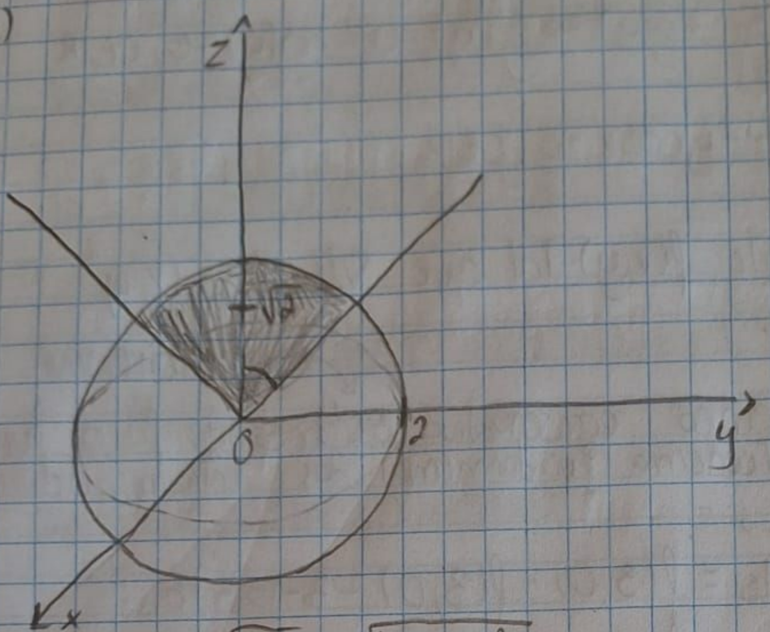
2023/2024

1-

a)



$$\int_{-2}^{2}\int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}}\int_{\sqrt{x^2+y^2}}^{\sqrt{4-x^2-y^2}} 1\,dz\,dy\,dx$$

b)

$$\int_{0}^{2\pi}\int_{0}^{2}\int_{\rho}^{\sqrt{4-\rho^2}} \rho\,dz\,d\rho\,d\theta$$

c)

$x^2+y^2=4-z^2$

$z=\sqrt{x^2+y^2} \Leftrightarrow z^2=x^2+y^2$

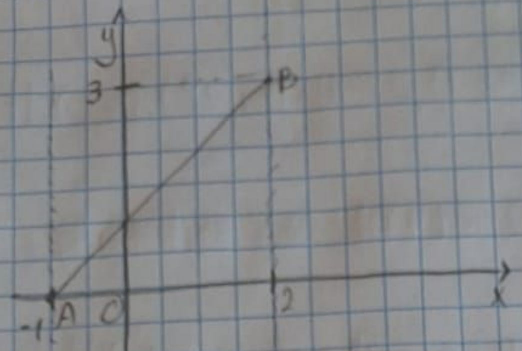
Logo $z^2=4-z^2 \Leftrightarrow z^2=2 \Leftrightarrow z=\pm\sqrt{2} \Rightarrow z=\sqrt{2}$

$\cos(\varphi)=\frac{\sqrt{2}}{2} \Leftrightarrow \varphi=\cos^{-1}\left(\frac{\sqrt{2}}{2}\right) \Leftrightarrow \varphi=\frac{\pi}{4}$

$$\int_{0}^{2\pi}\int_{0}^{2}\int_{0}^{\frac{\pi}{4}} r^2 \operatorname{sen}\varphi\, d\varphi\, dr\, d\theta$$

2)

a)



$C_2: tB + (1-t)A = t(2,3) + (1-t)(-1,0) = (2t,3t) + (t-1,0) = (3t-1,3t) = \ell(t)$

$\ell_2'(t) = (3,3) \neq 0 \ \forall t \in \text{int}([0,1])$

$L(C) = \int_0^1 \|\ell_2'(t)\| \, dt = \int_0^1 \sqrt{3^2+3^2} \, dt = \int_0^1 \sqrt{18} \, dt = 3\sqrt{2}$

b)

$C_1: \ell(t) = (t, t^2-1), \ t \in [-1,2]$

$\ell_1'(t) = (1, 2t) \neq \vec{0}, \ \forall t \in \text{int}([-1,2])$

$\|\ell'(t)\| = \sqrt{1+4t^2}$

Sendo $f(x,y) = x^2y^4$, temos:

$$\int_{-1}^{2} f(t, \ell(t)) \|\ell'(t)\| \, dt = \int_{-1}^{2} t^2(t^2-1)^4 \sqrt{1+4t^2} \, dt$$

c)

$f(x,y) = x^3y - x^2 + g(y)$

$\frac{\partial f}{\partial y}(x,y) = x^3 + g'(y) \Rightarrow g'(y) = 2y \Rightarrow g(y) = y^2$

Logo $f(x,y) = x^3y - x^2 + y^2 + c, \ c \in \mathbb{R}$

Como existe $f(x,y)$ tal que $\nabla f(x,y) = \vec{F}(x,y)$ logo $\vec{F}(x,y)$ é um campo conservativo

