THE CHINESE UNIVERSITY OF HONG KONG

Moiré Effect

Chan Kin Ching

香港中文大學電子工程學系

DEPARTMENT OF ELECTRONIC ENGINEERING

Moiré Effect

Author : Chan Kin Ching

Student ID : 1155110652

Supervisor : Prof. Blu T.

Associate Examiner : Prof. Li HS

A project report presented to the
Chinese University of Hong Kong
In partial fulfillment of the
Degree of Bachelor of Engineering

Department of Electronic Engineering
The Chinese University of Hong Kong

April, 2022

1. Abstract

This project is about investigating the Moiré Effect. Two main topics will be discussed, including extracting Moiré patterns from an image, removing Moiré patterns from a photograph, etc. Though, the Moiré Effect is omnipresent, it is not paid attention by most of the people and they have only little understanding about the effect even they have noticed its existence. In the first part of the thesis, it is shown that the Moiré effect occurs whenever any high frequency repeating patterns are overlapped. The Moiré Effect is seen almost everywhere in our daily life, such as taking photographs of the screen of a computer. To remove the Moiré patterns from an image, it is essential to first get the information of the Moiré patterns and extract it. Then, an optimal method will be investigated to remove the Moiré patterns from the image. In this project, the above topics will be discussed.

2. Acknowledgements

Professor Blu Thierry

3. Table of Contents

4. Introduction

Though the Moiré Effect is sometimes useful, such as performing simple encoding mentioned in the first part of the project. One the other hand, as mentioned above, the Moiré Effect is everywhere in our daily life, while some are undesirable, especially when it appears in taking photographs of a screen. Removing the Moiré Effect from a photograph is an important job to show a clear image without Moiré patterns. As a result, this part of the project aims to remove the Moiré patterns from an image and obtain a Moiré-free clear image. To perform the task, the nature and properties of the Moiré patterns have to be investigated in order to understand well how the patterns can be removed. The Fourier Transform of Moiré patterns is discussed in the first part of the project, and it may be possible to remove the Moiré patterns with the Fourier Transform.

Figure 1 Moiré patterns formed by cages in zoos [1]

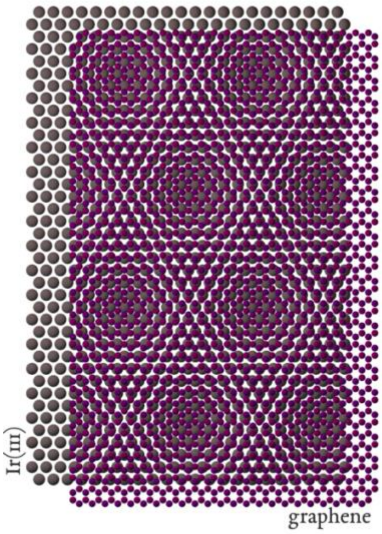

Figure 2 Moiré patterns formed by overlapping graphene sheets [2]

5. <u>Background</u>

In the first part of the report, the nature and formation of the Moiré Effect have been discussed. The effect is then used to encrypt an image, which is only able to be decoded with the exactly identical Moiré patterns. The Fourier Transform of the Moiré patterns is also demonstrated. In this part of the project, the Fourier Transform will be applied on the extraction and the removal of the Moiré patterns. As mentioned before, Moiré Effect occurs when high frequency repetitive patterns overlap. It is inevitable to see Moiré patterns when taking photo of repeated patterns, such as the computer screen, or some designs of clothes, as the sampler of the camera and the object are combined with repeating grids. It is not an easy job to remove the Moiré patterns from the photograph.

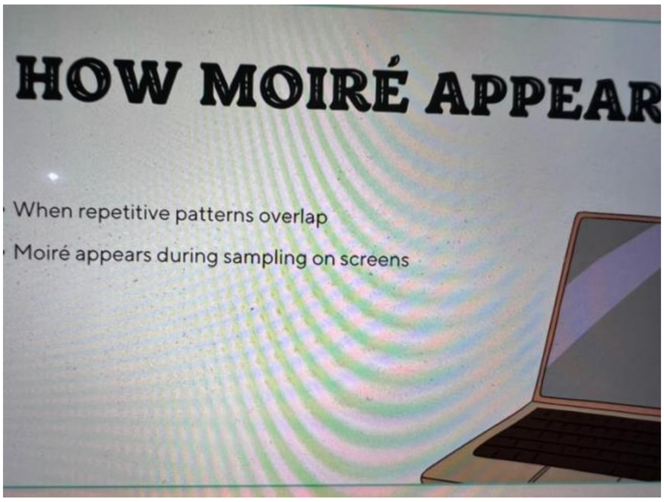

Figure 1 Moiré patterns when taking photo of a screen

i. Image Model Assumption

To start with, an image model is proposed to simplify the image into an equation. Suppose any image can be represented by the following equation.

$$I(x, y) = I_0 + Re(Ae^{iw_x x + iw_y y})$$

where I_0 is a constant and the exponential represents the changing Moiré patterns. The w contains information about the frequency and the orientation of the Moiré patterns.

