THE CHINESE UNIVERSITY OF HONG KONG

Moire Effect

Chan Kin Ching

香港中文大學電子工程學系

DEPARTMENT OF ELECTRONIC ENGINEERING

Moiré Effect

Author : Chan Kin Ching

Student ID : 1155110652

Supervisor : Prof. Blu T.

Associate Examiner : Prof. LI HS

A project report presented to the
Chinese University of Hong Kong
In partial fulfillment of the
Degree of Bachelor of Engineering

Department of Electronic Engineering
The Chinese University of Hong Kong

November, 2020

1. Abstract

The project is about the understanding of the Moiré Effect. Various aspects about the effect will be discussed, including the formation and the cause of the effect, different forms of the effect, the use, and the ways to avoid and remove the unfavorable ones. Though the Moiré Effect appears almost everywhere in our daily life, most people are not quite aware of it and have little understanding about the effect even they have noticed it. The Moiré Effect is formed when high frequency repeated patterns are overlapped, no matter the two patterns are on the same plane or not. It is seen almost everywhere in our daily life, some do not matter while some are undesirable, such as taking photo with some repeated patterns of high frequency. Some algorithm used to remove the undesired Moiré Pattern will also be discussed. Though the Moiré Effect is sometimes unwanted, it can be used in image hiding and to encrypt a message in the Moiré Patterns. Despite the fact that the Moiré Effect may be unwanted and unavoidable in our daily life, it can still be useful in some areas. In this project, the above will mainly be focused.

2. Acknowledgements

Professor Blu Thierry

Vladimir Saveljev

Sung-Kyu Kim

Jaisoon Kim

3. Table of Contents

4. Introduction

The Moiré Effect is a physical phenomenon formed by repeated patterns with high frequencies. Wherever there are high frequencies repeating patterns overlapping, the Moire Effect occurs. It is present everywhere in our daily life, such as fences on the street, architectures, taking photograph of an electronic device, etc. Though the effect is so omnipresent, it is uneasy to fully understand it, with various forms and aspects. The term "Moiré", which was originated from French, was a type of textile, traditionally of silk but now of cotton or synthetic fiber, with a rippled or "watered" appearance. Since the patterns of the textile are so similar but imperfect, the spacing in between creates the characteristic pattern which remains after the fabric dries. [1]

Fig. 1 Moire Effect in architectures, M3 Architects' Brisbane Girls Grammar School [2]

In this project, mainly two papers, “Moiré effect in displays: a tutorial” and “Image hiding based on circular geometric moiré”, are focused and investigated. In “Moiré effect in displays: a tutorial”, the Moiré Effect is briefly introduced. Examples of the formation of the Moiré effect are given and the exact equations of different forms of Moiré effect, including coplanar two-line gratings, sinusoidal grating, cylindrical Moiré, 2-D Fourier transform are discussed. In “Image hiding based on circular geometric moiré”, an application of the Moire effect is demonstrated – visual cryptography. Circular Moire grating is used to encrypt an image in the algorithm. Exact location of the center point and exact value of the amplitude of the oscillation are needed in order to decode the image.

5. Background

Moire Patterns are formed by overlapping repeated patterns with high frequencies. It can be caused by various forms, either on the same plane or non-coplanar ones, for grid lines, circular patterns, or even dots.

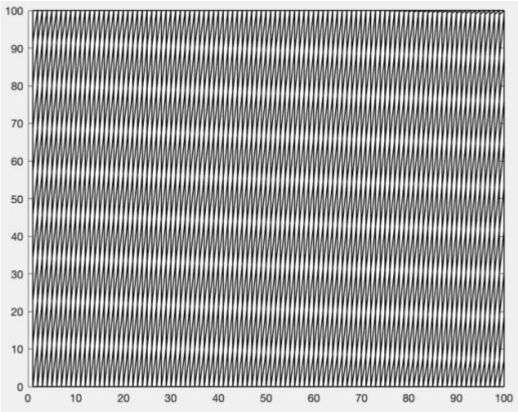

Fig. 2 Grid lines Moire pattern

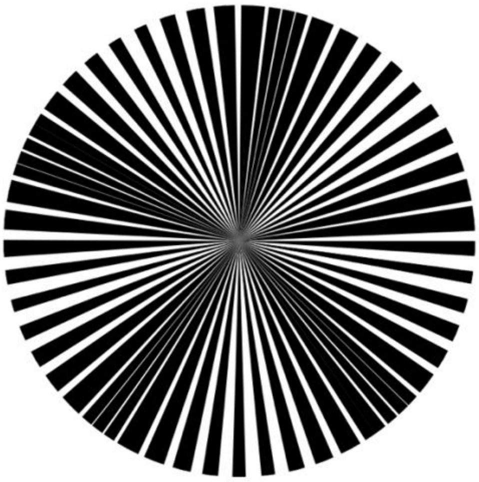

Fig. 3 Circular Moire pattern [3]

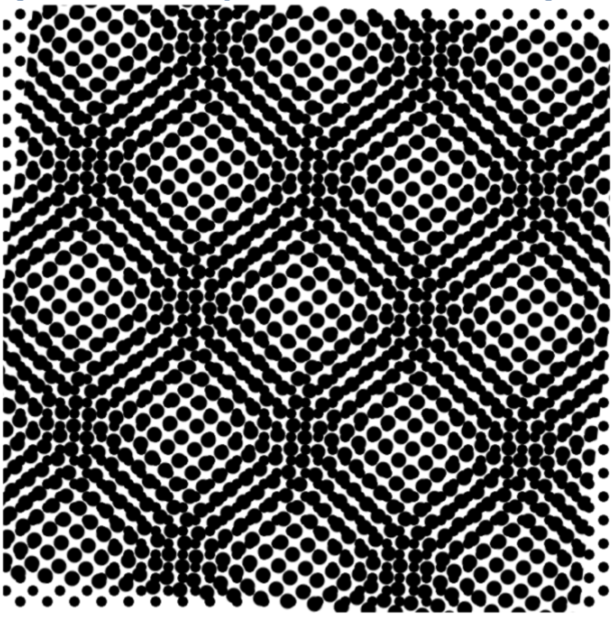

Fig. 4 Dots Moire pattern

Moire pattern will change rapidly even when the original patterns only move a bit. The patterns are highly affected by the angle and the period of the two overlapping patterns.

