DMDE Exercise 5

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January 2022

1 Problem 1: Type Ia Supernovae

1.1 Part a: Observed distance modulus vs redshift

$$\mu = m_V - M_V = 5log_{10}\left(\frac{d_L}{10pc}\right) \quad (1)$$

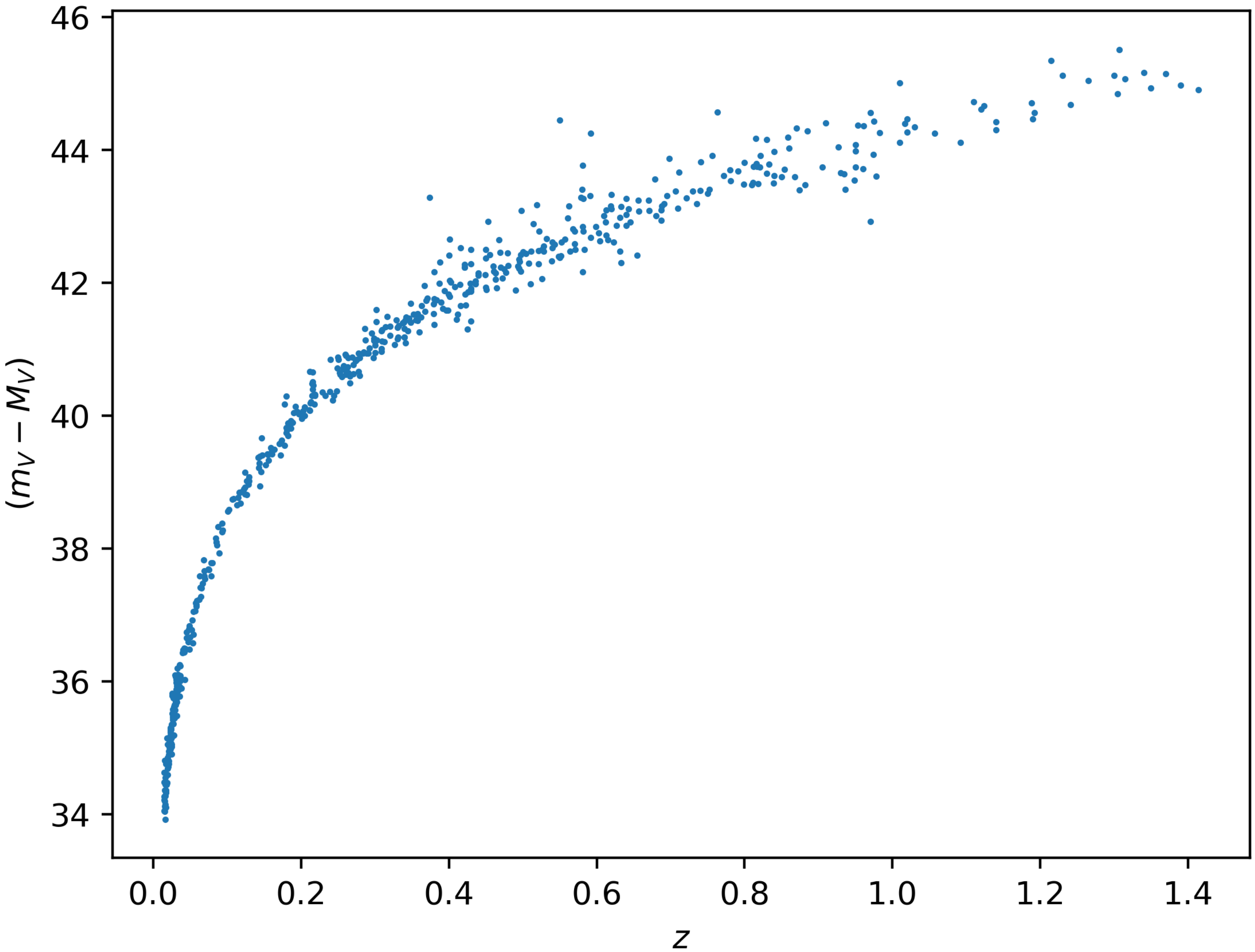

Figure 1: Observed distance modulus μ of a set of Type Ia Supernovae as a function of redshift z, with an assumed absolute magnitude of M_V=-19.6 for Type Ia Supernovae.