For an example Moiré pattern shown below,

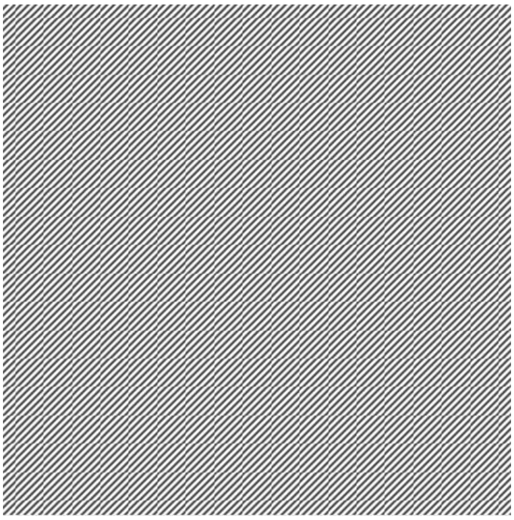

Figure 2 A Moiré pattern with 45-degree slant lines

can be represented by the following equation.

$$I(x, y) = 0.7 + Re((-0.2 + 0.4i)e^{iw_x x + iw_y y}) \qquad (1)$$

which values of the coefficient are set for the ease of identification in the later stage.

ii. Using maximum points of 2D Fourier Transform to find the coefficients

The Fourier Transform of a Moiré pattern has 3 maximum spikes in total, while the center spike represents the image, and the pixels around the 2 maximum spikes around the center spike represents the Moiré patterns. In the 1-dimensional case, it can be represented by the Fourier Transform of a square wave.

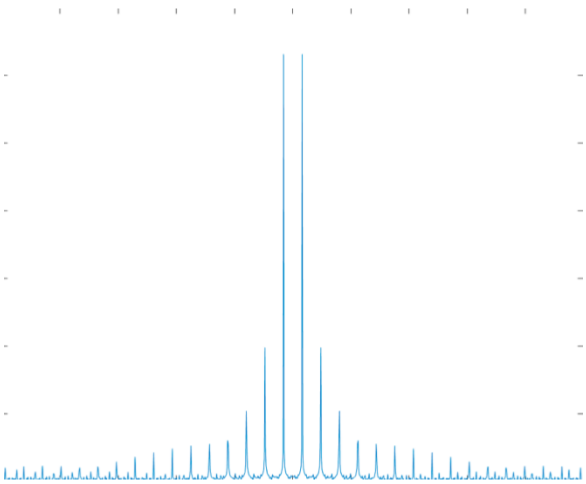

Figure 3 Fourier Transform of a Square wave

For the example Moiré patterns shown in Fig.4 above, its Fourier Transform is displayed below in Fig.5.

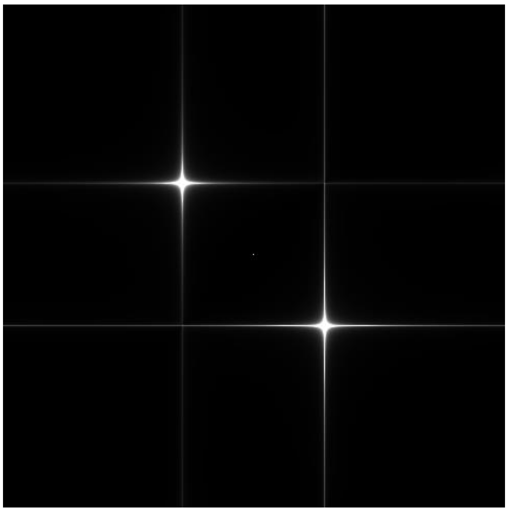

Figure 4 Fourier Transform of the Moiré pattern in Fig.3

The locations of the maximum points is used to find w_x and w_y, using the below equations.

$$w_x = \frac{2\pi x}{N} \tag{2}$$

$$w_y = \frac{2\pi y}{N} \tag{3}$$

where x and y is the coordinate of the maximum point. As the maximum points are parallel to each other, only one maximum point location is needed in calculating the w_x and w_y. In this case, the calculated w_x and w_y are both 0.884, while the calculated I_0 is 0.7, which are similar to the expected value. Yet, the calculated A is 0.331 + 0.0917i, which is completely different from the expected value of A.

iii. Minimize the error with Taylor expansion

As stated above, there are great error when calculated the coefficient A when using the above method. Another method has to be developed to calculate the accurate value of the parameters in the equation. Taylor expansion is used to minimize the error in the calculation. First, the values of the solution are approximated and calculated in the equation. The error of the value is used to adjust the approximated values in each iteration to reduce the error. An accurate value of the coefficients can be calculated in just few iterations.