i. Linear Moire grids

For linear Moire girds, with gridlines of the following two families [4],

$$y = mT_1 \tag{1}$$

$$x \sin \alpha + y \cos \alpha = nT_2 \tag{2}$$

where m, n are integers enumerating the lines, T_1, T_2 are the period of the two families, and α is the tilted angle. Then, the formed Moire patterns with the intersections of (1) and (2) will be,

$$-xT_1 \sin \alpha + y(T_2 - T_1 \cos \alpha) = T_1T_2(m - n) \tag{3}$$

The tangent of the Moire lines can be obtained by,

$$\tan \theta = \frac{-T_1 sin\alpha}{T_2 - T_1 cos\,\alpha}$$

$$= \frac{-\sin\alpha}{\frac{T_2}{T_1} - cos\alpha} \tag{4}$$

and the period of the Moire lines will be,

$$T_m = \frac{T_1 T_2}{\sqrt{T_1^2 - 2T_1 T_2 \cos\alpha + T_2^2}}$$

$$= \frac{T_2}{\sqrt{1 - 2\frac{T_2}{T_1}\cos\alpha + \left(\frac{T_2}{T_1}\right)^2}} \qquad (5)$$

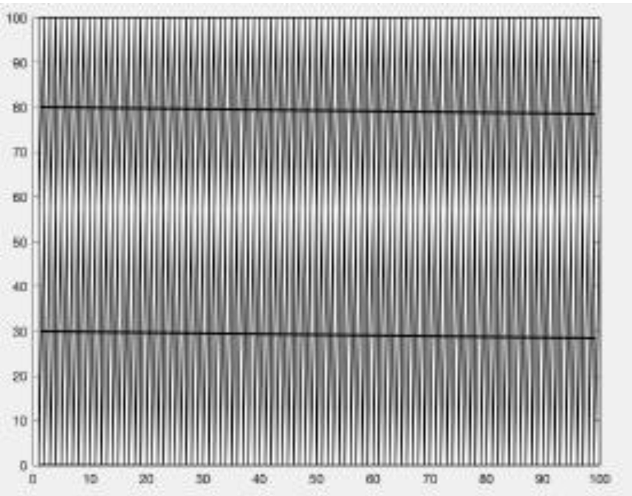

Fig. 5 Moire lines indicated by black lines joining the intersections of the original linear gridlines

ii. Image Hiding with circular Moire Patterns

Moire patterns can be used to hide visual information (pictures, text, etc.) in an image. An algorithm is implemented that the location for the secret message will have Moire patterns of different frequencies comparing with other locations in the image [5].

Normal circular Moire patterns are generated with the following equation,

$$M(y) = \frac{1}{2} + \frac{1}{2}\cos\left(\frac{2\pi}{\lambda}y\right) \qquad (6)$$

y is the longitudinal coordinate, λ is the pitch of the grating. M(y) value ranges from 0 to 1, correspond to black and white. The time-averaged Moire grating will be

$$M_T(x,y) = \lim_{n\to\infty}\frac{1}{n}\sum_{k=0}^{n-1}\frac{1}{2}\left(1+\cos\left(2\,\pi/\lambda\left(\varphi+\operatorname{asin}\left(\frac{2\pi k}{n}\right)\right)\right)\right) \qquad (7)$$

where $\varphi =$

$$\varphi = \begin{cases} \arctan\left(\frac{y_0-y}{x-x_0}\right), & \text{if } x \neq x_0 \\ \frac{\pi}{2}, & \text{if } (x = x_0)\ \&\ (y < y_0) \\ \frac{3\pi}{2}, & \text{if } (x = x_0)\ \&\ (y > y_0) \\ 0, & \text{if } (x = x_0)\ \&\ (y = y_0) \end{cases} \qquad (8)$$

The location for the hidden message will be similar to the lower part of the following image,

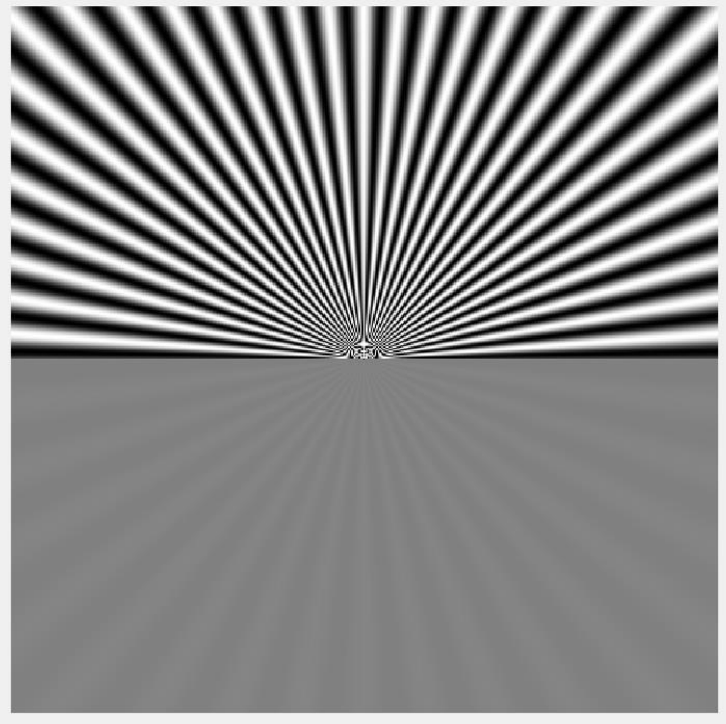

Fig. 6 time-averaged image of the circular Moire pattern

Thus, the secret message can be revealed and seen by human eyes without the help of any instruments. To decode the hidden message, an exact value of the central point and α have to be selected. The secret message cannot be revealed whenever there is a slight difference in any one of the both parameters.

