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Derivation of cosmic acceleration and the cosmological constant in the local universe

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Observed type Ia supernova luminosities [3,4] have revealed accelerating Hubble expansion thereby indicating an unknown energy fluid fills space or, alternatively, that the excellence of general relativity on the solar scale is not matched on the cosmic scale. The latter alternative could mean that a deeper understanding of space-time physics is appropriate for resolving "dark energy" and related problems (e.g., tension in the Hubble parameter measurements [16]). The cosmological constant in general relativity has been recalled as possibly germane to cosmic acceleration [5]. However, a satisfactory relativistic explanation of this parameter has not been given. Here it is shown that light speed postulated to be inward-infinite along lookback distance stemming from any given point in space-time—substituting for Einstein's isotropic light-speed along epochal distance—yields an outward-increasing cosmic time dilation which, when "rotated" into epochal space and inserted into the Lorentz transformation, gives a linearly increasing cosmic acceleration consistent with Hubble's law. This *leading order* result—in agreement with supernova type Ia magnitude data in the local universe ($z \lesssim 0.3$)—adds to previous knowledge by giving relativistic relationships for cosmic acceleration and the corresponding cosmological constant. Follow-on investigation based on *empirically consequential* lookback time is anticipated of "too fast" cosmic-structure dynamics (e.g., of wide binary stars [7], spiral galaxies [8], and galaxy clusters [9]).

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I. INTRODUCTION

Einstein proposed the cosmological constant [1] in general relativity to offset the gravitation of distributed matter thereby modeling a stable (not expanding or contracting) Universe—i.e., Milky Way Galaxy believed at the time to comprise the full extent of the universe. Twelve years later, however, he retracted the postulate as "his biggest blunder" following Hubble's (definitive) 1929 discovery [2] of "universes" (galaxies) outside the Milky Way receding at increasing speed with distance. But near the close of the 20th century, measurements of supernova type Ia (SNe Ia) luminosity-distance versus redshift [3,4] determined that cosmic acceleration has dominated cosmic deceleration to about half of lookback time. The initially ill-fated cosmological constant was soon recalled as potentially germane to cosmic acceleration while not, however, derivable from first principles.

NASA has identified the cosmological constant as one of three exploratory directions for explaining cosmic acceleration along with "…some strange kind of energy-fluid" that fills space and "…something wrong with Einstein's theory of gravity" comprising the other two [5]. The three taken together are called "dark energy" by the agency. (See [6] for an overview of the dark energy problem.) The present work does not specifically address "…some strange kind of energy-fluid," and it doesn't offer a correction to "…something wrong with Einstein's theory of gravity." Einstein's special relativity will, however, be revised to enable an explanation of cosmic acceleration.

The present work gives a relativistic derivation of cosmic acceleration—albeit with a major change at the foundation: this change, in its first step, is to go deeper than Einstein's *isotropic* light-speed in any given inertial system by adopting *anisotropic* light-speed in the same inertial system. This introduces nothing new to relativity physics other than new terms (e.g., in the Maxwell equations of electromagnetic function) to retain empirically accurate modeling of physics behavior [10].

In the second step, photons incident on any space-time point along its lookback path are postulated to propagate at effectively infinite speed (and reflect back at c/2 thereby satisfying the "round-trip axiom [10]") exclusive of all other photon speeds incident on the point along the same path. This step—giving a special case of the anisotropic light-speed condition—also brings nothing new to relativity physics. ..2230..

Within Hubble space expansion, however, exclusively infinite light-speed inward reveals empirically consequential *cosmic time-dilation* that increases outward along the lookback path. "Rotating" into epochal time and plugging into the Lorentz transformation then gives cosmic acceleration upon linearization (by radial differentiation). Overarching the entire development herein is the *leading-order* stipulation limiting redshift to $z \lesssim 0.3$.

II. INWARD INFINITE LIGHT-SPEED IN THE HUBBLE EXPANSION

Rather than just rely on the detailed demonstration [10] that Einstein's isotropic synchronization gauge is a special case within alternative (anisotropic) gauges—all "wholly equivalent to SRT in predicting empirical facts"—the infinite light-speed condition (inward or outward) in any given inertial system is derived below by examining the timeline of a single photon in normal (same path) reflection between parallel mirrors at displacement Δx moving along the *x*-axis at speed *v*.

