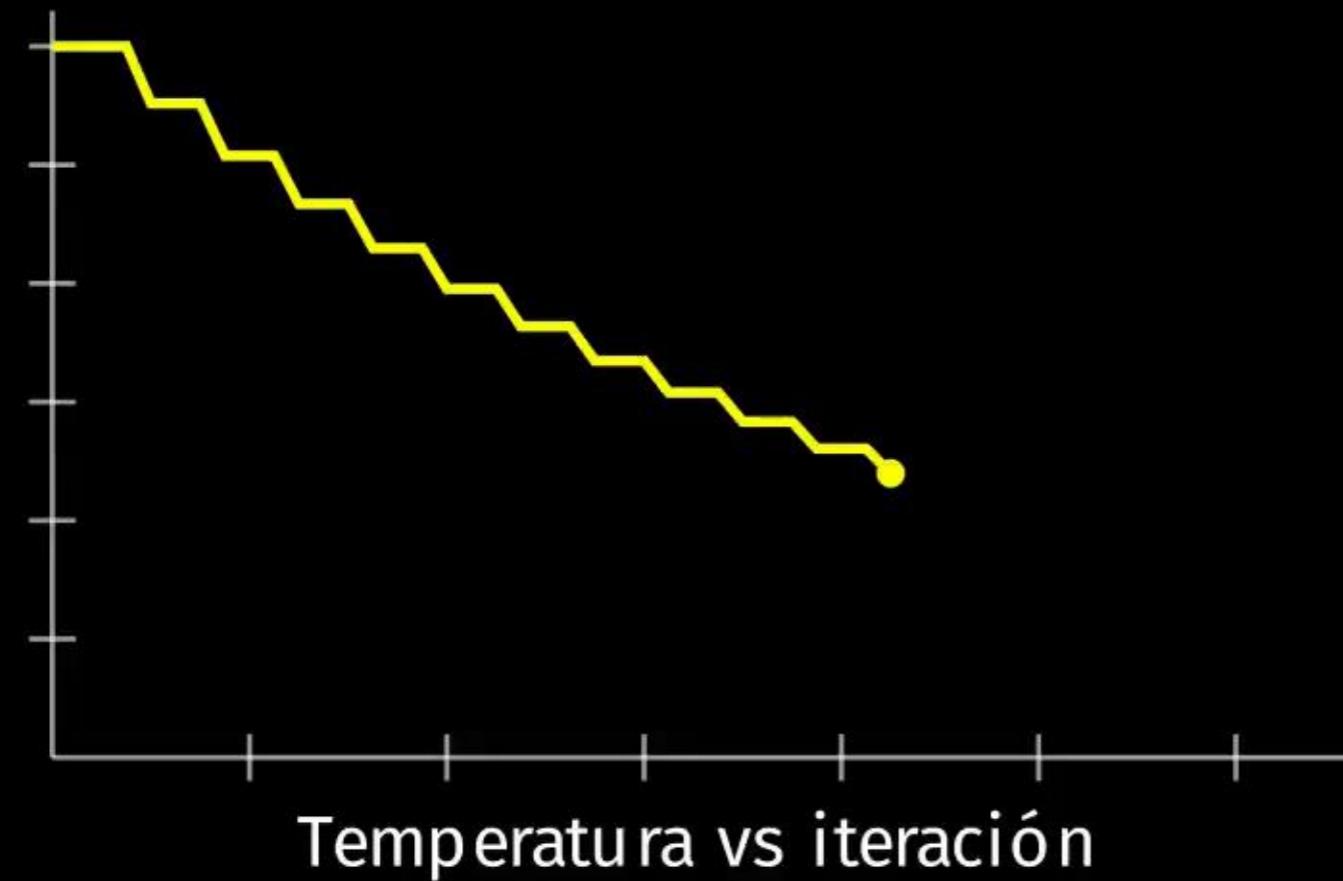
Temperatura: **0.869** Conflictos: **3** Incumbente: **2**  $\Delta$ : **+1**

# Simulated Annealing – Graph Coloring



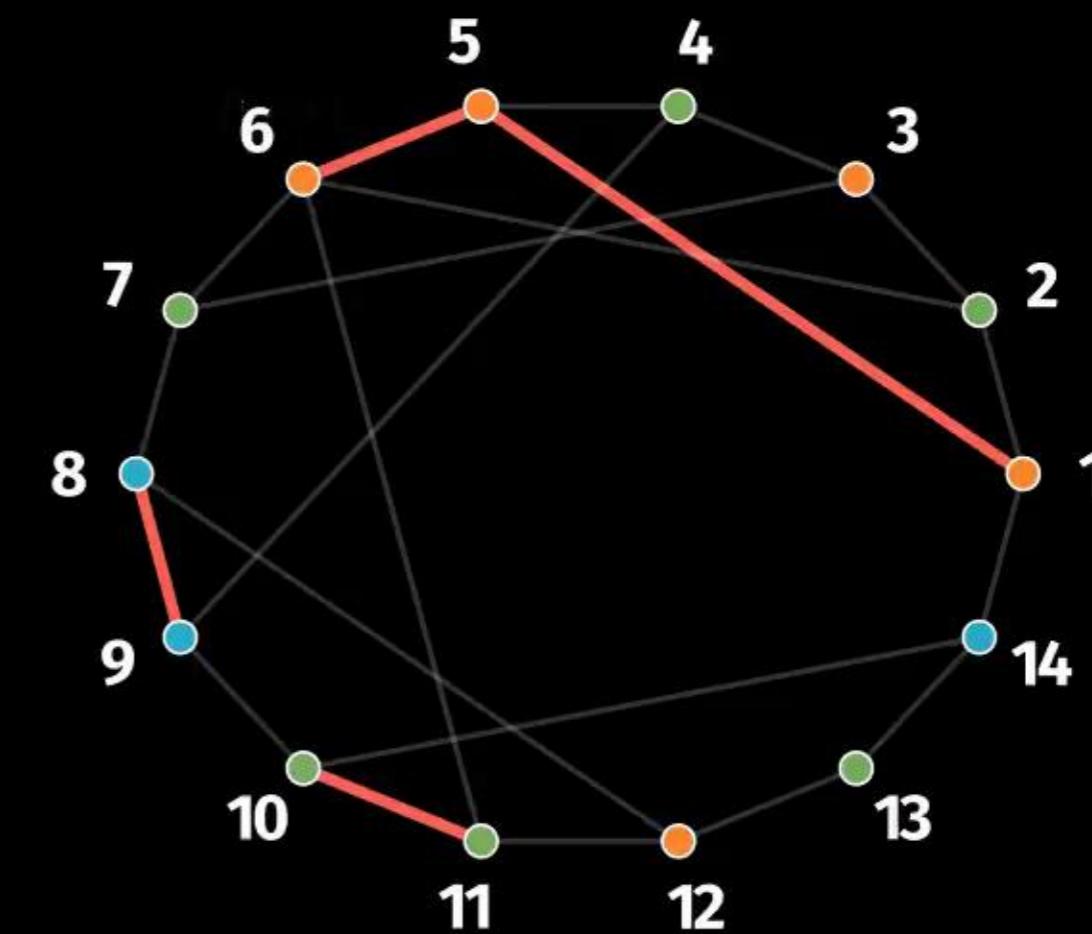
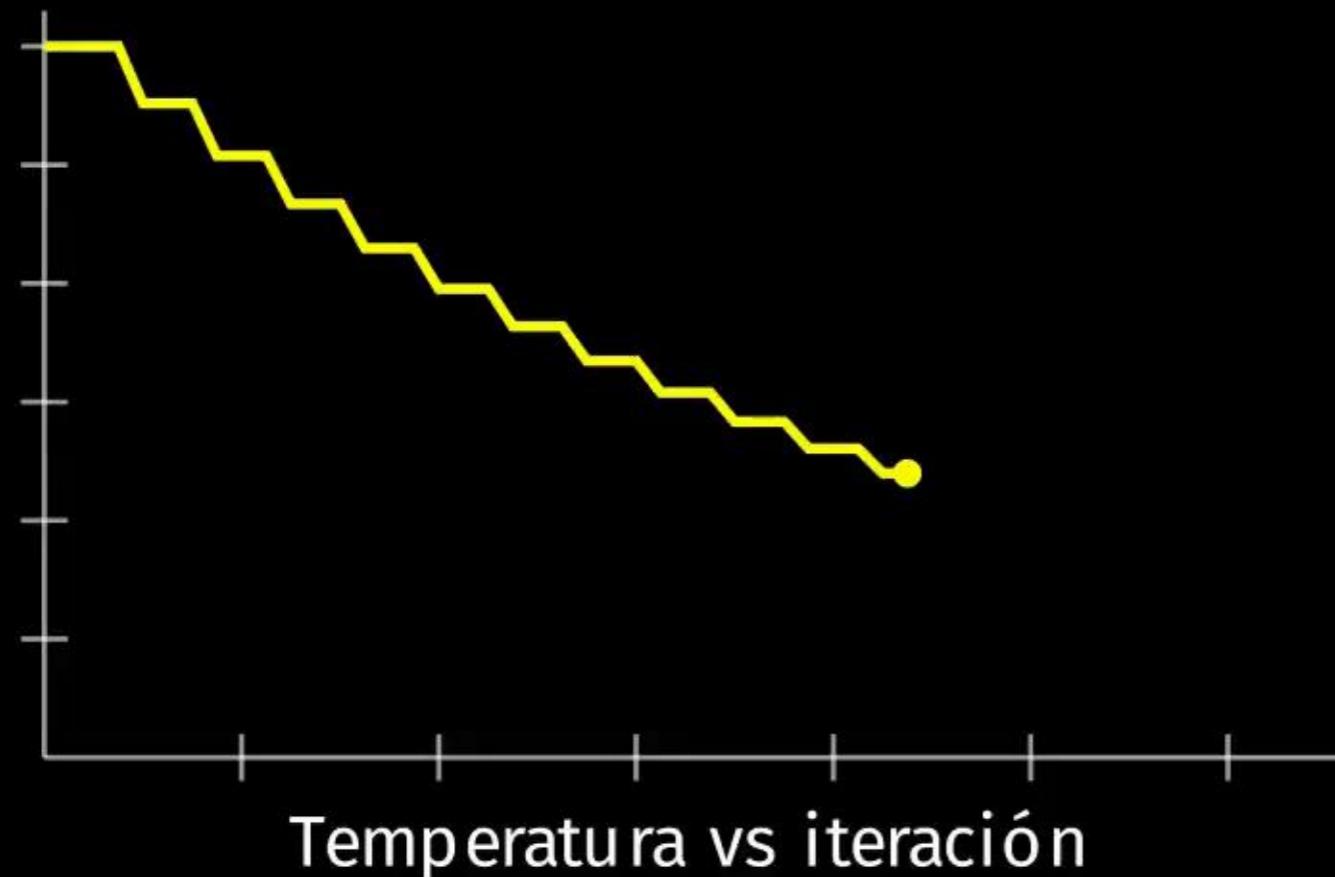
Temperatura: **0.869** Conflictos: **3** Incumbente: **2**  $\Delta$ : **+3**

# Simulated Annealing – Graph Coloring



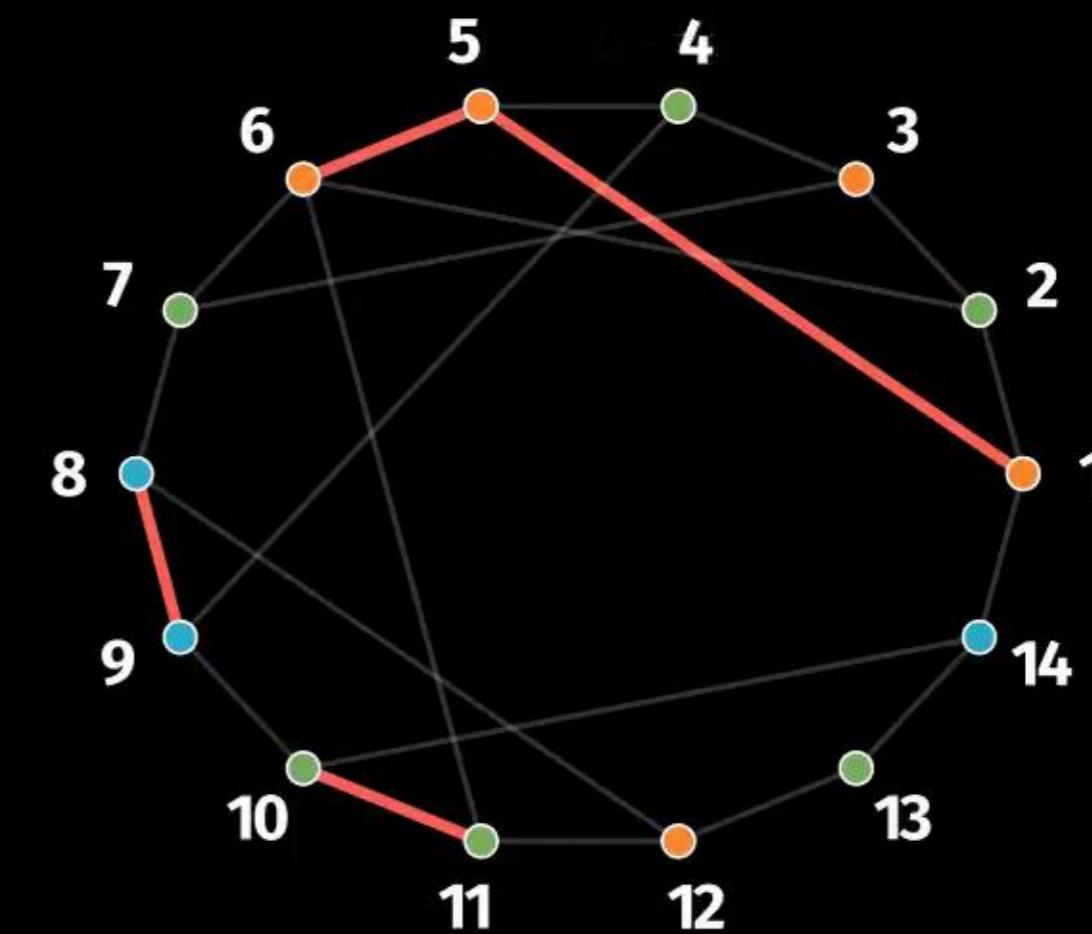
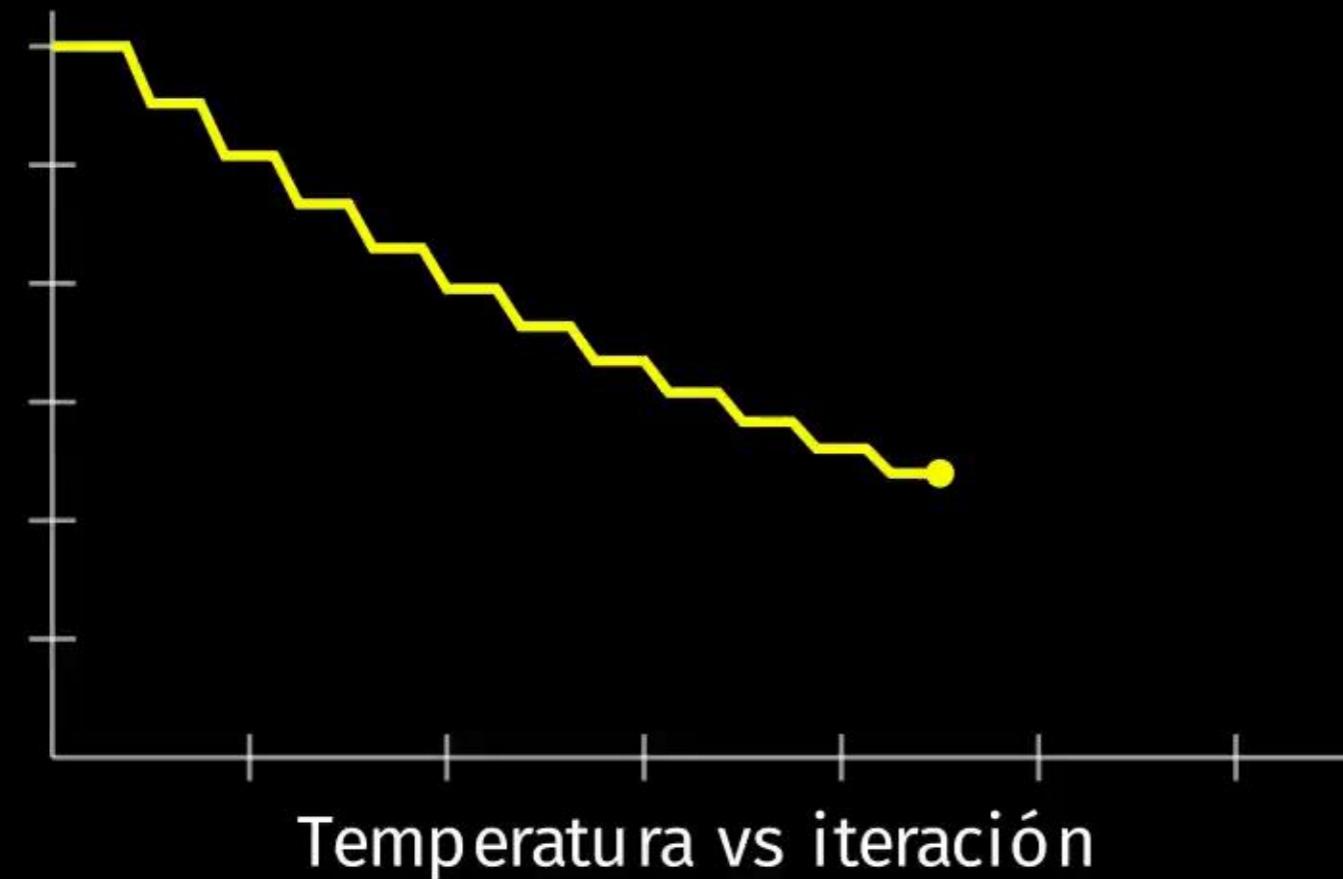
Temperatura: **0.799** Conflictos: **3** Incumbente: **2**  $\Delta$ : **+0**