1.2 Part b: Distance moduli for different cosmological models

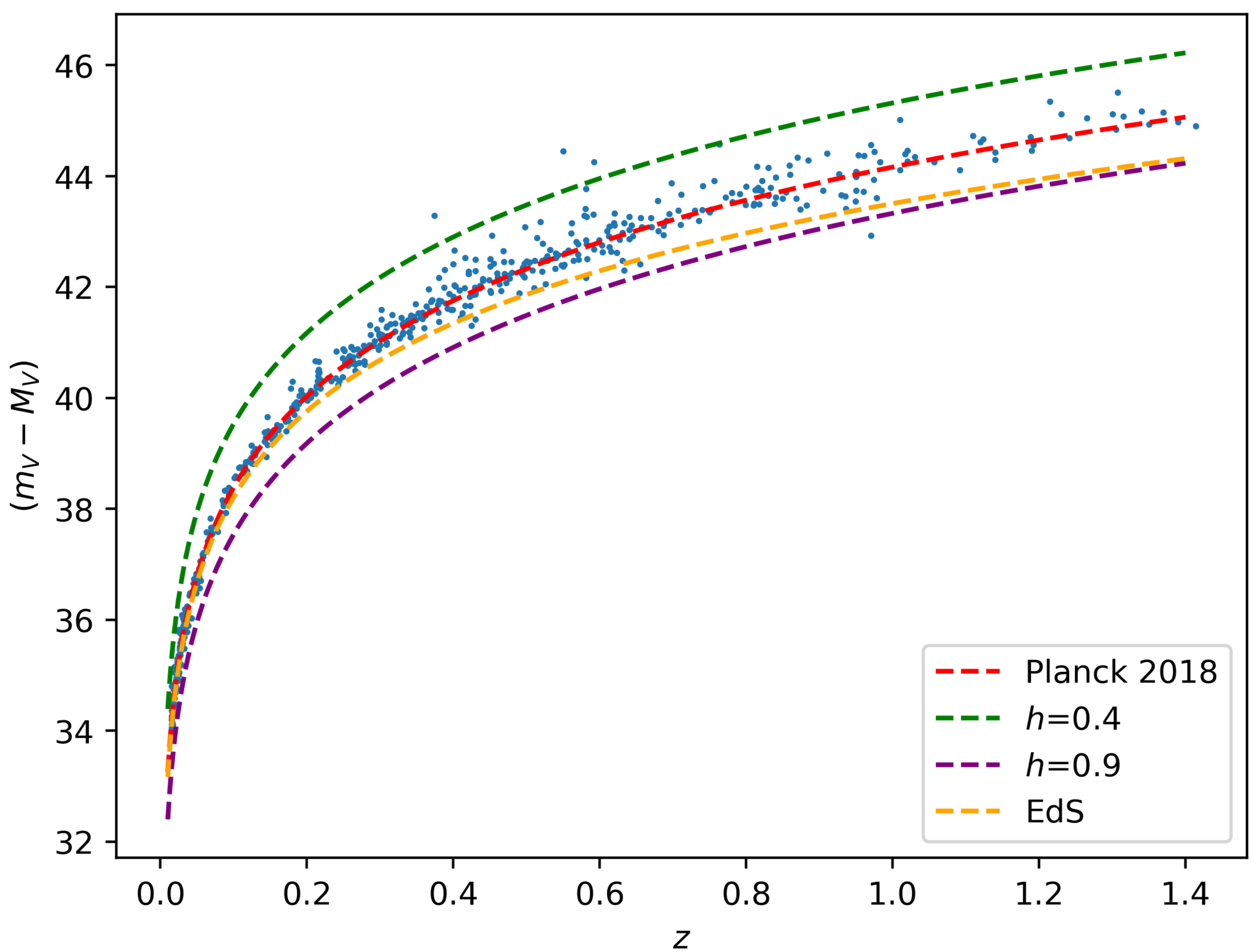

Figure 2: Theoretical models of the observed distance modulus μ for four different cosmological models. The four models used are the Planck 2018 parameters, a Universe with h=0.4, a Universe with h=0.9, both with Ω_m=0.3, Ω_Λ=0.7, and also the Einstein de Sitter Universe. The data points represent the same observed distance modulus as part (a).

1.3 Part c: Interpretation

Figure 2 plots the observed distance modulus as a function of redshift for a sample of Type Ia Supernovae. The theoretical relationship between distance modulus and redshift for four cosmological models are also shown. The plot demonstrates that the cosmological model with measurements of the cosmological parameters from the CMB from the Planck collaboration in 2018 provides a good fit to the data. This is expected as the Planck 2018 data provides some of the best measurements of the cosmological parameters available. However, the Hubble constant in the Planck 2018 is H_0=67.66 $km\ Mpc^{-1}\ s^{-1}$, and the Hubble constant derived from measurements of the cosmic distance ladder such as with Type Ia SNe is higher, usually yielding results about H_0=73 $km\ Mpc^{-1}\ s^{-1}$. Also shown in Figure 2 are some models which we know not to be good models of the Universe. The green curve shows a cosmological model with a Hubble parameter of H_0=40 $km\ Mpc^{-1}\ s^{-1}$ and Ω_m=0.3, Ω_Λ=0.7. The purple curve shows a cosmological model with a Hubble parameter of H_0=90 $km\ Mpc^{-1}\ s^{-1}$ and Ω_m=0.3, Ω_Λ=0.7. One can see that neither model provides a good fit of the data but that the value of the Hubble parameter that provides a good fit to the data lies between those values. The yellow curve shows an Einstein-de Sitter Universe, which deviates significantly from the data at high redshifts because in this model, the expansion of the Universe is not accelerating.

