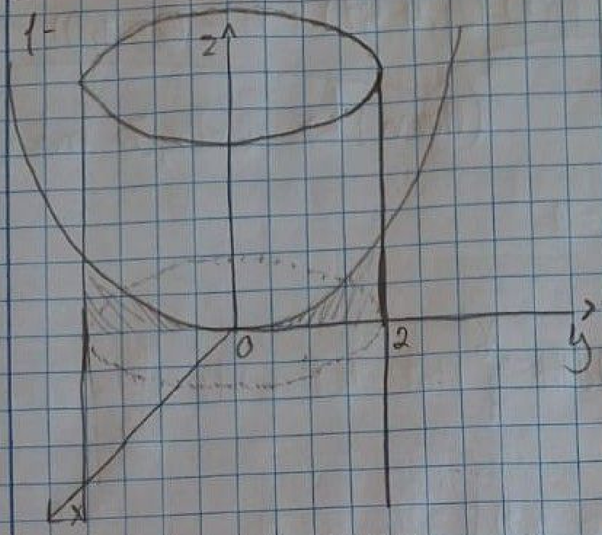
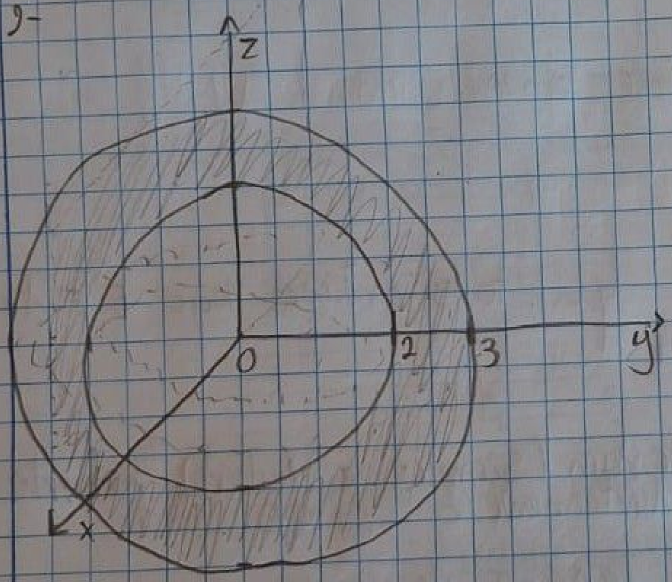
2022/2023-

1-



$$\int_0^2\int_0^{2\pi}\int_0^{\rho^2} f(\rho\cos\theta, \rho \operatorname{sen}\theta, z)\,\rho\, dz\, d\theta\, d\rho$$

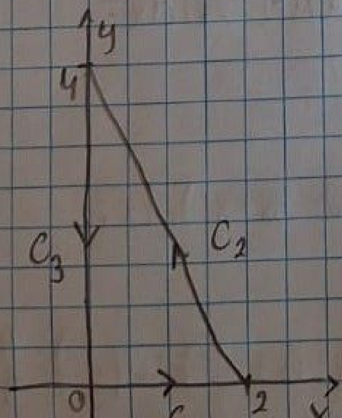
2-



$$\int_0^{\pi}\int_0^{\pi}\int_2^3 f(r\cos\theta \operatorname{sen}\varphi, r \operatorname{sen}\theta \operatorname{sen}\varphi, r\cos\varphi)\, r^2 \operatorname{sen}\varphi\, dr\, d\theta\, d\varphi$$

3-

a)



$C_1: tB + (1-t)A = t(2,0) + (1-t)(0,0) = (2t, 0) = \ell_1(t),\ t \in [0,1]$

$C_2: t(0,4) + (1-t)(2,0) = (2-2t, 4t) = \ell_2(t),\ t \in [0,1]$

$C_3: t(0,0) + (1-t)(0,4) = (0, 4-4t) = \ell_3(t),\ t \in [0,1]$

$$\left.\begin{aligned} \ell_1'(t) &= (2,0) \neq \vec{0} \\ \ell_2'(t) &= (-2,4) \neq \vec{0} \\ \ell_3'(t) &= (0,-4) \neq \vec{0} \end{aligned}\right\} \forall t \in \operatorname{int}([0,1])$$

$$L(C) = L(C_1) + L(C_2) + L(C_3) = \int_0^1 (2 + \sqrt{20} + 4)\, dt = 6 + \sqrt{20}$$

b) $\vec{F}(x,y) = (1+xy, x-y)$

$W = \int_C F\, ds$ $\quad \underbrace{\ell_1(t) = (0, 4t) \quad \ell_2(t) = (2t, 4-4t) \quad \ell_3(t) = (2-2t, 0)}_{\text{(Provar)} \ \hookrightarrow \text{orientação indireta}}$