Using this method with just 2 iterations, the parameters w_x, w_y, I_0 can be exactly found. The answer for A is also the same with the expected one, which is -0.2 + 0.4i.

iv. Removing Moiré patterns from an image

After extracting and getting the information of the Moiré patterns of an image, attempts are done to remove the Moiré patterns with various methods.

First, simple image filters such as averaging filters, Gaussian filters, Laplacian filters, etc. are tried on a Moiré image. Some filters such as averaging filters work fine to remove the Moiré patterns of the image, while the Gaussian filters show the best result.

Yet, using averaging filters and Gaussian filters will cause blurring on the image, which is not able to see a clear image. Fourier Transform is tried on the image to attempt removing the Moiré patterns. Since the maximum points of the Fourier Transform of an image containing Moiré patterns represent the patterns, setting the points to 0 can eliminate the Moiré patterns in the image. The pixels of the two corners of the maximum points are set to 0, while the other pixels are kept the same, as shown in Fig.6 below.

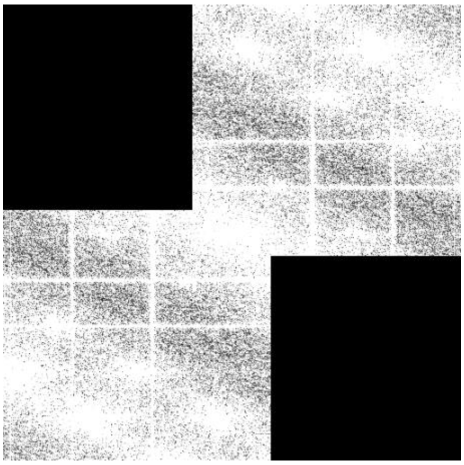
Figure 5 Removing the corners pixels of the Fourier Transform

Although some information of the Moiré patterns may contain in the pixels around the centre spike, a part of the Moiré patterns can be effectively removed by removing the other pixels. Less blurring is needed when applying the Gaussian filters after processing the Fourier Transform of the image.

In addition, RGB channel separation and Principal component analysis are tried to perform separately on a real image. Because of the sampling method of the three channels of RGB, the three channels may contain different information about the Moiré patterns. Investigating the three channels is possible to remove the Moiré patterns. On the other hand, the Principal component analysis can summarize the information content of the image with a smaller sets of summary images. It is also possible to extract the Moiré patterns of a real image.

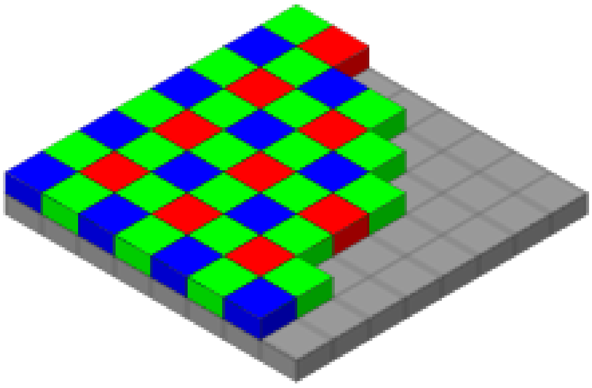

Figure 6 Arrangement of colour filters of a digital image sensor [3]

6. Methodology and Implementation

i. Image Model Assumption

As stated in equation 1, any Moiré image can be represented by the image model.

```
moireImage=I0+real(A*exp(1i*wx0*x+1i*wy0*y));
```

A Moiré image will be shown as Fig.3, letting the coefficients as the same as those in part (5).

ii. Using maximum points of 2D Fourier Transform to find the coefficients

First, the value of the centre pixels is set to 0, in order to find the surrounding maximum points.

```
origin = rows/2+1;

J(origin,origin) = 0;
```

The location of the maximum point is found using the max() function.

```
[x, y] = find(ismember(J, max(J(:))));
```

where x = 185 and y = 185.

w_x and w_y are then calculated using equation 2 and equation 3.

```
wx = jx(2) - origin;

wy = jy(2) - origin;

wx = 2 * pi * wx/H

wy = 2 * pi * wy/K
```

The values calculated are $w_x = 0.884$ and $w_y = 0.884$, which is very close to the expected value 0.880, having an error of about 0.5%.

