Advanced Econometrics: Home Assignment 2

Suggested solution for problematic parts and comments

2023

Problem 1: GMM

- The moment conditions were not derived correctly in some cases. The correct moment conditions are:

$$\begin{aligned} E[Y_t] &= \mu \\ E[Y_t^2] &= \mu^2 + \sigma^2(1 + \theta_1^2 + \theta_2^2) \\ E[Y_t Y_{t-1}] &= \mu^2 + \sigma^2(\theta_1 + \theta_1\theta_2) \\ E[Y_t Y_{t-2}] &= \mu^2 + \sigma^2\theta_2 \\ E[Y_t Y_{t-3}] &= \mu^2 \end{aligned}$$

- For the estimation we need to use the data in the following form:

```
estimation_data = cbind(data[4:n],
                        data[4:n]^2,
                        data[4:n] * data[3:(n-1)],
                        data[4:n] * data[2:(n-2)],
                        data[4:n] * data[1:(n-3)])
```

- Many solutions did not include the interpretation of coefficient significance. The estimates were all significant except for the μ.

Problem 2: Delta method

- The gradient has the following form:

$$\begin{bmatrix} \frac{1}{\beta_0+\beta_1^2} \\ \frac{2\beta_1}{\beta_0+\beta_1^2} - \frac{1}{2\sqrt{\beta_1}} \end{bmatrix}$$

- The variance-covariance matrix needs to be adjusted. Since our quantity Φ includes only β_0 and β_1 we need to remove the third column and the third row.

```
vcov[-3, -3]
```

- In the estimation you need to specify the correct expression. The expression does not correspond to our model's x_1 and x_2, it is only a notation. If you are not sure, it always helps to read the documentation in r. Also, the correct coefficients need to be used. Since Φ includes β_0 and β_1, only the first two coefficients are used.

```
deltamethod( ~ log(x1 + x2^2)-sqrt(x2),
             coef(model)[1:2],
             vcov(model)[-3, -3])
```

- Lastly, you are supposed to fit the model and estimate the variance of Φ based on the estimated parameters taken from the model, the true values are there to generate the data.

Problem 3: Bootstrap

- You were asked to plot the exponential q-q plot. There are many options for how to do that, you can use for example **qqexp()** function, or **plot()** function where you specify the theoretical quantiles of exponential distribution with $\lambda = 2$.
- The theoretical variance is $1/\lambda^2$.
- In many cases the discussion was missing. Pay attention to the setup and what exactly you are asked.

Problem 4: Endogeneity

- The model we are trying to estimate does not include variable x_2. This omitted variable is causing the endogeneity.
- Relevant instrumental variables are: z_1, z_2, and z_4, But only z_1 is valid, because z_2, and z_4 are included in the equation defining x_2.
- Our estimated model does not include x_2, which would therefore fall into the error term. Thus when checking the correlations with the error term of our model, we need to acknowledge, that x_2 is also a part of it.
- The 2SLS is a consistent estimator of both β_1 and β_3. The OLS is a consistent estimator only for β_3.

- x_2 should not be included in the model. We are interested in estimating the model without x_2.