iii. Fourier Series and Fourier Transform

The square wave,

f(x) = 0 , when x = 2kπ

= 1 , when x = (2k + 1) π

where k is an integer (9)

can be represented by Fourier Series,

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n \cos nx + b_n \sin nx)$$

$$\approx \frac{1}{2} + \frac{2}{\pi}\sin x + \frac{2}{3\pi}\sin 3\pi + \frac{2}{5\pi}\sin 5\pi \qquad (10)$$

as shown in the following figure,

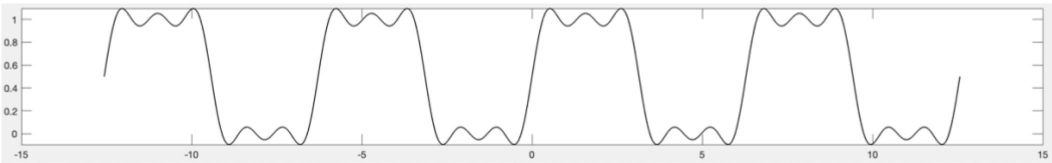

Fig. 7 Fourier Series Representation of f(x)

For any 1-D Moire pattern, 3 minimum point around the center can be obtained when Fourier transform is performed on it.

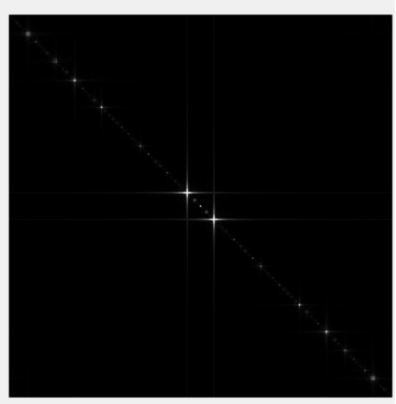

Fig. 8 Fourier transform of a 45-degree tilted Moire pattern

When the tangent of the Moire pattern is changed, the location and the rotation angle of the two surrounding points will be different.

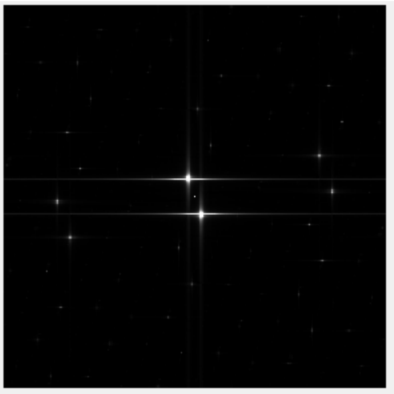

Fig. 9 Fourier transform of a 20-degree Moire pattern

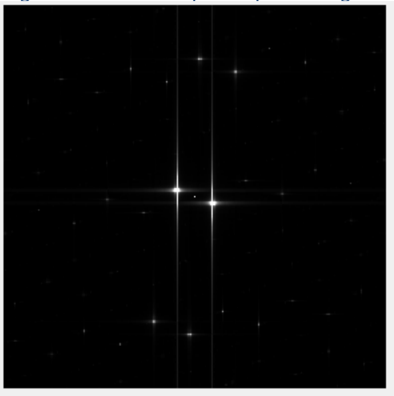

Fig. 10 Fourier transform of a 70-degree Moire pattern

The separation of the two neighbouring points will also change when the period of the Moire patterns is changed.

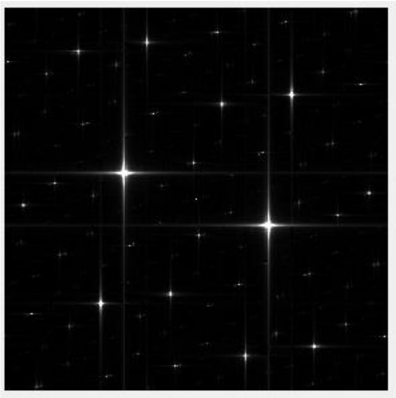

Fig. 11 Fourier transform of a Moire pattern of period of 5

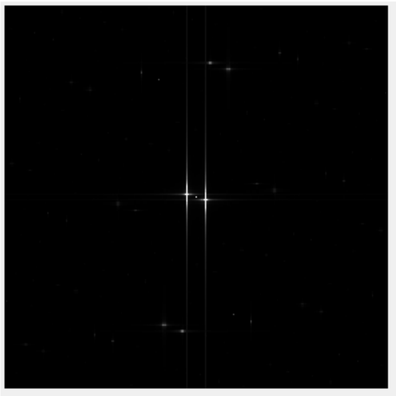

Fig. 12 Fourier transform of a Moire pattern of period of 40

By Fourier transform, the period and orientation of the Moire pattern can be obtained from the location and orientation of the corresponding maximum of the Fourier transform.