# Simulated Annealing – Graph Coloring



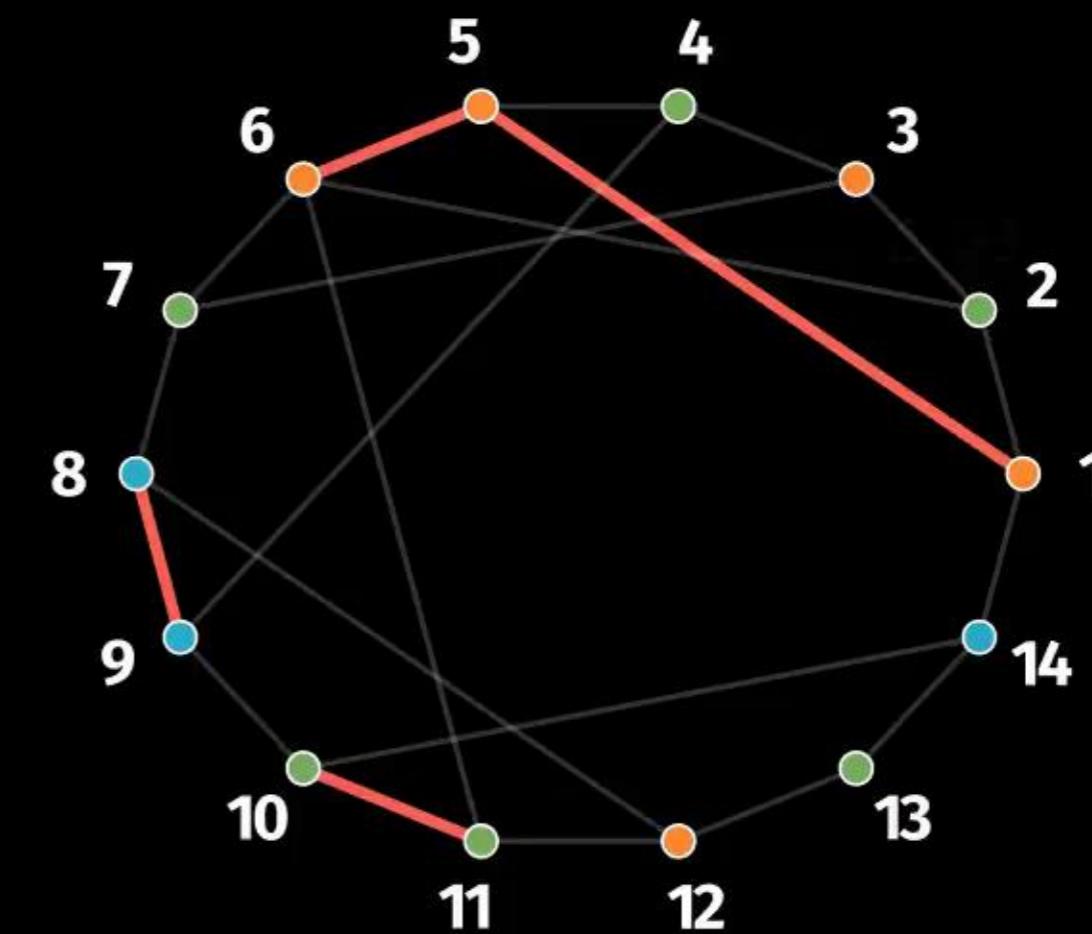
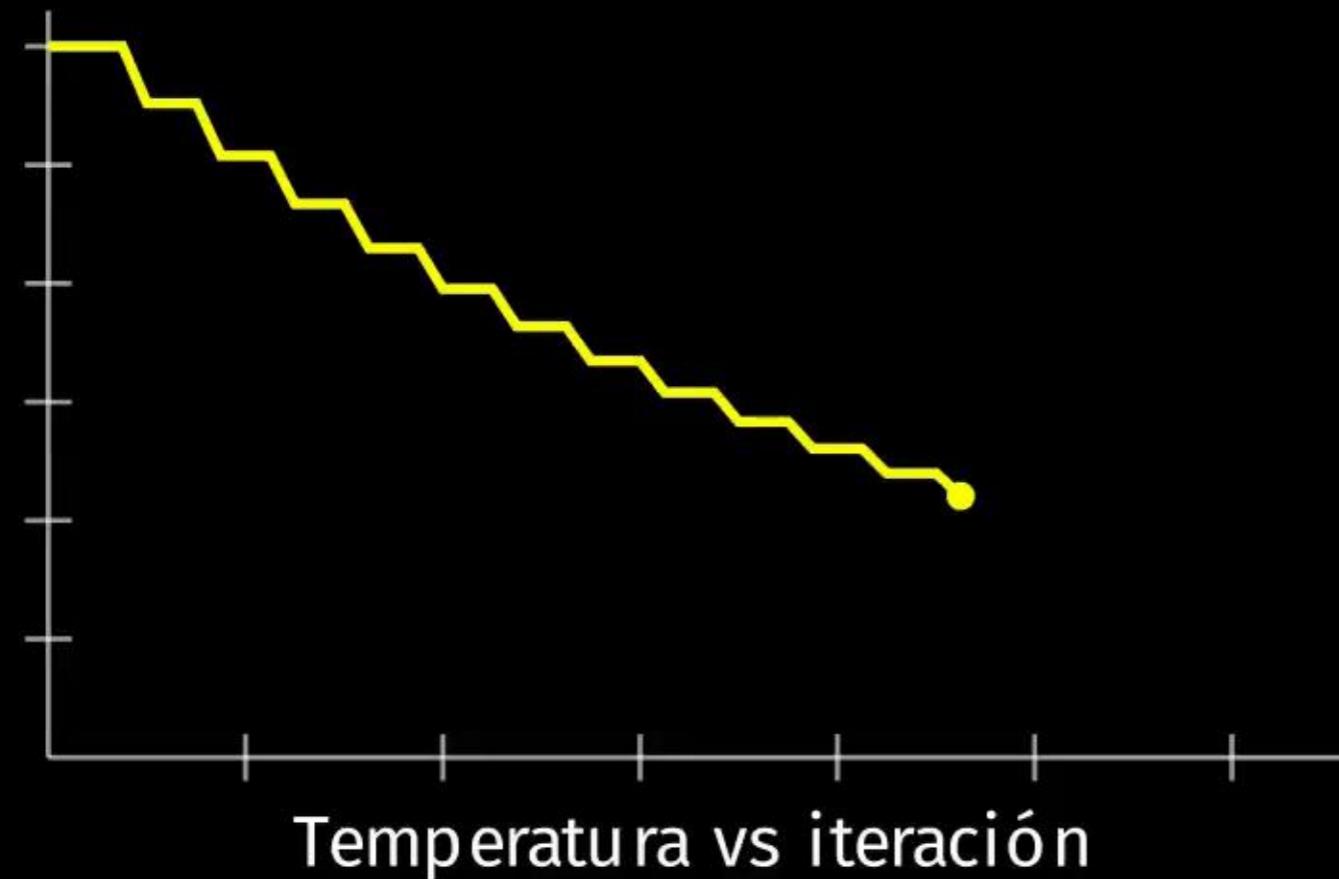
Temperatura: **0.799** Conflictos: **4** Incumbente: **2**  $\Delta$ : **+1**

# Simulated Annealing – Graph Coloring



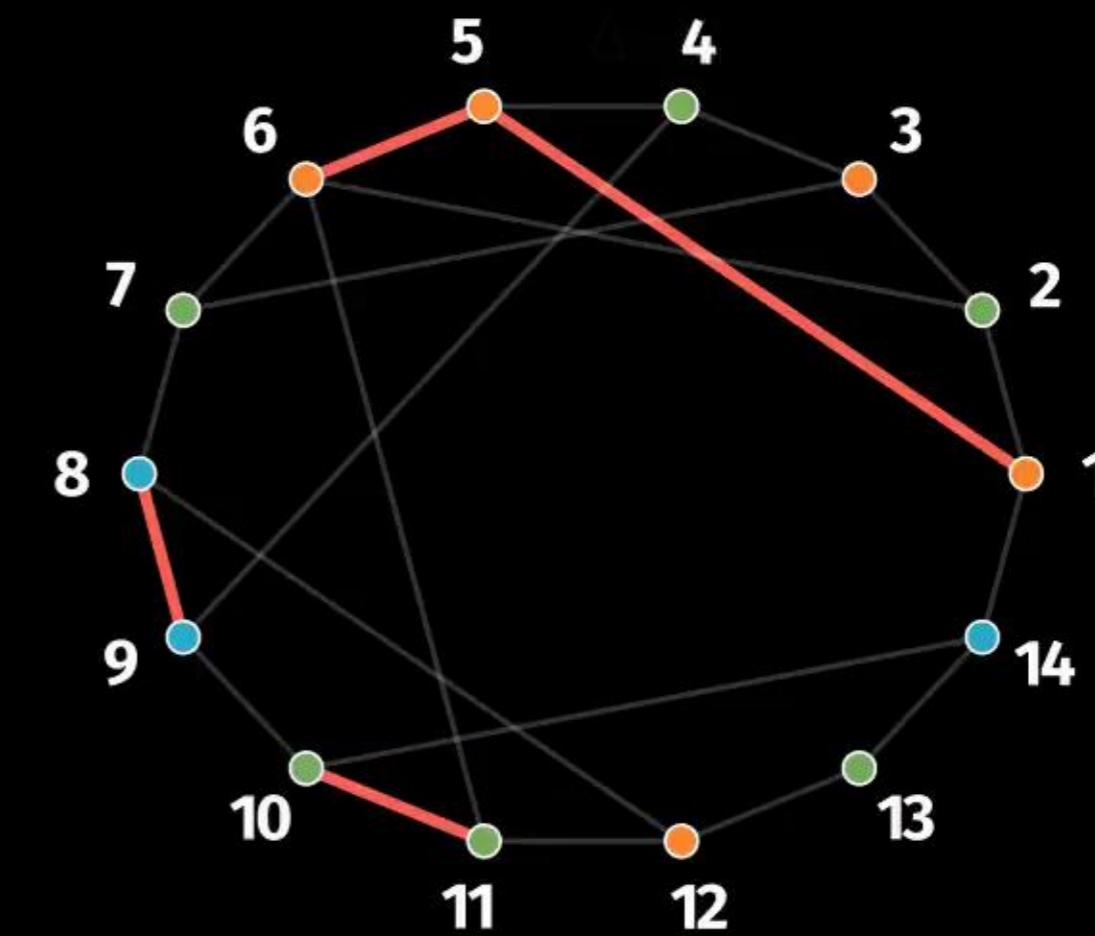
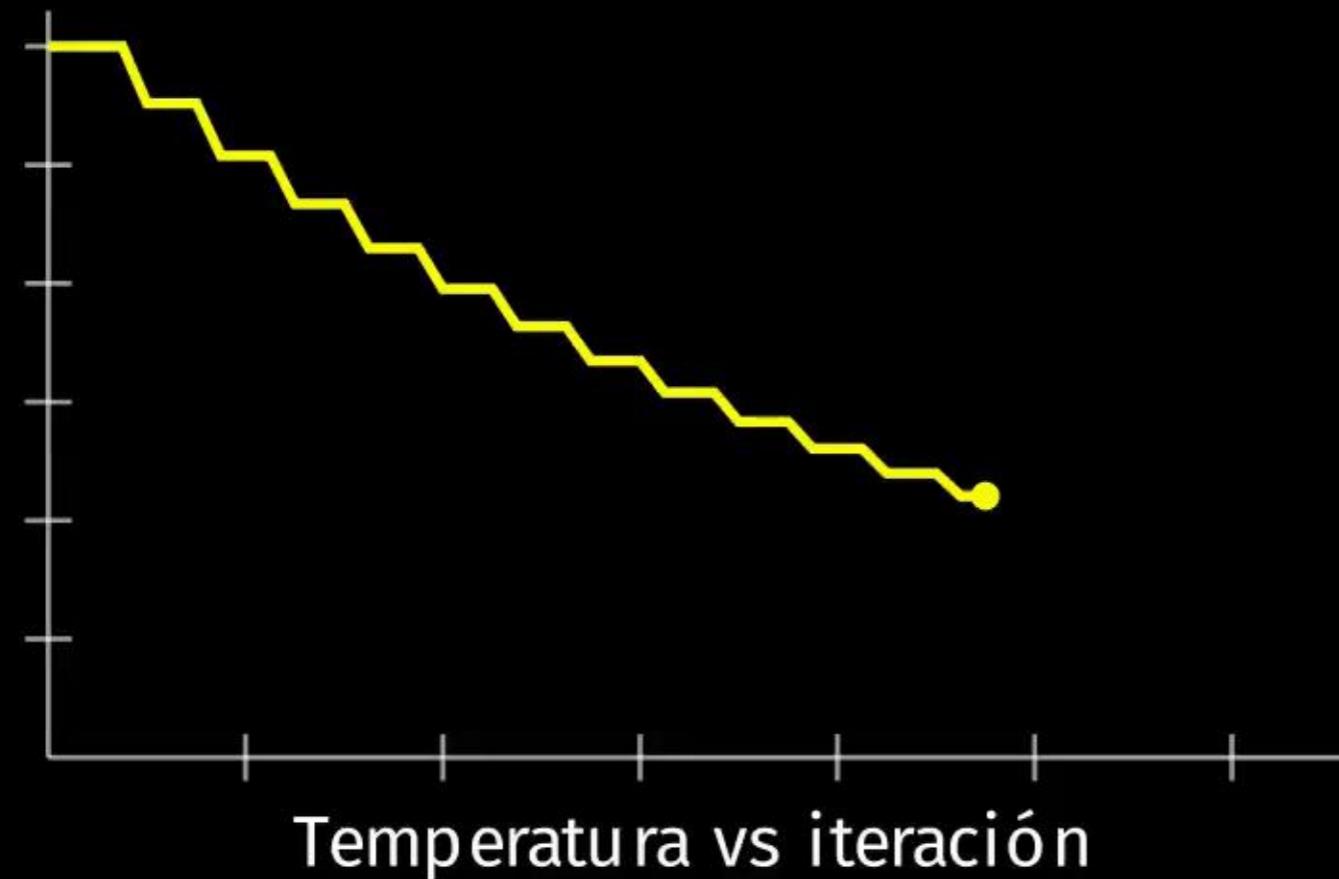
Temperatura: **0.799** Conflictos: **4** Incumbente: **2**  $\Delta$ : **+1**