2 Problem 2: The CMB

2.1 Part a: Power spectra

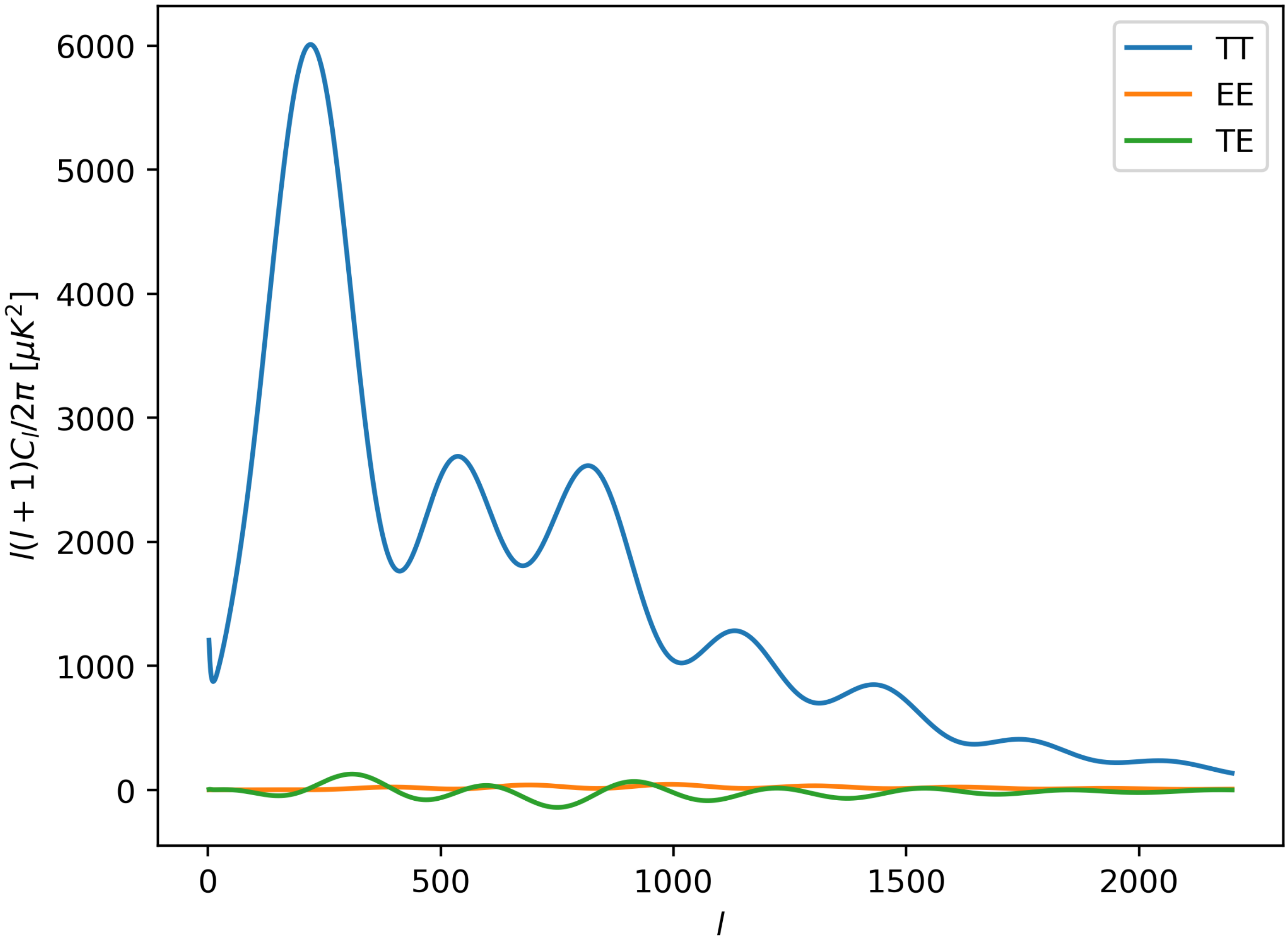

Figure 3: Temperature and polarization power spectra (TT,EE,TE), vs multipole moment (l) calculated using CAMB solver, using default cosmological parameters.