Sendo $f(x,y)$ a função potencial

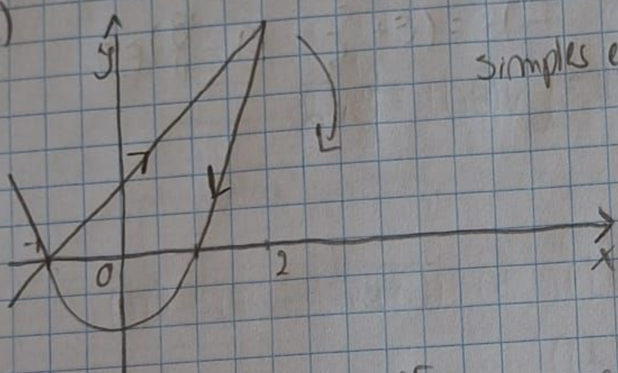
d) $F(x,y)=(3yx^2-2x, 2y+x^3)$ $\quad \rho(t)=(3t-1, 3t)$

$W=\int_C F\,ds=\int_0^1 (3(3t)(3t-1)^2-2(3t-1), 2(3t)+(3t-1)^3)\cdot(3,3)\,dt=$

$=3\int_0^1 (81t^3-54t^2+9t-6t+2+27t^3-27t^2+15t-1)\,dt$

$=3\int_0^1 (108t^3-81t^2+18t+1)\,dt=3\left(27t^4-27t^3+9t^2+t\right)\Big|_{t=0}^{t=1}=30$

e)



Como podemos ver a curva é simples e fechada:

Como C é uma curva fechada e F é um campo conservativo então:

$\int_C F\,ds=0$

$\frac{\partial F_1}{\partial x}(x,y)=6yx-2$ $\quad \frac{\partial F_1}{\partial y}(x,y)=3x^2$

$\frac{\partial F_2}{\partial x}(x,y)=3x^2$ $\quad \frac{\partial F_2}{\partial y}(x,y)=2$

... logo F é de classe C^1

3- $\rho(u,v)=(v, 3\cos u, 3\operatorname{sen} u)$, $\rho:[0,2\pi]\times[2,3]$

$\frac{\partial \vec{\rho}}{\partial u}(u,v)\times\frac{\partial \vec{\rho}}{\partial v}(u,v)=(0, 3\cos u, 3\operatorname{sen} u)\neq 0 \quad \forall (u,v)\in[0,2\pi]\times[2,3]$

$A(S)=\int_2^3\int_0^{2\pi} \|(0, 3\cos u, 3\operatorname{sen} u)\|\,du\,dv=\int_2^3\int_0^{2\pi} 3\,du\,dv=$

$=\int_2^3 6\pi\,dv=6\pi$

4 - $\vec{r}(u,v) = (\cos u \operatorname{sen} v, \operatorname{sen} u \operatorname{sen} v, \cos v)$ $\vec{r}: [0, 2\pi] \times [0, \pi] \to \mathbb{R}^3$

$$\frac{\partial \vec{r}}{\partial u}(u,v) \times \frac{\partial \vec{r}}{\partial v}(u,v) = (-\operatorname{sen}^2 v \cos u, -\operatorname{sen}^2 v \operatorname{sen} u, -\operatorname{sen} v \cdot \cos v) \neq$$

$$\neq \vec{0} \quad \forall t \in ([0, 2\pi] \times [0, \pi])$$

$$\left\| \frac{\partial \vec{r}}{\partial u}(u,v) \times \frac{\partial \vec{r}}{\partial v}(u,v) \right\| = \operatorname{sen} v$$

$\frac{\partial F_1}{\partial x}(x,y,z) = 1$ $\frac{\partial F_1}{\partial y}(x,y,z) = 0$ $\frac{\partial F_1}{\partial z}(x,y,z) = 0$

$\frac{\partial F_2}{\partial x}(x,y,z) = 0$ $\frac{\partial F_2}{\partial y}(x,y,z) = 1$ $\frac{\partial F_2}{\partial z}(x,y,z) = 0$

$\frac{\partial F_3}{\partial x}(x,y,z) = 0$ $\frac{\partial F_3}{\partial y}(x,y,z) = 0$ $\frac{\partial F_3}{\partial z}(x,y,z) = 1$

Como C é uma curva de Jordan regular com orientação positiva e F é de classe C^1 com $S \cup C \subseteq E$

... logo F é de classe C^1

Podemos aplicar o teorema de Gauss

$$\operatorname{div} \vec{F}(x,y,z) = 3$$

$$\int_C F \, ds = \int_{-1}^{1} \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_{\sqrt{1-x^2-y^2}}^{\sqrt{1-x^2-y^2}} 3 \, dz \, dy \, dx = \int_0^{2\pi} \int_0^{\pi} \int_0^1 3r^2 \operatorname{sen} \varphi \, dr \, d\varphi \, d\theta =$$

$$= \int_0^{2\pi} \int_0^{\pi} \operatorname{sen} \varphi \, d\varphi \, d\theta = \int_0^{2\pi} (-\cos \varphi) \Big|_{\varphi=0}^{\varphi=\pi} d\theta = \int_0^{2\pi} 2 \, d\theta = 4\pi$$