On the Relationship of Semiconductor Yield and Reliability

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***Abstract*—Traditionally, semiconductor reliability has been estimated from the life tests or accelerated stress tests at the completion of manufacturing processes. Recent research, however, has been directed to reliability estimation during the early production stage through a relation model of yield and reliability. Because the relation model depends on the assumed density distribution of manufacturing defects, we investigate the effect of the defect density distributions on the predicted reliability, for a single-area device without repair and for a two-area device with repair, respectively. We show that for any device, reliability functions preserve an ordering of yield functions. It is also pointed out that the repair capability improves only yield but not reliability, resulting in a large value of the factor that scales from yield to reliability. In order to achieve a reliable device, therefore, we suggest to improve yield and to perform the device test such as burn-in if the scaling factor is large.**

***Index Terms*—Conditional reliability, defect density distribution, nonfatal defects, scaling factor.**

I. Introduction

IN general, reliability is defined as the probability of a device to perform its function satisfactorily under a specified environmental condition for a predetermined period of operating time [1], [11]. Traditionally, semiconductor reliability has been estimated from the life tests or accelerated stress tests of the complete devices. Because device testing is getting more expensive, time consuming, and less capable of identifying the failure causes, a new methodology is necessary to assure reliability during the early production stage [12]. Such a need together with a recent recognition of the correlated improvements between yield and reliability observed in the experimental data has led to research in developing the relation models of yield and reliability. The basic theory behind the relation models goes as follows.

In semiconductor manufacturing, defects are created in a device. Each defect in the device affects yield or reliability, depending on its size and location. A defect that is of sufficient size and occurs in a place where it can result in an immediate device failure is called the fatal defect [5], [14], the killer defect [2], or the yield defect [8], [12]. For example, a pinhole defect is fatal if it occurs in the overlap region between two conductors that cross each other [5]. Such a fatal defect is detected at manufacturing yield test. On the other hand, a defect that is either too small or located in a position that does not cause an immediate failure is called the nonfatal defect [5], [14], the latent defect [2], or the reliability defect [8], [12]. For example, a small defect that locally reduces the line width or spacing is a nonfatal defect [15]. A nonfatal defect may or may not cause device failure in the field, depending on the operating time and the environmental condition. Because yield is defined by the probability that a device selected at random has no fatal defects [2], [5], [12], only fatal defects are of interest and nonfatal defects are of no consequence in yield models. For over 40 years, yield models have been suggested by selecting different distributions for the defect density, where the defect density is the number of fatal defects per unit area. If yield models are extended to consider nonfatal defects, then it is possible to estimate and improve reliability in an actual process rather than at the completion of manufacturing process. It should be noted that reliability in this regard is often referred to as the extrinsic reliability [6], [11].

Previously, there have been some attempts to extend yield models to estimate reliability. At first, a simple time-independent Poisson reliability was obtained from the Poisson yield model, assuming directly that the number of nonfatal defects is proportional to the number of fatal defects following a Poisson distribution [6], [13], [18], [19]. Later, the Poisson reliability was extended to the negative binomial reliability to consider defect clustering [2]. By noting that such a time-independent reliability is not consistent with reliability estimated from the time-to-first-failure data which is commonly used, expressions for time-dependent reliability have been developed that are applicable for the gate oxide devices [7], [8], [10]. All these models implicitly assumed that the number of fatal defects is independent of the number of nonfatal defects in a device. Recently, a multinomial distribution has been considered for a single-area device without repair in order to model the number of nonfatal defects, the number of fatal defects that fail during burn-in, and the number of nonfatal defects that are released to field operation [9]. From a feature of the multinomial distribution, the number of fatal defects and the number of nonfatal defects in a device are negatively correlated if the total number of defects in a device is fixed. Analytical results are obtained to explain the correlated improvement among yield, burn-in, and reliability, considering the general distribution for defect density.

This paper consists of two parts for further studying of the interaction of semiconductor yield and reliability. In the first part, because explicit relationships between yield and reliability depend on the assumed distribution for defect density, we revisit a single-area device without repair in order to investigate the

Manuscript received March 24, 2004; revised April 15, 2005. This work was supported by the Natural Sciences and Engineering Research Council of Canada (Project STPGP 234802-00) and by the National Science Foundation (Projects DMI-0300032 and DMI-0243409).

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Digital Object Identifier 10.1109/TSM.2005.852110

effect of the selected defect density distribution on reliability prediction. For simplicity of explanation, a burn-in step is not considered. Then in the second part of the paper, we consider a device consisting of two areas. An example of a two-area device includes a pinhole monitor that has a high defect density along the edge of a pattern and a low defect density in the middle. Another example is given by a redundant memory chip whose part of area is often repairable. For the two-area device, the location of a defect must be considered as well as the type of a defect. Thus, we introduce a statistical hierarchical model which is flexible in handling different device types and/or different situations. Through the proposed hierarchical model, the interaction between yield and reliability for a two-area device is investigated with a consideration of the repair capability.