# Simulated Annealing – Graph Coloring



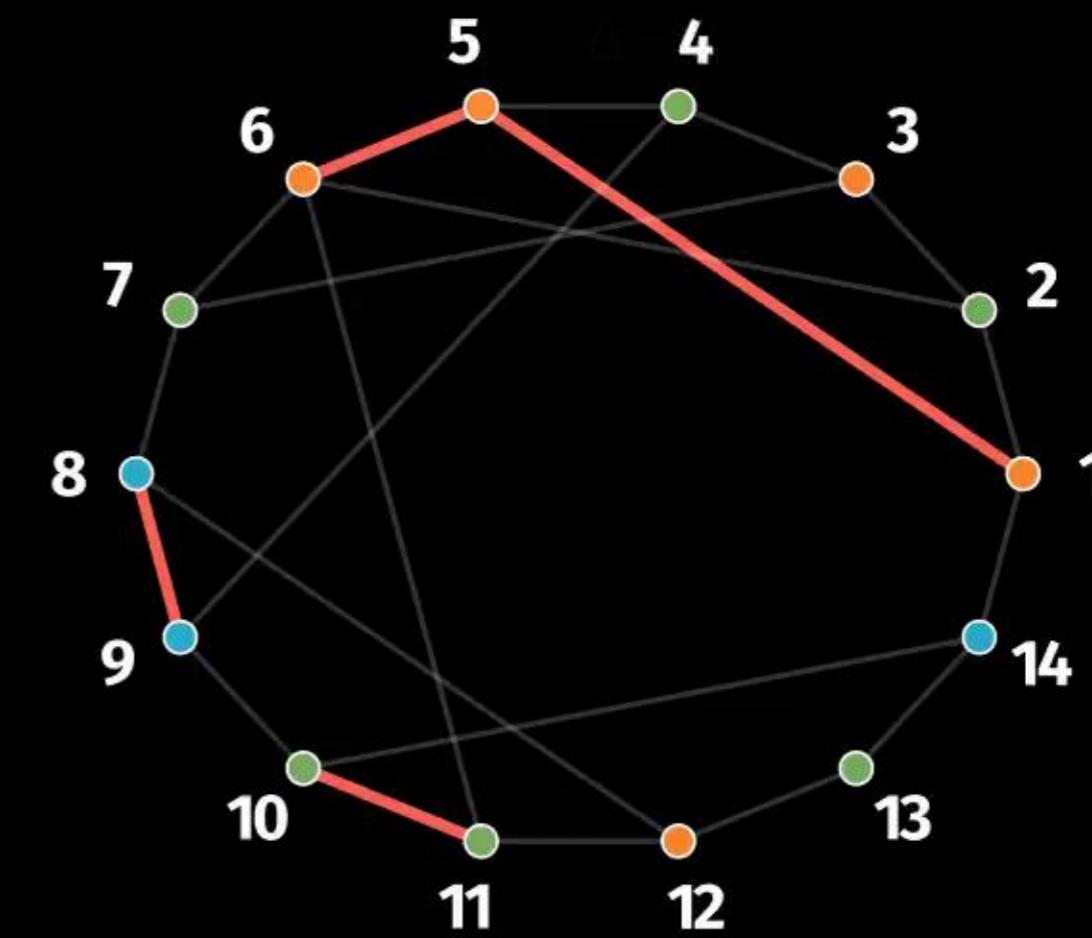
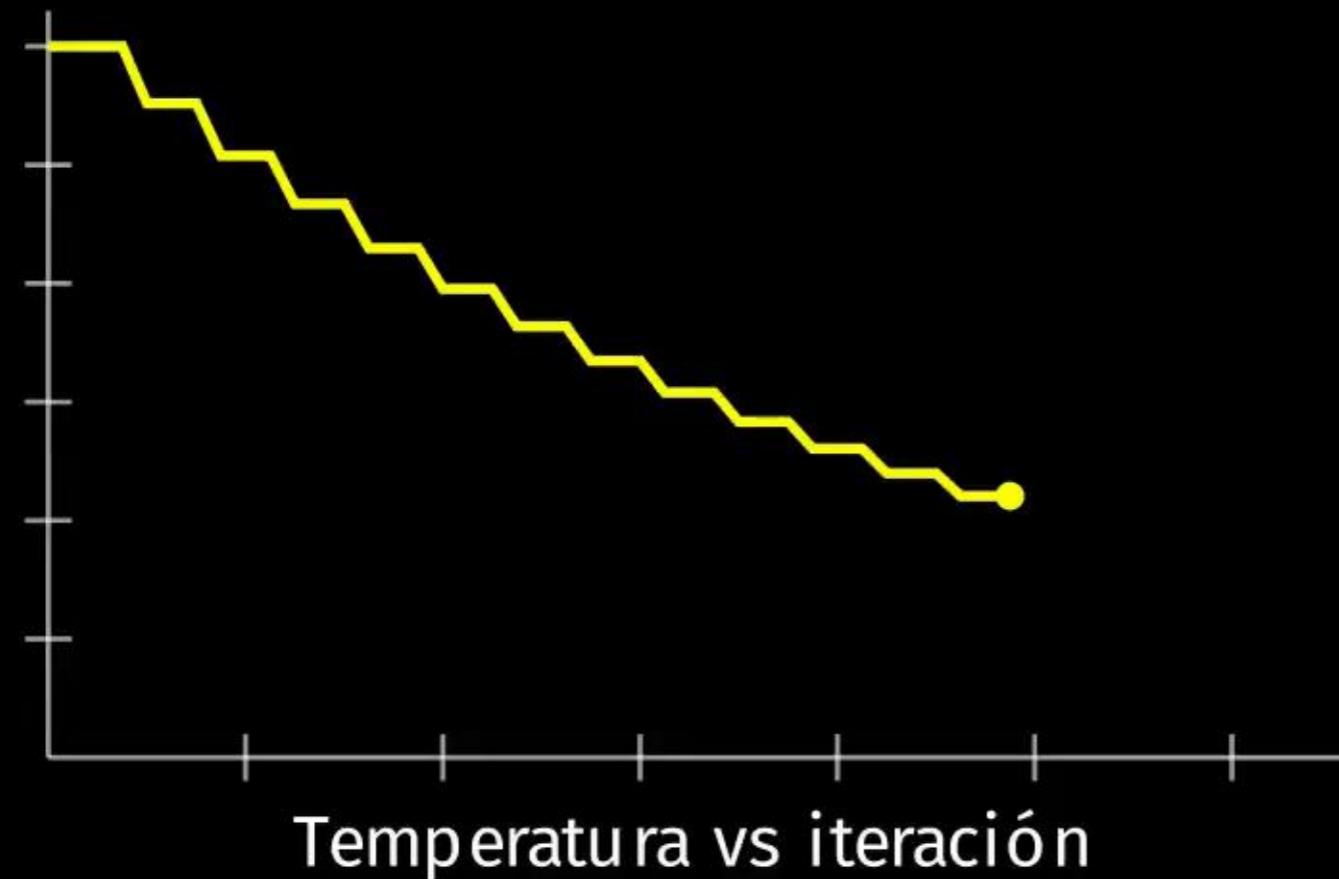
Temperatura: **0.735** Conflictos: **4** Incumbente: **2**  $\Delta$ : **+3**

# Simulated Annealing – Graph Coloring



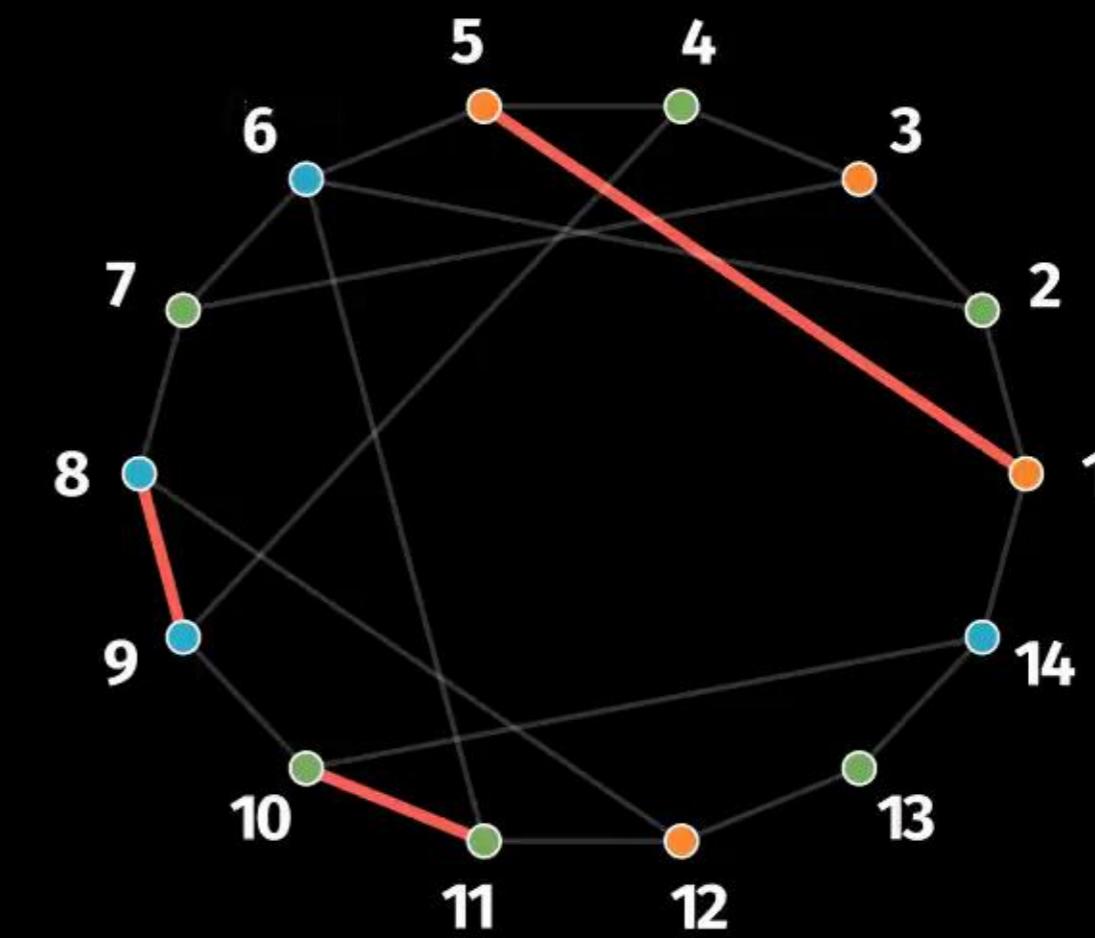
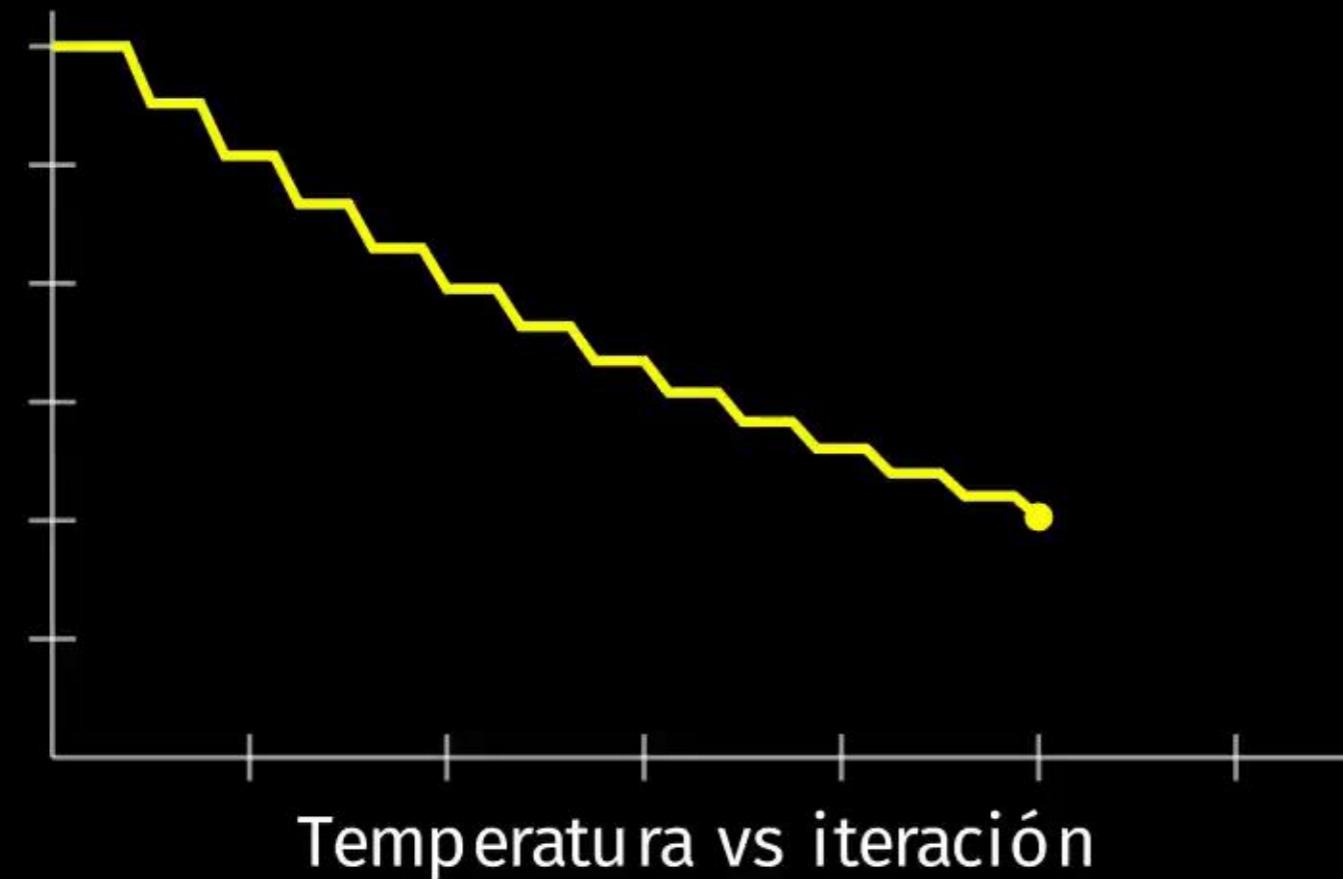
Temperatura: **0.735** Conflictos: **4** Incumbente: **2**  $\Delta$ : **+2**

