Strategic games on Xemya

Tomasz Kowalski

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Ultimatum

Main players:

- Apollonia: a weak country.
- Tysq: a great power.

In the background (all big powers):

- Cocquavin, Albania, Ruritania, Sipango, Ammer-Ku.

Tysq issues an ultimatum to Apollonia: do as we say, or else.

- Tysq has excellent military equipment in good condition and large numbers.
- Apollonia has powerful allies Cocqauvin and Albania (clearly stronger than Tysq, or so it seems) and an expectation that they will fulfill their committments.
- How should Apollonia respond? Accept or reject?

Oracle

First payoff matrix

- Apollonia's moves: accept (A) or reject (R).
- Tysq's moves: war (W) or peace (P).

Four possible outcomes:

	W	P
A	(M, m)	(K, k)
R	(N, n)	(L, ℓ)

with payoffs: upper case for Apollonia, lower case for Tysq.

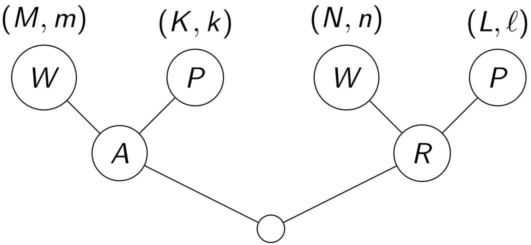
Rough estimates of payoff values:

- $M < N < K < L$
- $m < k, \ell < n$

Simultaneous vs sequential games

- Simultaneous: pick a strategy and play.
- Sequential: adjust your strategy to opponent's moves.

Our case is clearly sequential: ultimatum $\rightsquigarrow$ Apollonia's acceptance or rejection $\rightsquigarrow$ Tysq's response.



Conditional strategies: What would we do, if they did X? And what, if they did Y?

Second payoff matrix: conditional strategies

- Apollonia's moves: accept (A) or reject (R).
- Tysq's moves:
  - $\frac{W}{W}$ – war regardless of Apollonia's move,
  - $\frac{P}{P}$ – peace regardless of Apollonia's move,
  - $\frac{W}{P}$ – war if Apollonia accepts, peace if Apollonia rejects,
  - $\frac{P}{W}$ – peace if Apollonia accepts, war if Apollonia rejects.

Here is the new payoff matrix:

	$\frac{W}{W}$	$\frac{P}{P}$	$\frac{W}{P}$	$\frac{P}{W}$
A	(M, m)	(K, k)	(M, m)	(K, k)
R	(N, n)	(L, ℓ)	(L, ℓ)	(N, n)

Note that the payoff values do not change. We still have $M < N < K < L$ and $m < k$, $\ell < n$.

Best responses

1. If T. plays $\frac{W}{W}$, A.'s best response is R, because $M < N$.
2. If T. plays $\frac{P}{P}$, A.'s best response is R, because $K < L$.
3. If T. plays $\frac{W}{P}$, A.'s best response is R, because $M < L$.
4. If T. plays $\frac{P}{W}$, A.'s best response is A, because $N < K$.

Apollonia does not have a move that is always better. In technical terms: Apollonia does not have a dominant strategy.

1. If A. plays A, T.'s best response is either $\frac{P}{P}$ or $\frac{P}{W}$, as $k > m$.
2. If A. plays R, T.'s best response is either $\frac{W}{W}$ or $\frac{P}{W}$, as $n > \ell$.

Tysq does not have a dominant strategy. But Tysq has a strictly dominated strategy: a strategy that is never a best response to anything, namely, $\frac{W}{P}$. Such a strategy should never be played by a rational player!

Third payoff matrix: Nash equilibria

- Apollonia's moves: accept (A) or reject (R).
- Tysq's moves:
  - $\frac{W}{W}$ – war regardless of Apollonia's move,
  - $\frac{P}{P}$ – peace regardless of Apollonia's move,
  - $\frac{P}{W}$ – peace if Apollonia accepts, war if Apollonia rejects.

Here is the payoff matrix:

	$\frac{W}{W}$	$\frac{P}{P}$	$\frac{P}{W}$
A	(M, m)	(K, k)	$(K, k)^\star$
R	$(N, n)^\star$	(L, ℓ)	(N, n)

Nash equilibria:

The starred outcomes are such that no player has a strict incentive to move away from one, if the other player is kept fixed.

Fourth payoff matrix and expected utilities

The column $\frac{P}{P}$ is unstable: either one or the other player will have a strict incentive to move away from it. The strategy $\frac{P}{P}$ should not be played by a rational player! So, the final payoff matrix is:

	$\frac{W}{W}$	$\frac{P}{W}$
A	(M, m)	$(K, k)^\star$
R	$(N, n)^\star$	(N, n)

- Decide on a strategy by calculating its expected utility.

Let p be the probability of Tysq playing $\frac{W}{W}$. Then, the probability of Tysq playing $\frac{P}{W}$ is $1 - p$.

Apollonia's expected utilities:

- $EU_A = pM + (1-p)K$ (e.u. of playing A)
- $EU_R = pN + (1-p)N = N$ (e.u. of playing R)

How to choose a strategy

The strategy Apollonia should choose, according to game-theoretic wisdom (and common sense), is to

- play A if $EU_A > EU_R$,
- play R if $EU_A < EU_R$,
- play a randomised mix of A and R if $EU_A = EU_R$.

Recall that $M < N < K < L$. Let

- $S = K - M$ (the value of peace),
- $H = N - M$ (the price of honour).

Now $S > H$, so $0 < \frac{H}{S} < 1$. By simple calculations, we obtain:

- play A if $\frac{H}{S} < 1 - p$,
- play R if $\frac{H}{S} > 1 - p$,
- play mix if $\frac{H}{S} = 1 - p$.

Oracle's questions

- The first question is about p. This can be quite direct.
- The second question is about $\frac{H}{S}$. How big is H in comparison to S? This must be asked in a roundabout way if the oracle does not want to give an introductory lecture on game theory.

What really happened on Xemya?

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I know, because I am from Xemya myself. You probably have guessed that. You are also not mistaken if you think I am from Apollonia. But the real story is not for today.

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Thank you!