This paper is organized as follows. Section II presents the explicit relationships between yield and reliability for a single-area device without repair. We investigate how the selected defect density distribution affects the estimated reliability, and show analytically that the reliability functions preserve the ordering of yield functions by the selected defect density distributions. We also show that the conditional reliability that incorporates the yield information is not less than reliability, implying that a device with no fatal defects is more likely to have no nonfatal defects. In Section III, a hierarchical model is used to model manufacturing defects in a two-area device. The expressions for yield, reliability and conditional reliability are obtained and the effect of repair on yield and reliability is also discussed. Finally, conclusions are given in Section IV.

II. Single-Area Device Without Repair

In semiconductor manufacturing, devices are inspected at various test points during the process and the number of defects is counted. Let N be the number of manufacturing defects introduced during fabrication on a device of area A. Let $D = N/A$, where D is the density of defects per unit area. Let $D_0 = E(D)$ and $\lambda = E(N) = AD_0$.

Imagine an experiment that observes a defect in a device and records if the defect is fatal or nonfatal. Suppose that the experiments are performed N times and the N outcomes are independent with each other. Let N_{f} and N_{nf} be the number of fatal and nonfatal defects in a device, respectively. Let θ be the probability of a defect being fatal. In practice, the value of θ depends on the defect size distribution and the device feature [15]. Then

$$N_{\mathrm{f}}|N \sim \mathrm{binomial}(N, \theta)$$

where we use notation such as $X|Y \sim \mathrm{binomial}(Y, p)$ to mean that the conditional distribution of X given $Y = y$ is binomial with parameters y and p.

A. Yield

The manufacturing yield Y is the probability of zero fatal defects in a device [5], [11], [12], and is derived by

$$Y = \sum_{n=0}^{\infty} \Pr(N_{\mathrm{f}} = 0|N = n) \Pr(N = n) = E[(1-\theta)^N]. \quad (1)$$

The exact form of yield expression depends on the distribution used for N. The simple Poisson yield model is obtained by assuming that the defect density D is constant and equals to D_0 and N follows a Poisson distribution with parameter $\lambda = AD_0$. Thus, the Poisson yield is given by

$$Y_{\mathrm{Poisson}} = \sum_{n=0}^{\infty} (1-\theta)^n \frac{e^{-\lambda}\lambda^n}{n!} = e^{-\lambda_y}$$

where λ_y is the mean number of fatal defects in a device. Note that $\lambda_y = E(N_{\mathrm{f}}) = \lambda\theta$.

Because the Poisson yield is too pessimistic when defects are clustered in certain areas, the compound Poisson distribution often gives a better fit

$$Y_{\mathrm{compound}} = \sum_{n=0}^{\infty} (1-\theta)^n \int_0^{\infty} \frac{e^{-AD}(AD)^n}{n!} f(D) dD \quad (2)$$

where $f(D)$ is the distribution of D. Several functions such as a symmetric triangle, exponential, uniform, and gamma have been suggested in the yield literature as the defect density distribution [4], [5], [12]. If D follows a uniform distribution, (2) reduces to

$$\begin{aligned} Y_{\mathrm{uniform}} &= \sum_{n=0}^{\infty} (1-\theta)^n \int_0^{2D_0} \frac{e^{-AD}(AD)^n}{n!} \frac{1}{2D_0} dD \\ &= \frac{1}{2\lambda_y}(1 - e^{-2\lambda_y}). \end{aligned}$$

If D has a triangle distribution that is used to approximate a normal distribution, the resulting yield model is called the Murphy's, which is given by [16]

$$\begin{aligned} Y_{\mathrm{Murphy}} &= \sum_{n=0}^{\infty} (1-\theta)^n \Bigg[\int_0^{D_0} \frac{e^{-AD}(AD)^n}{n!} \frac{D}{D_0} dD \\ &\qquad + \int_{D_0}^{2D_0} \frac{e^{-AD}(AD)^n}{n!} \left(2 - \frac{D}{D_0}\right) \\ &\qquad \times \frac{1}{D_0} dD \Bigg] \\ &= \left(\frac{1 - e^{-\lambda_y}}{\lambda_y} \right)^2. \end{aligned}$$

For the exponential distribution of D, the yield model is called the Seeds' and given by [17]

$$\begin{aligned} Y_{\mathrm{Seeds}} &= \sum_{n=0}^{\infty} (1-\theta)^n \int_0^{\infty} \frac{e^{-AD}(AD)^n}{n!} \frac{e^{-D/D_0}}{D_0} dD \\ &= (1 + \lambda_y)^{-1}. \end{aligned}$$

When D follows a gamma distribution, the resulting model is called the negative binomial yield

$$\begin{aligned} Y_{\mathrm{nb}} &= \sum_{n=0}^{\infty} (1-\theta)^n \int_0^{\infty} \frac{e^{-AD}(AD)^n}{n!} \frac{1}{\Gamma(\alpha)\beta^{\alpha}} \\ &\quad \times D^{\alpha-1} e^{-D/\beta} dD \\ &= \left(1 + \frac{\lambda_y}{\alpha}\right)^{-\alpha} \end{aligned}$$

