Financial Econometrics I: Homework 1

February 26, 2024

Instructions:

- The contact person for all agenda regarding this homework is Josef Kurka, e-mail: josef.kurka@fsv.cuni.cz
- Please form groups of two students. If you have trouble finding a colleague, write me an e-mail, and I will match you with others having the same problem.
- Submit one file for the whole homework per group, the file needs to contain names of both students. Submit by SIS in the module "Study group roster" (Studijní mezivýsledky) - Financial Econometrics I - Lecture - HW1.
- Deadline for submissions is **Tuesday, March 12, 2024, 23:59.** Any late submissions will be awarded zero points.
- Your solution should have a form of Jupyter Notebook with R source-code. Code should be properly commented, interpretations of results as well as theoretical derivations should be written in markdown cells. This is the only file you need to send. If you prefer not to write formulas in LaTeX, you can send PDF with your derivations and interpretations in additional file and R code in Jupyter Notebook.
- Your solution should be as concise as possible, but all the important steps need to be explained. Solutions that are not commented properly can not be awarded the full amount of points.

Problem 1:
(6 points)

From the file *symbols.csv* choose 1 of the 10 Sectors (Industrials, Financials, Health Care, etc.). Download the prices for all the stocks belonging to the corresponding Sector for the period 07/2015 - 12/2023. Exclude the stocks that are not available in the *quantmod* package. Check that your data contains all the desired symbols (include this check in your output).

1. Compute the log-returns and simple returns for all the stocks. Save these to *lrets* and *rets* objects respectively. From now on, you will work with the logarithmic returns.
2. Compute the sample mean, variance, skewness, excess kurtosis, minimum and maximum of the series of logarithmic returns for each of the stocks in your sample. Display these in a nicely readable manner.
3. Try to devise one Figure that plots all time series of returns in your sample.
4. Discard the symbols, where you don't have valid data (non-missing, non-NA) for at least 80% of the dates in the sample period (use the stocks with the most observations as the benchmark for the sample period). For each symbol in your dataset, keep only the dates where you have valid data for all of the remaining symbols, i.e. you will have N time-series with matching timestamps. Now compute the mean logarithmic return for each date. The result should be a time-series with one (mean) log-return for each date.
5. Estimate the parameters of the stable distribution for the mean returns computed in 4. Please, comment.
6. Plot the histogram of the mean returns and compare to the densities of normal distribution, and stable distribution with the fitted parameters from the previous step. Please, comment.

Problem 2:
(5 points)

Consider 2 processes.
The first one is given by the formula

$$p_t = \mu + p_{t-1} + \epsilon_t + \epsilon_{t-1},$$

where ϵ_t is an i.i.d $N(0, 4)$ distributed sequence.

The second process is given by

$$r_t = r_{t-1} + \epsilon_t,$$

where ϵ follows

$$\epsilon_t = -\epsilon_{t-1} + \eta_t,$$

where η_t is an i.i.d $N(0, 1)$ distributed sequence, $cov(\eta_t, \epsilon_{t-k}) = 0$ for all t, and k.

1. Compute theoretical mean, and variance for both processes p_t and r_t. Is any of the processes stationary in terms of mean and variance? Explain.
2. Compute $Cov(\epsilon_t, \epsilon_{t-1})$ for both processes.
3. Simulate 1000 realizations of length 500 for both processes p_t and r_t (i.e. simulate a random realization of each process of length $T = 500$, repeat 1000 times) with following parameters $\mu = 0.2$, $p_0 = r_0 = 15$, $\epsilon_0 = 0$.

Plot the results, and comment please. Compute (analytically) $E(p_{200})$, and $E(r_{400})$.