# Simulated Annealing – Graph Coloring



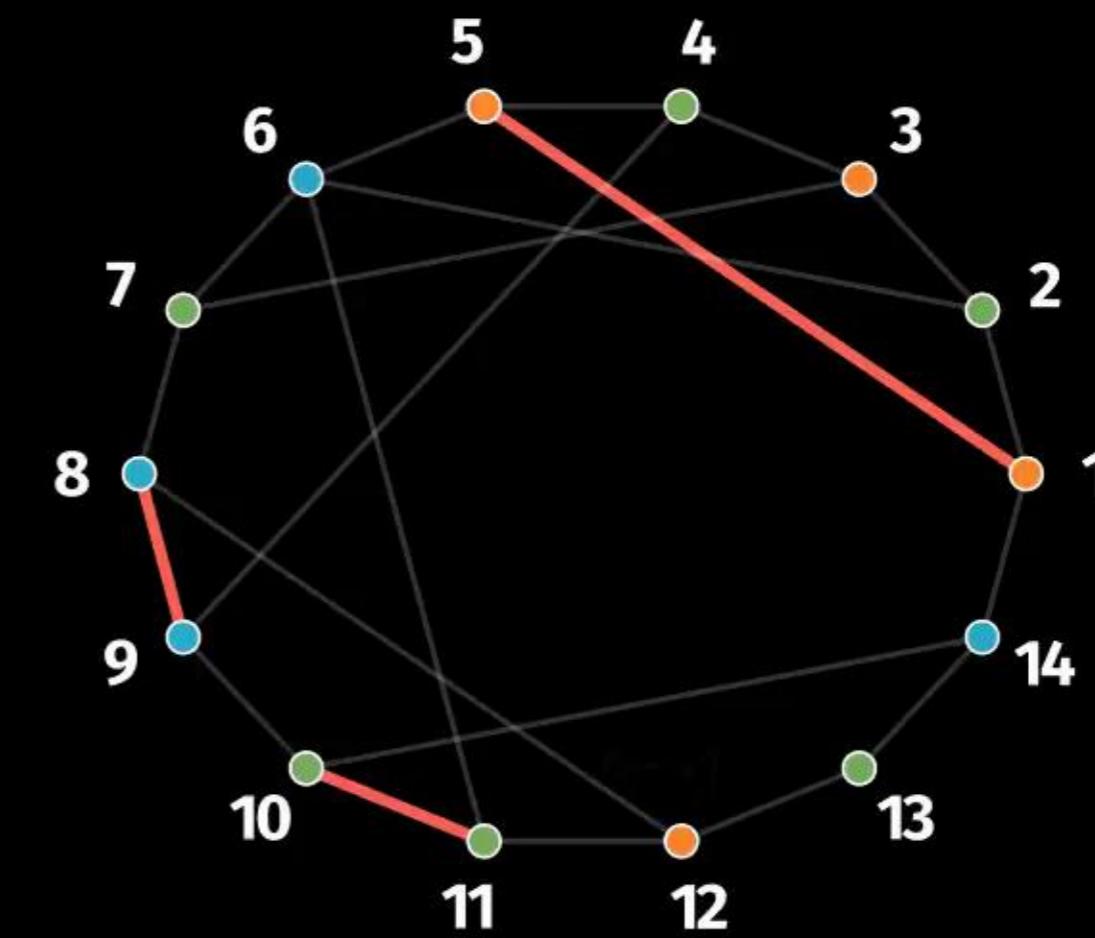
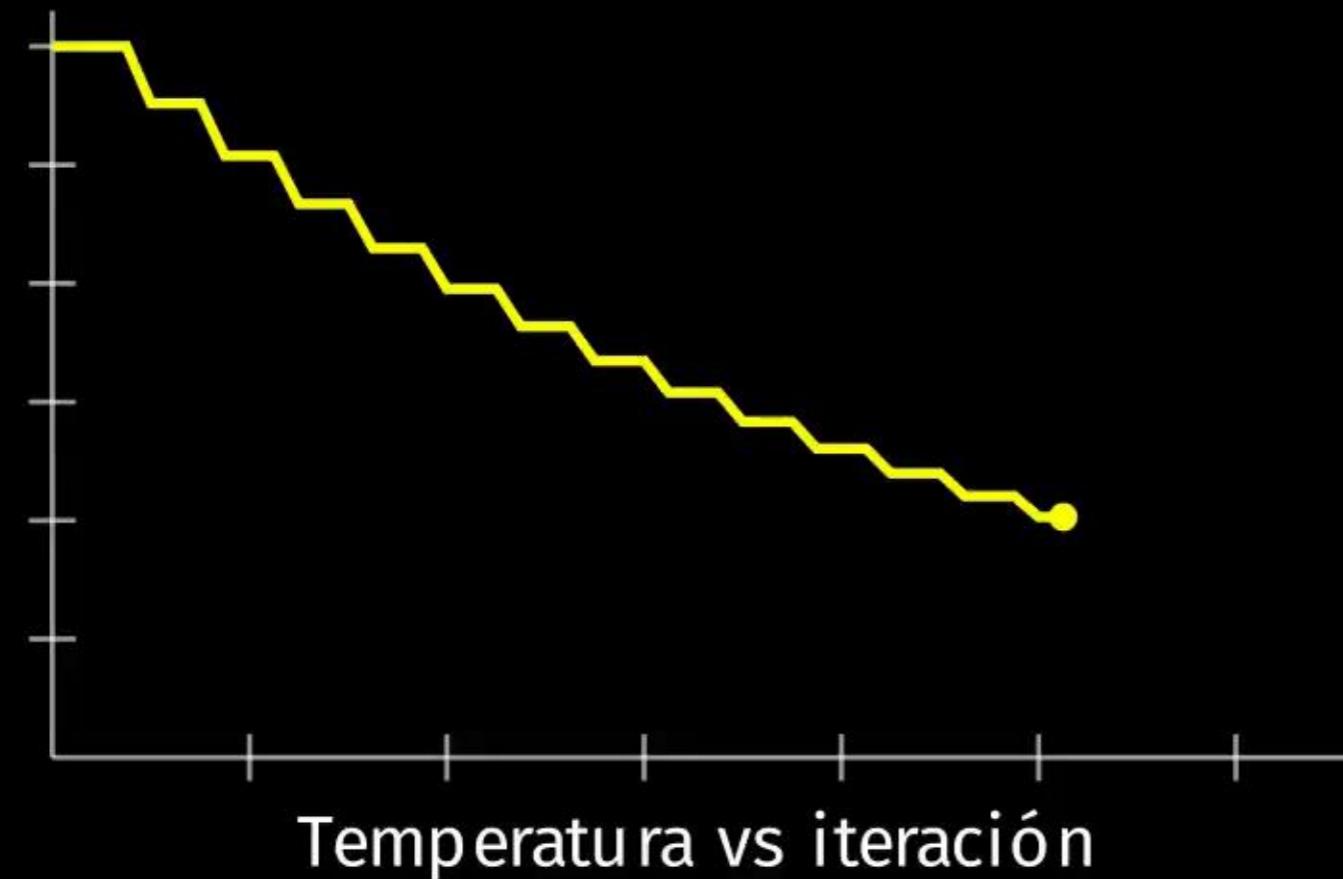
Temperatura: **0.735** Conflictos: **4** Incumbente: **2**  $\Delta$ : **+2**

# Simulated Annealing – Graph Coloring



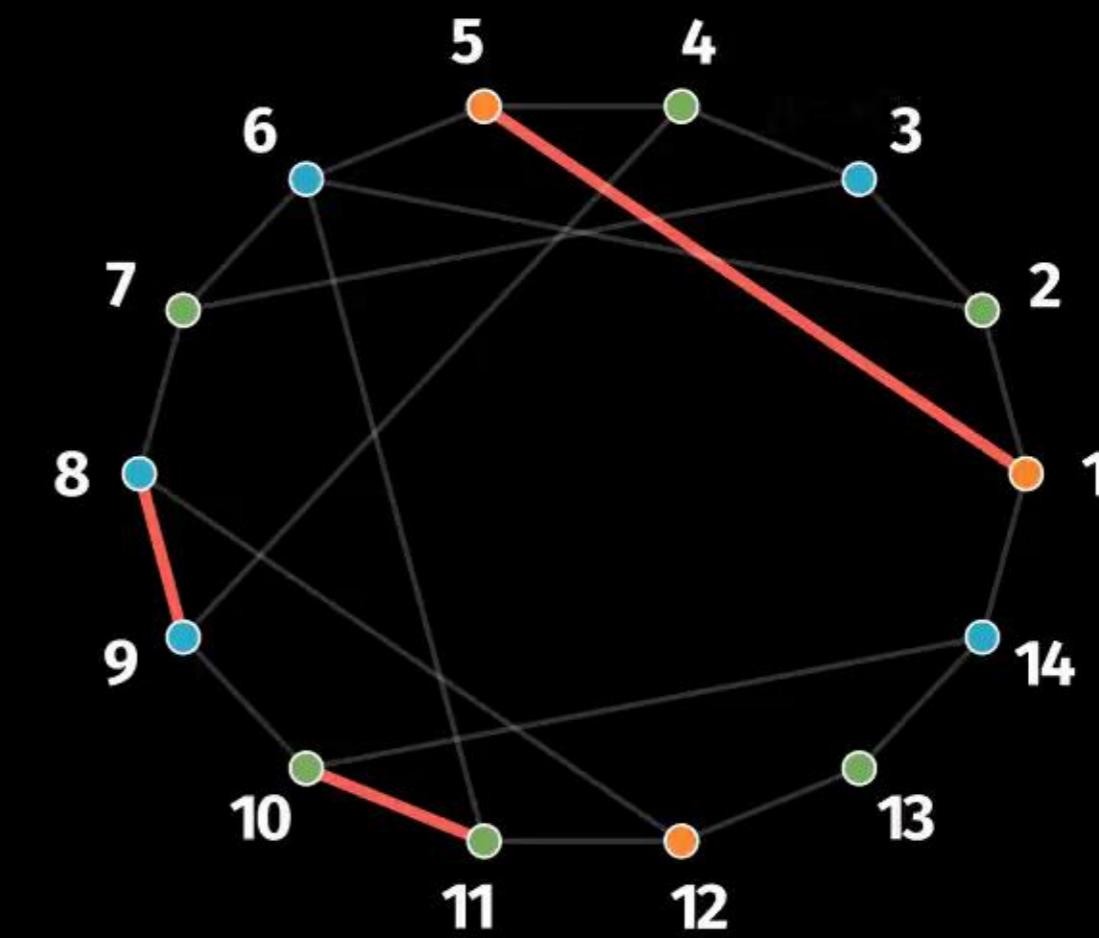
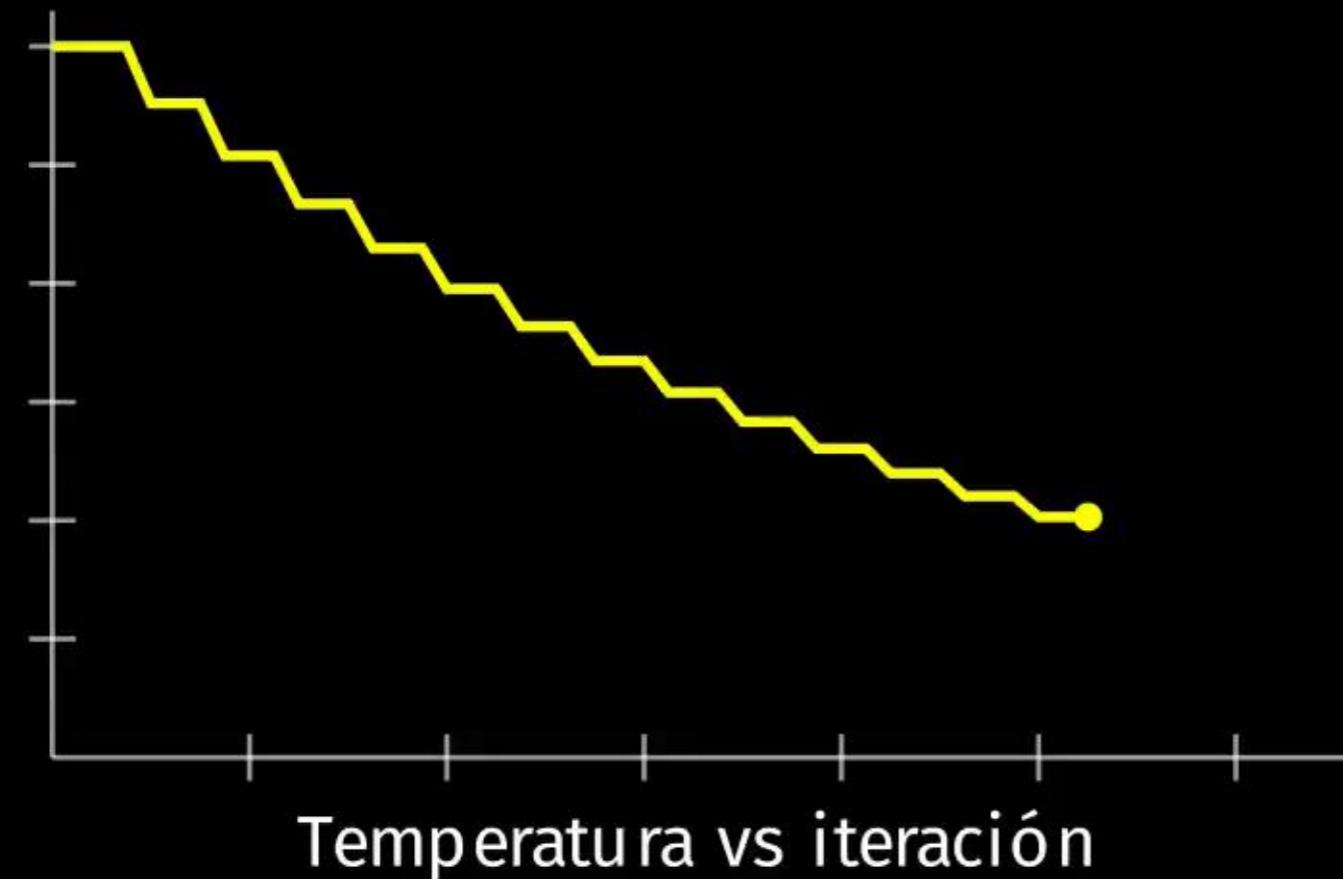
Temperatura: **0.677** Conflictos: **3** Incumbente: **2**  $\Delta$ : **-1**

# Simulated Annealing – Graph Coloring



Temperatura: **0.677** Conflictos: **3** Incumbente: **2**  $\Delta$ : **+1**

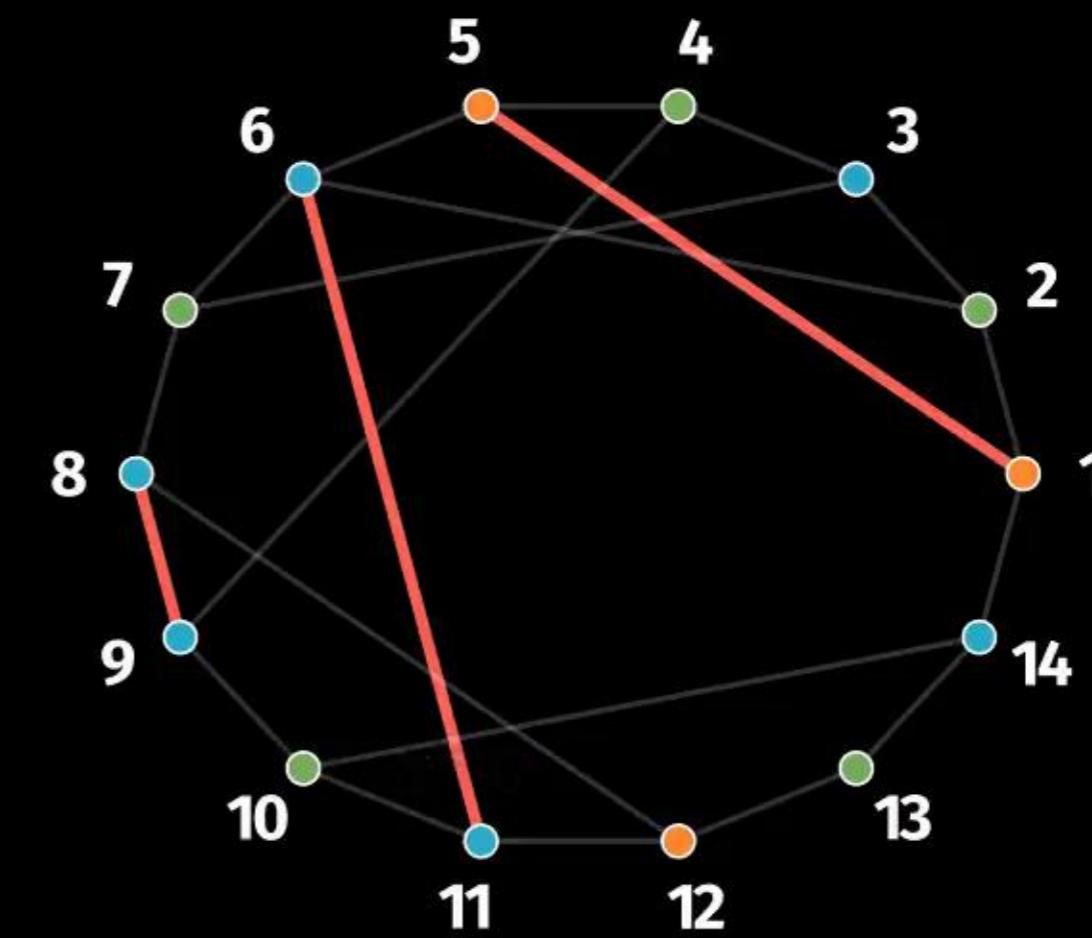
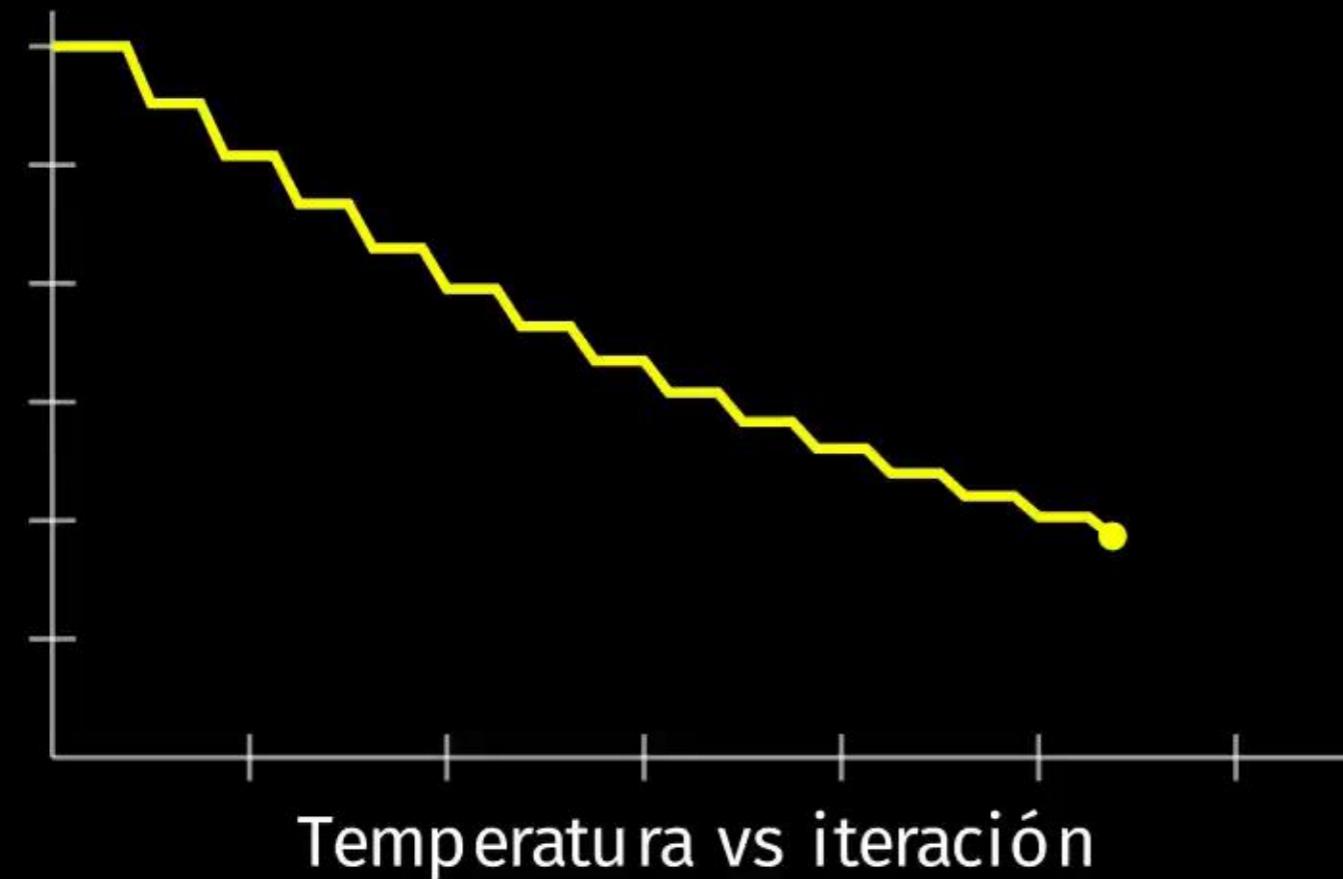
# Simulated Annealing – Graph Coloring