2.2 Part b: Power spectra

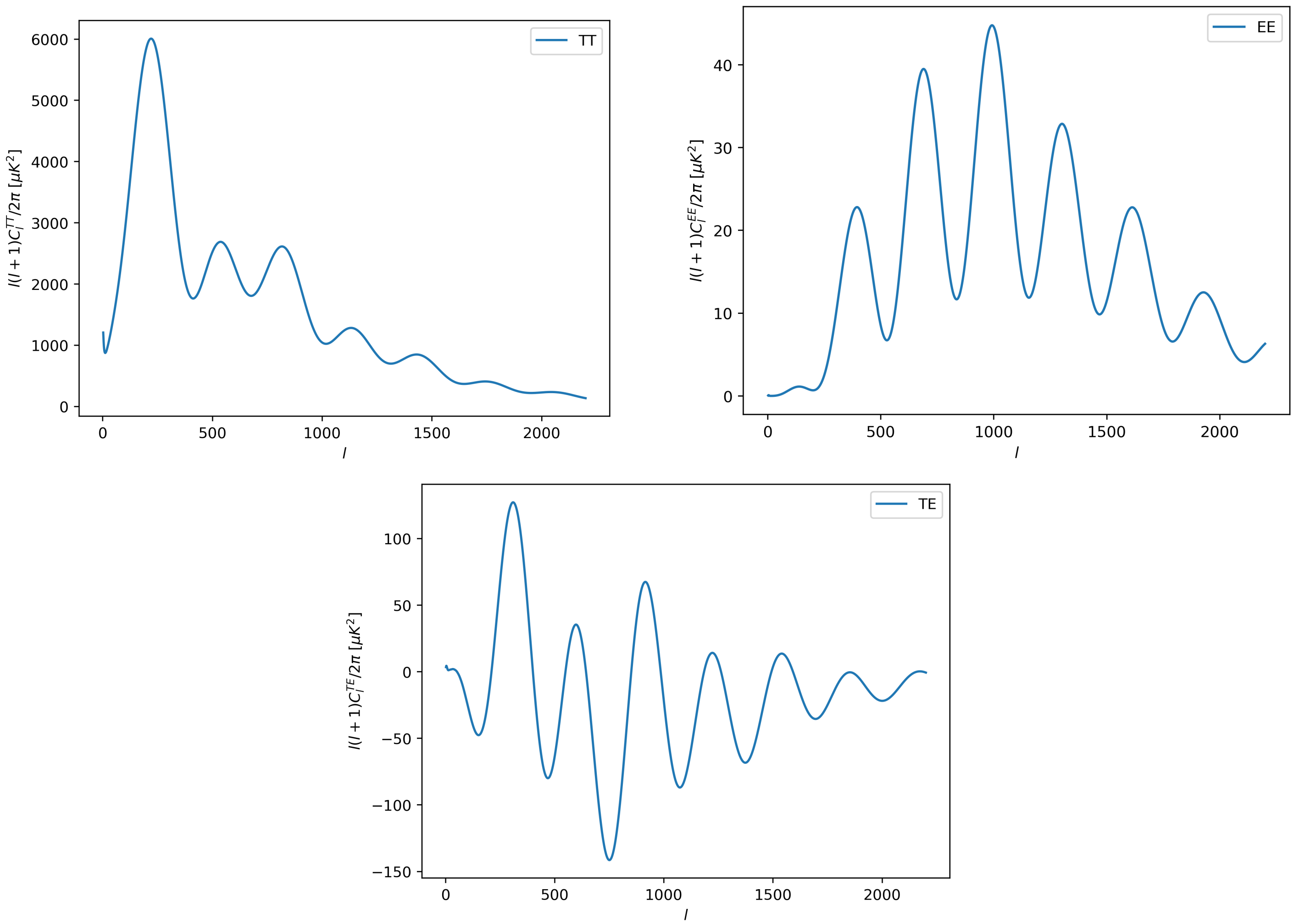

Figure 4: *Left:* Temperature power spectrum of the CMB. *Right:* EE-polarization power spectrum of the CMB. *Middle:* TE-polarization power spectrum of the CMB. Power spectra calculated using CAMB solver and the default parameters.