Photon round-trip time between the mirrors is

$$\begin{aligned}\Delta t &= \Delta t_F + \Delta t_R \\ &= \Delta x/(c - v) + \Delta x/(c + v). \end{aligned} \tag{1}$$

Considering only the retrograde flight path for now and recalling the Lorentz transformation,

$$x' = (x - \beta ct)/\gamma \tag{2a}$$
$$t' = (t - (\beta/c)\, x)/\gamma \tag{2b}$$
$$y' = y \tag{2c}$$
$$z' = z \tag{2d}$$

instandard notation with $\beta = v/c$ and $\gamma = (1-\beta^2)^{1/2}$, the elapsed time in the moving inertial system during a single transit along the retrograde path may be expressed as

$$\Delta t_R' = \Delta x'\,(1 - \beta)/c \tag{3}$$

where, from (2b) at $t = 0$ and $x = \Delta x$,

$$t'_{x=\Delta x} = (0 - (\beta/c)\,\Delta x)/\gamma = -(v/c^2)\,\Delta x', \tag{4}$$

while—invoking Einstein's stipulated isotropic (c=constant) light-speed in all inertial frames—

$$t'_{x'=0} = \Delta x'/c \tag{5}$$

when the retrograde photon arrives at x'=0. Combining (4) and (5) gives (3), and

$$c_R' = \Delta x'/\Delta t_R' = c/(1-\beta) \tag{6}$$

for inward light-speed. In the limit $\beta \to 1$, inward light-speed becomes infinite—the essential principle of the present work. A similar procedure gives $c_F' = c/(1+\beta)$ in the forward direction, with $c_F' = c/2$ in the limit $\beta \to 1$. Note that infinite light-speed and zero elapsed time along the retrograde path leaves $\Delta t_F' = \Delta x'/(c/2) = 2\Delta x'/c$ for the elapsed round trip time, in agreement with the round-trip axiom.

Because the Lorentz transformation is compliant with the empirical facts from laboratory experiments, it follows that one-way infinite light-speed must be similarly compliant. In this development, special relativity is the specific mathematical formulation that exactly eliminates terms due to anisotropic light-speed as, for example, the emergent extra terms in the Maxwell equations of electromagnetic theory [10].

The foregoing development is, however, insufficient for establishing inward infinite light-speed as *scientifically* important (i.e., empirically consequential). More to the point, within non-expanding space, as assumed above, anisotropic light-speed complies with the empirical facts but brings nothing to the table that is *empirically* new to physics. Within *expanding space*, however, exclusively infinite light-speed inward to every space-time point advances a relativistic explanation of cosmic acceleration. Derivation of cosmic acceleration and the corresponding cosmological constant (i.e., by way of the Friedmann acceleration equation) is given in the next section followed by comparison of theory versus measurement.

III. DERIVATION OF COSMIC ACCELERATION

Inward infinite light-speed as the principal condition of the present leading order derivation is represented by

$$\underline{t} = t - r(t)/c, \tag{7}$$

where $r(t)$ is the epochal distance from a fundamental observer at $r = 0$ to a remote fundamental observer moving with the Hubble flow, and $\underline{t}$ is *empirically consequential* lookback time along the lookback path.

An immediate result of (again) empirically consequential lookback time is the emergence of a newly revealed outward-increasing time-dilation (along the lookback path)—obtained by differentiating (7) with-respect-to epochal time giving

$$\begin{aligned} \mathrm{d}\underline{t}/\mathrm{d}t &= 1 - (\mathrm{d}r/\mathrm{d}t)/c \\ &= 1 - rH/c. \end{aligned} \tag{8}$$

To demonstrate consistency with the Hubble law, (8) must be "rotated" into the baseline epoch. In this step, $\mathrm{d}t'/\mathrm{d}\underline{t} = 1 + rH/c$ is applied to $\mathrm{d}\underline{t}/\mathrm{d}t$, giving

$$\begin{aligned} \mathrm{d}t'/\mathrm{d}t &= (\mathrm{d}t'/\mathrm{d}\underline{t})\,(\mathrm{d}\underline{t}/\mathrm{d}t) \\ &= 1 - (rH/c)^2. \end{aligned} \tag{9}$$

Here seen is the "$\sim r^2$" dependence of (leading order) cosmic time dilation which is necessary in conjunction with the Lorentz transformation to preserve Hubble's Law (i.e., by way of cosmic acceleration linearly increasing with distance).