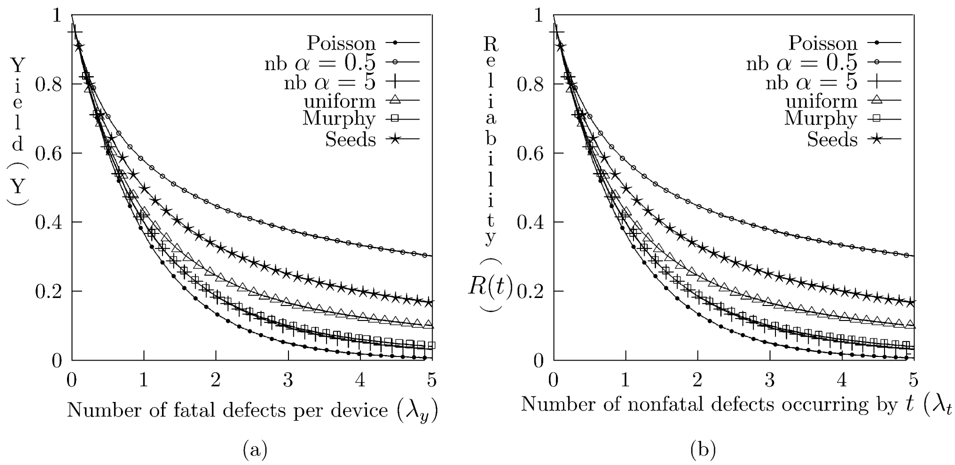

Fig. 1. Yield and reliability in numbers of fatal and nonfatal defects, respectively. (a) Yield versus number of fatal defects. (b) Reliability versus number of nonfatal defects.

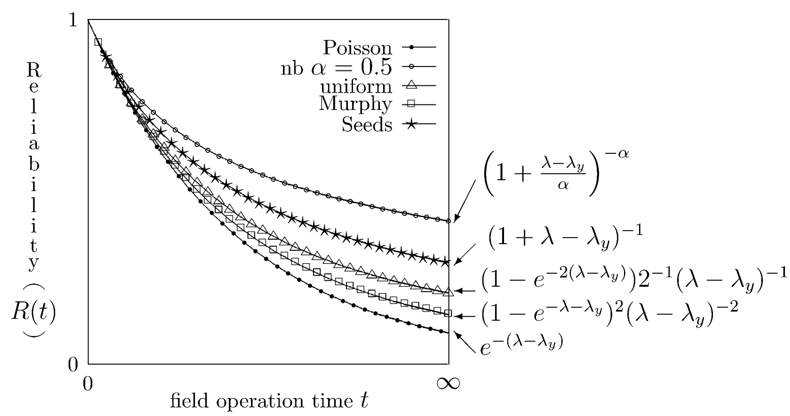

Fig. 2. Reliability as a nonincreasing function of field operating time.

where α is referred to as the clustering factor. By varying the value of α, the negative binomial yield covers the whole range of yield estimations. A smaller value of α leads to greater variation in defect density and implies a higher clustering. It is noted that if $\alpha = 1$ then $Y_{\text{nb}} = Y_{\text{Seeds}}$ and if $\alpha \to \infty$ then $Y_{\text{nb}} \to Y_{\text{Poisson}}$. The practical range of α is known to be 0.3 to 5.0 [12]. The parameters of the defect density distributions must be estimated from sample data using statistical methods such as method of moments [3], [4]. Numerical results have shown that yield functions are ordered by the selected defect density functions [4], [12]. See Fig. 1(a), where the Poisson yield and the negative binomial yield at a small value of α give the lowest and highest yields, respectively.

B. Reliability and Failure Rate

In general, reliability is defined after the completion of the manufacturing process and in the beginning of field operation. Thus, reliability is independent of yield information, assuming that any devices that are released to field have passed the manufacturing yield test implying that no fatal defects exist. Therefore, only the number of nonfatal defects is of interest in reliability analysis, and the corresponding reliability is expressed for a fixed operating time t by

$$\begin{aligned} R(t) &= \sum_{m=0}^{\infty} \Pr(N_{\text{nf}} = m, \text{ none of } m \text{ occur by time } t) \\ &= \sum_{n=0}^{\infty}\sum_{m=0}^{n} \Pr(N_{\text{nf}} = m, \text{ none of } m \text{ occur} \\ &\qquad \text{by time } t \mid N = n) \Pr(N = n) \\ &= E[1 - p_t(1-\theta)]^N \end{aligned} \tag{3}$$

where p_t is the probability that a nonfatal defect fails before time t in field operation. In (3), the explicit form of reliability depends on the distribution of N. By comparing (1) and (3), we see that reliability expressions are of the same form with the yield expressions just by replacing λ_y by λ_t, where $\lambda_t = \lambda p_t(1-\theta)$. Therefore, the reliability expression for a selected defect density distribution can be obtained from the corresponding yield expression. Fig. 1(b) shows reliability models for different defect density distributions. It is clear from Fig. 1 that the accuracy of estimated yield as well as reliability depends on the model selected to describe the density distribution of manufacturing defects.

