2024/2025

1-

a) $\begin{cases} x^2+y^2=3u+2v \\ xy^2+y=uv \end{cases} \Leftrightarrow \begin{cases} x^2+y^2-3u-2v=0 \\ xy^2+y-uv=0 \end{cases}$

Então, temos $\vec{F}(x,y,u,v)=(x^2+y^2-3u-2v,\ xy^2+y-uv)$

$\frac{\partial F_1}{\partial x}(x,y,u,v)=2x \quad \frac{\partial F_1}{\partial y}(x,y,u,v)=2y \quad \frac{\partial F_1}{\partial u}(x,y,u,v)=-3 \quad \frac{\partial F_1}{\partial v}(x,y,u,v)=-2$

$\frac{\partial F_2}{\partial x}(x,y,u,v)=y^2 \quad \frac{\partial F_2}{\partial y}(x,y,u,v)=2xy+1 \quad \frac{\partial F_2}{\partial u}(x,y,u,v)=-v \quad \frac{\partial F_2}{\partial v}(x,y,u,v)=-u$

Todas as funções são contínuas pois são somas ou/e produtos de funções constantes e funções projeção (ambas funções contínuas).

Então, F é de classe C^1 e, por isso, é contínua, logo também é contínua numa vizinhança do ponto $(2,0,0,2)$

$\vec{F}(2,0,0,2)=(2^2+0^2-3.0-2.2,\ 2.0^2+0-0.2)=\vec{0}$

$\det(J\vec{F}_{(u,v)}(2,0,0,2))=\begin{vmatrix} -3 & -2 \\ -2 & 0 \end{vmatrix}=-4\neq 0$

Portanto, podemos aplicar o T.F.I e concluir que existe $\vec{f}$ tal que $(u,v)=\vec{f}(x,y)$ numa vizinhança de $(x,y,u,v)=(2,0,0,2)$

b) $Jf(2,0)=-(JF_{(u,v)}(2,0,0,2))^{-1}\cdot JF_{(x,y)}(2,0,0,2)$

$J(F_{(u,v)}(2,0,0,2))^{-1}=\begin{bmatrix} -3 & -2 \\ -2 & 0 \end{bmatrix}^{-1}=\frac{1}{-4}\begin{bmatrix} 0 & 2 \\ 2 & -3 \end{bmatrix}$

$J(F_{(x,y)}(2,0,0,2))=\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}$

Logo $Jf(2,0)=-\left(-\frac{1}{4}\right)\begin{bmatrix} 0 & 2 \\ 2 & -3 \end{bmatrix}\cdot\begin{bmatrix} 4 & 0 \\ 0 & 1 \end{bmatrix}=\frac{1}{4}\begin{bmatrix} 0 & 2 \\ 8 & -3 \end{bmatrix}=\begin{bmatrix} 0 & 1/2 \\ 2 & -3/4 \end{bmatrix}$

c) Como $\vec{f}(x,y)=(u(x,y),v(x,y))$

$\frac{\partial u}{\partial x}(x,y)=0 \quad \frac{\partial v}{\partial y}(x,y)=-\frac{3}{4}$

Então, $\operatorname{div}\vec{f}(x,y)=0+\left(-\frac{3}{4}\right)=-\frac{3}{4}$

2-

a) $\frac{\partial f}{\partial x}(x,y) = 4-9x^2$

$\frac{\partial f}{\partial y}(x,y) = 6-2y$

Logo, para descobrir os pontos críticos temos:

$\begin{cases} 4-9x^2 = 0 \\ 6-2y = 0 \end{cases} \Leftrightarrow \begin{cases} x = \pm\frac{2}{3} \\ y = 3 \end{cases}$ Então, os pontos críticos são: $(\frac{2}{3},3)$ e $(-\frac{2}{3},3)$

$\frac{\partial^2 f}{\partial x^2}(x,y) = -18x$

$\frac{\partial^2 f}{\partial y \partial x}(x,y) = \frac{\partial^2 f}{\partial x \partial y}(x,y) = 0$