2.3 Part c: $\Omega_b h^2$

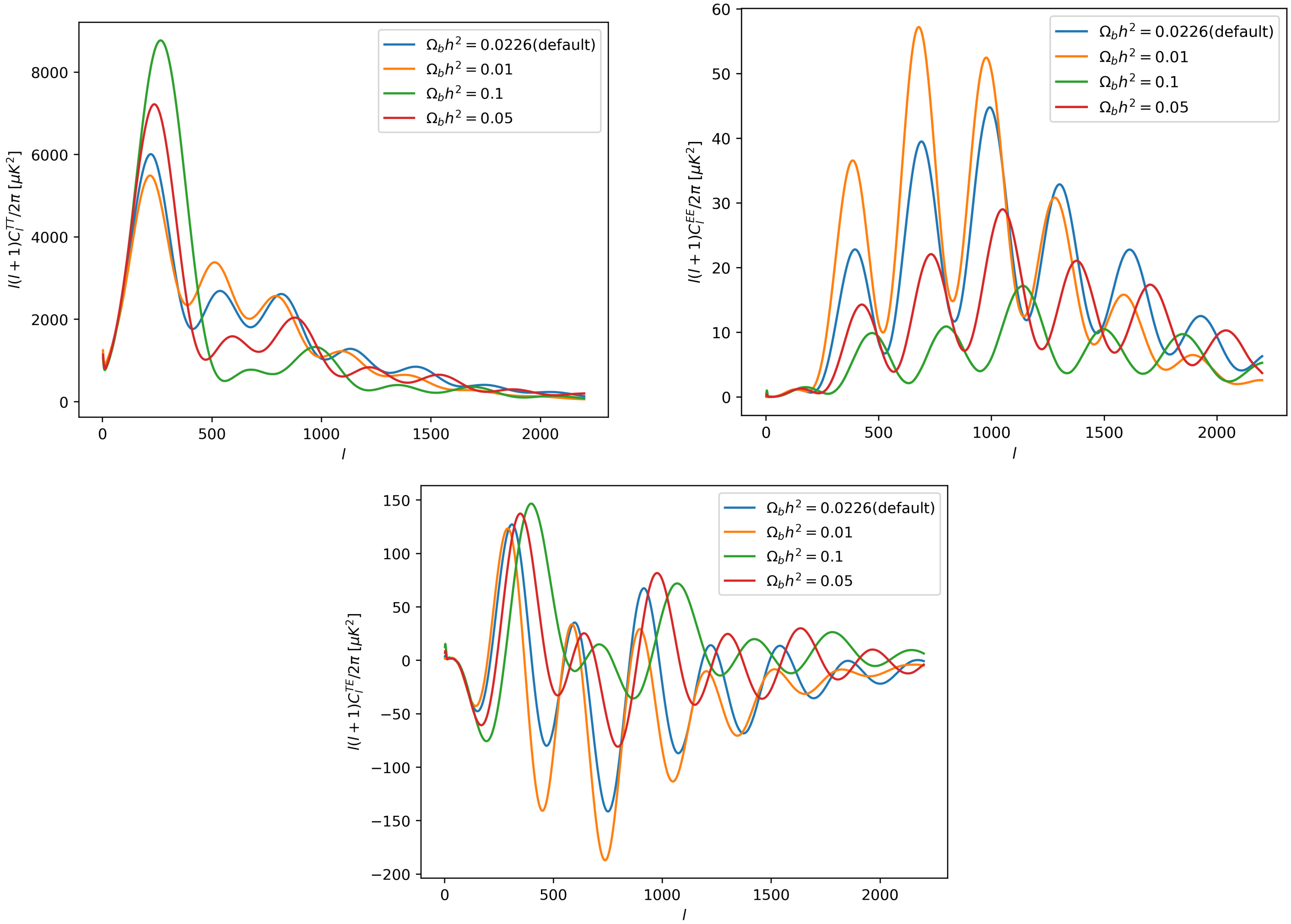

Figure 5: The temperature and polarization power spectra of the CMB for several values of the baryon density parameter $\Omega_b h^2$.

The left plot of Figure 5 shows the temperature power spectrum for a range of baron density parameters. The baryon density affects mainly the magnitude of the first peak in the temperature power spectrum and also affects the position and magnitude of the other peaks. Larger baryon density increases the magnitude of the first peak. The right hand plot of Figure 5 shows the EE-polarization spectrum of the CMB. The magnitude of the first three peaks of the EE-polarization spectrum are decreased by larger baryon density parameters. With larger baryon density parameters, the magnitude difference between the first 3 and last three peaks is also lessened. The baryon density parameter also somewhat affects the positions of the peaks. The middle plot of Figure 5 shows the TE-polarization spectrum of the CMB for the same range of baryon density parameters. Here the baryon density parameter has a significant effect on the size and position of the peaks and troughs in the spectrum. With a larger baryon density parameter, the peaks occur at larger l, and the first peak is larger in magnitude. With smaller a baryon density parameter, the peaks occur at smaller l. With smaller baryon density, the troughs at small l are quite large.