Movimiento aceptado

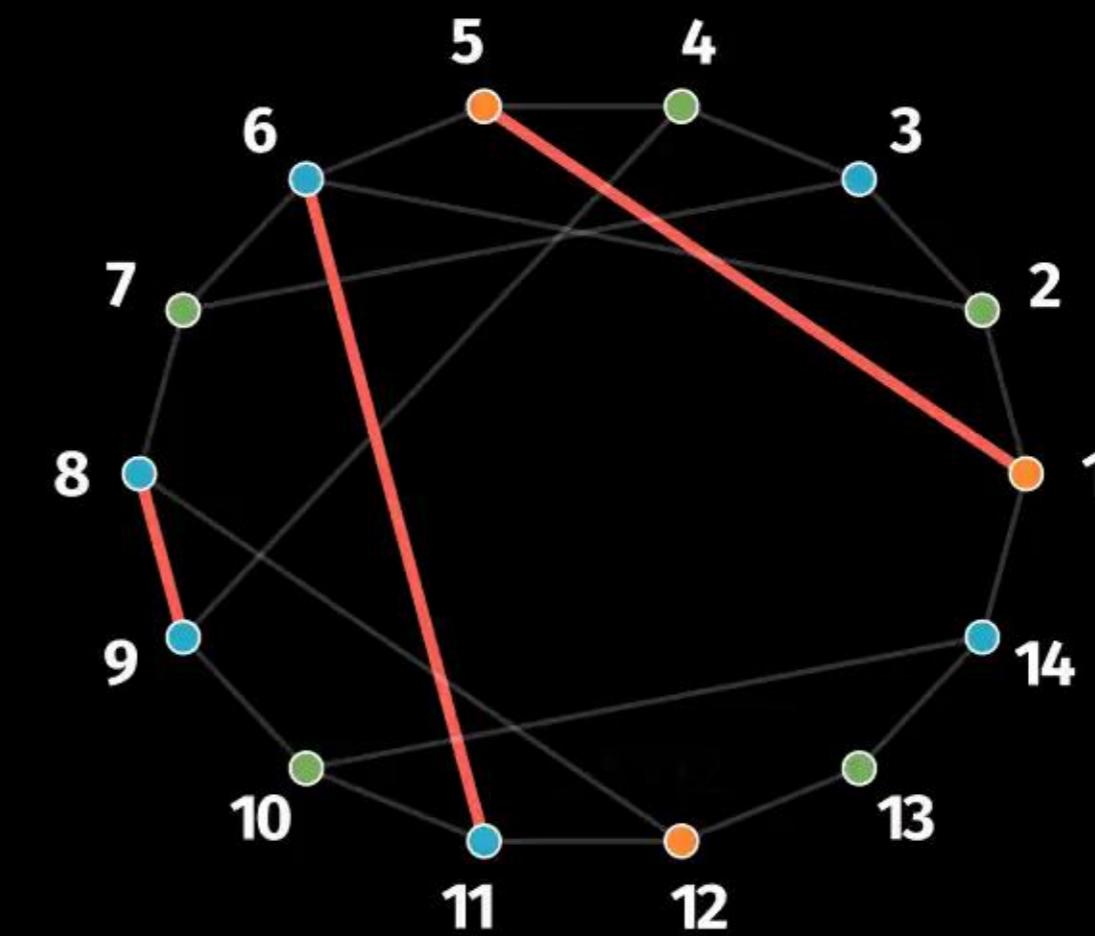
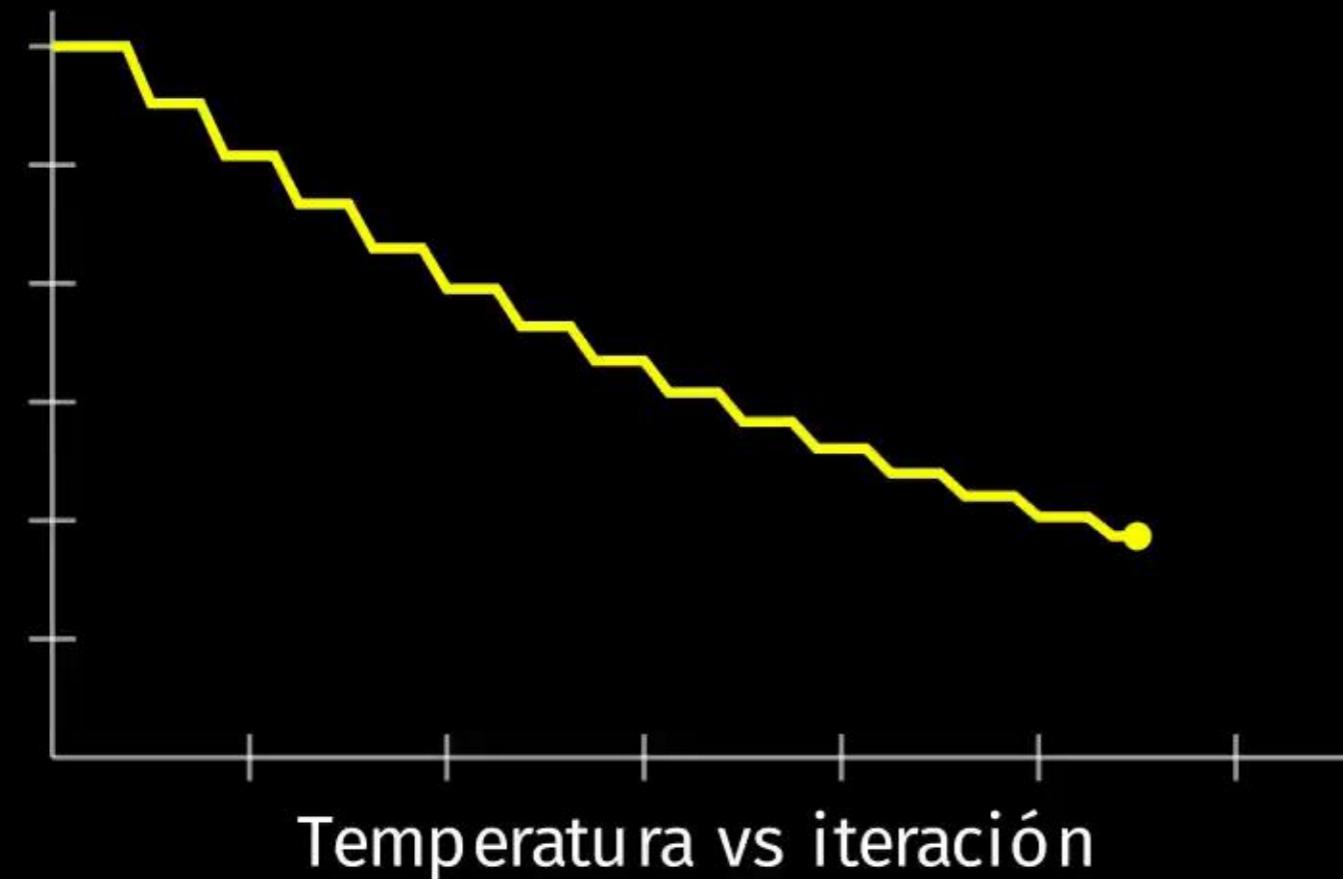
Temperatura: **0.677** Conflictos: **3** Incumbente: **2**  $\Delta$ : **+0**

# Simulated Annealing – Graph Coloring



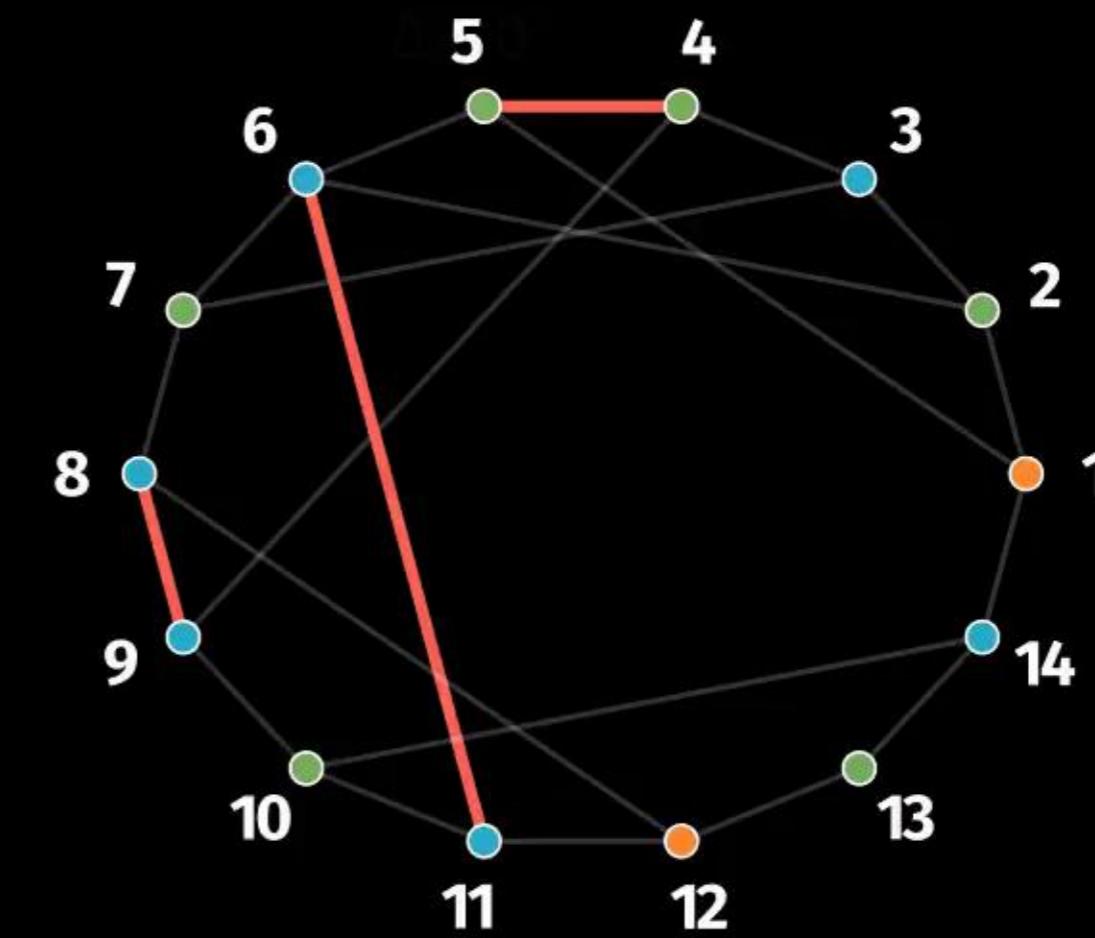
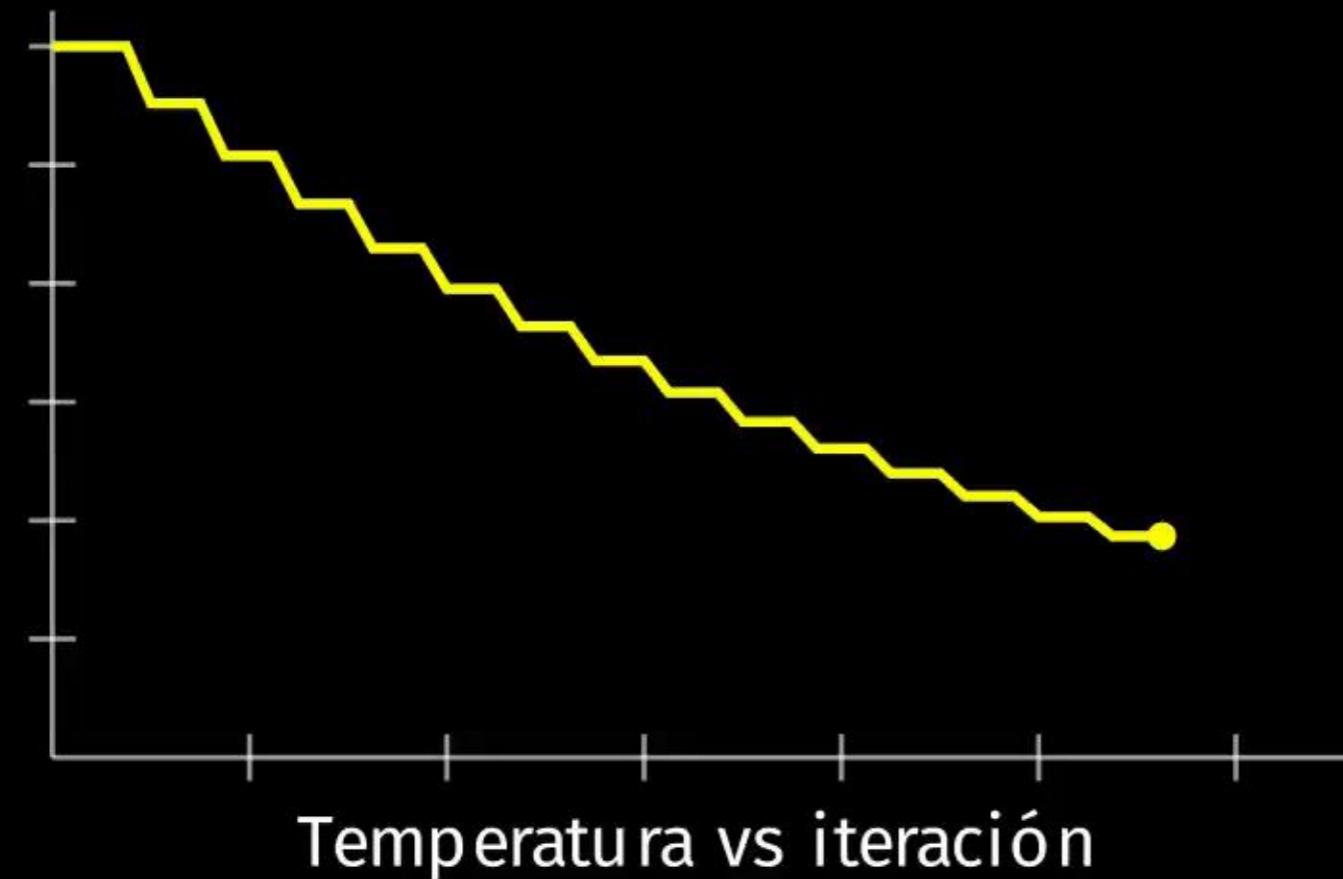
Temperatura: **0.622** Conflictos: **3** Incumbente: **2**  $\Delta$ : **+0**