6. Methodology and Implementation

In this project, Matlab is used to do the simulation of the Moire Effect.

i. Linear Moire grids

For the grid Moire pattern, the vertical lines are generated with the following code,

```
>> x = 1:100;

>> plot([x; x], [max(x) * zeros(1, ...

>> length(x))*min(ylim); ...

>>   max(x) * ones(1, length(x))*max(ylim)], ...

>>          "Color", [0,0,0], "LineWidth", 1)
```

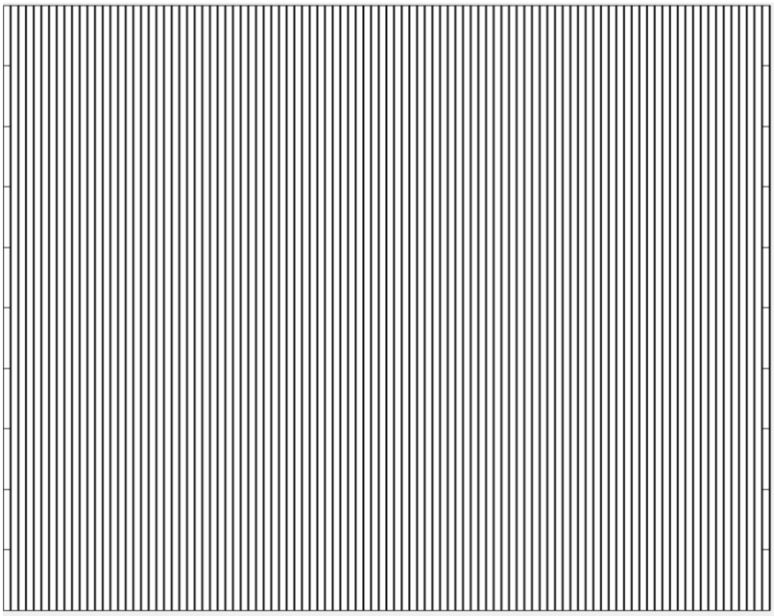

Fig. 13 Plotting vertical lines

Then add the overlapping inclined liens using the following code,

```
>> angle = 90 - 2;

>> slope = tan(deg2rad(angle));

>> c = abs(1 / cos(deg2rad(angle)));

>> for m=-slope * 100:100

>>     k = slope * x + c * m;

>>     y = k;

>>     y(k > max(x)) = max(x);

>>     y(k < 0) = 0;
```

```
>>     hold on

>>     plot(x, y, "LineWidth", 1, "Color", ...

>>                 [0,0,0])

>> end
```

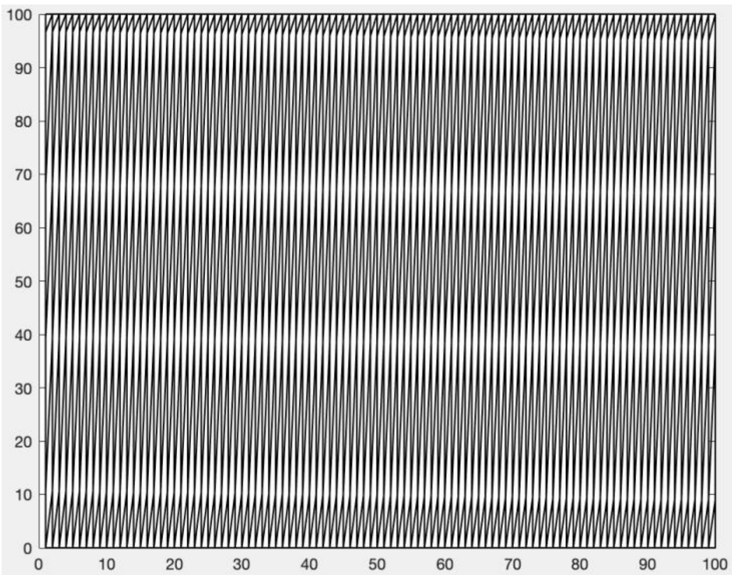

Fig. 14 Moire Patterns generated when the tilted lines are added on the original girds

ii. Image Hiding with Circular Moire Patterns

Circular Moire Patterns that have different period on the upper half and lower half of the image have the following different λ and angle values,

```
>> if x < rows / 2

>>                 lambda1 = 0.1;
```

```
>>      a1 = 0.0383;

>> else

>>      lambda1 = 0.18;

>>      a1 = 0.0689;

>> end
```

To generate the circular Moire patterns, the following function is implemented,

M(x,y) = (1 + cos(2 * pi / lambda1 * (atan((y0 - y)/(x - x0)) + a1 * sin(2 * pi * k / 1)))) / 2 (11)

where k is 0. The Time-averaged image can be generated with the following code,

```
>> for k=0:n-1

>>   M(x,y) = M(x,y) + (1 + cos(2 * pi / ...
```

```
>>   lambda1 * (angle ...

>>   + a1 * sin(2 * pi * k / n)))) / 2;

>> end

>> M(x,y) = M(x,y) / n;
```

The hidden message can then be revealed.

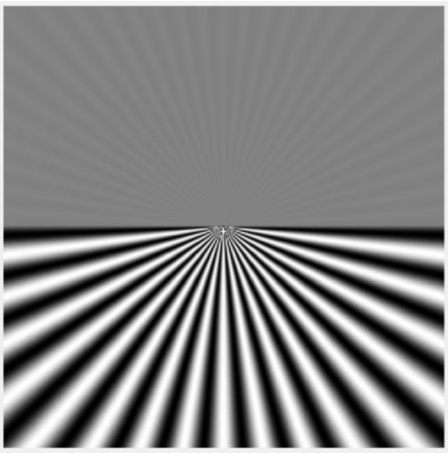

Fig. 15 Time-averaged image to reveal the secret message (the upper part of the image)

