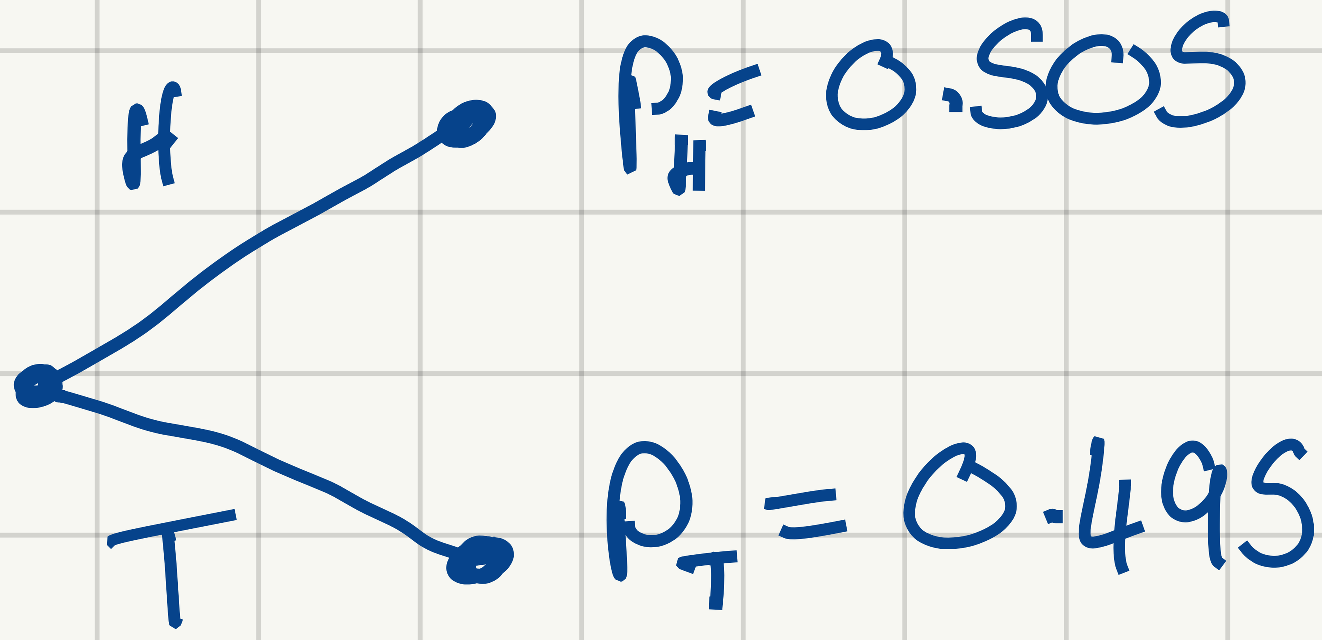
Bayesian Inference

Tom Carron

a)



b) Fair coin

$$P(O_n h \mid \text{Fair}) = \left(\tfrac{1}{2}\right)^n$$

c) Double headed

$$P(O_n h \mid dh) = 1$$

d) Initial priors

$$P(O_n h \mid \text{initial}) = (0.505)^n$$

e)
$$P(B|A) = \frac{P(O_n h \mid \text{Fair}) \times P(\text{Fair})}{P(O_n h)}$$

where $P(O_n h) = P(O_n h \mid \text{Fair}) \times P(\text{Fair}) + P(O_n h \mid dh) \times P(dh)$

$$P(O_n h) = \left(\tfrac{1}{2}\right)^n (0.99) + 1(0.01)$$

$$P(B|A) = \frac{\left(\frac{1}{2}\right)^n \times (0.99)}{\left(\frac{1}{2}\right)^n \times (0.99) + 0.01}$$

(f) for $n = 2$

$$P(B|A) \simeq 0.961$$

for $n = 7$

$$P(B|A) \simeq 0.436$$

For $n = 7$, we have less than a 50% chance that the model is an acceptable description of the system.

Thus we would consider the coin to be more unfair than previously thought, and that the provided prior information is unreliable.