Given a specific operating condition, we have $p_0 = 0$ and p_t is nondecreasing in t. As a result, we have $R(0) = 1$ and $R(t)$ is nonincreasing in t. In the limiting case of $t \to \infty$, $R(t)$ is the probability that a device has no nonfatal defects. Fig. 2

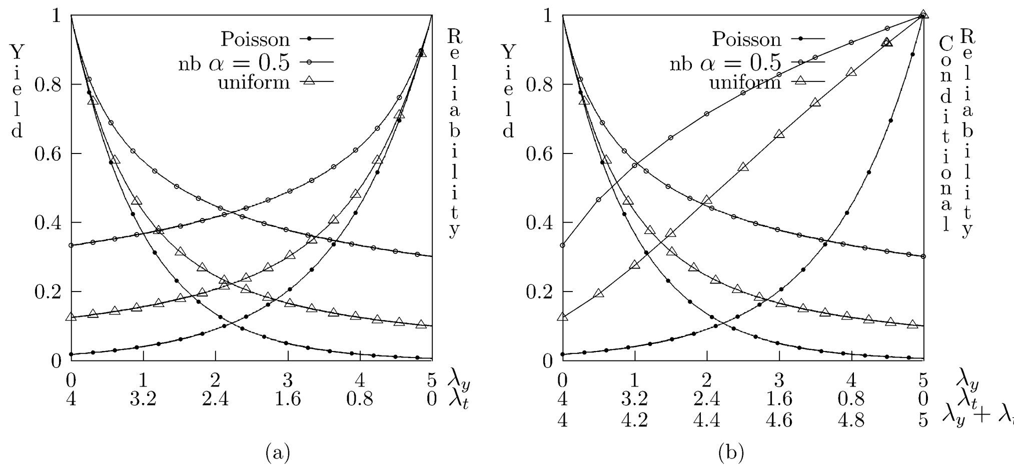

Fig. 3. Tradeoff of yield and reliability for a fixed number of defects and ($p_t = 0.8$ and $\lambda = 5$). (a) Yield versus reliability. (b) Yield versus conditional reliability.

compares reliability as a function of operating time for different defect density distributions. For any $t \geq 0$, reliability functions preserve the ordering of yield functions by the selected defect density distributions.

The device failure rate can be derived from (3) using a definition of [1]

$$h(t) = \frac{-d \ln R(t)}{dt}.$$

Depending on the selected distribution of defect density, we have the failure rate functions of

$$\begin{aligned} h_{\text{Poisson}}(t) &= \lambda(1-\theta)\frac{dp_t}{dt} \\ h_{\text{uniform}}(t) &= \left(\frac{1}{\lambda_t} - \frac{2e^{-2\lambda_t}}{1-e^{-2\lambda_t}}\right) h_{\text{Poisson}}(t) \\ h_{\text{Murphy}}(t) &= 2\left(\frac{1}{\lambda_t} - \frac{e^{-\lambda t}}{1-e^{-\lambda_t}}\right) h_{\text{Poisson}}(t) \\ h_{\text{Seeds}}(t) &= \frac{1}{1+\lambda_t} h_{\text{Poisson}}(t) \\ h_{\text{nb}}(t) &= \frac{\alpha}{\alpha+\lambda_t} h_{\text{Poisson}}(t). \end{aligned}$$

We can analytically show that reliability functions for the defect density distributions are ordered. For any $t \geq 0$ and $\alpha < 1$, it is not difficult to show that

$$\begin{aligned} h_{\text{nb}}(t) &< h_{\text{Seeds}}(t) < h_{\text{uniform}}(t) \\ &< h_{\text{Murphy}}(t) < h_{\text{Poisson}}(t) \end{aligned}$$

implying that

$$\begin{aligned} R_{\text{nb}}(t) &> R_{\text{Seeds}}(t) > R_{\text{uniform}}(t) \\ &> R_{\text{Murphy}}(t) > R_{\text{Poisson}}(t). \end{aligned}$$

Otherwise, if $1 < \alpha < (\lambda_t(1-e^{-2\lambda_t} - 2\lambda_t e^{-2\lambda_t})/(\lambda_t(1-e^{-2\lambda_t}) - (1-e^{-2\lambda_t} - 2\lambda_t e^{-2\lambda_t}))$, then

$$\begin{aligned} h_{\text{Seeds}}(t) &< h_{\text{nb}}(t) < h_{\text{uniform}}(t) \\ &< h_{\text{Murphy}}(t) < h_{\text{Poisson}}(t). \end{aligned}$$

Otherwise, if $(\lambda_t(1-e^{-2\lambda_t} - 2\lambda_t e^{-2\lambda_t})/(\lambda_t(1-e^{-2\lambda_t}) - (1-e^{-2\lambda_t} - 2\lambda_t e^{-2\lambda_t})) < \alpha < (2\lambda_t(1-e^{-\lambda_t} - \lambda_t e^{-\lambda_t})/(\lambda_t(1-e^{-\lambda_t}) - 2(1-e^{-\lambda_t} - \lambda_t e^{-\lambda_t}))$

$$\begin{aligned} h_{\text{Seeds}}(t) &< h_{\text{uniform}}(t) < h_{\text{nb}}(t) \\ &< h_{\text{Murphy}}(t) < h_{\text{Poisson}}(t). \end{aligned}$$