iii. Fourier Series and Fourier Transform

Suppose a 1-D Moire pattern as followed,

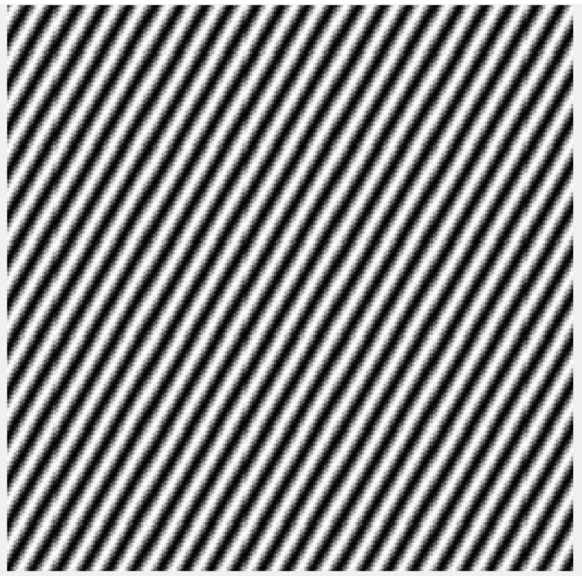

Fig. 16 1-D Moire pattern with tangent of 60-degree

can be generated with the following code,

```
>> rowVector = (1 : rows)';

>> period = 10;

>> amplitude = 1;

>> offset = 1 - amplitude;

>> cosVector = amplitude * (1 + cos(2 * pi * ...
```

```
>> rowVector / period))/2 + offset;

>> moireImage = repmat(cosVector, [1, columns]);

>> rotate = imrotate(moireImage,60);

>> moireImage = imcrop(rotate, [rows/2 rows/2 ...

>> rows/2 rows/2]);
```

2-D Fourier transform is performed on the image by fft2 and the zero-frequency component is shifted to the center of the image by fftshift. The following image will be obtained.

Fig. 17 Fourier transform of the Moire patterns generated in Fig. 16

After setting the low-high range of the image, the following adjusted image can be obtained.

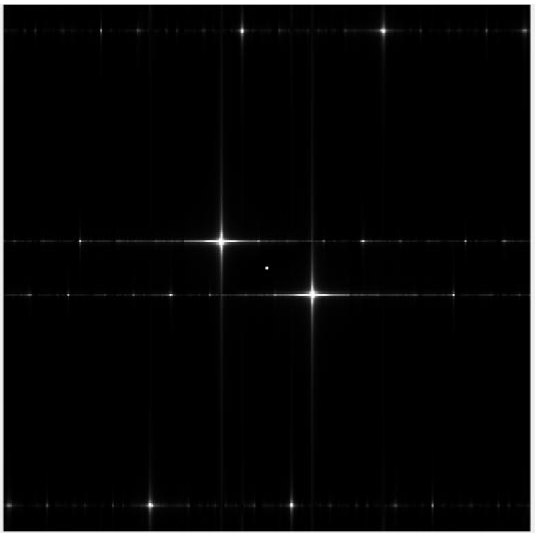

Fig. 18 Adjusted image of Fig. 17

The location of the three minimum point around the center can be obtained.

7. <u>Simulation Results</u>

The simulation involved in this project is done by Matlab R2021b.

i. Linear Moire grids

The period of the Moire lines increases rapidly even when there is only slight change in the orientation of one of the grids.