# Simulated Annealing – Graph Coloring



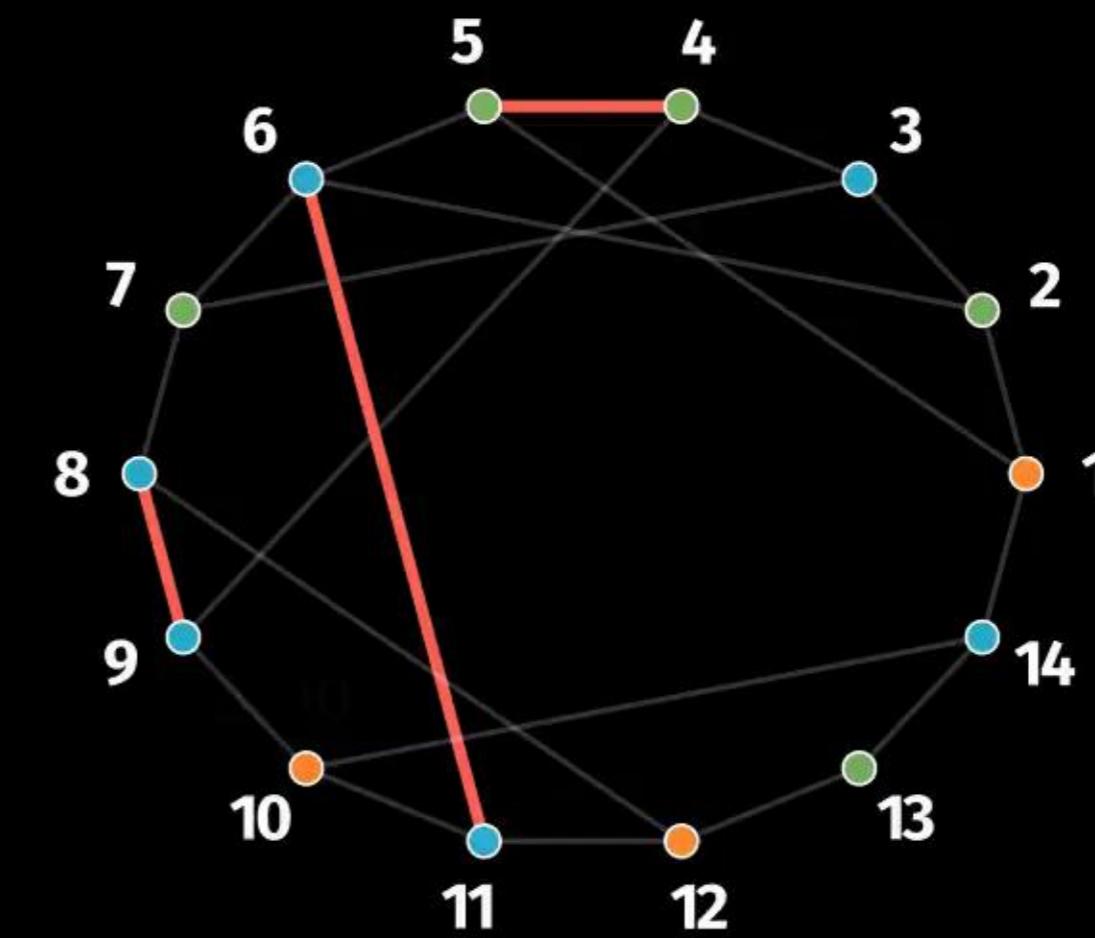
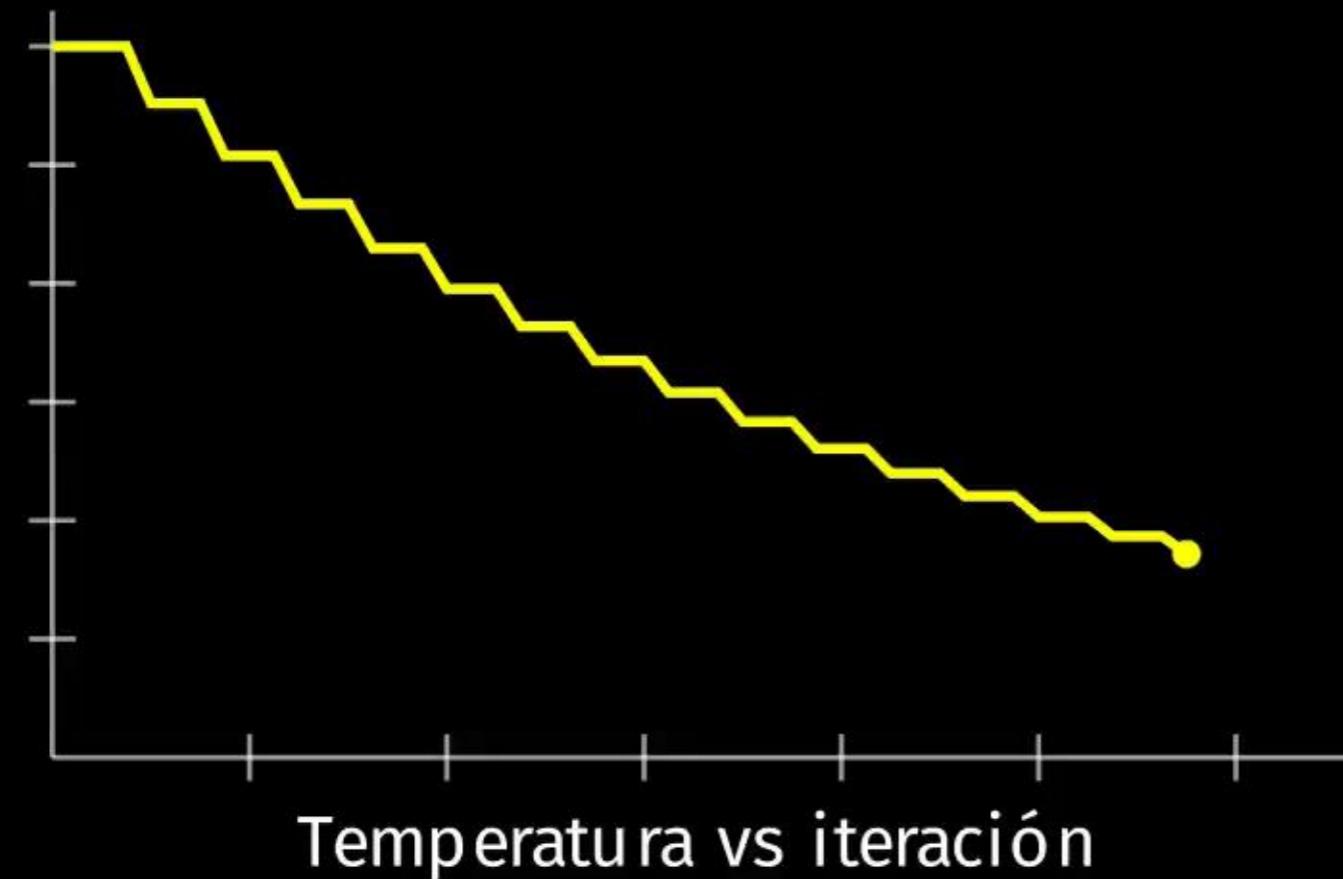
Temperatura: **0.622** Conflictos: **3** Incumbente: **2**  $\Delta$ : **+2**

# Simulated Annealing – Graph Coloring



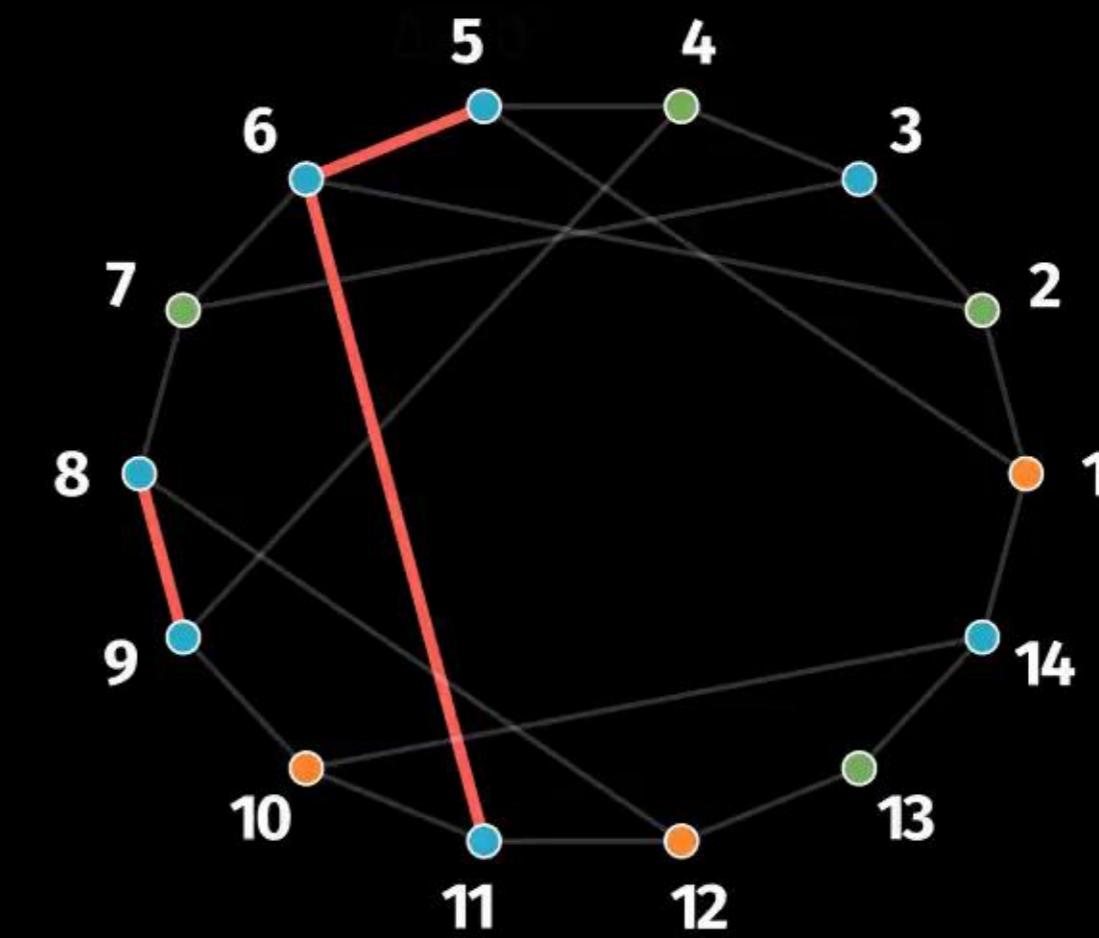
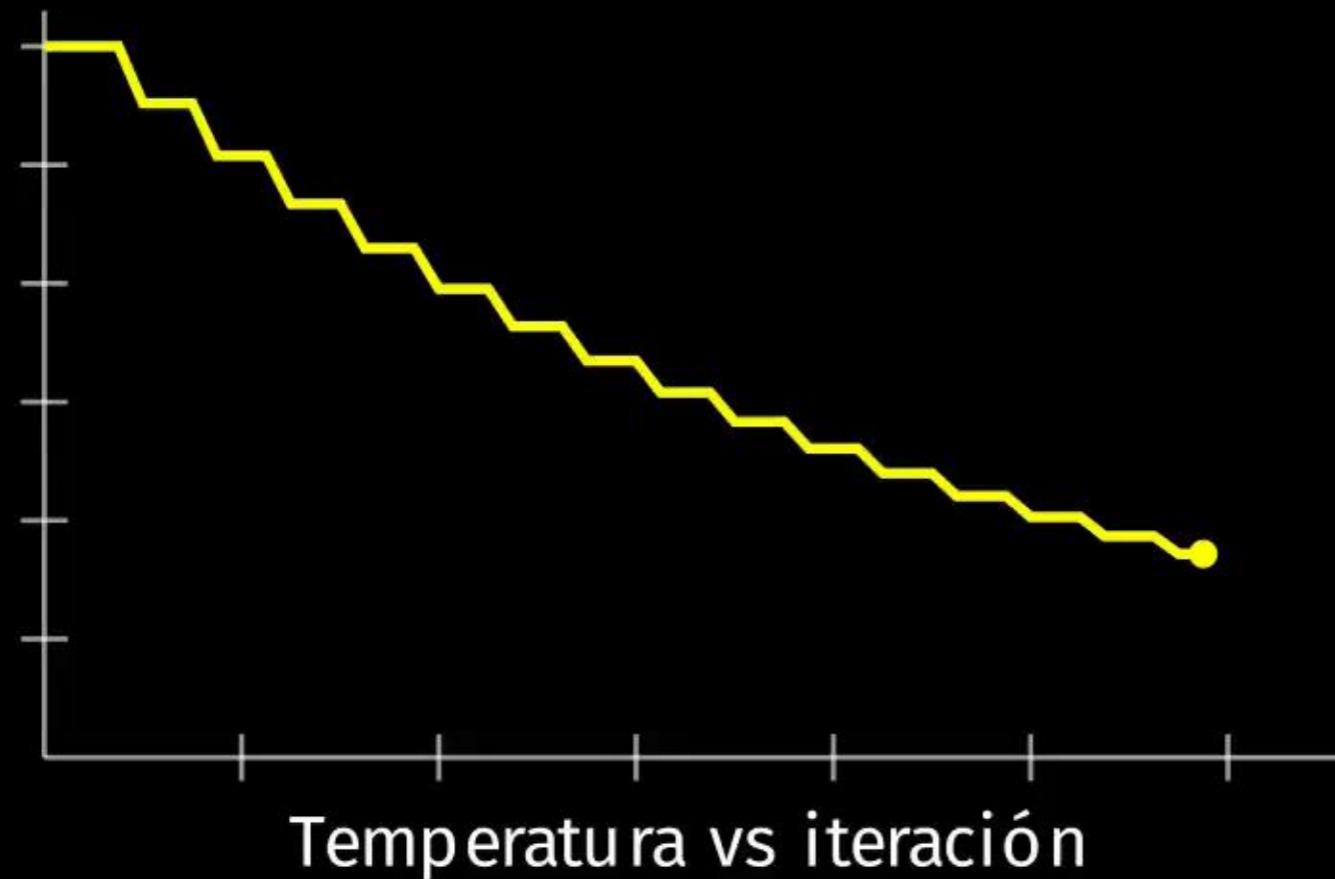
Temperatura: **0.622** Conflictos: **3** Incumbente: **2**  $\Delta$ : **+0**

