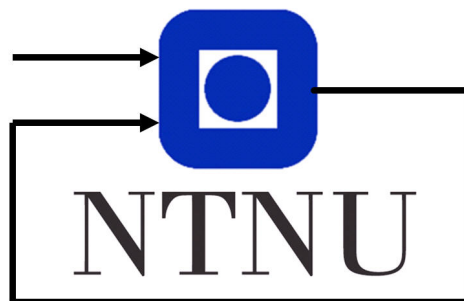


Prepratory work for
TTK4115 - Linear System Theory
Lab day 4

Group 11
Håvard Olai Kopperstad
Khuong Huynh
Tomas Nils Tellier

November 2022



Department of Engineering Cybernetics

2.4 Part IV Kalman filter

2.4.1 Problem 1 - Discretization

The following discrete time model of the system is given:

$$\begin{aligned} \mathbf{x}[k+1] &= \mathbf{A}_d \mathbf{x}[k] + \mathbf{B}_d \mathbf{u}[k] + \mathbf{w}_d[k] \\ \mathbf{y}[k] &= \mathbf{C}_d \mathbf{x}[k] + \mathbf{v}_d[k] \\ \mathbf{w}_d &\sim N(\mathbf{0}, \mathbf{Q}_d), \quad \mathbf{v}_d \sim N(\mathbf{0}, \mathbf{R}_d) \end{aligned}$$

And the subsequent full state-space model \mathbf{x} is to be used:

$$\mathbf{x} = \begin{bmatrix} p \\ \dot{p} \\ \dot{e} \\ e \\ \lambda \\ \dot{\lambda} \end{bmatrix}$$

With this extended state space model we get the system in continuous time:

$$\mathbf{A}_c = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ \frac{L_2}{J_\lambda} & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{B}_c = \begin{bmatrix} 0 & 0 \\ 0 & \frac{l_p K_f}{J_p} \\ 0 & 0 \\ \frac{l_h K_f}{J_e} & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{C}_c = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

This model is then discretized by using the “c2d” function in MATLAB, and we obtain the following matrixes:

$$\mathbf{A}_d = \begin{bmatrix} 1 & 0.002 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0.002 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.002 \\ 0.0012 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{B}_d = \begin{bmatrix} 0 & 0 \\ 0 & 0.0011 \\ 0 & 0 \\ 0.0002 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\mathbf{C}_d = \begin{bmatrix} 0 & 0 & 1 & 0.001 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0.001 \end{bmatrix}$$

2.4.2 Problem 2 - experimentation

When choosing values along the diagonal for \mathbf{Q}_d , the relation between the given element in the \mathbf{Q}_d matrix and \mathbf{R}_d matrix will decide how much the Kalman filter will be willing to change its priori estimate towards the measured value. A higher value in \mathbf{Q}_d will result in a higher variance in \mathbf{P} , meaning that the system will trust the measurements more and thus be more willing to change its value towards the measured value. A lower value in \mathbf{Q}_d will lead to the opposite result, meaning that the system will trust the model more and thus less willing to change its value towards the measured value.

A test-plan for different values for the \mathbf{Q}_d matrix for the Kalman filter that are to be tested:

Case	Scaling		Hypothesis
	\mathbf{Q}_d	\mathbf{R}_d	
1.1	1	1	The system is stable and goes to reference. No s.d.
1.2	100	1	Noisy estimations if there is noisy measurements
1.3	$\frac{1}{100}$	1	Estimations will follow the model, giving more stable estimations, but might give higher error.
1.4	∞	1	The estimations will be equal to the measurements
1.5	0	1	The estimations will not be affected by the measurements