Prepratory work for
TTK4115 - Linear System Theory
Lab day 3

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Part III - Luenberger observer

2.3.1 Problem 1 - Extended state-space formulation

$$\begin{bmatrix} \dot{p} \\ \ddot{p} \\ \dot{e} \\ \ddot{e} \\ \ddot{\lambda} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ -\frac{L_2}{J_\lambda} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} p \\ \dot{p} \\ e \\ \dot{e} \\ \dot{\lambda} \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{l_p K_f}{J_p} \\ 0 & 0 \\ \frac{l_h K_f}{J_e} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{V}_s \\ V_d \end{bmatrix}$$

2.3.2 Problem 2 - Observability

When continuously measuring a state over time, it is possible to calculate its derivative. So by observing e, we can derive an expression for $\dot{e}$. And by observing $\dot{\lambda}$, we can derive an expression for $\ddot{\lambda}$. Since $\ddot{\lambda}$ is an expression of p, the expression can be rearranged to find p and its derivative $\dot{p}$. Thus the system is observable with measuring only two states: $\dot{\lambda}$ and e.

$$\ddot{\lambda} = -\frac{L_2}{J_\lambda} p$$

$$p = -\frac{J_\lambda}{L_2} \ddot{\lambda}$$

$$C = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathcal{O} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ \frac{-L_2}{J_\lambda} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & \frac{-L_2}{J_\lambda} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

2.3.3 Problem 3 - Angle measurement

The equations for p and e were found graphically, with the use of vectors and trigonometry. The following drawing shows a sketch of the helicopter seen from its side, with some elevation and zero pitch:

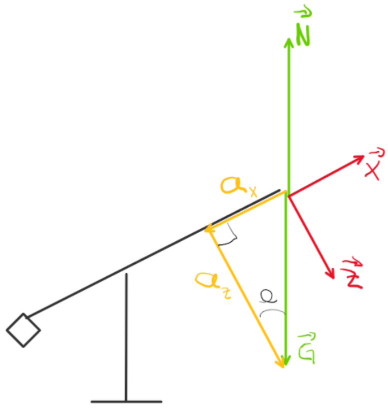

Figure 1: Helicopter model viewed into the y-axis of the fixed rotor frame

From this sketch, one can build a right angled triangle with the normal and acceleration forces:

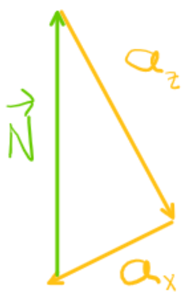

Figure 2: Restructured model illustrating the normal force and acceleration forces

If one turns its point of view of the model to look straight onto the a_z-axis, one gets the following sketch:

Note that the helicopter still has zero pitch.

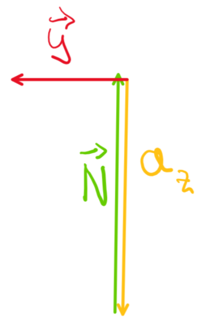

Figure 3: Model turned 90°to look straight into the a_z-axis

Now, if some pitch is introduced, one gets the following sketch:

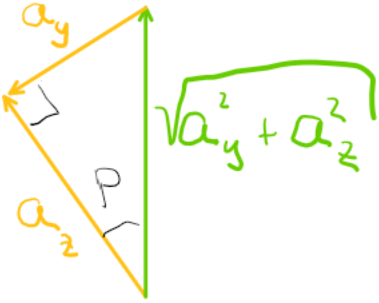

Figure 4: Restructured model with pitch

From this model, one can quite easily express the angles of pitch and elevation using the force measurements from the IMU:

$$p = \arctan\left(\frac{a_y}{a_z}\right)$$
$$e = \arctan\left(\frac{a_x}{\sqrt{a_y^2 + a_z^2}}\right)$$