$\frac{\partial^2 f}{\partial y^2}(x,y) = -2$

Então: $H(\frac{2}{3},3) = \begin{bmatrix} -12 & 0 \\ 0 & -2 \end{bmatrix}$ $d_1 < 0$, $d_2 > 0$

$H(-\frac{2}{3},3) = \begin{bmatrix} 12 & 0 \\ 0 & -2 \end{bmatrix}$ $d_1 > 0$, $d_2 < 0$

Logo: $(\frac{2}{3},3)$ é um máximo local

$(-\frac{2}{3},3)$ é um ponto sela

b) $f(1,2) = 5+4+6.2-4-3 = 14$

$\frac{\partial f}{\partial x}(1,2) = -5$ $\frac{\partial^2 f}{\partial x^2}(1,2) = -18$

$\frac{\partial f}{\partial y}(1,2) = 2$

Sendo $h_1 = (x-1)$ e $h_2 (y-2)$

Temos que:

$T_2(x,y) = f(1,2) + \frac{\partial f}{\partial x}(1,2)h_1 + \frac{\partial f}{\partial y}(1,2)h_2 + \frac{1}{2}\left(\frac{\partial^2 f}{\partial x^2}(1,2)h_1^2 + 2\frac{\partial^2 f}{\partial x \partial y}(1,2)h_1h_2\right) + \frac{\partial^2 f}{\partial y^2}(1,2)h_2^2 =$

$= 14 - 5h_1 + 2h_2 + \frac{1}{2}(18h_1^2 - 2h_2^2) = 14 - 5h_1 + 2h_2 + 9h_1^2 - h_2^2$

3-

a) $\frac{\partial f_1}{\partial x}(1,1)=2 \quad \frac{\partial f_1}{\partial y}(1,1)=1$

$\frac{\partial f_2}{\partial x}(1,1)=-2 \quad \frac{\partial f_2}{\partial y}(1,1)=-1$

Então, $\det(Df(1,1)) = \begin{vmatrix} 2 & 1 \\ -2 & -1 \end{vmatrix} = -2-(-2)=0$

Então, a afirmação é falsa pois $\det(Df(1,1))=0$

b) $d((1,0,2),(x,y,z)) = \sqrt{(x-1)^2+y^2+(z-2)^2}$, como $z=2+2x+y$ temos

$d((1,0,2),(x,y,z(x,y))) = \sqrt{(x-1)^2+y^2+(2x+y)^2}$

$f(x,y)=d^2=(x-1)^2+y^2+(2x+y)^2$

$\frac{\partial f}{\partial x}(x,y)=2(x-1)+4(2x+y)$

$\frac{\partial f}{\partial y}(x,y)=2y+2(2x+y)$

Então:

$$\begin{cases} 2(x-1)+4(2x+y)=0 \\ 2y+2(2x+y)=0 \end{cases} \Leftrightarrow \begin{cases} x-1+4x+2y=0 \\ y+2x+y=0 \end{cases} \Leftrightarrow \begin{cases} -3y=1 \\ x=-y \end{cases} \Leftrightarrow \begin{cases} y=-\frac{1}{3} \\ x=\frac{1}{3} \end{cases}$$

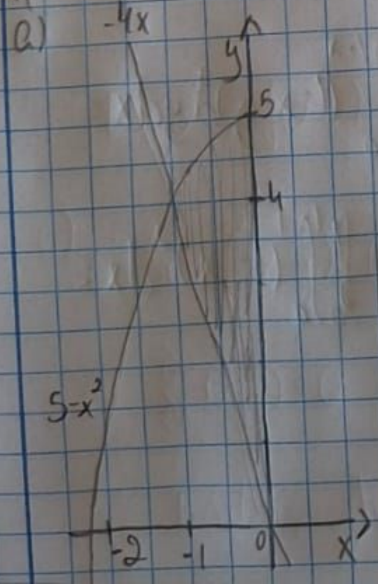
substituindo, por exemplo, no plano

$z=2+\frac{2}{3}-\frac{1}{3}=\frac{7}{3}$

Então, o ponto do plano mais próximo do ponto $(1,0,2)$ é o ponto $(1/3,-1/3,7/3)$ logo, a afirmação é verdadeira.