$W = W_1 + W_2 + W_3$

$W_1 = \int_0^1 (1, -4t).(0, 4)\, dt = -8$

$W_2 = \int_0^1 (1 + 2t(4-4t), 6t-4)(2, -4)\, dt = \frac{26}{3}$

$W_3 = \int_0^1 (1, 2-2t)(-2, 0)\, dt = -2$

$W = \frac{26}{3} - 2 - 8 = -\frac{4}{3}$

c) $\frac{\partial F_1}{\partial x}(x,y) = y \qquad \frac{\partial F_1}{\partial y}(x,y) = x$

$\frac{\partial F_2}{\partial x}(x,y) = 1 \qquad \frac{\partial F_2}{\partial y}(x,y) = -1$

$\Bigg\{$ ... todas contínuas logo F é de classe C^1

C é uma curva de Jordan (simples e fechada), parametrizada regularmente e orientada diretamente e C_{int} de D.

Então, podemos aplicar o Teorema de Green.

$$\int_C F\, ds = \int_0^2 \int_0^{4-2x} (1-x)\, dy\, dx = \int_0^2 (1-x)(4-2x)\, dx =$$

$$= \int_0^2 (4 - 6x + 2x^2)\, dx = 2\int_0^2 (2 - 3x + x^2)\, dx =$$

$$= 2\left(2 \times 2 - \frac{3}{2} \times 2^2 + \frac{1}{3} \times 2^3\right) = 2\left(4 - 6 + \frac{8}{3}\right) = 2\left(\frac{2}{3}\right) = \frac{4}{3}$$

O resultado é simétrico pois o Teorema de Green calcula utilizando a orientação direta

4-

a) $f(x,y) = xe^{3y} - y\cos x + g(y)$

$\frac{\partial f}{\partial y}(x,y) = 3xe^{3y} - \cos x + g'(y) \Rightarrow g'(y) = 0 \Rightarrow g(y) = c,\ c \in \mathbb{R}$

Logo, $f(x,y) = xe^{3y} - y\cos x + c,\ c \in \mathbb{R}$

Então como existe $f(x,y)$ tal que $\nabla f(x,y) = F(x,y)$ F é um campo conservativo

FALTOU PROVAR

b) Como F é um campo conservativo ($\in C^1$), sendo f a função potencial, podemos aplicar o Teorema fundamental das integrais de linha:

$$\int_C F \cdot ds = f(-3,0) - f(3,0) = -3 - 3 = -6$$

c) Sim, a afirmação está correta, se for uma curva fechada e F for campo conservativo então,

$$\int_\gamma F \cdot ds = 0$$

5-

$$\frac{\partial \vec{r}}{\partial u}(u,v) \times \frac{\partial \vec{r}}{\partial v}(u,v) = \begin{vmatrix} e_1 & e_2 & e_3 \\ \cos v & \operatorname{sen} v & 1 \\ -u\operatorname{sen} v & u\cos v & 0 \end{vmatrix} = (-u\cos v, -u\operatorname{sen} v, u) \neq$$

$$\neq \vec{0},\ \forall (u,v) \in \operatorname{int}([0,1] \times [0,2\pi])$$

Logo $\vec{r}(u,v)$ é uma parametrização regular

$$\int_0^{2\pi}\int_0^1 (1+u^2)\,\|(-u\cos v, -u\operatorname{sen} v, u)\|\,du\,dv =$$

$$= \int_0^{2\pi}\int_0^1 (1+u^2)\cdot\sqrt{2}\cdot u\,du\,dv = \int_0^{2\pi}\int_0^1 (\sqrt{2}\cdot u + \sqrt{2}u^3)\,du\,dv =$$

$$= \sqrt{2}\int_0^{2\pi} \frac{3}{4}\,dv = \frac{3\sqrt{2}}{2}\pi$$