2.3.4 Problem 4 - State estimator

By using the function "place" in MATLAB, the observer gain $\boldsymbol{L}$ was manually determined with pole placement. As mentioned in **2.3.2 Problem 2 - Observability** the system is only observable when measuring at least the two states $\dot{\lambda}$ and e. Thus when trying to place estimators poles when having a $\boldsymbol{C}$ matrix that results in an unobservable system, MATLAB will give out an error that the system is not controllable. As placing a pole of an state that is not observable would not make the system observable. This verifies the theoretical relation between observability and placing of estimator poles. For testing the observer with the different pole placements the **C** matrix is chosen to be the identity matrix for case 1.1 to 1.9. For case 1.10, **C** is chosen to what was derived in **2.3.2 Problem 2 - Observability** and for case 1.11 p is added to the matrix.

A test-plan for different choices for the poles of the estimator that are to be tested:

Case	Poles		
	p1 & p2	p3 & p4	p5
1.1, 1.1.1	$-10cos(25°) \pm i10sin(25°)$	$-10cos(12.5°) \pm i10sin(12.5°)$	-10
1.2	$-20cos(25°) \pm i20sin(25°)$	$-20cos(12.5°) \pm i20sin(12.5°)$	-20
1.3	$-100cos(25°) \pm i100sin(25°)$	$-100cos(12.5°) \pm i100sin(12.5°)$	-100
1.4	$-1cos(25°) \pm i1sin(25°)$	$-1cos(12.5°) \pm i1sin(12.5°)$	-1
1.5	$-4cos(25°) \pm i4sin(25°)$	$-4cos(12.5°) \pm i4sin(12.5°)$	-4
1.6	$-10cos(70°) \pm i10sin(70°)$	$-10cos(12.5°) \pm i10sin(12.5°)$	-10
1.7	$-100cos(80°) \pm i100sin(80°)$	$-100cos(12.5°) \pm i100sin(12.5°)$	-10
1.8	$-10cos(25°) \pm i10sin(25°)$	$-30cos(12.5°) \pm i30sin(12.5°)$	-20
1.9	$-30cos(25°) \pm i30sin(25°)$	$-10cos(12.5°) \pm i10sin(12.5°)$	-20
1.10	$-10cos(25°) \pm i10sin(25°)$	$-10cos(12.5°) \pm i10sin(12.5°)$	-10
1.11	$-10cos(25°) \pm i10sin(25°)$	$-10cos(12.5°) \pm i10sin(12.5°)$	-10

Case 1.1 - Poles placed on the circle $x^2 + y^2 = 10^2$

Expect the estimators to converge and that the system is stable.

Case 1.2 - Poles placed on the circle $x^2 + y^2 = 20^2$

Expect the estimators to converge and that the system is stable, but a more noisy estimator

Case 1.3 - Poles placed on the circle $x^2 + y^2 = 100^2$

Same as Case 1.2, but more noise.

Case 1.4 - Poles placed on the circle $x^2 + y^2 = 1^2$

Bad estimations as the time constant of the poles are to high, resulting in a slow estimator.

Case 1.5 - Poles placed on the circle $x^2 + y^2 = 4^2$

Bad estimations as the time constant of the poles are to high, but better than 1.4.

Case 1.6 - Poles placed on the circle $x^2 + y^2 = 10^2$, but with higher imaginary part

High overshooting and a slow estimator.

Case 1.7 - Poles placed on the circle $x^2 + y^2 = 100^2$

High overshooting and a quicker estimator than case 1.6.

Case 1.8 - Poles distrubed in the left half plane

A quicker estimator for elevation, and more noisy estimator for pitch

Case 1.9 - Poles distrubed in the left half plane

A quicker estimaton for pitch and more noisy estimator for elevation

Case 1.10 - Poles placed on the circle $x^2 + y^2 = 10^2$

Bad estimations, as the dependency $\ddot{\lambda}$ has on p has to be calculated using constants that might be inaccurate.

Case 1.11 - Poles placed on the circle $x^2 + y^2 = 10^2$

Better estimations than case 1.10, but still with some noise. Especially for the states that are not measured from the IMU.