Finally, if $\alpha > (2\lambda_t(1-e^{-\lambda_t} - \lambda_t e^{-\lambda_t})/(\lambda_t(1-e^{-\lambda_t}) - 2(1-e^{-\lambda_t} - \lambda_t e^{-\lambda_t}))$, then

$$\begin{aligned} h_{\text{Seeds}}(t) &< h_{\text{uniform}}(t) < h_{\text{Murphy}}(t) \\ &< h_{\text{nb}}(t) < h_{\text{Poisson}}(t). \end{aligned}$$

Though reliability is supposed to be independent of the yield information, if the number of defects in a device is fixed, then reliability decreases as yield increases. This is because as less fatal defects are removed from the manufacturing yield test, more nonfatal defects are released to field if the number of defects in a device is fixed. Mathematically, given $N = n$, the covariance of N_{f} and N_{nf} is $-n\theta(1-\theta)$ [3]. Fig. 3(a) illustrates the tradeoff between yield and reliability for a fixed number of defects in a device. For any combination of fatal and nonfatal defects, the Poisson and the negative binomial reliability give the lowest and highest yield and reliability, respectively.

The conditional reliability that incorporates the yield information may be of more interest to the manufacturer and the designer than reliability [2]. Consider the joint probability of

$$\begin{aligned} &\Pr(\text{a device passed the yield test, a device survives by } t) \\ &\quad = \Pr(N_{\text{f}} = 0, N - N_{\text{f}} = n, \text{ none of } n \text{ nonfatal occur} \\ &\qquad \text{by } t | N = n) \Pr(N = n) \\ &\quad = E[(1-\theta-p_t(1-\theta))^N]. \end{aligned} \tag{4}$$

The joint probability at $t = 0$ is yield, and is decreasing in time. The conditional reliability is defined by the ratio of (4) to (1):

$$\begin{aligned} R(t|\text{pass the yield test}) &= \Pr(\text{survives by } t | N_{\text{f}} = 0) \\ &= \frac{E[1-\theta-p_t(1-\theta)]^N}{E(1-\theta)^N}. \end{aligned} \tag{5}$$

Fig. 3(b) compares yield and conditional reliability. By comparing Fig. 3(a) and (b), we numerically see that the conditional reliability is not less than reliability for any given defect density distribution. In general, one can prove

$$R(t|\text{pass the yield test}) \geq R(t)$$

where the equality holds if and only if N follows a Poisson distribution. An approach for the proof is similar to [9, Th. 1]. This implies that a device without any fatal defects is more likely to have no nonfatal defects.

C. Yield-Reliability Relationship

Assuming the general distribution for N, (1) and (3) can be used to show that yield and reliability are improved together, and $R(t) \gtreqqless Y$ if and only if $\gamma_t \lesseqqgtr 1$ where $\gamma_t = \lambda_t/\lambda_y$ [9]. We see from (1) and (3) that two types of parameters affect yield as well as reliability: design-related parameters such as the device area A; and process-related parameters such as θ and defect density D. However, operation-related parameters such as temperature and voltage affect reliability through p_t but not yield. Therefore, yield contains part of the information necessary to predict reliability, but high yield does not necessarily imply high reliability unless the operating conditions are fixed.

The explicit yield-reliability relationships can be obtained only if the defect density distribution is selected. For different defect density distributions, the relationships are derived by

$$\begin{aligned} \ln R_{\text{Poisson}}(t) &= \gamma_t \ln Y_{\text{Poisson}} \\ R_{\text{nb}}^{-(1/\alpha)}(t) - 1 &= \gamma_t \left(Y_{\text{nb}}^{-(1/\alpha)} - 1\right) \\ \ln(1 - 2\lambda_r R_{\text{uniform}}(t)) &= \gamma_t \ln(1 - 2\lambda_y Y_{\text{uniform}}) \\ \ln(1 - \lambda_r \sqrt{R_{\text{Murphy}}(t)}) &= \gamma_t \ln(1 - \lambda_y \sqrt{Y_{\text{Murphy}}}) \\ R_{\text{Seeds}}^{-1}(t) - 1 &= \gamma_t (Y_{\text{Seeds}}^{-1} - 1). \end{aligned} \tag{6}$$

We see from (6) that the process yield can be scaled to field reliability with the ratio γ_t. Thus, in this paper, we refer to the ratio γ_t as the scaling factor from yield to reliability. Note that a larger value of γ_t gives a smaller value of reliability for a given yield value. Clearly, reliability and warranty cost can be controlled in an actual process by considering yield and the scaling factor. Because Y and λ_y are obtained from the inspection and the defect count, the only unknown parameter is p_t. The value of p_t may be directly calculated considering the details of the device layout and/or the defect size distribution. For example, it is given for gate oxide [8] that

$$p_t = \begin{cases} 0, & \text{if } 0 \leq t < \tau e^{(G/V)(w - x^*)} \\ \dfrac{\Pr\left(w - \frac{V}{G}\ln\frac{t}{\tau} \leq X \leq x^*\right)}{\Pr(X \leq x^*)}, & \text{if } \tau e^{(G/V)(w - x^*)} \leq t \leq \tau e^{(G/V)w} \\ 1, & \text{if } t \geq \tau e^{(G/V)w} \end{cases}$$

where X is a size of a defect selected at random; x^* is the critical size at which the oxide fails in the manufacturing test; w is the oxide thickness; τ is a constant (1×10^{-11} s; G is a constant (350 MV/cm); and V is the operating voltage across the oxide.