# Simulated Annealing – Graph Coloring



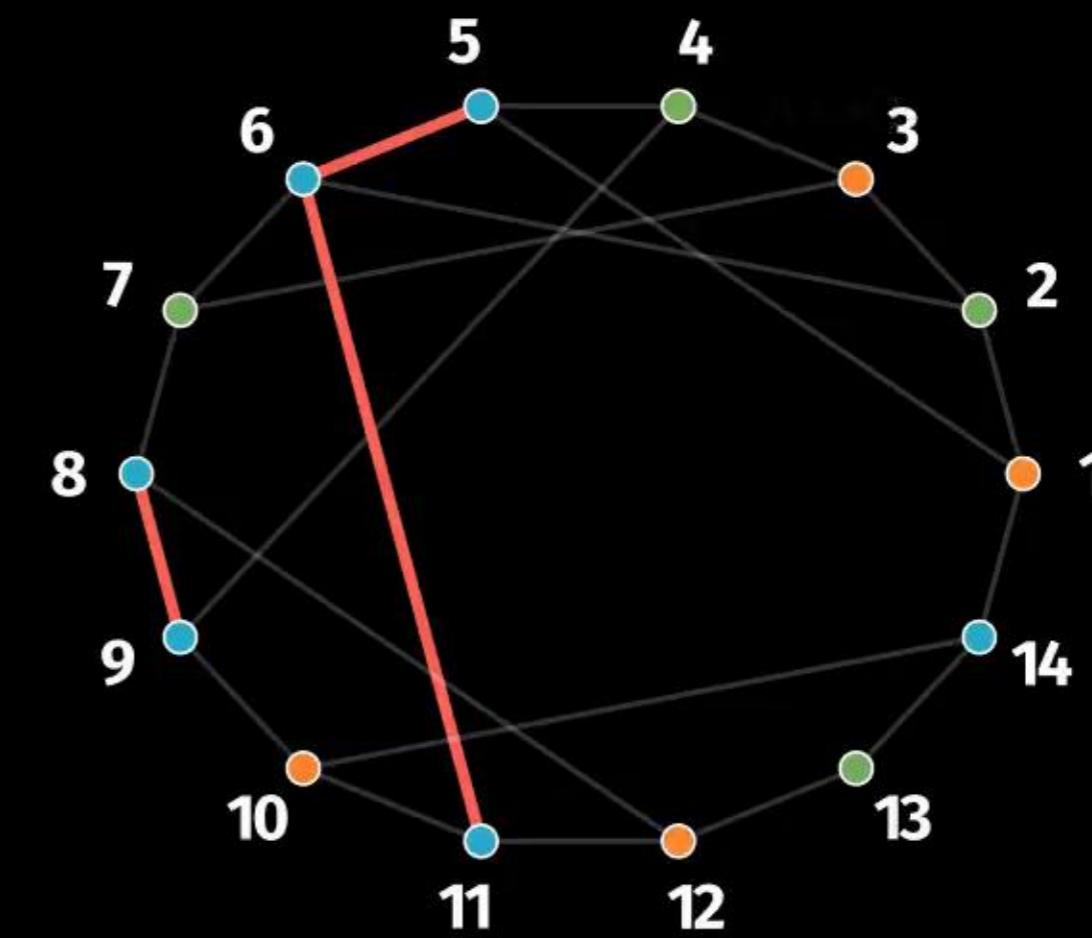
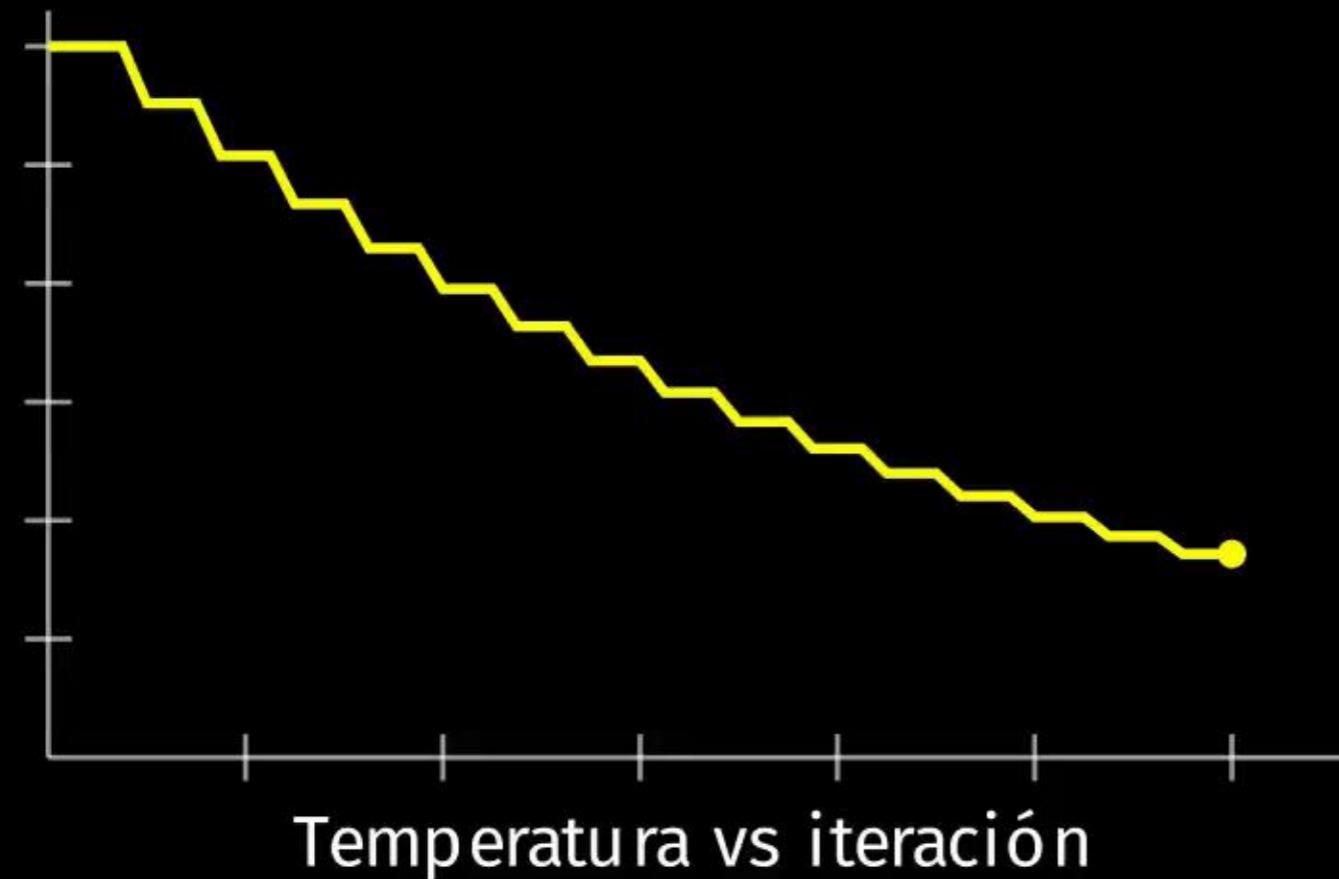
Temperatura: **0.573** Conflictos: **3** Incumbente: **2**  $\Delta$ : **+0**

# Simulated Annealing – Graph Coloring



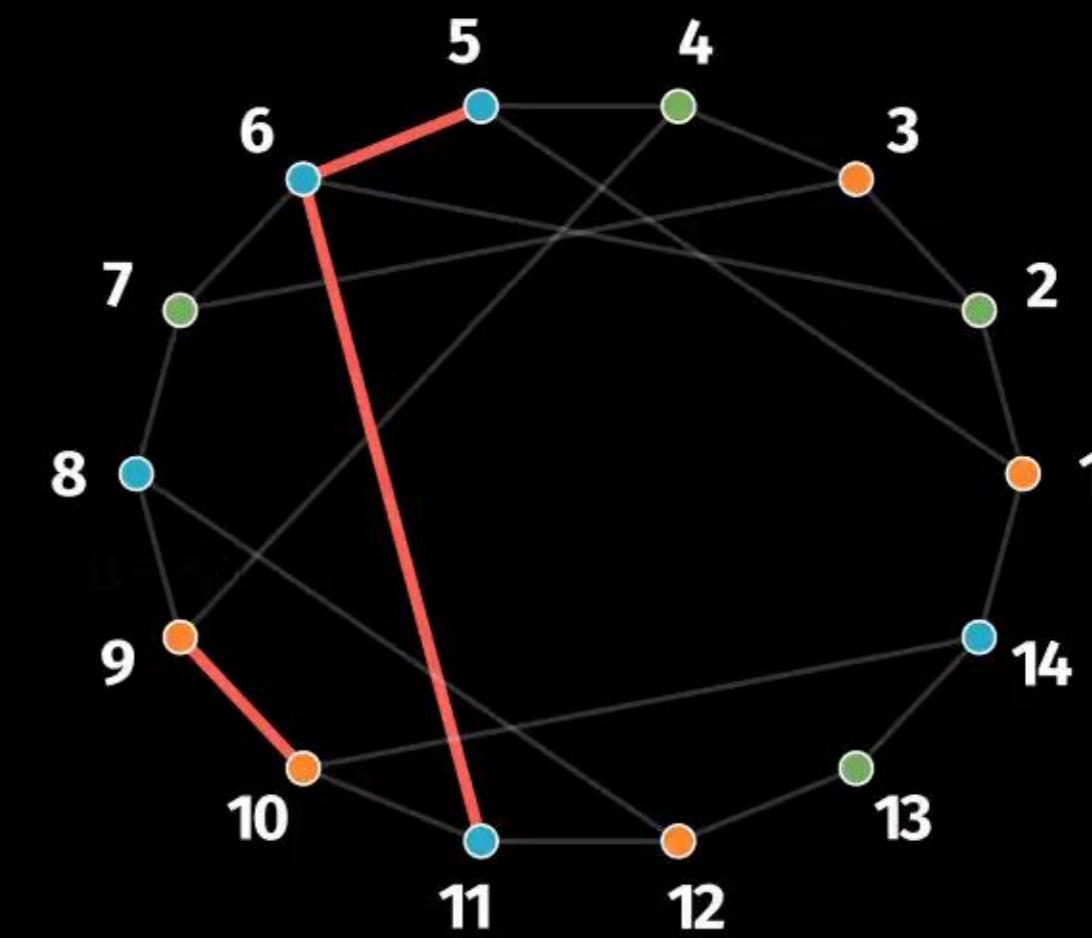
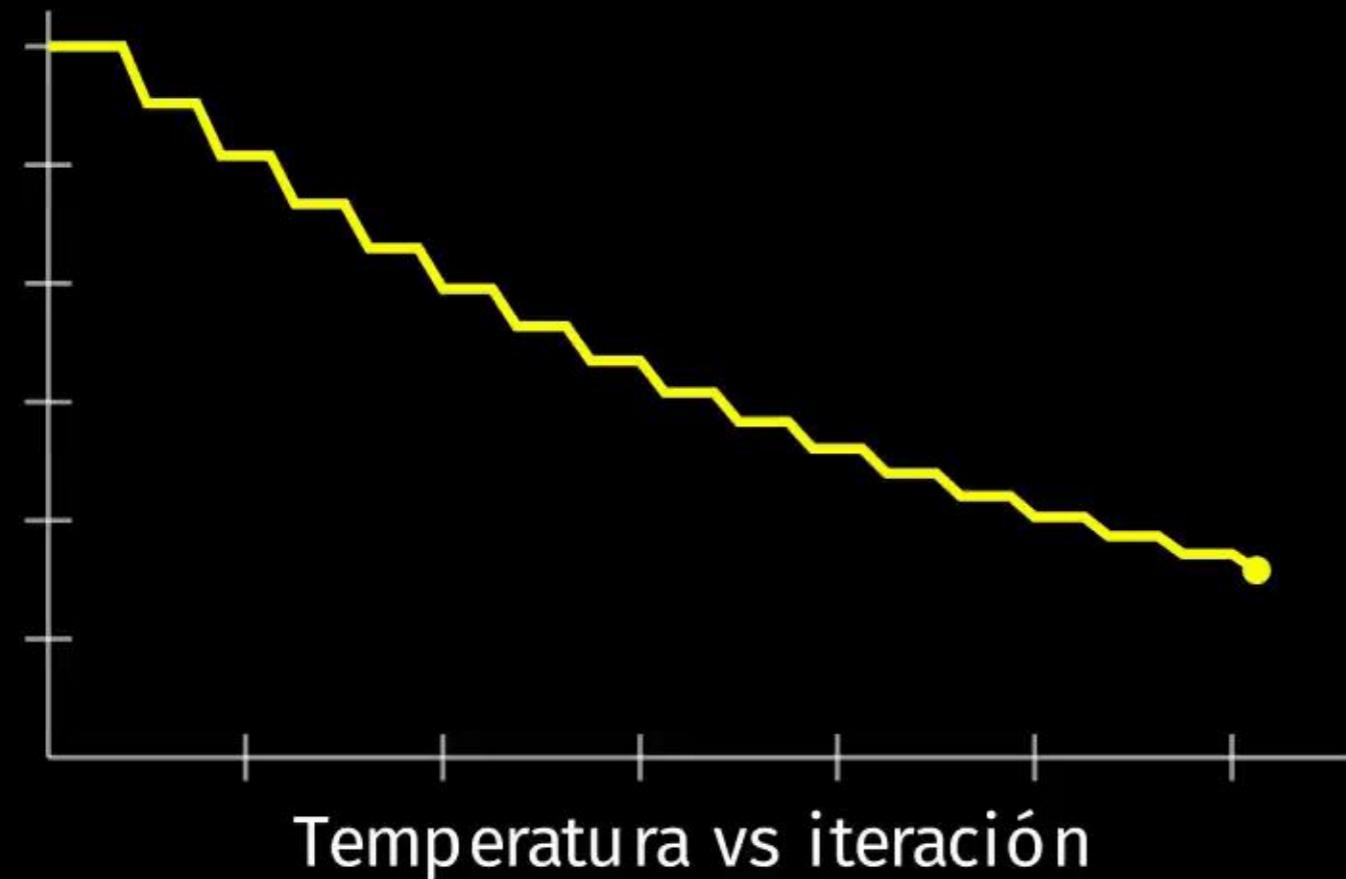
Temperatura: **0.573** Conflictos: **3** Incumbente: **2**  $\Delta$ : **+0**

# Simulated Annealing – Graph Coloring



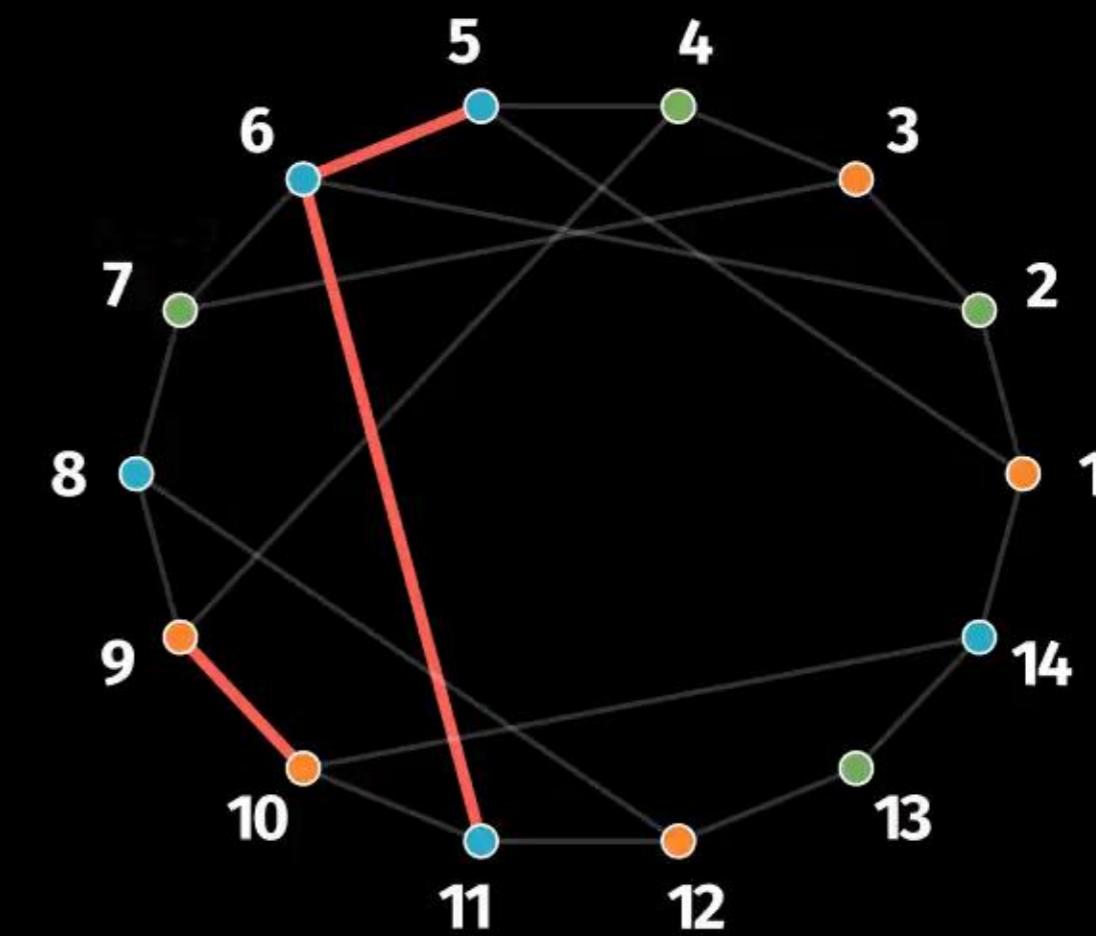
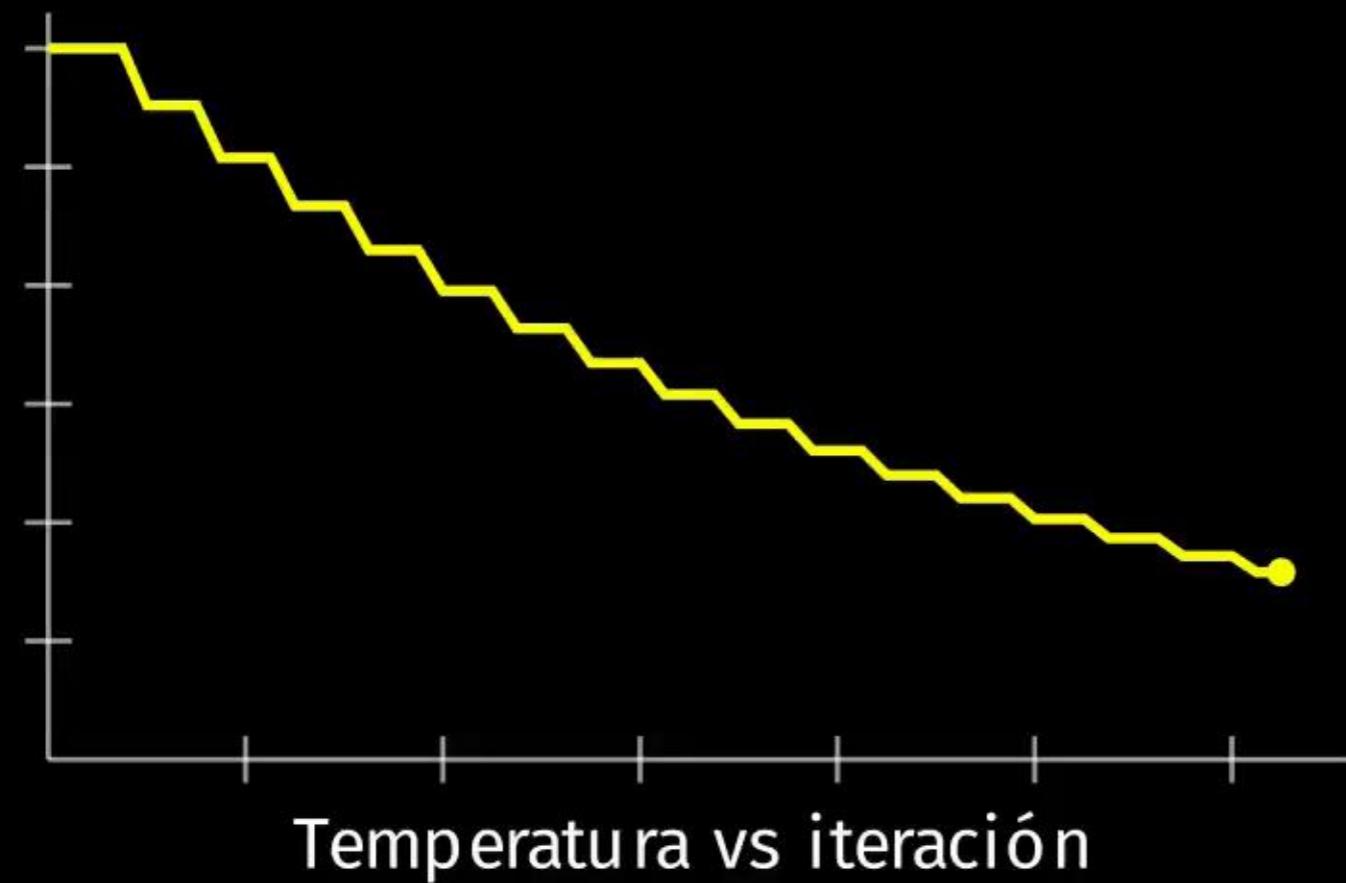
Temperatura: **0.573** Conflictos: **3** Incumbente: **2**  $\Delta$ : **+0**

