DMDE Exercise 3

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1 Problem 1: X-Ray mass profile

1.1 Part a:

The gas density profile of a cluster is given by

$$\rho_{gas}(r) = \rho_{gas}(0)\left(1+\frac{r^2}{r_c^2}\right)^{-1} \tag{1}$$

while the temperature profile is given by

$$T_{gas}(r) = T_{gas}(0)\left(1+\frac{r}{7r_c}\right)^{-1.6} \tag{2}$$

From the Gas and Temperature profiles, the total mass profile can be determined as follows.

$$M_{tot}(<r) = -\frac{k_B T_{gas}(r) r}{G\mu m_p}\left(\frac{dln\rho_{gas}(r)}{dlnr}+\frac{dlnT_{gas}(r)}{dlnr}\right) \tag{3}$$

For y=f(x), x>0,y>0,

$$\frac{dlny}{dlnx} = \frac{x}{y}\frac{dy}{dx}$$

$$\frac{dln\rho_{gas}}{dlnr} = \frac{r}{\rho_{gas}}\frac{d\rho_{gas}}{dr} = r\frac{d}{dr}\left(1+\frac{r^2}{r_c^2}\right)^{-1} = -\frac{2r^2}{r_c^2+r^2} \tag{4}$$

$$\frac{dlnT_{gas}}{dlnr} = \frac{r}{T_{gas}}\frac{dT_{gas}}{dr} = \frac{r}{T_{gas}}\frac{d}{dr}\left(1+\frac{r}{7r_c}\right)^{-1.6} = -\frac{1.6r}{7r_c+r} \tag{5}$$

$$M_{tot}(<r) = \frac{k_B T_{gas}(0) r}{G\mu m_p}\left(1+\frac{r}{7r_c}\right)^{-1.6}\left(\frac{2r^2}{r_c^2+r^2}+\frac{1.6r}{7r_c+r}\right) \tag{6}$$