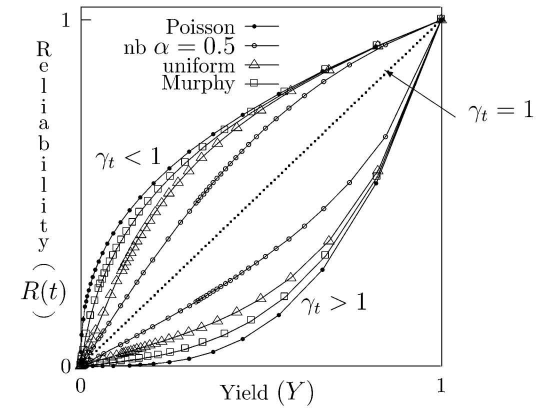

Fig. 4. Reliability as a function of yield for different values of scaling factor.

Suppose that a fixed yield value is used to estimate reliability for the different defect density distributions. Fig. 4 compares reliability functions that are estimated from a fixed yield value. For small t-values, the Poisson, Murphy, and uniform reliability models give larger reliability estimates than the negative binomial reliability model with a small value of α, and vice versa for large t-values. Intuitively, this is because if the same yield value is used to predict different reliability functions, the Poisson, Murphy, and uniform reliabilities are overestimated in that order or the negative binomial reliability is underestimated, noting that the negative binomial yield is larger than other yields.

From the explicit relationships, we note that reliability increases in yield for a fixed scaling factor, and reliability decreases in the scaling factor for a fixed yield. Therefore, in order to achieve a target reliability level, yield must be improved first. Then if the value of scaling factor is large, a substantial improvement of reliability can be obtained from the screening method such as burn-in. For the effectiveness of burn-in, refer to [8], [9].

III. Two-Area Device With Repair

Suppose that the device consists of two area components A_1 and A_2, where $A = A_1 + A_2$. We assume further that repair is not possible in area 1, but within area 2, fatal defects can be independently repaired with probability P_{repair}. In this case, a two-stage hierarchical model [3] can be used to consider both the location of defects (i.e., area 1 or area 2) and the type of defects (i.e., fatal or nonfatal).

Let N_1 and N_2 be the number of defects occurring in the areas 1 and 2, respectively.

Thus, $N = N_1 + N_2$. Given N, we assume in the first stage that

$$N_1 | N \sim \text{binomial}(N, P_1) \tag{7}$$

where P_1 is the probability that a given defect occurs in area 1 with $P_1 = A_1/A$. It follows immediately that $N_2|N \sim \text{binomial}(N, P_2)$ where P_2 is the probability that a given defect occurs in area 2 and $P_2 = 1 - P_1$. Now each defect in area 1 is either fatal or nonfatal depending on its location and size. Thus, let $N_{1:\text{f}}$ and $N_{1:\text{nf}}$ be the number of fatal and nonfatal defects occurring in area 1, respectively.

Because $N_{1:\mathrm{f}}$ depends on N_1 and $N_{2:\mathrm{f}}$ depends on N_2, we assume in the second stage that

$$N_{1:\mathrm{f}}|N_1 \sim \mathrm{binomial}(N_1, \theta_1) \tag{8}$$

and

$$N_{2:\mathrm{f}}|N_2 \sim \mathrm{binomial}(N_2, \theta_2)$$

where θ_1 and θ_2 are the probability of a defect in area 1 and in area 2 being fatal, respectively.

A. Yield

From (7) and (8), we can show that

$$N_{1:\mathrm{f}}|N \sim \mathrm{binomial}(N_1, \theta_1 P_1).$$

Then, yield of area 1 is obtained by

$$Y_1 = \sum_{n=0}^{\infty} \Pr(N_1 = 0|N = n)\Pr(N = n) = E[(1-\theta_1 P_1)^N].$$

Similarly, yield of area 2 is given by

$$\begin{aligned} Y_2 = \sum_{n=0}^{\infty}\sum_{k=0}^{n} &\Pr(N_{2:\mathrm{f}} = k,\ \text{all } k \text{ fatal defects} \\ &\text{are repaired}|N = n)\Pr(N = n) \\ = E[1 - &\theta_2 P_2(1 - P_{\mathrm{repair}})]^N. \end{aligned}$$