Coefficient I_0 and A is further calculated using mldivide operator \,

```
A = ones(H * K,1);

B = zeros(H);C = zeros(H);

for x=1:H

   for y=1:K

      pow = 1i * wx * x + 1i * wy * y;

      B(x,y) = exp(pow);

      C(x,y) = exp(-pow);

   end

end

I = reshape(moireImage,H*K,1);

M = [ones(H * K,1),B(:),C(:)];

X = M\I
```

where I_0 and A is X(1) and 2*X(2), which are calculated as 0.7 and 0.331 + 0.0917i respectively. The calculated I_0 is the same as the expected one, but the calculated A is completely different from that of the expected value -0.2 + 0.4i.

iii. Minimize the error with Taylor expansion

In each iteration, the error is used to fine tune the approximation of the solution.

```
J=exp(i*x(:)*w(1,:)+i*y(:)*w(2,:));
a=J\I(:);
Imin(:)=J*a;
dw=0*w;
for k=1:K
    A=[ones(M*N,1),i*x(:),i*y(:)].*J(:,k);
    da=A\(I(:)-Imin(:));
    dw(:,k)=da(2:3)/(a(k)+da(1));
end
err(j)=norm(prevImin(:)-Imin(:))/sqrt(M*N);
w=w+real(dw);
prevImin=Imin;
```

where a(1) is the value of I_0 and the 2*a(2) is the value of A. They are calculated as 0.7 and -0.2 + 0.4i in two iterations, which is exactly the same as expected.

iv. Removing Moiré patterns from an image

Different image filters are used in each attempt to remove the Moiré patterns from the image. Below are some examples of the filters used.

a. Averaging Filter

```
boxKernel = ones(10,10);

conv2(revealed_image, boxKernel, 'valid');
```

b. Gaussian Filter

```
gauss =

imgaussfilt(revealed_image,2.5,Padding="symmetric",FilterDomain='frequency');
```

c. Median Filter

```
medfilt2(revealed_image,[8,8]);
```

d. Laplacian Filter

```
fspecial('laplacian');
```

Among all filters, Gaussian filters work the best to remove the Moiré patterns of the image, but the image will be blurred. To obtain a clearer image, the Fourier Transform of the image is processed such that the corners of the maximum points are set to 0, as shown in Fig.6.

The location of the maximum points is calculated with the following method.

```
jx = param.w(1,3)*H/(2*pi);jy = param.w(2,3)*K/(2*pi);

centrex = H/2;centrey = K/2;
```

The pixels starting from the half of the maximum points are set to 0.

```
ft(1:centrex+round(jx/2),1:centrey+round(jy/2)) = 0;

ft(centrex-round(jx/2):H,centrey-round(jy/2):K) = 0;
```

The Fourier Transform is then used to obtain the original image, which has less Moiré patterns than before.

In addition, RGB channel separation and Principal component analysis are used to remove the Moiré patterns.

To separate the RGB channels,

```
[R,G,B] = imsplit(moireImage0);
```

while separating the image with principal component analysis,

```
coeff = pca(X);

Itransformed = X * coeff;
```

Three different images are obtained in the both methods.

7. Simulation/Experimental Results

The simulation involved in this project is done by MATLAB R2022a.

i. Image Model Assumption

Moiré patterns with different frequencies and in different orientations can be generated by the image model stated in equation (1).

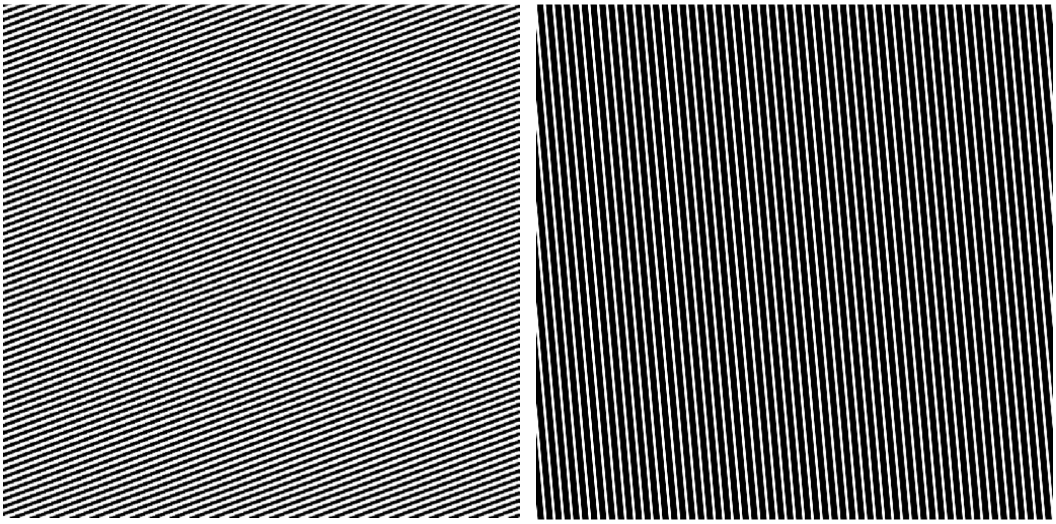

Figure 7 Random Examples of Moiré patterns generated by the image model

ii. Using maximum points of 2D Fourier Transform to find the coefficients

For the Fourier Transform example below, the 2 maxima are found at (185,185) and (329,329) where the top left corner of the image is (0,0).