1.2 Part b:

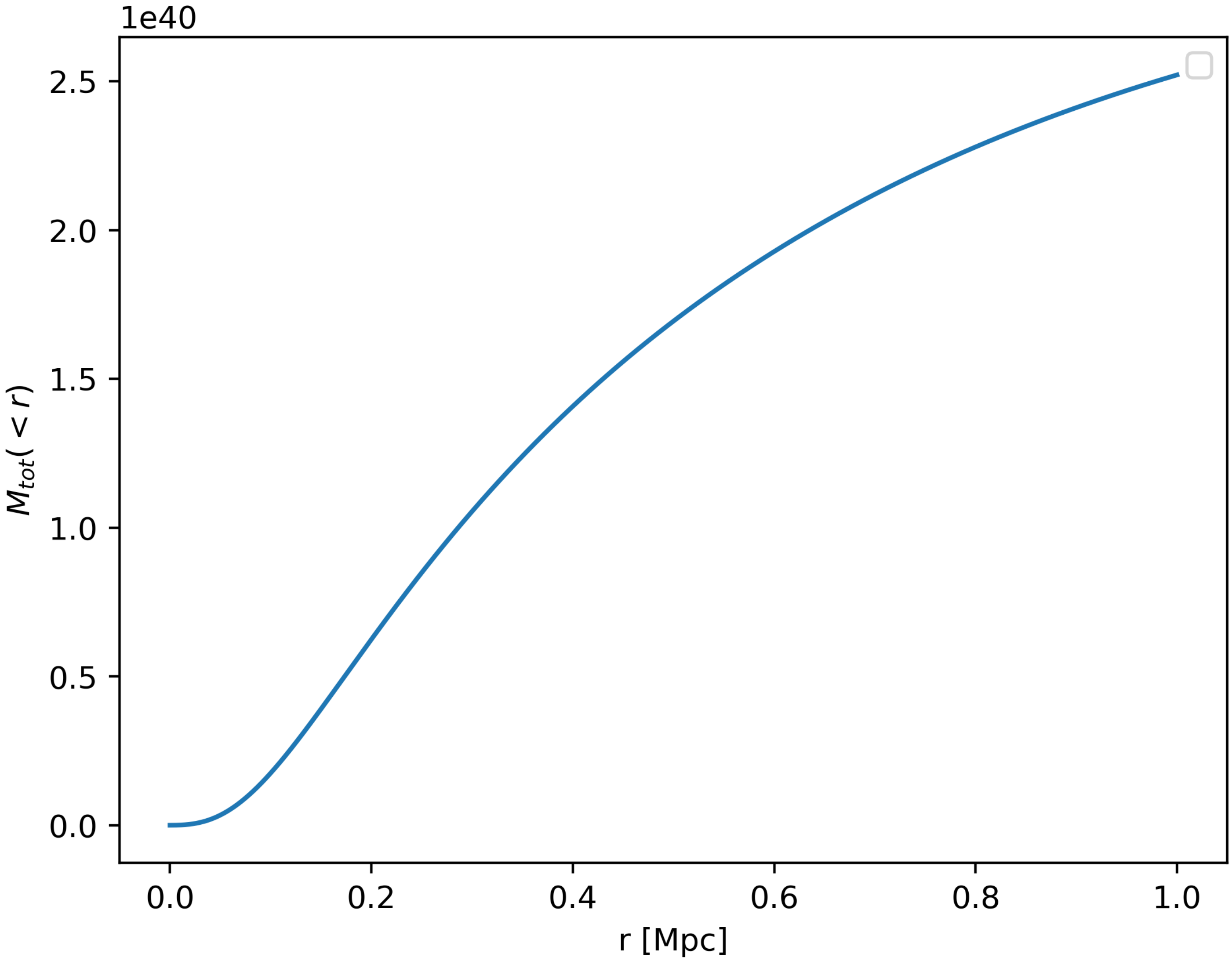

Figure 1: $M_{tot}(< r)$ with r_c=0.15 Mpc and $T_{gas}(0)$=3keV, calculated using equation (6). The shape of the plot is as one would expect, with the total mass increasing with increasing radius, however the increase is not linear.The increase is slow at low radii but then the increase is more significant between 0.1 and 0.4Mpc, and then the increase tapers off to a lower rate at larger radii.

1.3 Part c:

2 Problem 2: Galaxy Cluster Mass

2.1 Part a:

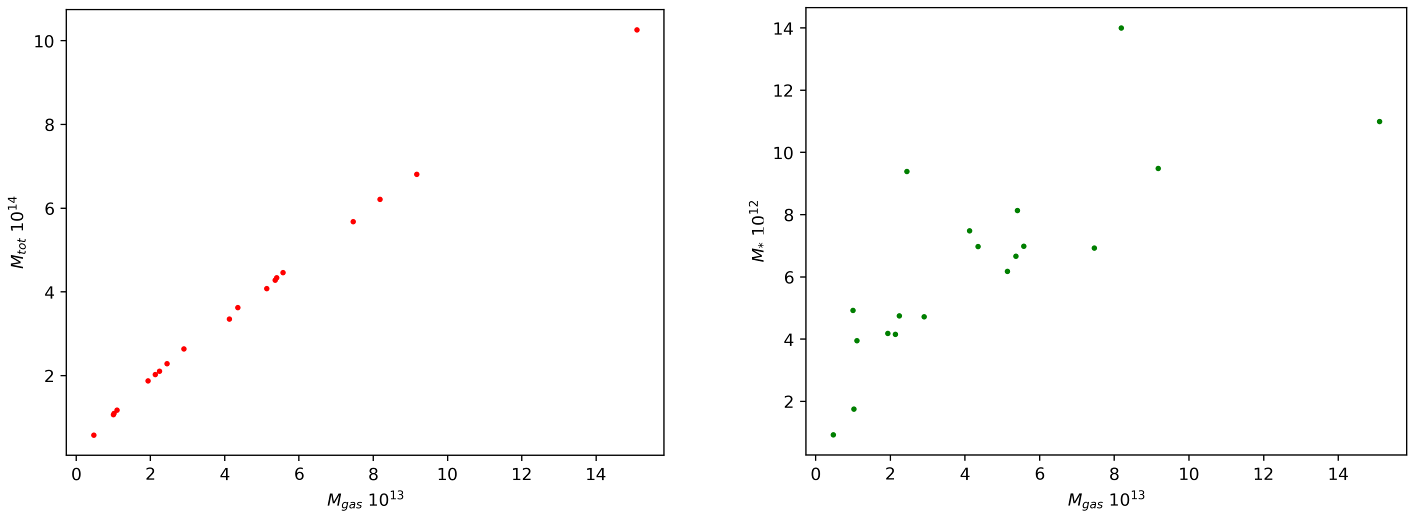

Figure 2: M_{tot} and M_* vs M_{gas}. *Left:* M_{tot} vs M_{gas}. There is a clear linear relationship between the total mass and the gas mass. The gas mass is a small fraction of the total mass (10-15%). *Right:* M_* vs M_{gas}. There is a correlation between the gas mass and the stellar mass but it is a more varied relationship than the total mass and the gas mass. These plots show that measuring the gas mass is a good way to estimate the total mass of a cluster.

