Prepratory work for TTK4115 - Linear System Theory Lab day 4

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2.4 Part IV Kalman filter

2.4.1 Problem 1 - Discretization

The following discrete time model of the system is given:

$$
\begin{aligned}
\boldsymbol{x}[k+1] &= \boldsymbol{A_d}\boldsymbol{x}[k] + \boldsymbol{B_d}\boldsymbol{u}[k] + \boldsymbol{w_d}[k] \\
\boldsymbol{y}[k] &= \boldsymbol{C_d}\boldsymbol{x}[k] + \boldsymbol{v_d}[k] \\
\boldsymbol{w_d} \sim N(\boldsymbol{0}, \boldsymbol{Q_d}), &\qquad \boldsymbol{v_d} \sim N(\boldsymbol{0}, \boldsymbol{R_d})
\end{aligned}
$$

And the subsequent full state-space model $\boldsymbol{x}$ is to be used:

$$
\boldsymbol{x} = \begin{bmatrix} p \\ \dot{p} \\ \dot{e} \\ \dot{e} \\ \lambda \\ \dot{\lambda} \end{bmatrix}
$$

With this extended state space model we get the system in continuous time:

$$
\boldsymbol{A_c} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{L_2}{J_\lambda} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}
$$

$$
\boldsymbol{B_c} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{l_p K_f}{J_p} \\ 0 & 0 \\ \frac{l_h K_f}{J_e} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}
$$

$$
\boldsymbol{C_c} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
$$

This model is then discretized by using the “c2d” function in MATLAB, and we obtain the following matrixes:

$$
\boldsymbol{A_d} = \begin{bmatrix} 1 & 0.002 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.002 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.002 \\ 0.0012 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}
$$

$$B_d = \begin{bmatrix} 0 & 0 \\ 0 & 0.0011 \\ 0 & 0 \\ 0.0002 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$C_d = \begin{bmatrix} 0 & 0 & 1 & 0.001 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.001 \end{bmatrix}$$

2.4.2 Problem 2 - experimentation

When choosing values along the diagonal for $\boldsymbol{Q_d}$, the relation between the given element in the $\boldsymbol{Q_d}$ matrix and $\boldsymbol{R_d}$ matrix will decide how much the Kalman filter will be willing to change its priori estimate towards the measured value. A higher value in $\boldsymbol{Q_d}$ will result in a higher variance in $\boldsymbol{P}$, meaning that the system will trust the measurements more and thus be more willing to change its value towards the measured value. A lower value in $\boldsymbol{Q_d}$ will lead to the opposite result, meaning that the system will trust the model more and thus less willing to change its value towards the measured value.

A test-plan for different values for the $\boldsymbol{Q_d}$ matrix for the Kalman filter that are to be tested:

Case	Scaling		Hypothesis
	$\boldsymbol{Q}_d$	$\boldsymbol{R}_d$	
1.1	1	1	The system is stable and goes to reference. No s.d.
1.2	100	1	Noisy estimations if there is noisy measurements
1.3	$\frac{1}{100}$	1	Estimations will follow the model, giving more stable estimations, but might give higher error.
1.4	∞	1	The estimations will be equal to the measurements
1.5	0	1	The estimations will not be affected by the measurements