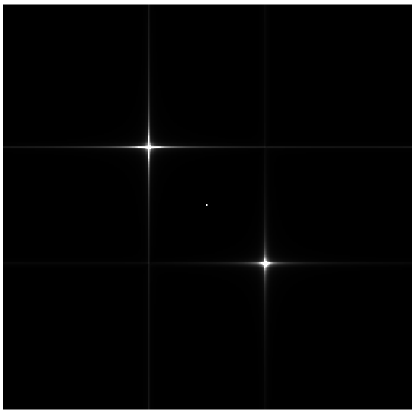

Figure 8 Fourier Transform of a Moiré image

The w calculated is 0.884, using the equation (2) and equation (3), which is similar to the expected value of w of 0.88. On the other hand, the value of I_0 calculated is 0.7, which is the same as the original value, but the value of A is 0.331 + 0.0917i, which is completely different from the original value of -0.2 + 0.4i. The calculated values are used to generate a new Moiré pattern. The newly generated Moiré pattern is shown below.

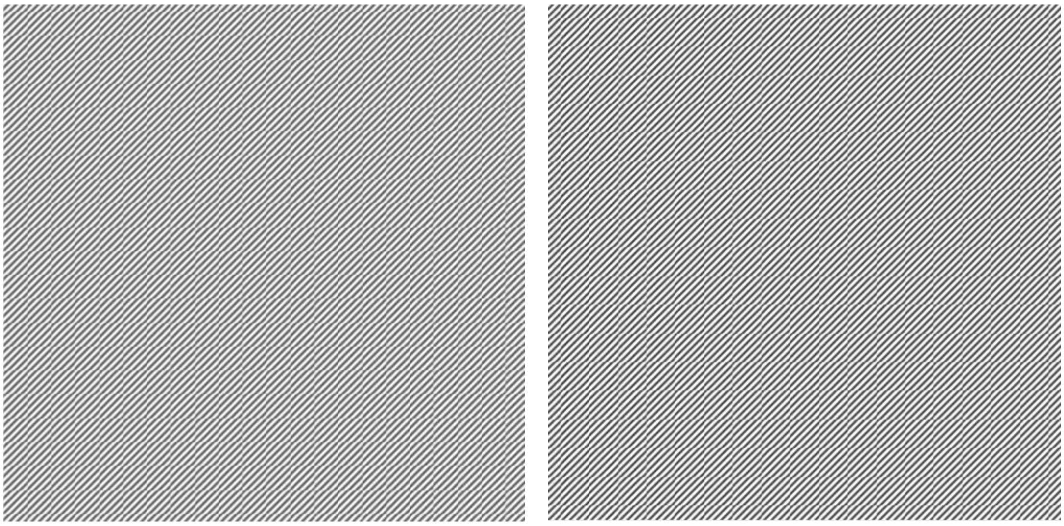

Figure 9 The newly generated Moiré pattern and the original Moiré pattern respectively

The two images look the same. Yet, there are differences in the phase of the two images. The new Moiré pattern is unable to decode the previously encoded image and show a mixture of Moiré patterns.