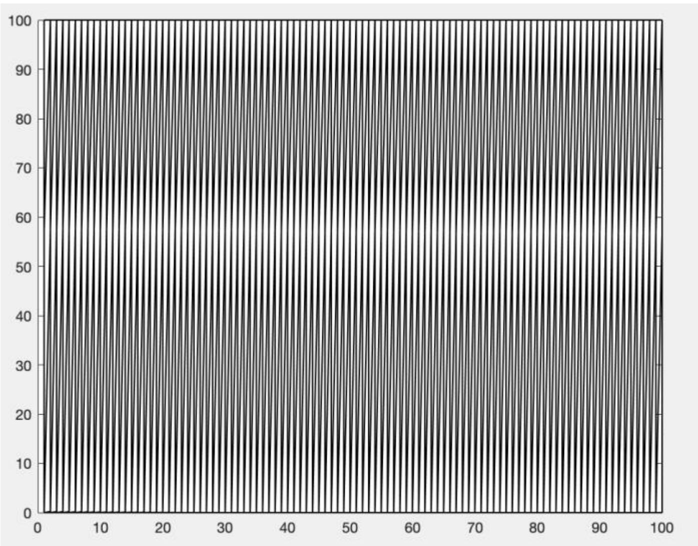

Fig. 19 Moire patterns with 1-degree rotation

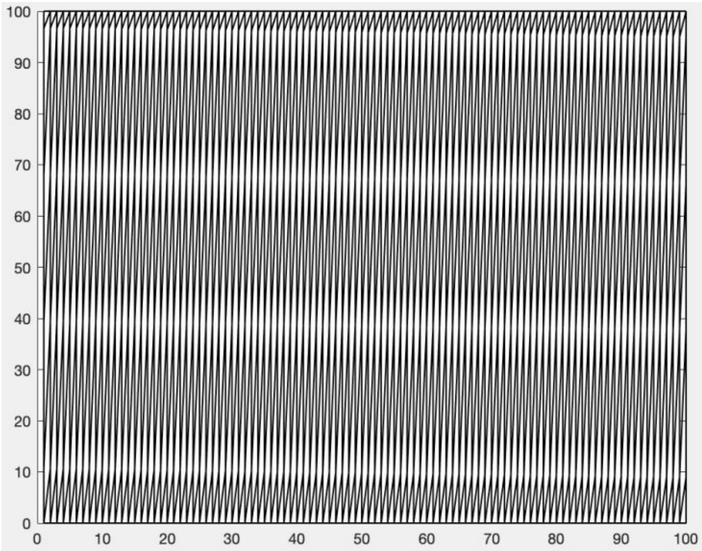

Fig. 20 Moire patterns with 2-degree rotation

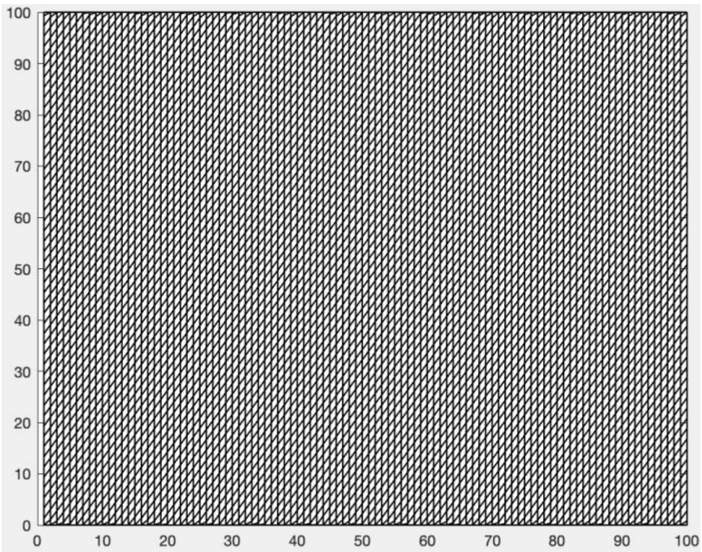

Fig. 21 Moire patterns with 30-degree rotation

ii. Image Hiding with Circular Moire Patterns

The original image with two different frequencies is generated.

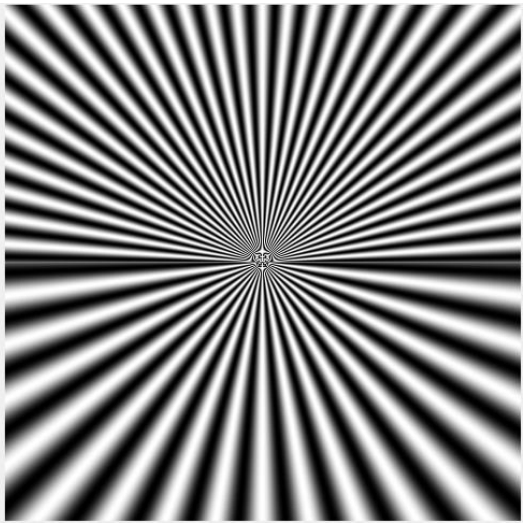

Fig. 22 Circular Moire Patterns with different frequencies for the upper and lower part of the iamge

The time-averaged image will be able to reveal the message when the exact parameters are correctly selected.

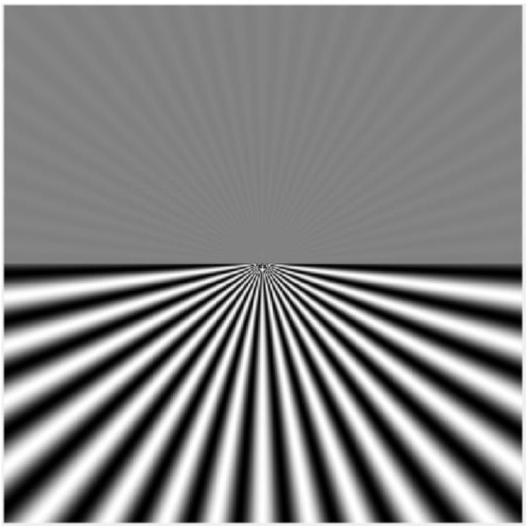

Fig. 23 Time-averaged image if the parameters of the upper half of the image is selected

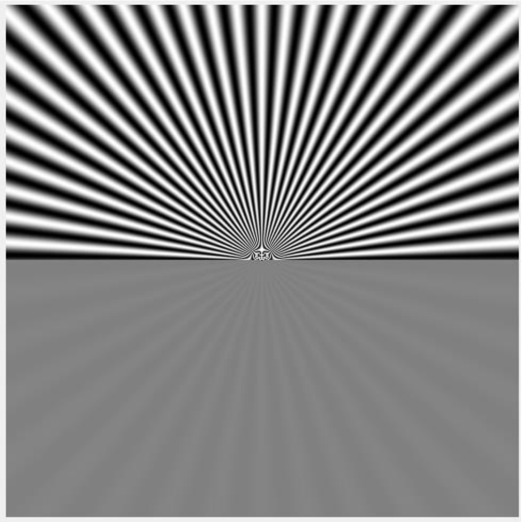

Fig. 24 Time-averaged image if the parameters of the lower half of the image is selected

When any of the parameters are not correctly selected, the hidden secret cannot be revealed.

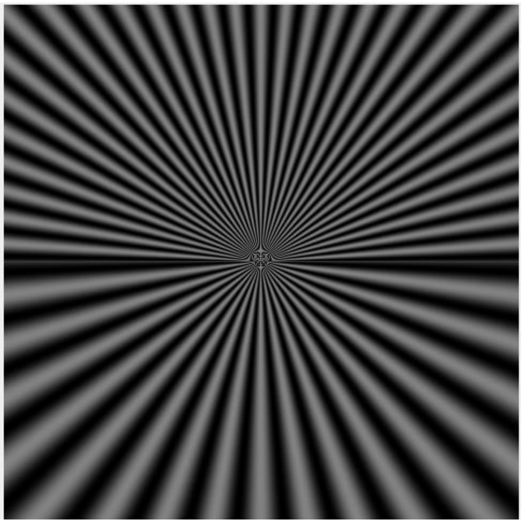

Fig. 25 Time-averaged image if any of the parameters is not correct

iii. Fourier Series and Fourier Transform

The square wave stated in (9) can be approximated by Fourier Series in (10).