2.2 Part b:

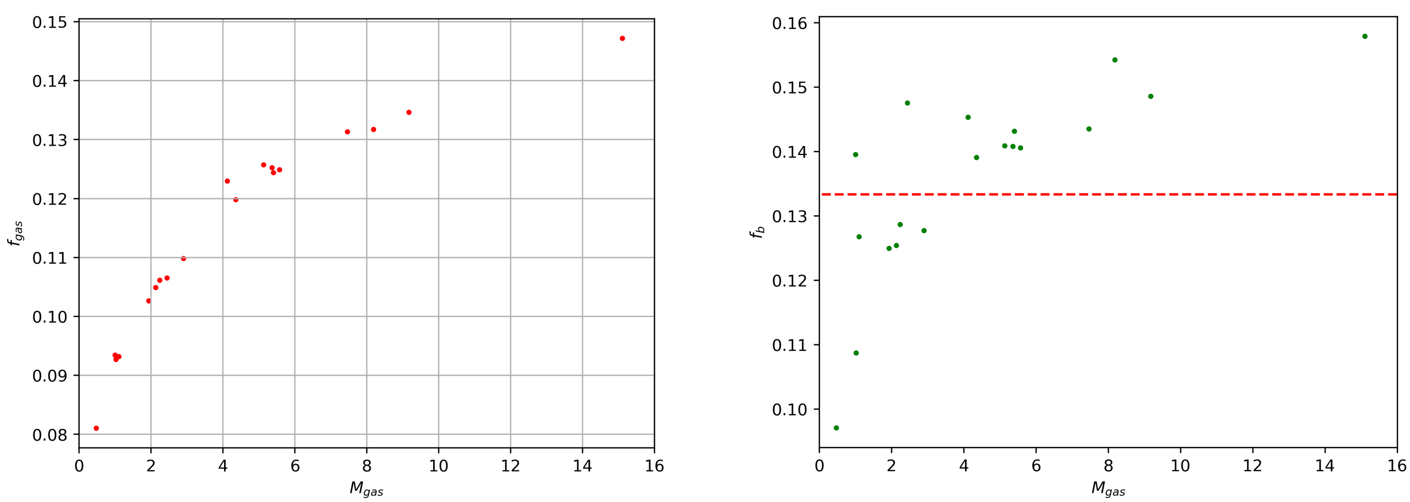

Figure 3: f_{gas} and f_b vs M_{gas}. *Left:* f_{gas} vs M_{gas}. The gas mass fraction is proportional to the gas mass. *Right:* f_b vs M_{gas}. The baryon fraction is proportional to the gas mass. The red dotted line shows the universal baryonic mass fraction for comparison.

2.3 Part c:

Yes, one can estimate the total Mass from measuring the gas Mass, however this would be a very approximate method. We know that the total mass is related to the X-Ray gas mass for typical clusters as $M_{gas} = (0.1 - 0.15)M_{tot}$. We can also relate the Gas mass to the total mass through the gas fraction $f_{gas} = \frac{M_{gas}}{M_t ot}$ where the gas fraction can be estimated from the best fit comsological parameters. $f_{gas} \approx \frac{\Omega_b}{\Omega_m}$.

2.4 Part d:

We have a universal baryonic mass fraction $f_b \approx \frac{\Omega_b}{\Omega_m} \approx \frac{0.04}{0.3} = \frac{2}{15} \approx 0.1\dot{3}$. This can be seen plotted as the dotted line in the right panelof Figure 3. The baryonic fraction of clusters is plotted in the same panel. As the plot shows, the baryonic fraction of the clusters is close to that of the Universe, but there is a correlation with the gas mass of the cluster. Taking the mean of this sample we get $\bar{f}_b = 0.136$. Calculating the mean baryonic fraction of a large sample of clusters could yield a reasonable assumption for the mass distribution of the Universe.

3 Code

The python script used to produce the results in this report is available at `https://github.com/tomcarron/DMDE_Ex3.git`