2.4 Part d: $\Omega_c h^2$

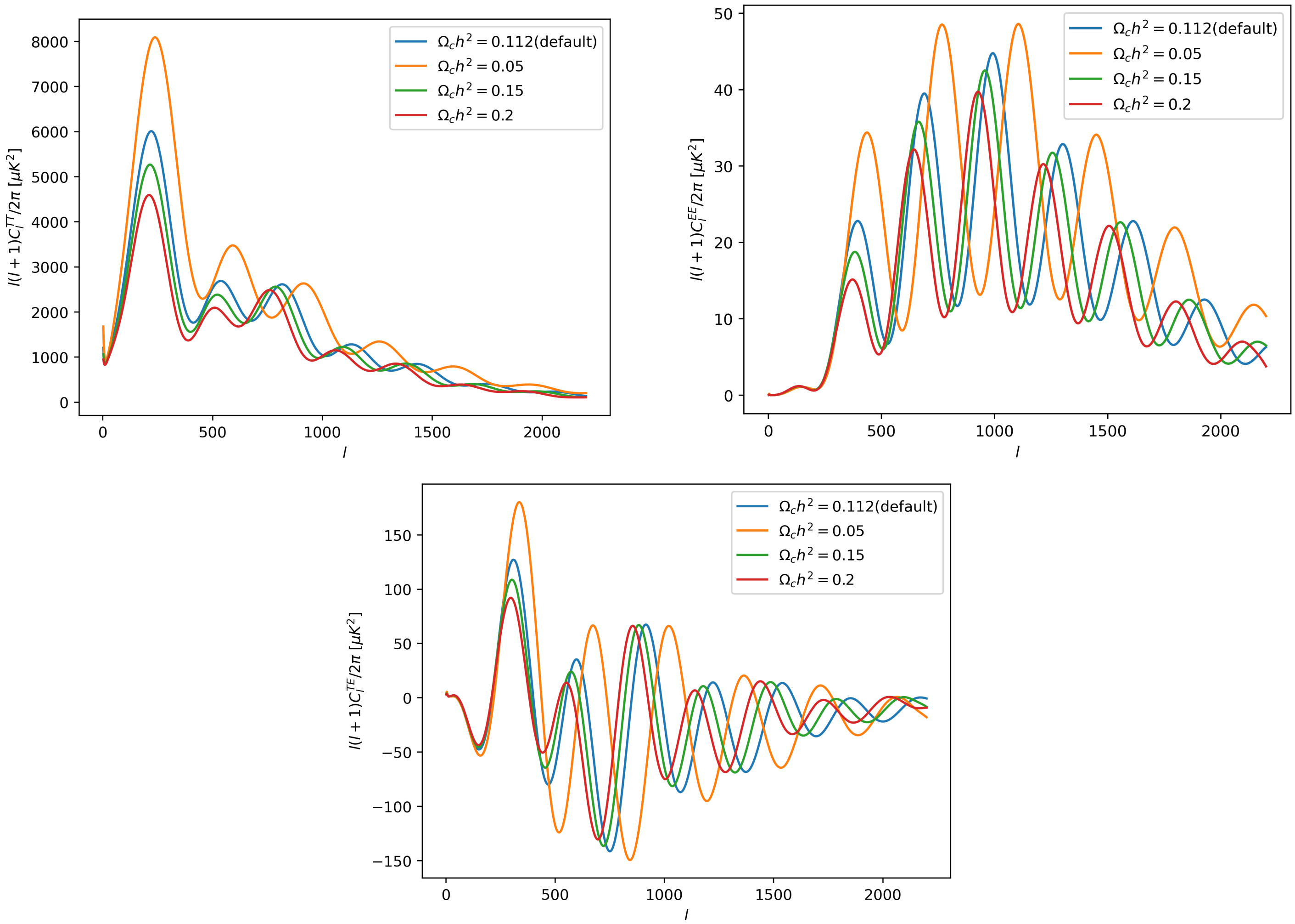

Figure 6: The temperature and polarization power spectra of the CMB for several values of the cold dark matter density parameter $\Omega_c h^2$.

The left plot of Figure 6 shows the Temperature power spectrum of the CMB for several values of the density parameter of Cold Dark Matter $\Omega_c h^2$. For larger $\Omega_c h^2$, the magnitude of the peaks in the temperature power spectrum decrease. The positions of the peaks are affected slightly at larger l. The right plot of Figure 6 shows the EE-polarization power spectrum for varying $\Omega_c h^2$. Larger values of $\Omega_c h^2$ decrease the magnitude of power spectrum peaks and shift the peaks to slightly smaller l (larger angular size). The middle plot of Figure 6 shows the TE-polarization power spectrum of the CMB for several values of $\Omega_c h^2$. Larger values of $\Omega_c h^2$ decreases the magnitudes of the peaks and troughs in the spectrum and shifts the peaks to smaller slightly smaller l also.

