Computational Geometry: Theory and Experimentation (2023)
LP Exercises

Peyman Afshani

Focus:

Some details of implementing LP and MbC Algorithm
Solving some theoretical questions about LP
and using LP

1 Implementation Questions

Task 1 (optional). Implement 1D LP. The input is values $a_1, \cdots, a_n$ and $b_1, \cdots, b_n$ and c The output should be the value of x that minimizes cx while $a_i x \leq b_i$ for all $1 \leq i \leq n$.

Task 2 (optional). Make sure your implementation can return infeasible, or unbounded. How would you implement this?

Task 3. Think about the implementation of the 2D LP. How do you initialize the solution?

Task 4 (optional). Think of a "good" heuristic to initialize the initial answer for the bride-finding LP in the MbC algorithm.

Task 5. Think about the MbC algorithm and the formulation of the bridge-finding subroutine as a LP. In the MbC algorithm, we would like to find the bridge, i.e., the line segment s between one point p on the left and another point q on the right.

However, the LP formulation finds a **line** ℓ. Explain how in practice knowing ℓ will not help us find points p and q.

Task 6. Consider again the MbC algorithm and the related LP. Answer the following questions.

- When (if it can) the LP be infeasible?
- When (if it can) the LP be unbounded?
- When (if it can) the LP be degenerate?
- Can any of these situations be avoided?

2 Theoretical Questions

Consider the following algorithm.

Algorithm 1 An algorithm for randomly permuting an array A

procedure SHUFFLE(A, n)
  for i from 1 to n **do**
    $r \leftarrow$ random int between 1 and n
    Swap $A[i]$, $A[r]$
  end for
end procedure

Task 7. Show that this algorithm does not compute a uniform random permutation of the input array.

Note that in a uniform random permutation, every permutation has probability exactly $\frac{1}{n!}$.

Task 8. Two point sets P and Q are said to be *separable* if their convex hulls are disjoint. What is the fastest algorithm you can build that can decide whether two point sets are separable?