A. Lorentz transformation within the Hubble expansion

Substituting (2a) into (2b) and rearranging gives

$$\gamma t' = \gamma^2 t - v/c^2\ (r - \beta ct), \tag{10}$$

where r replaces x for spherical symmetry. In applying (10) within the Hubble flow, an observer is imagined to move with the flow at distance r. After a short time-interval δt at time $t = t_0 = 0$ in the present epoch, (10) becomes

$$\gamma_0\, \delta t' = \gamma_0^2\, \delta t - a_0\, \delta t/c^2\, r \tag{11}$$

to leading order, where Maclaurin expansions of $\gamma\,\delta t'$, $\gamma^2\,\delta t$, and $a\,\delta t$ bring negligible higher order terms (i.e., order $(\delta t)^2$). Taking the time derivative gives

$$\gamma_0\, \mathrm{d}t'/\mathrm{d}t = \gamma_0^2 - a_0/c^2\, r. \tag{12}$$

For small (marginally relativistic) Hubble flow speeds, the Lorentz factor is well represented by $\gamma_0 = 1 - 1/2\,(rH_0/c)^2$, and (12) may be written

$$(1 - 1/2\,(rH_0/c)^2)\,\mathrm{d}t'/\mathrm{d}t = 1 - (rH_0/c)^2 - a_{\mathrm{CA}}/c^2\,r \tag{13}$$

as the form of the Lorentz transformation for determining cosmic acceleration, where a_{CA} now represents pure cosmic acceleration in place of a_0. Substituting (9) and simplifying to leading order gives

$$1/2\,(rH_0/c)^2 = a_{\mathrm{CA}}/c^2\,r. \tag{14}$$

Linearizing via radial differentiation then yields cosmic acceleration:

$$\begin{aligned} a_{\mathrm{CA}} &= c^2\,\mathrm{d}/\mathrm{d}r\,\left(1/2\,(rH_0/c)^2\right) \\ &= rH_0^2. \end{aligned} \tag{15}$$

Because a_{CA} is directly dependent on H_0, cosmic acceleration may be considered emergent within the Hubble expansion.

B. Cosmological constant

Matter-based cosmic deceleration is absent from the foregoing derivation, and the time-derivative of Hubble's law accordingly allows $a_{\mathrm{CA}} = rH^2 = rH^2 + r\mathrm{d}H/\mathrm{d}t$ thereby giving $\mathrm{d}H/\mathrm{d}t = 0$. Then Friedmann's acceleration equation [11,12],

$$\begin{aligned} \mathrm{d}H/\mathrm{d}t + H^2 &= (\mathrm{d}^2a/\mathrm{d}t^2)/a \\ &= -\,8\pi G(\rho + 3p/c^2)/3 + \Lambda c^2/3, \end{aligned} \tag{16}$$

becomes

$$H^2 = (\mathrm{d}^2a/\mathrm{d}t^2)/a = \Lambda c^2/3 \tag{17}$$

for the empty universe (ρ, $p = 0$) giving

$$\Lambda = 3H^2/c^2 \tag{18}$$

for the cosmological constant (i.e., constant within the epoch to leading order). Here, since cosmic acceleration is emergent within the Hubble expansion as noted above, it follows that the cosmological constant is similarly emergent.

IV. Theoretical vs. measured type Ia supernova magnitudes

Figure 1 shows a graph of median residuals of observed and binned SNe Ia magnitudes "m" less their corresponding magnitudes at 10 parsecs "M" plotted against redshift z [13] with present theory given by the dashed line. Here $\Delta(\mathrm{m}{-}\mathrm{M}) = -\,5\,\log_{10}(l_2/l_1)$ gives the theoretical SNe Ia magnitude residual vs. z in the leading order range, where, after the time increase $\Delta t = r/c = z/H_0$, l_2 is the epochal distance accounting for cosmic acceleration, and l_1 is the epochal distance accounting for Hubble flow exclusive of acceleration.