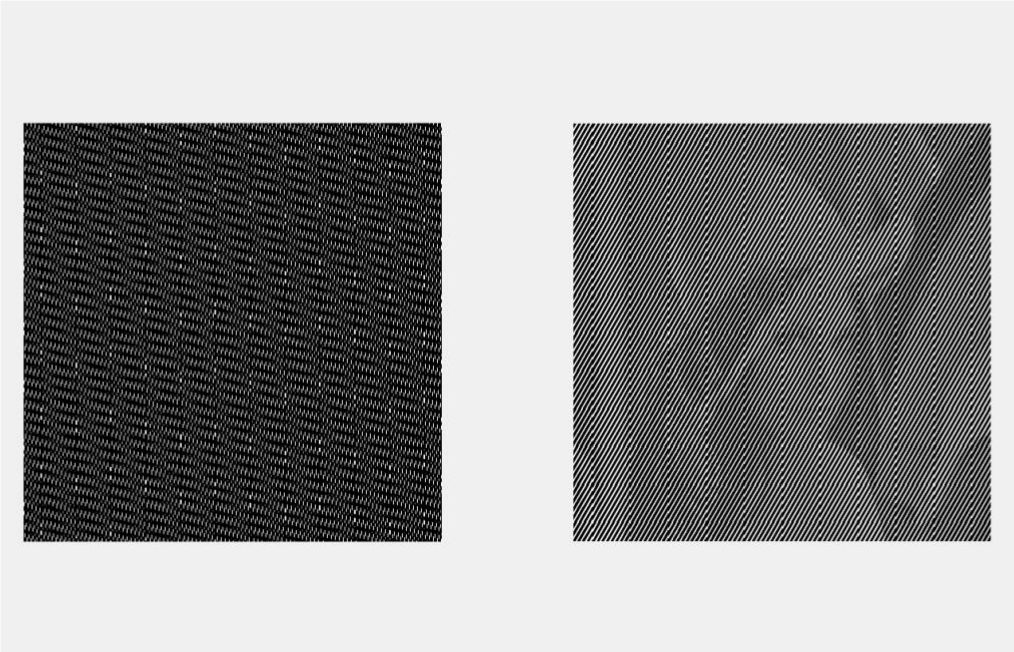

Figure 10 Moiré patterns with inaccurate parameters cannot decode on the right like the expected one on left

iii. Minimize the error with Taylor expansion

With this new method, the exact values of all the parameters can be calculated.

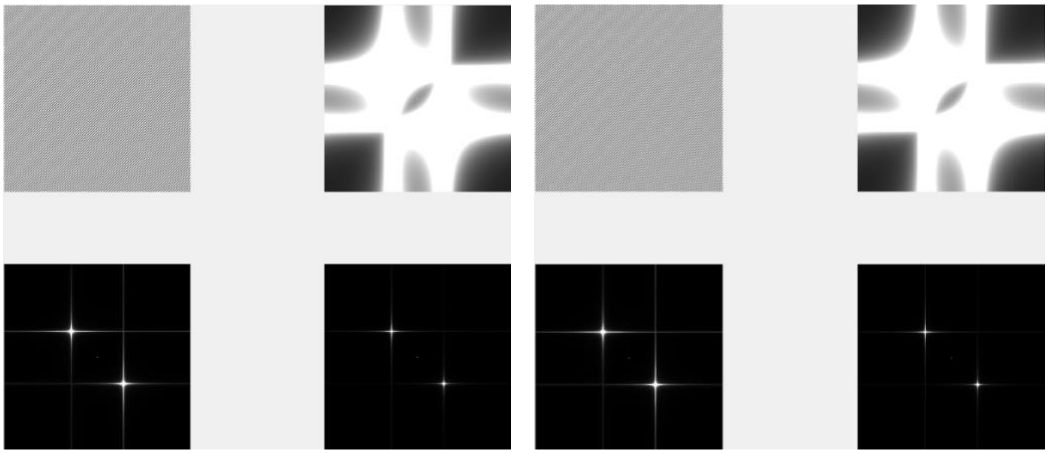

Figure 11 Original patterns are on the left and the regenerated patterns are on the right

The Moiré pattern, its Fourier Transform, adjustment with the min-max range of the image are shown. The left one is the original Moiré pattern while the right one is the Moiré pattern using the calculated values. Both show the exact same patterns in the four illustrations of the Moiré pattern. This shows that the value calculated by this method is highly accurate in achieving the information of the Moiré pattern.

With this regenerated Moiré pattern, the decoded image can be encoded and the result is exactly the same as that decoded by the original image.

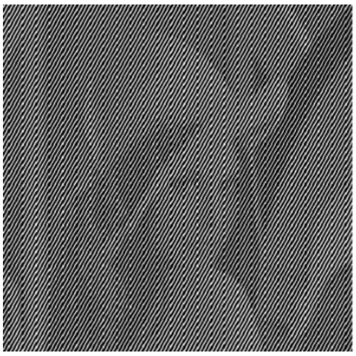

Figure 12 Decoded image with the calculated pattern

iv. Removing Moiré patterns from an image

The results of the demonstrated filters are shown below.

a. Averaging Filter

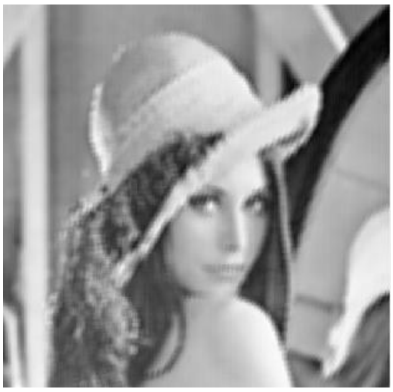

Figure 13 Result of averaging filters

b. Gaussian Filter

Figure 14 Result of gaussian filters

c. Median Filter

Figure 15 Result of median filters

d. Laplacian Filter

Figure 16 Result of Laplacian filters

Among the above filters, the Gaussian filter perform the best in eliminating Moiré patterns and producing the clearest image. Yet, blurring is inevitable when using the Gaussian filters. The Fourier Transform is first used to reduce the Moiré patterns, to reduce the size of the Gaussian filter needed to get a clearer image.