2.5 Part e: Hubble parameter (h)

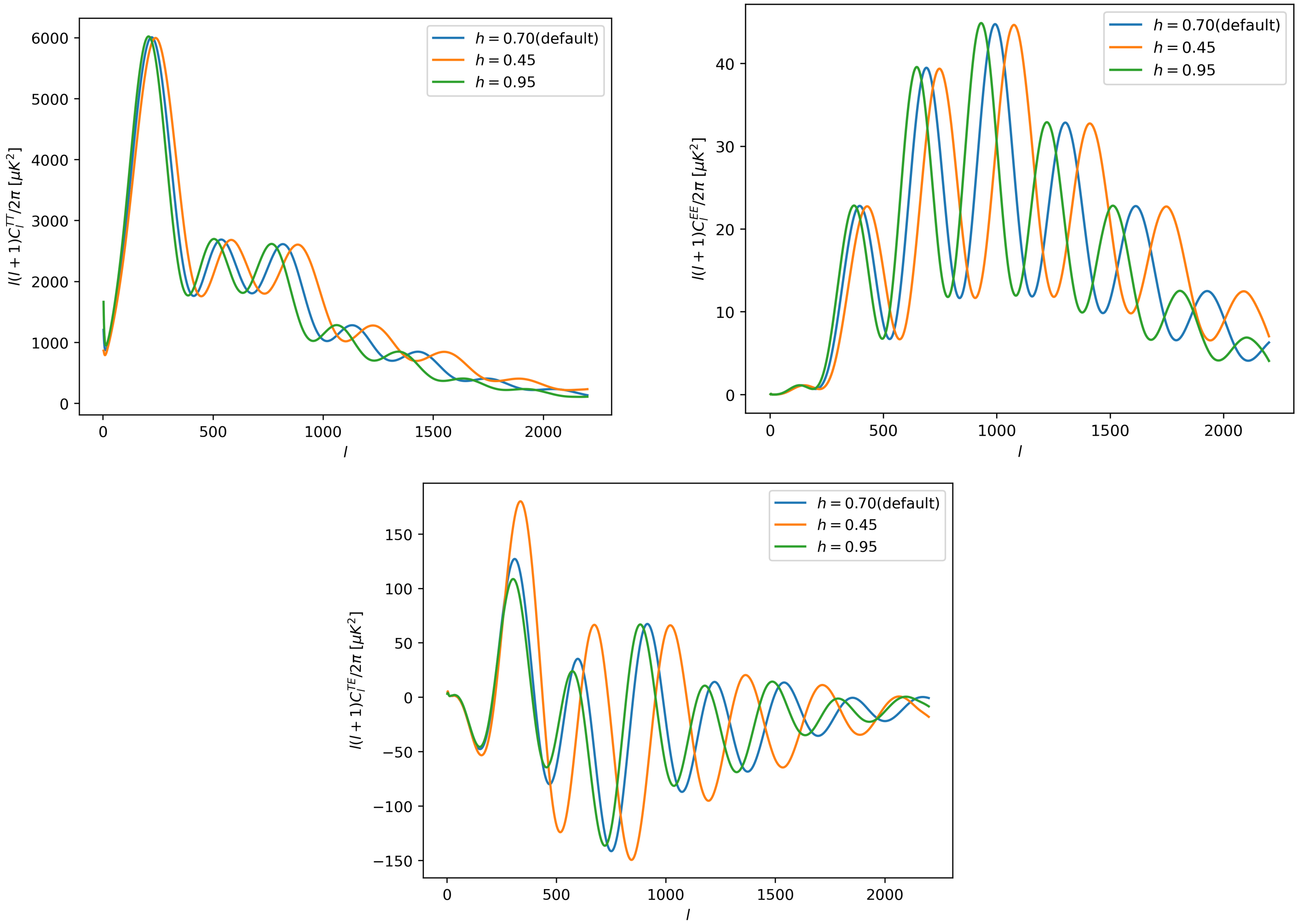

Figure 7: The temperature and polarization power spectra of the CMB for several values of the Hubble parameter H_0. The dimensionless Hubble parameter h is used here for simplicity. H_0=100 h km Mpc^{-1} s^{-1}.

Figure 7 shows the effect of the Hubble constant on the temperature and polarization power spectra. The left plot shows the temperature power spectrum, the Hubble parameter does not affect the magnitude of the peaks much but it affects the position of the peaks, with a larger Hubble parameter shifting the spectrum to smaller l, i.e larger angular scales. The right plot shows the EE-polarization power spectrum. The Hubble parameter her also does not affect the magnitude of the peaks, but larger values of the Hubble parameter also shift the spectrum to smaller l and larger angular scales. The middle plot shows the TE-polarization spectrum, again for different values of the Hubble parameter. Here, decreasing the value of the Hubble parameter decreased the magnitude of the peaks, particularly for small values of l, and it also shifts the spectrum to smaller values of l, i.e larger angular scales.