If $P_{\mathrm{repair}} = 0$, then Y_1 and Y_2 are of the same form. In general, because $P_{\mathrm{repair}} > 0$, $Y_2 \geq Y_1$ for $P_1 = P_2$ and $\theta_1 = \theta_2$ (see Fig. 5). Since the whole device yield is the probability that both areas have no fatal defects, we can show that

$$\begin{aligned} Y_{\mathrm{device}} = \sum_{n=0}^{\infty}\sum_{k=0}^{\infty} &\Pr(N_{1:\mathrm{f}} = 0,\ N_{2:\mathrm{f}} = k, \\ &\text{all } k \text{ fatal repaired}|N=n)\Pr(N=n) \\ = E[1 - &\theta_1 P_1 - \theta_2 P_2(1 - P_{\mathrm{repair}})]^N \end{aligned} \tag{9}$$

using

$$(N_{1:\mathrm{f}}, N_{2:\mathrm{f}})|N \sim \mathrm{multinomial}(N, \theta_1 P_1, \theta_2 P_2).$$

It immediately follows from [9, Th. 1] that

$$Y_{\mathrm{device}} \geq Y_1 Y_2$$

and the equality holds if and only if N has a Poisson distribution. This model was previously considered in yield literature [14], but assuming that $N_{1:\mathrm{f}}$ and $N_{2:\mathrm{f}}$ are independent. Note that we allow for the dependence between the number of defects in two areas that are in the same device, implying that devices with large numbers of defects in area 1 will have large numbers of defects in area 2. Because of the positive interaction of the number of fatal defects in the two areas, the device yield is greater than the product of each area yield. Fig. 5 compares the yield components.

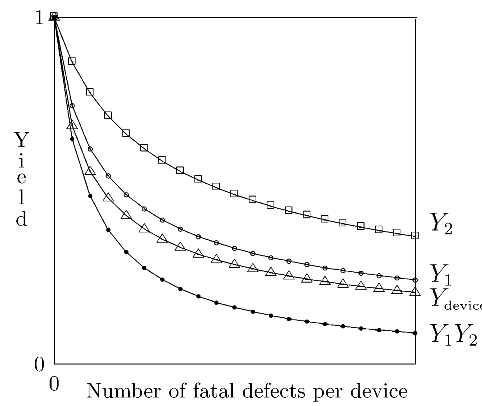

Fig. 5. Comparison of yield components at $P_1 = P_2$ and $\theta_1 = \theta_2$.

B. Reliability

Let $p_{1:t}$ and $p_{2:t}$ be the probability of a nonfatal defect occurring before time t in areas 1 and 2, respectively. Note that though we assume that nonfatal defects in each area have the identical time-to-failure distributions, this can be easily generalized to allow for the different time-to-failure distributions by extending a two-stage hierarchical model to a three-stage hierarchical model. All the subsequent derivations are similar.

It follows that

$$\begin{aligned} R_1(t) &= \sum_{k=0}^{\infty} \Pr(N_{1:\mathrm{nf}} = k,\ \text{none of } k \text{ nonfatal defects} \\ &\qquad\qquad \text{occur by time } t) \\ &= \sum_{n=0}^{\infty}\sum_{k=0}^{n} (1 - p_{1:t})^k \Pr(N_{1:\mathrm{nf}} = k|N=n)\Pr(N=n) \\ &= E[1 - p_{1:t}P_1(1-\theta_1)]^N, \\ R_2(t) &= E[1 - p_{2:t}P_2(1-\theta_2)]^N \end{aligned}$$

and

$$\begin{aligned} R_{\mathrm{device}}(t) &= \sum_{n=0}^{\infty}\sum_{n_2=0}^{n}\sum_{n_1=0}^{n-n_2} (1-p_{1:t})^{n_1}(1-p_{2:t})^{n_2} \\ &\quad \times \Pr(N_{1:\mathrm{nf}} = n_1,\ N_{2:\mathrm{nf}} = n_2,\ N = n) \\ &= E[1 - p_{1:t}P_1(1-\theta_1) - p_{2:t}P_2(1-\theta_2)]^N. \end{aligned} \tag{10}$$

Again it holds from [9, Th. 1] that

$$R_{\mathrm{device}}(t) \geq R_1(t)R_2(t)$$

for any fixed time $t \geq 0$, with the equality holding when N has a Poisson distribution. Each of $R_1(t)$, $R_2(t)$ and $R_{\mathrm{device}}(t)$ preserves the ordering of yield functions in the selected defect density distributions.

Manufacturers often accept relatively high defect densities if an area of a device is repairable, because the repair capability can increase the yield. See (9). However, repair cannot improve reliability, as can be seen in (10). If P_{repair} increases in (6), then yield increases but the scaling factor also increases, resulting in a decreased value of the predicted reliability for a given yield.

$$\begin{aligned}
&R_{\text{device}}(t|\text{pass the yield test})\\
&\quad=\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\Pr\big(N_{1:\text{nf}}=k,\ \text{none of } k \text{ nonfatal occur by } t,\\
&\qquad\qquad N_{2:\text{nf}}=j,\ \text{none of } j \text{ nonfatal occur by } t|N_{1:\text{f}}=0,\\
&\qquad\qquad N_{2:\text{f}}=i,\ \text{all } i \text{ fatal repaired}\big)\\
&\quad=\frac{E[1-\theta_1P_1-\theta_2P_2(1-P_{\text{repair}})-p_{1:t}P_1(1-\theta_1)-p_{2:t}P_2(1-\theta_2)]^N}{E[1-\theta_1P_1-\theta_2P_2(1-P_{\text{repair}})]^N}.
\end{aligned}$$

This can be used to explain why low reliability is observed even with high yield, for a device that is repairable.