By setting the corners of the Fourier Transform to 0, the Moiré patterns can be reduced.

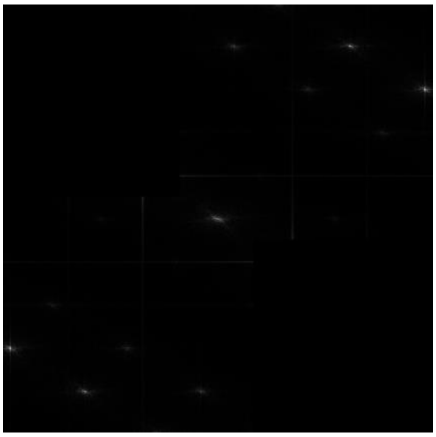

Figure 17 The Fourier Transform after setting the corners to 0

Comparing the original image and the processed image, Moiré patterns in the processed image are much less than the original one.

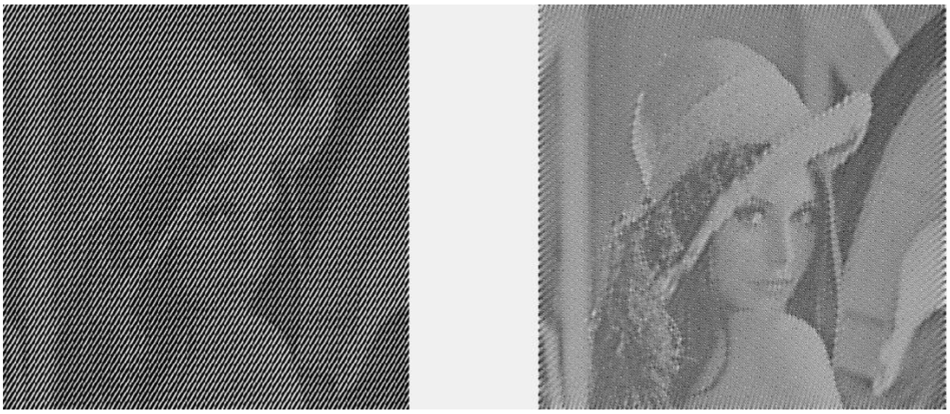

Figure 18 Original image on the left and processed image on the right

The processed image can then be filtered by a smaller Gaussian filter to obtain a less blurring image.

Figure 19 Original image with Gaussian filter on the left while the new processed image with smaller Gaussian filter on the right

Smaller Gaussian filter is needed to eliminate most of the Moiré patterns using the Fourier-processed image and the blurring is reduced to obtain a clearer image.

For a real image, the RGB channel separation and principal component analysis can remove some of the Moiré patterns.