4-

a)



$$\int_{-1}^{0}\int_{-4x}^{5-x^2} f(x,y)\,dy\,dx = \int_0^4\int_{-1/4y}^{0} f(x,y)\,dx\,dy + \int_4^5\int_{-\sqrt{5-y}}^{0} f(x,y)\,dx\,dy$$

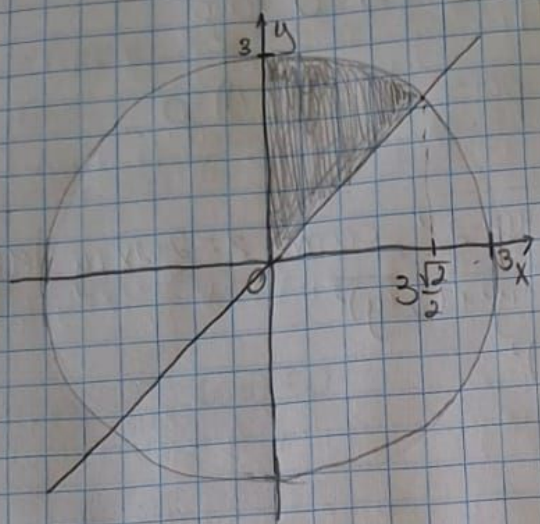
$y=5-x^2 \Leftrightarrow x=\pm\sqrt{5-y}$

$y=-4x \Leftrightarrow x=-\frac{1}{4}y$

b) $\int_{-1}^{0}\int_{-4x}^{5-x^2} dy\,dx = \int_{-1}^{0}(5-x^2+4x)\,dx = \left(5x-\frac{x^3}{3}+2x^2\right)\Big|_{x=-1}^{x=0} =$

$= -\left(-5-\frac{(-1)}{3}+2\right) = -\left(-3+\frac{1}{3}\right) = \frac{8}{3}$

5-
a)



sendo $y=x$

$x^2+x^2=9 \Leftrightarrow x=\pm 3\frac{\sqrt{2}}{2}$

$$\int_0^{3\frac{\sqrt{2}}{2}}\int_x^{\sqrt{9-x^2}} x\,dy\,dx$$

b)

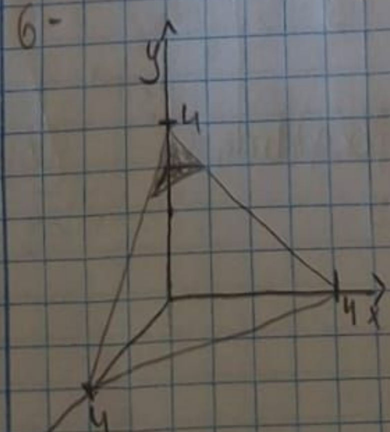
$$\int_0^3\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \rho^2\cos\theta\,d\theta\,d\rho$$

c) $x_c = \frac{m_y}{m}$

$$m=\int_0^3\int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \rho^2\cos\theta\,d\theta\,d\rho$$

$$m_y=\int_0^3\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \rho^3\cos\theta$$

6-



$\int_0^1\int_0^{1-x}\int_3^{4-x-y} dz\,dy\,dx = \int_0^1\int_0^{1-x}(1-x-y)\,dy\,dx =$

$= \int_0^1\left((1-x)^2-\frac{1}{2}(1-x)^2\right)dx = \frac{1}{2}\int_0^1(1-x)^2\,dx =$

$= -\frac{1}{2}\left(\frac{(1-x)^3}{3}\right)\Big|_{x=0}^{x=1} = \frac{1}{6}$