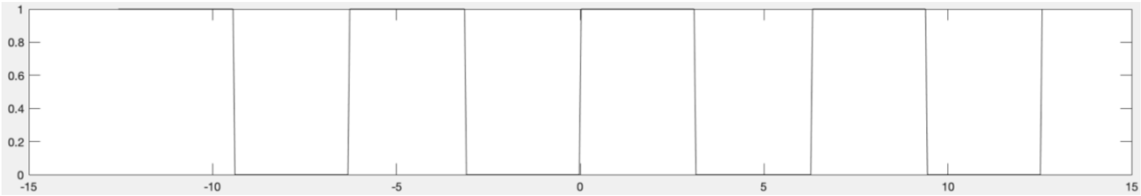

Fig. 26 Plot of (9)

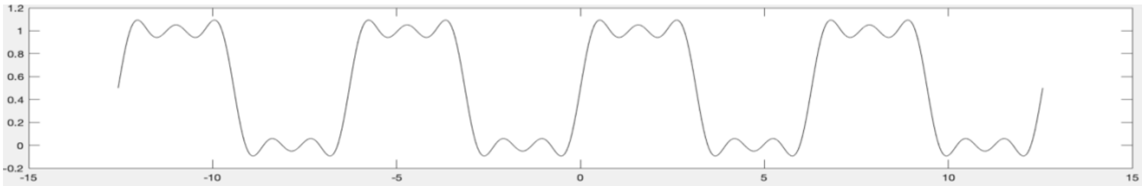

Fig. 27 Plot of (10), the Fourier Series Representation of (9)

For 2-D image, a Moire pattern with a 45-degree tangent is first generated.

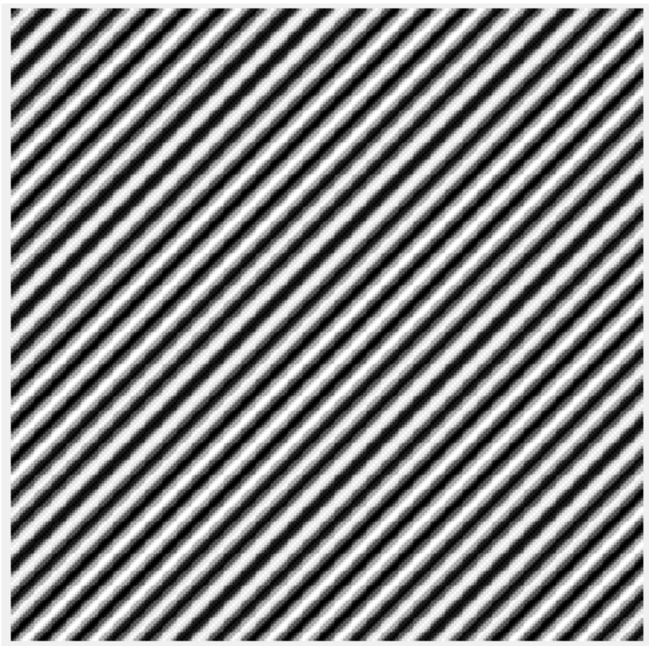

Fig. 28 1-D Moire pattern tilted with 45-degree

After performing Fourier transform to fig. 28, the following image is obtained.

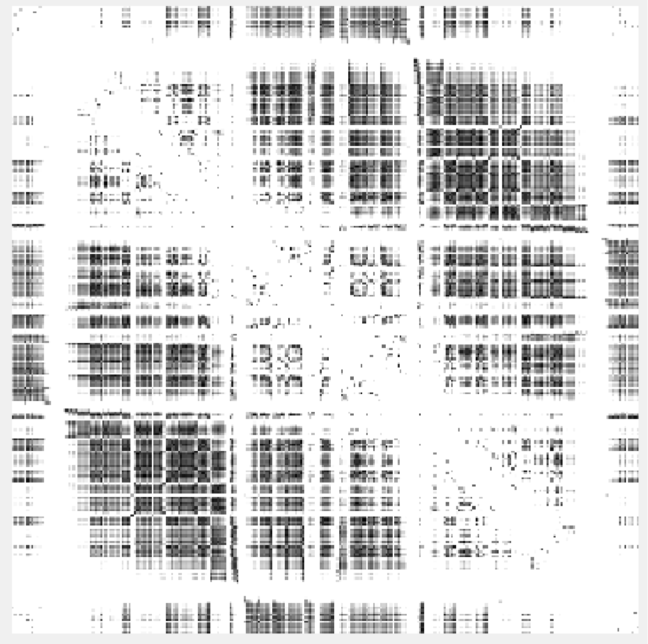

Fig. 29 Fourier transform of fig. 28

The zero-frequency component of the Fourier transform of the image is then shifted to the center of the image.

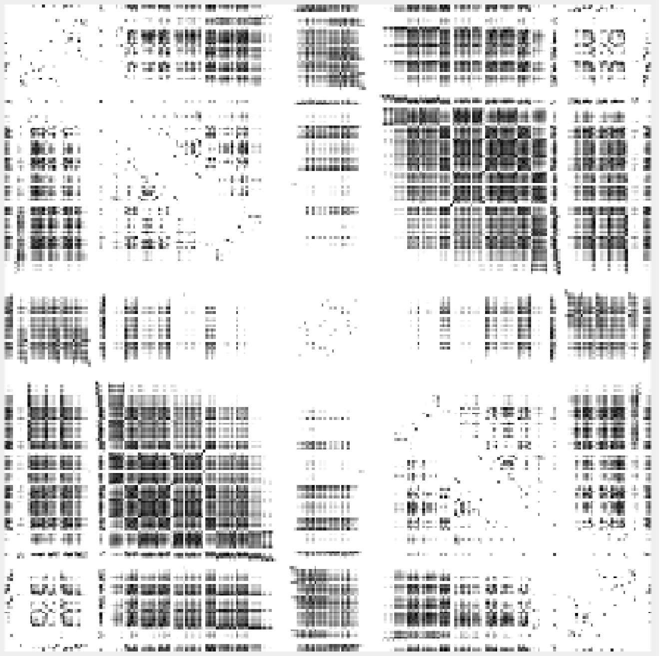

Fig. 30 zero-frequency component of the Fourier transform being shifted to the center

The magnitude of the image is then normalized to the range of 0 to 600. The minimum points are easier to be located.

