## Data Analysis in Astronomy and Physics (SoSe22)

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## Exercise Set 8

Due: 9:30 30 May 2022

Discussion: 13:00 3 June 2022

Online submission at via ILIAS in the directory Exercises / Übungen -> Submission of Exercises / Rückgabe des Übungsblätter

## 1. Error Calculations [40 points]

Suppose we are viewing a binary orbit face-on. The primary star has mass  $2.19^{+0.43}_{-0.41}M_{\odot}$ , luminosity  $60.8^{+1.3}_{-1.2}L_{\odot}$ , and effective temperature  $6595^{+53}_{-58}K$ . The secondary star has mass  $1.62^{+0.26}_{-0.32}M_{\odot}$ , luminosity  $3.2^{+0.7}_{-0.8}L_{\odot}$ , and effective temperature  $4284^{+78}_{-73}K$ . The distance to the binary is determined to be 5pc. Compute the following properties including error.

hint: recall the solar values (without error)

$$M_{\odot} = 1.989 \times 10^{30} kg,$$

$$m_{\odot} = -26.74$$
,

$$L_{\odot} = 3.828 \times 10^{26} \frac{J}{s}$$

$$R_{\odot} = 6.955 \times 10^8 m$$
, and

$$T_{eff,\odot} = 5780K$$

a) What is the total mass of the binary? What is the reduced mass? 10 points

hint: the reduced mass of  $M_1$  and  $M_2$  is  $\mu = \frac{M_1 M_2}{M_1 + M_2}$ 

b) What is the radius of each star? 10 points

hint: recall the equation for the luminosity of a star  $L=4\pi R^2\sigma T_{eff}^4$ , where  $\sigma$  is the Stefan-Boltzmann constant

c) What is the flux coming from each star? What is the total flux? What is the apparent magnitude of the binary system? 20 points

hint: the flux is determined by  $F = \sigma T_{eff}^4$ , while apparant magnitude is given by  $m = -2.5log_{10}\left(\frac{F}{F_{\odot}}\right) + m_{\odot}$ 

## 2. PCA using covariance [60 points]

In this problem we will redo the Principal Component Analysis (PCA) as presented by Francis & Wills (1999) on a set of quasar data. The paper can be downloaded from this link. The data is available on the website as datafile: quasar.dat. The main result from the paper is shown in their Table 3:

Table 3. Results of Eigenanalysis – The Principal Components<sup>a</sup>

	PC1	PC2	PC3	PC4	PC5
Eigenvalue	6.4505	2.8157	1.5879	0.6257	0.5698
Proportion	0.496	0.217	0.122	0.048	0.044
Cumulative	0.496	0.713	0.835	0.883	0.927
Variable	PC1	PC2	PC3	PC4	PC5
$\log L_{1216}$	0.053	0.535	-0.123	-0.029	-0.405
$\alpha_x$	0.295	-0.198	0.079	0.485	-0.155
$\widetilde{\text{FWHM H}\beta}$	-0.330	0.077	-0.357	-0.082	-0.141
$\text{FeII/H}\beta$	0.341	-0.140	0.003	-0.487	-0.212
$\log \text{ EW } [\text{O III}]$	-0.310	0.016	0.255	0.394	-0.095
log FWHM CIII]	-0.198	0.077	-0.623	0.054	0.402
$\log EW Ly\alpha$	-0.177	-0.502	-0.006	-0.143	0.033
log EW CIV	-0.336	-0.262	0.048	-0.050	-0.303
$CIV/Ly\alpha$	-0.342	0.062	0.025	-0.074	-0.584
log ÉW CIII]	-0.262	-0.413	-0.124	-0.176	-0.008
Si III]/C III]	0.342	-0.149	-0.018	-0.311	-0.116
$NV/Ly\alpha$	0.231	-0.050	-0.573	0.107	-0.288
$\lambda 1400/\mathrm{Ly}\alpha$	0.223	-0.351	-0.225	0.441	-0.216

We now carry through a PCA on the data of the quasar sample given in Francis & Wills (1999).

- a) Read the paper! Remove data rows that have missing data. Create a table of the original data and compute mean value and standard deviation of each column. 10 Points
- b) Take the original data and put it into normalized or weighted form, so that the effect of different units is effectively removed. Normalize by the standard deviation! 10 Points
- c) Visually inspect the data after the normalization by plotting each column (x: data index, y: data value). Confirm (by eye) that each component is about normally distributed. 10 Points
- d) Construct the covariance matrix. This is a  $13 \times 13$  symmetric matrix. 10 Points

$$C_{ij} = \sigma_i \sigma_j$$

- e) Compute the eigenvalues and eigenvectors of the covariance matrix. Plot the Eigenvalues against their number (index). Recreate Table 3 from Francis & Wills (1999). 10 Points
- f) Compute errors of the eigenvalues with a bootstrap analysis or jackknife. Use sample size of 10000. Plot the distributions for the first 5 eigenvalues. 10 Points