The conditional reliability for area 1 that incorporates the yield information is given by

$$\begin{aligned}
&R_1(t|\text{pass the yield test})\\
&\quad=\Pr(\text{survives by } t|N_{1:\text{f}}=0)\\
&\quad=\frac{E[1-\theta_1P_1-p_{1:t}(1-\theta_1)P_1]^N}{E(1-\theta_1P_1)^N}
\end{aligned}$$

from the fact that

$$(N_{1:\text{f}},N_{1:\text{nf}})|N\sim\text{multinomial}(N,\theta_1P_1,(1-\theta_1)P_1).$$

Similarly, the conditional reliability for area 2 is given by

$$\begin{aligned}
&R_2(t|\text{pass the yield test})\sum_{i=0}^{\infty}R_2(t|N_{2:\text{f}}=i,\ \text{all } i\\
&\qquad\qquad\text{fatal defects in area 2 are repaired})\\
&\quad=\sum_{i=0}^{\infty}\sum_{k=0}^{\infty}\Pr(N_{2:\text{nf}}=k,\ \text{none of } k \text{ nonfatal}\\
&\qquad\qquad\text{occur by } t|N_{2:\text{f}}=i,\ \text{all } i \text{ fatal repaired})\\
&\quad=\frac{E[1-\theta_2P_2(1-P_{\text{repair}})-p_{2:t}P_2(1-\theta_2)]^N}{E[1-\theta_2P_2(1-P_{\text{repair}})]^N}.
\end{aligned}$$

From

$$\begin{aligned}
&(N_{1:\text{f}},N_{1:\text{nf}},N_{2:\text{f}},N_{2:\text{nf}})|N\\
&\quad\sim\text{multinomial}(N,\theta_1P_1,(1-\theta_1)P_1,\theta_2P_2,(1-\theta_2)P_2)
\end{aligned}$$

the conditional reliability for the entire device is expressed by the equation at the top of the page. It follows from [9, Th. 1] that

$$\begin{aligned}
R_1(t|\text{pass the yield test})&\geq R_1(t)\\
R_2(t|\text{pass the yield test})&\geq R_2(t)\\
R_{\text{device}}(t|\text{pass the yield test})&\geq R_{\text{device}}(t).
\end{aligned}$$

C. *Yield-Reliability Relationship*

The explicit yield and reliability relationships for different defect density distributions can be obtained from (6) by letting $\lambda_y=\lambda\theta_1P_1$ and $\lambda_t=\lambda(1-\theta_1)P_1p_{1:t}$ for area 1 and by letting $\lambda_y=\lambda\theta_2P_2(1-P_{\text{repair}})$ and $\lambda_t=\lambda(1-\theta_2)P_2p_{2:t}$ for area 2, respectively. Similarly, the relationship between the whole device yield and reliability can be obtained from (6) by letting $\lambda_y=\lambda[\theta_1P_1+\theta_2P_2(1-P_{\text{repair}})]$ and $\lambda_t=\lambda[(1-\theta_1)P_1p_{1:t}+(1-\theta_2)P_2p_{2:t})]$. The yield functions can be scaled to reliability functions using the corresponding scaling factor.

IV. Conclusion

Reliability and yield modeling is essential for a successful semiconductor manufacturing process. Traditionally, reliability modeling has been separate from yield modeling: yield models are based on the number of fatal defects, and reliability models are obtained from parametric distributions, such as exponential or Weibull, fitted to the time-to-first-failure data obtained after the manufacturing process is complete. Nowadays, it is very important to predict semiconductor reliability at an early stage of the production process because market competition becomes more intense and time frames for product development are shrinking. One recent approach to reliability prediction at early stages is using a relation model of yield and reliability since experimental results on various semiconductor devices indicate the correlated improvement of yield and reliability.

Though the explicit relationships between yield and reliability depend on the selected density distribution of manufacturing defects, no research has been done to study the effects of defect density distributions on the reliability predicted from yield. In the first part of this paper, we show for a single-area device that the reliability functions preserve the ordering of yield functions by the selected defect density distributions. After deriving the explicit relation models of yield and reliability for various defect density distributions, we conclude that yield can be scaled to obtain field reliability with a scaling factor for any given defect density distribution.

In the second part of the paper, we introduce a hierarchical model which has the flexibility in modeling different situations of devices. With an example of two-area device that allows for repair in one area, the interaction between yield and reliability has been further investigated. We pointed out through the model that the repair capability improves yield but not reliability, resulting in a large value of scaling factor. Therefore, for a repairable device, reliability can be very low even with high yield.

In order to get a reliable device, yield must be improved and additional screening method such as burn-in can be considered for a device with a large value of the scaling factor.

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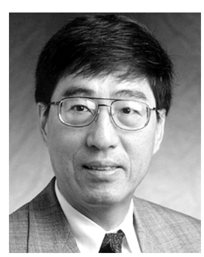

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