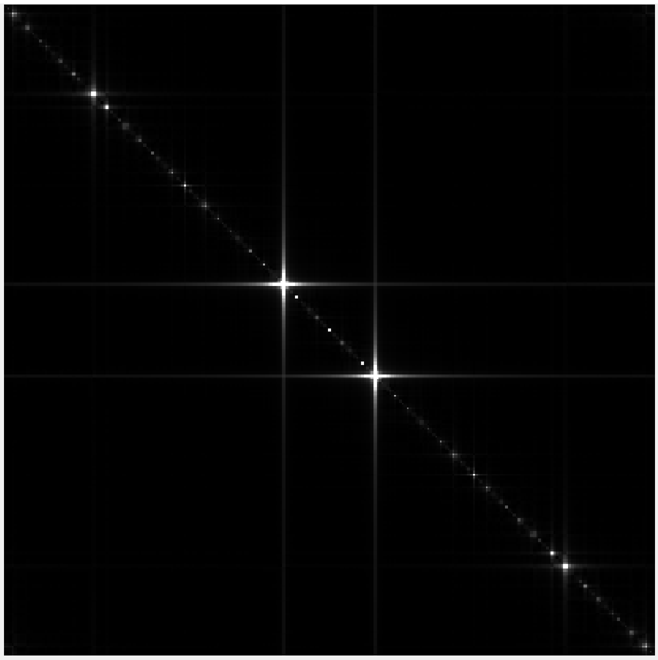

Fig. 31 Normalized Fourier Transform of Moire pattern of Fig. 28

8. Discussions

Moire Effect occurs when there are high frequency repeated patterns, no matter what form the patterns are in, 1-D or 2-D, coplanar or in 3-D space, grids, circular or dotted patterns. The superposition of the patterns forms Moire patterns, which have huge change whenever one of the patterns moves. As shown in Fig. 19 and Fig. 20, even there’s only a slight change in the orientation angle of 1 degree, the period of the Moire patterns decreases almost by half and the number of Moire lines increases from 2 to almost 4, with the greatly decreased separation between the Moire lines, as shown in equation (4) and (5).

Moire patterns can be used as image hiding by changing the frequency. By using circular Moire patterns, the hidden message can be revealed by the time-averaged algorithm in equation (7) when the location of the center point and the frequency of the Moire pattern are finely tuned. When the parameters are not correctly selected, the hidden secret cannot we revealed, as shown in Fig. 25.

When 2-D Fourier transform is performed on a linear Moire pattern, three minimum point can be located around the centre of the image, as shown in Fig. 31. The location of the three point represents the period and the orientation of the linear Moire girds that changing either of the parameter will result in the shift in location of the surrounding points.

9. Conclusion

The Moire Effect appears everywhere in the modern world. Sometimes it is used as an artistic effect in architectures, but sometimes it appears in photographs, which may ruin the photos. Despite that the Moire Effect is sometimes undesired and viewed as “illusion” in people’s daily life, the effect can be useful in image cryptography, one of the applications of the Moire Effect focused in this paper.

10. References

[1] “Moiré pattern,” Wikipedia, 12-Oct-2021. [Online]. Available: https://en.wikipedia.org/wiki/Moir%C3%A9_pattern. [Accessed: 20-Nov-2021].

[2] phuyaké, 2008.

[3] Molecular expressions: Light and color - moire patterns: Interactive tutorial. [Online]. Available: https://micro.magnet.fsu.edu/primer/java/scienceopticsu/moirepatterns/index.html. [Accessed: 21-Nov-2021].

[4] V. Saveljev, S.-K. Kim, and J. Kim, “Moiré effect in displays: A tutorial,” Optical Engineering, vol. 57, no. 03, p. 1, 2018.

[5] C. Bulucea, Recent advances on Applied Mathematics: Proceedings of the 14th WSEAS international conference on applied mathematics (math '09): Puerto de la Cruz, Tenerife, Canary Islands, Spain, December 14-16, 2009. WSEAS Press, 2009.

11. Appendices

(10): $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx$

$= \frac{1}{\pi}\int_{0}^{\pi} dx$

$= \frac{1}{\pi}[x]_0^{\pi}$

$= 1$

a$_n$= $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos nx\,dx$

$= \frac{1}{\pi}\int_{0}^{\pi} \cos nx\,dx$

$= \frac{1}{\pi}\left[-\frac{\sin nx}{n}\right]_0^{\pi}$

$= \frac{1}{\pi}\cdot 0 = 0$

a$_n$= $\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin nx\,dx$

$= \frac{1}{\pi}\int_{0}^{\pi} \sin nx\,dx$

$= \frac{1}{\pi}\left[-\frac{\cos nx}{n}\right]_0^{\pi}$

= 0, when n is even; $\frac{2}{n\pi}$ when n is odd

f(x) = $\frac{1}{2}$a$_0$ + $\sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$

$\approx \frac{1}{2} + \frac{2}{\pi}\sin x + \frac{2}{3\pi}\sin 3\pi + \frac{2}{5\pi}\sin 5\pi$