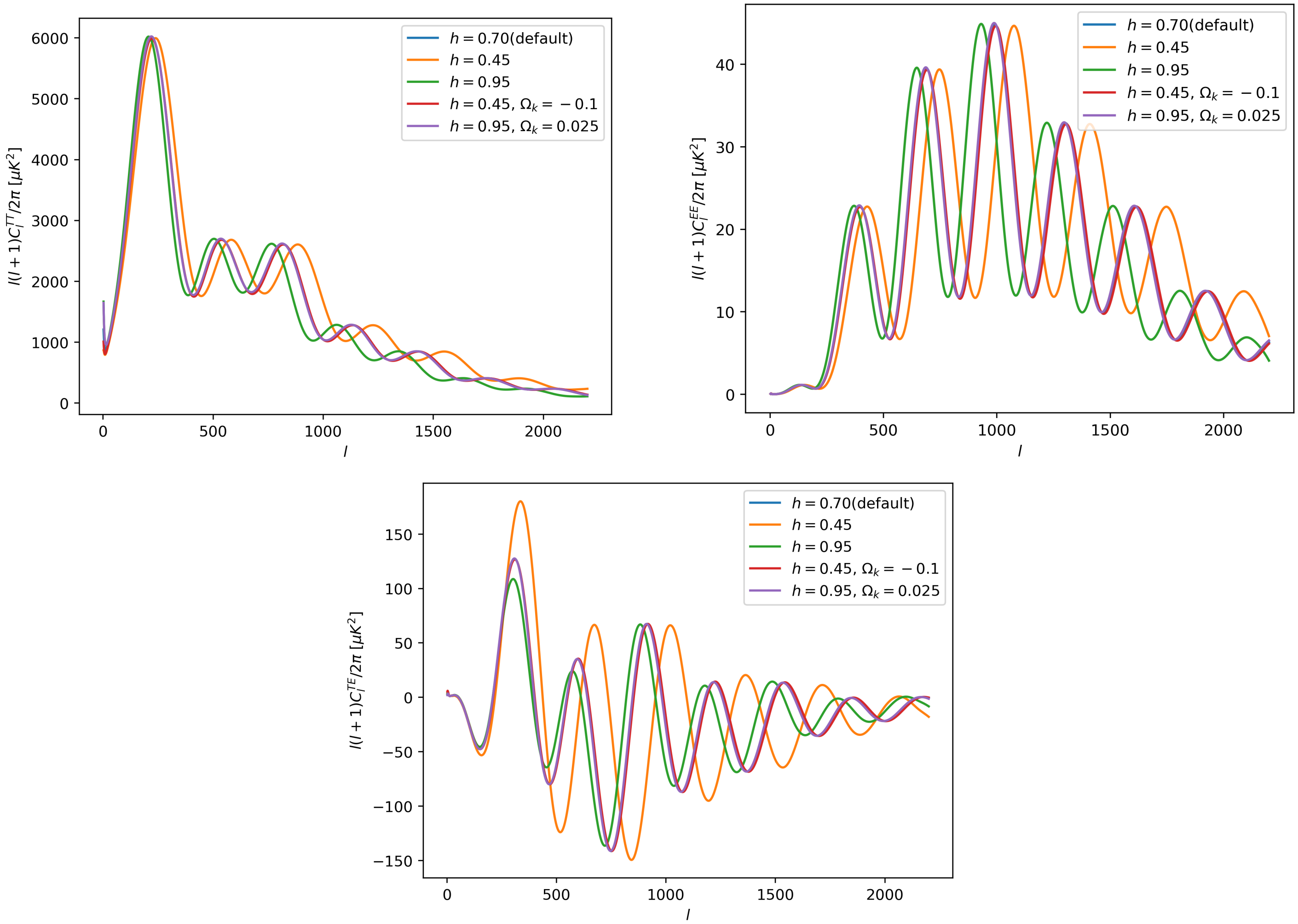

Figure 8: Temperature and polarization power specrta of the CMB for varying Hubble parameter and curvature Ω_k.

Using the same spectra from Figure 7, values of the curvature density parameter Ω_k were found for $h = 0.95$ and $h = 0.45$ that recreated the default plot with $h = 0.70$ and $\Omega_k = 0$. For the case of $h = 0.95$, it was found that a curvature density parameter of $\Omega_k = 0.025$ recreated the acoustic peaks of the default spectrum. For this case, the other important density parameters were $\Omega_m = 0.149850$ and $\Omega_\Lambda = 0.825150$. This model is represented by the purple spectra in Figure 8 and is a very close approximation to the default spectra. For the case of $h = 0.45$, it was found that a curvature density parameter of $\Omega_k = -0.1$ recreated the acoustic peaks of the default spectrum. For this case, the other important density parameters were $\Omega_m = 0.667852$ and $\Omega_\Lambda = 0.432148$. This model is represented by the red spectra in Figure 8 and is a very close approximation to the default spectra. This shows that very different Universe models, one dominated by Λ and one by matter can show similar power spectra in the CMB with the inclusion of curvature.

3 Code

The python script, data sets and plots used for the analysis in this exercise sheet are available at https://github.com/tomcarron/DMDE_Ex5.git