In this comparison, cosmological redshift as a photon "stretches" during transit through the Hubble flow is re-interpreted as time-dilation at its source. Here, finite light-speed, isotropic or anisotropic, incident on any space-time point is set aside and replaced by *empirically consequential* inward infinite light-speed (with c/2 outward), the effect of which is below detection within the laboratory and Solar System but evident on the cosmic scale. Defined either way, redshift is $z = rH_0/c$ to leading order, with $H_0 = 74$ km s^{-1} mpc^{-1} [14].

Present theory, also evaluated for $H_0 = 74$ km sec^{-1} mpc^{-1}, is seen to agree with the median SNe Ia magnitude residuals over the $0.01 < z < 0.3$ range. Cosmic deceleration, accounting for standard gravitation assuming uniform matter-density across the local universe ($z \lesssim 0.3$), drops the (pure) cosmic acceleration (dashed) curve ≈ 3% for baryonic matter alone (0.46E–30 gm/cm^3) ( ) and ≈ 18% for combined baryonic and dark matter (2.86E–30 gm/cm^3) ( ). In calculating the decreases, the Schwarzschild solution [15] $a = -\,GM/r^2$ is recalled. Substituting $M = 4/3\ \pi r^3\rho_0$ then gives cosmic deceleration, $a_{\mathrm{M}} = -\ 4/3\ \pi G\rho_0 r$, for uniformly distributed matter, baryonic or baryonic and dark matter combined.

Net cosmic acceleration—i.e., pure cosmic acceleration decreased 18% by cosmic deceleration—agrees with the SNe Ia residual magnitude data over the $0.01 < z < 0.3$ range, however the LambdaCDM curve $(\Omega_{\mathrm{M}}, \Omega_\Lambda) = (0.3, 0.7)$ runs above the median residuals data over the same range. The former employs the Lorentz transformation in deriving cosmic acceleration while the latter employs general relativity to infer specific dark energy and the corresponding cosmic acceleration within the Planck-

Mission observed flat universe. This departure suggests that relativity theory, excellent on the solar scale, could be advanced on the cosmic scale.