# Simulated Annealing – Graph Coloring



Temperatura: **0.527** Conflictos: **3** Incumbente: **2**  $\Delta$ : **+0**

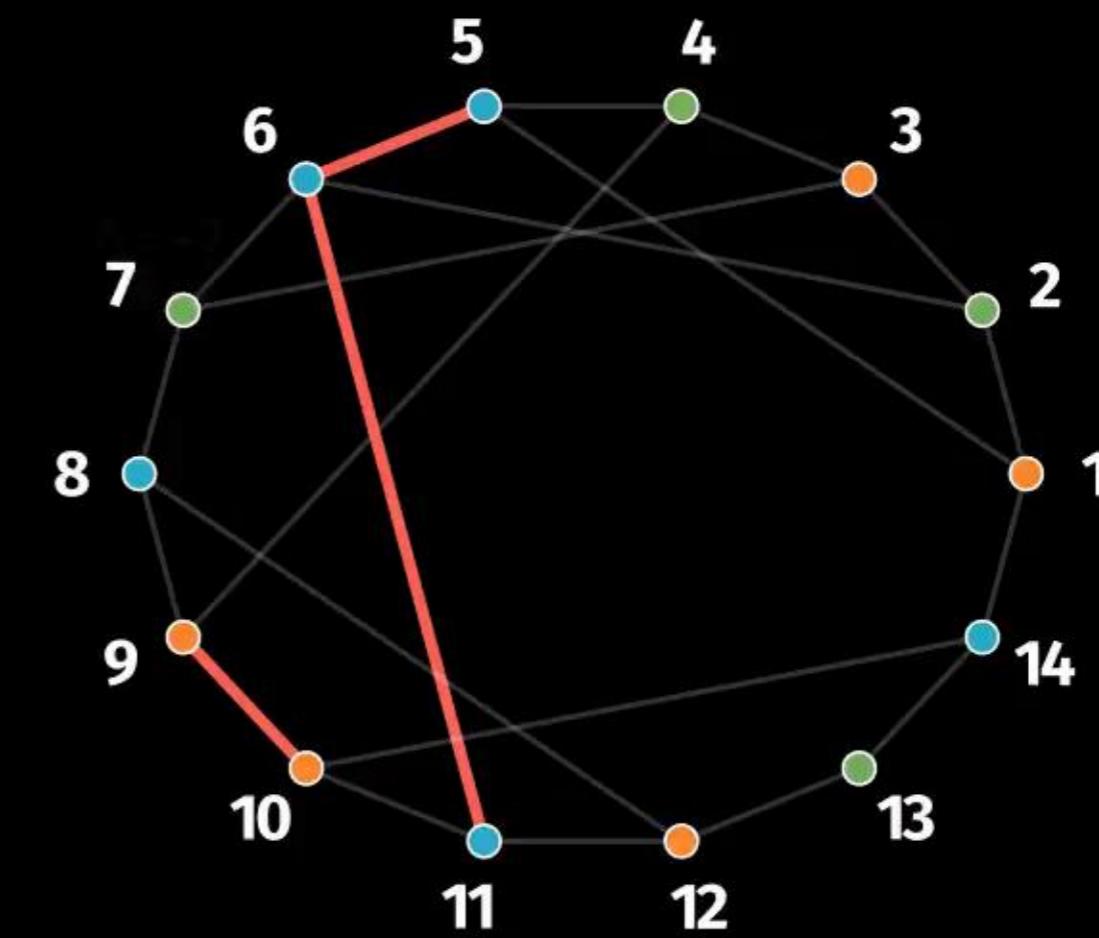
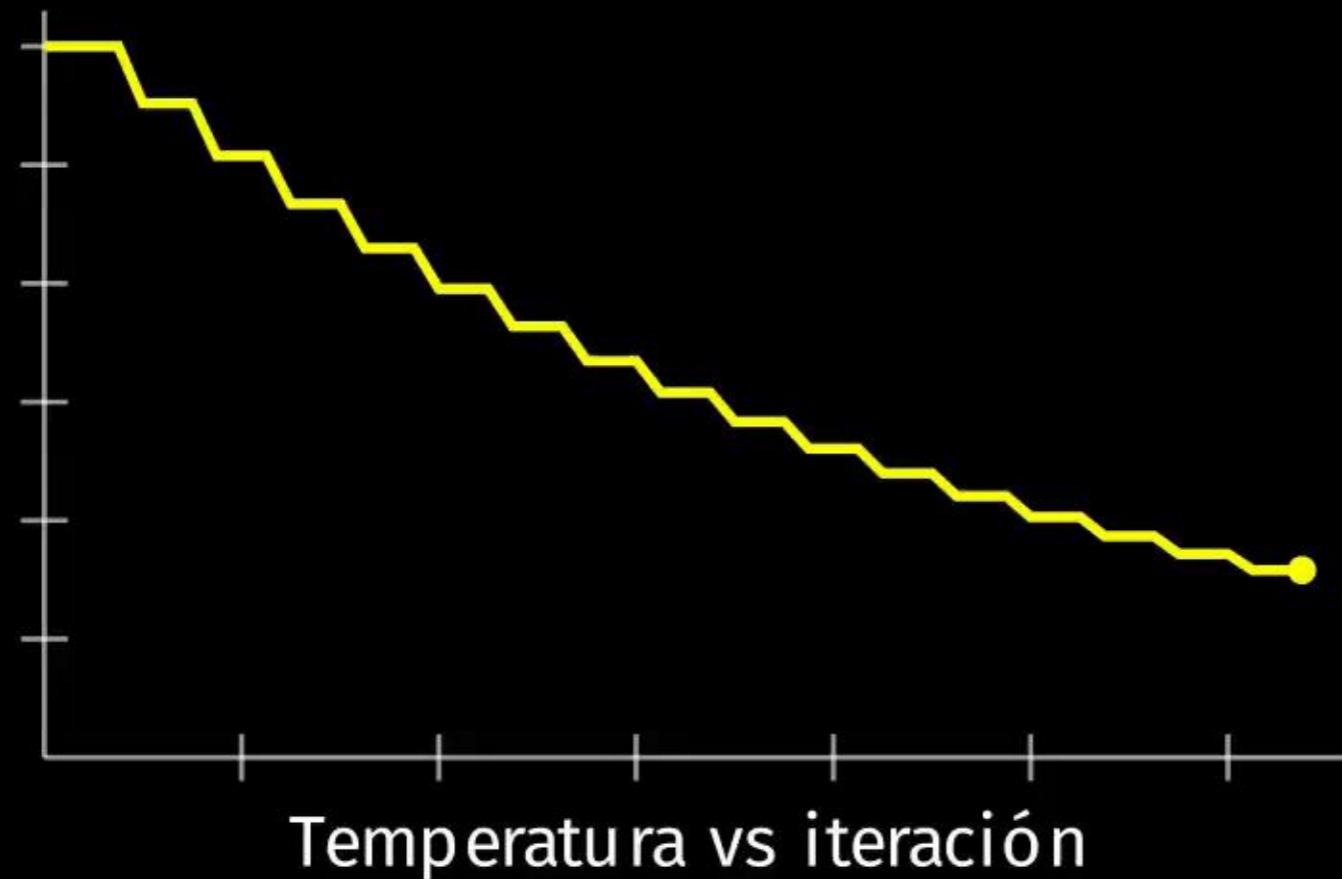
# Simulated Annealing – Graph Coloring



## Movimiento rechazado

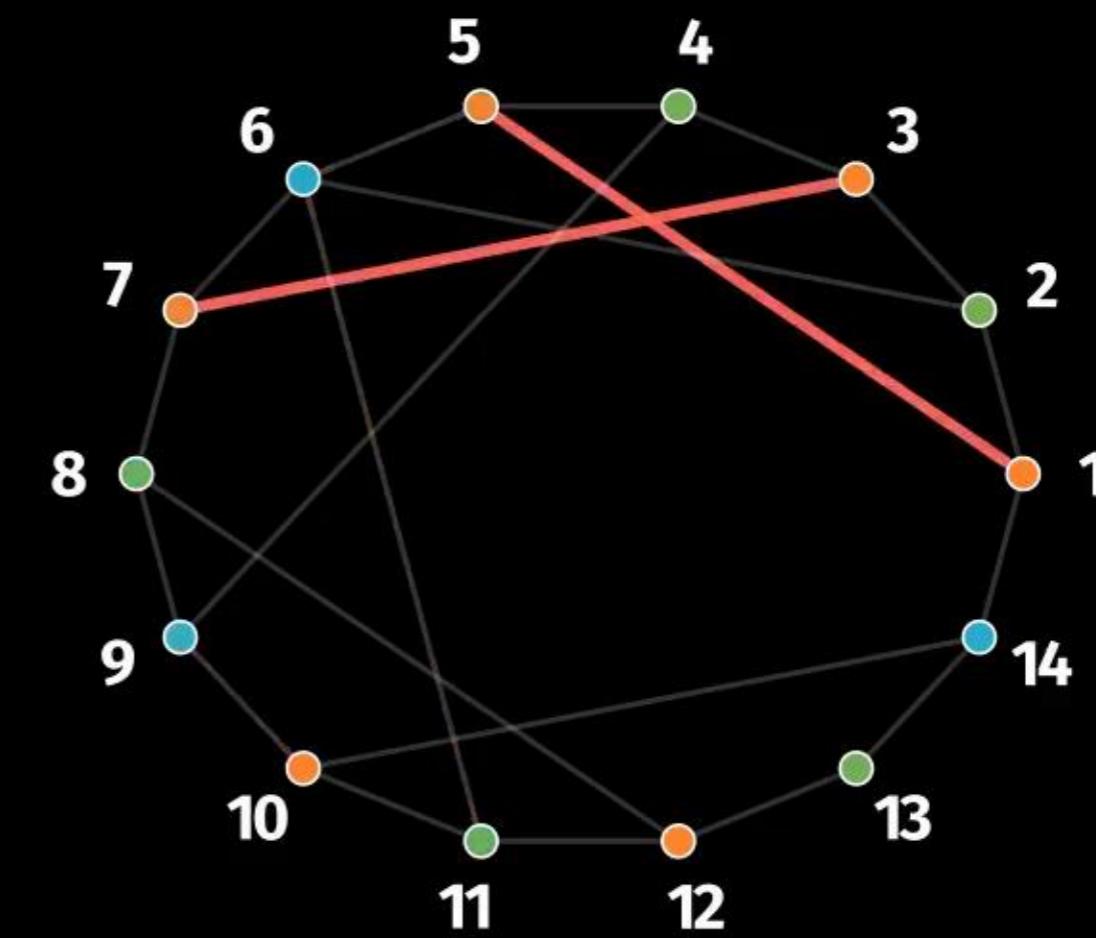
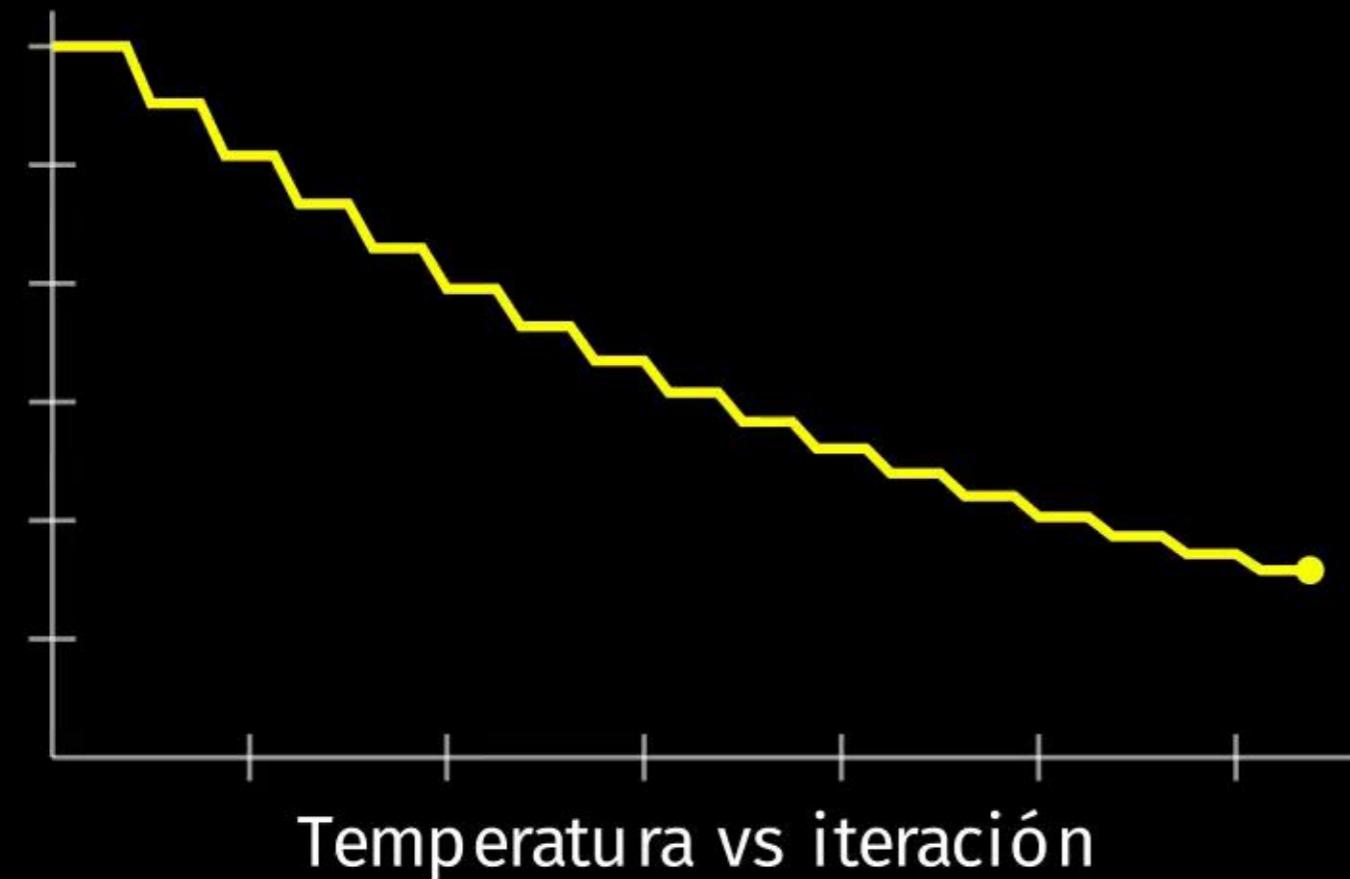
Temperatura: **0.527** Conflictos: **3** Incumbente: **2** Δ: **+2**

# Simulated Annealing – Graph Coloring



Temperatura: **0.527** Conflictos: **3** Incumbente: **2**  $\Delta$ : **+2**

# Simulated Annealing – Graph Coloring



Temperatura: **0.485** Conflictos: **2** Incumbente: **2**  $\Delta$ : **+2**