Figure 20 Original image

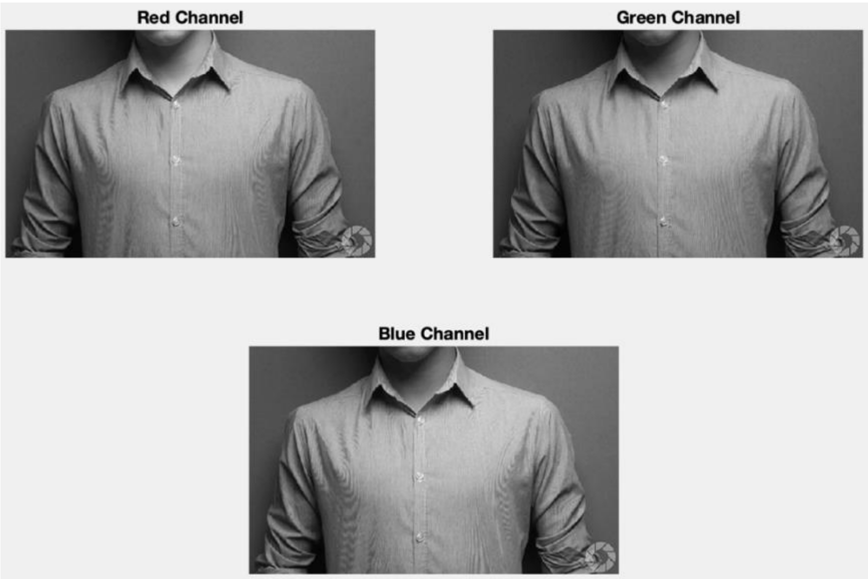

Figure 21 RGB channel separation of the image

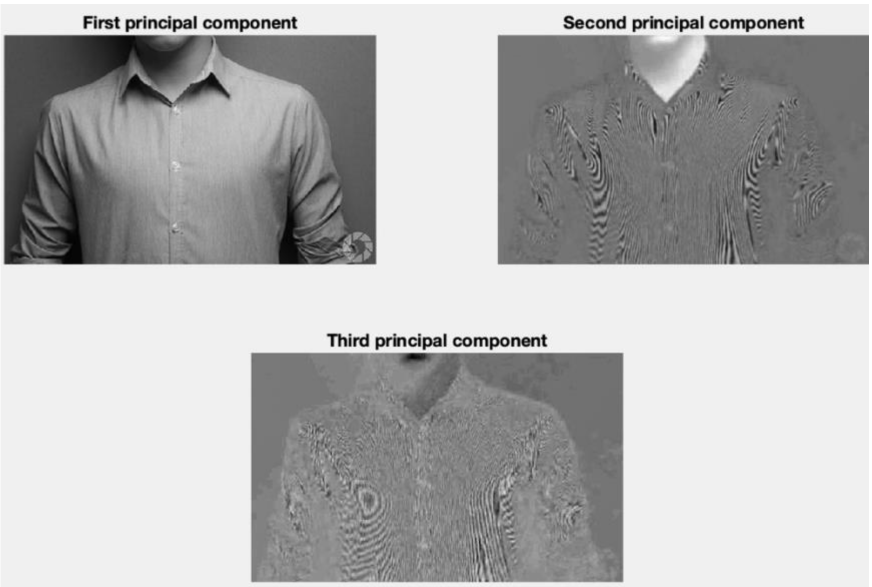

Figure 22 Principal component analysis of the image

Different color channels show different Moiré patterns and the RED channel has the least Moiré patterns. On the other hand in the principal component analysis, the first principal component shows the least Moiré patterns.

8. Discussions

For a real image, the Moiré patterns are not uniform and are changing slowly along the pixels. If the above global Fourier Transform analysis method is used directly on a real image, Moiré patterns cannot be eliminated effectively as the information of the Moiré patterns calculated is varying in different pixels of the image. Thus, the method can be performed block-wise in the image such that the image is split into multiple square blocks and each block is processed with the Fourier Transform analysis method to eliminate its Moiré patterns. A full image with Moiré patterns eliminated can be obtained after combining all the blocks back.

On the other hand, the Moiré patterns removal using RGB channel separation and principal component analysis are possible methods to remove the Moiré patterns from a real image. Further investigations are needed to eliminate most of the Moiré patterns in the image. The information of Moiré patterns may be stored in different RGB channels and principal components. The three images obtained can be further processed to get a clear image with any Moiré patterns.

9. <u>Conclusion</u>

To sum up, it is important to get highly accurate information of the Moiré patterns to understand the Moiré patterns and extract it from an image. With just a small error of even 0.1%, there can be a great difference in the phase of the Moiré patterns and it will be impossible to extract the correct Moiré patterns with the incorrect parameters. To remove the Moiré patterns from an image, multiple methods can be used, including averaging filters, Gaussian filters, etc. Yet, these simple filters applying on the image will cause blurring, which is not ideal for obtaining a clear image. The extracted information of the Moiré patterns can be used to eliminate the patterns with the Fourier Transform. This method is possible to reduce the Moiré patterns of an image and it causes less Gaussian blurring. Some other methods may also be possible to remove the Moiré patterns in a real image, including RGB channel separation and principal component analysis. However, the complicated algorithm behind needs to be further investigated to precisely remove the Moiré patterns of a real image.

10. References

1. Wikipedia. (2006). The Bayer arrangement of colour filters on the pixel array of an image sensor. photograph. Retrieved April 3, 2022, from https://en.wikipedia.org/wiki/Bayer_filter#/media/File:Bayer_pattern_on_sensor.svg.

2. Wikipedia. (2007). Moiré pattern seen over a cage in the San Francisco Zoo. photograph. Retrieved April 1, 2022, from https://en.wikipedia.org/wiki/Moiré_pattern#/media/File:Moiré_fringes_IMG_3712.jpg.

3. Wikipedia. (2020). When graphene is grown on the (111) surface of iridium. photograph. Retrieved April 1, 2022, from https://en.wikipedia.org/wiki/Moir%C3%A9_pattern#/media/File:Graphene_moire_on_Ir(111)_-_schematic.svg.