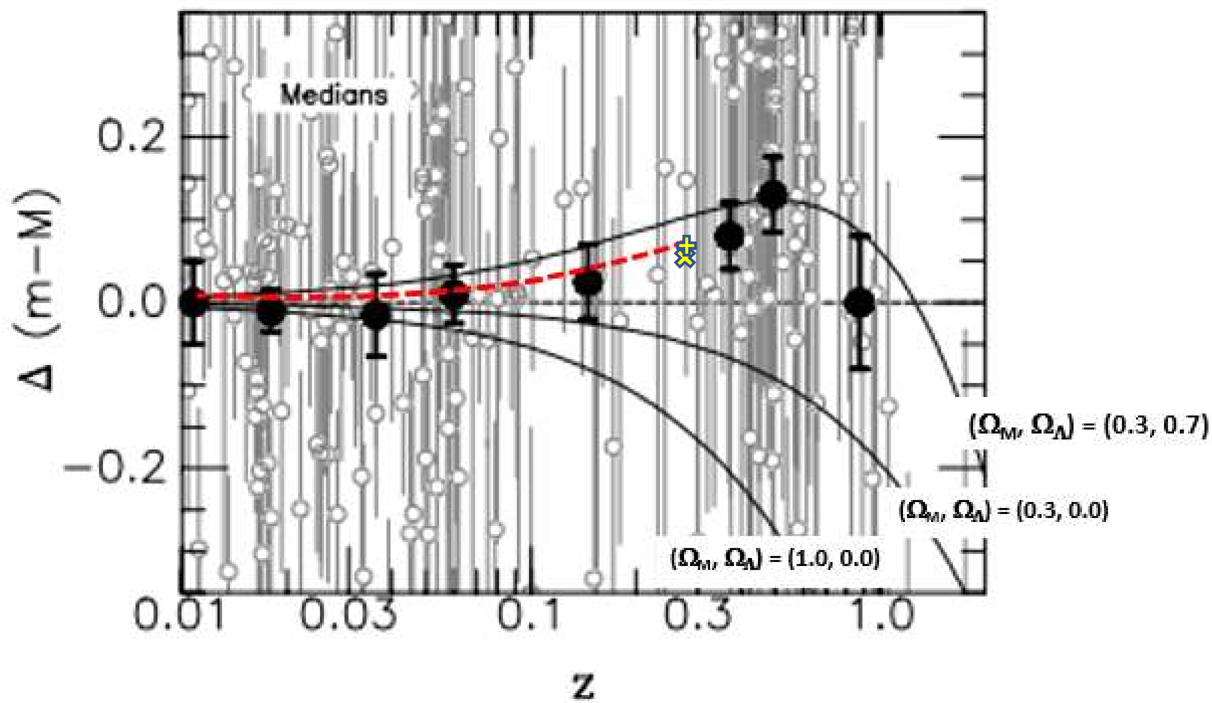

FIG. 1. Present theory and LambdaCDM curves compared with data on a residual Hubble diagram. Median residuals with ± 68% uncertainties of measured and binned type Ia supernova magnitudes (m), each reduced by the supernova magnitude at 10 parsecs distance (M), are plotted against the log of redshift (z). The data are seen to rise to the cosmic acceleration versus deceleration crossover at $z \approx 0.5$ and then descend to zero residual at $z \approx 0.9$. The dashed line shows present theory for cosmic acceleration, accurate to leading order $z \approx 0.1$ in the local universe with reduced accuracy to $z = 0.3$, with the effect of baryonic matter and baryonic plus dark matter given by ✚ and ✖ respectively. Theory accords with measurements to $z \approx 0.3$ beyond which deceleration from encompassed matter approaches and soon dominates acceleration. (Original figure from Tonry J. L., et al. Cosmological Results from High-z Supernovae. *ApJ, 594, 1*. (2003))

V. DISCUSSION

Light-speed is not merely anisotropic in the present work but inward infinite toward every point in the (cosmic) space-time manifold with c/2 outward along the same path—exclusive of all other photon velocities inward or outward along the path. This property by itself is absent empirical consequence and accordingly has no immediate scientific importance. Within the Hubble expansion, however, it gives an outward increasing time dilation along lookback distance and time that, upon "rotation" into epochal space-time and substitution into the Lorentz transformation, yields the (local universe) cosmic acceleration $a_{CA} = rH_0^2$ and the corresponding cosmological constant $\Lambda = 3H_0^2/c^2$. Theoretical SNe Ia residual magnitudes based on $a_{CA} = rH_0^2$ are in accordance with the measured residual magnitudes over the local universe, $0.01 < z < 0.3$.

It is of course appropriate to address the empirical and theoretical implications of the present work. An immediate implication of cosmic time dilation along lookback time and distance is the possible existence of a related time dilation on the galaxy (and galaxy cluster) scale that either explains the "too-fast" dynamics of these entities or reduces the amounts and distributions of dark matter required to satisfactorily model their dynamics. Here, continuing work over the past 4-5 years to the present suggests the relationship exists.

Returning to the (leading order) local universe scale, inward infinite light-speed suggests that each observer—instantaneously "connected" to all other observers at corresponding lookback times and distances—resides at one of an infinitude of *empirically consequential* lookback origins. This, in turn, recalls the multiverse understanding of physical reality. Here we take note of the *classical* character of the present theory—i.e., exclusive of quantum mechanical effects and entirely deterministic from time-zero forward despite the multiverse implication of each observer seeing all others *adjacently* earlier in time. In other words from another perspective, any given lookback state of the local universe determines all other (local universe) states forward and backward in time. But quantum mechanics—absent from the present developments except for "early-on" conceptual reflections—may accommodate the counter-intuitive multiverse impli-cation of the present work within its own considerably counter-intuitive, while increasingly validated, twentieth-century physics.

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