Theoretical and Computational Star Formation

Exercise 1

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1 Jeans Analysis

1.1 Jeans Radius and Jeans Mass

Consider two clouds in the interstellar medium. One cloud consists purely of molecular Hydrogen (H_2), has a temperature of 20 K, and a volume density of 10^{-18} g/cm^3. The other cloud consists purely of neutral Hydrogen (H), has a temperature of 100 K, and a volume density of 10^{-23} g/cm^3. Table 1 gives the Jeans radii and Jeans masses calculated for these two clouds.

Cloud	Jeans Radius [m]	Jeans Mass [kg]
H_2	41.8	$3.07 \cdot 10^{-10}$
H	$3.86 \cdot 10^4$	$2.41 \cdot 10^{-6}$

Table 1: Jeans masses and Jeans Radii for both clouds.

Figure 1 shows the Jeans mass as a function of density for the molecular Hydrogen cloud (H_2) and the neutral atomic Hydrogen cloud (H). The Jeans mass is inversely proportional to the density of the cloud, i.e at low densities, the Jeans mass is large, and at high densities the Jeans mass is small.

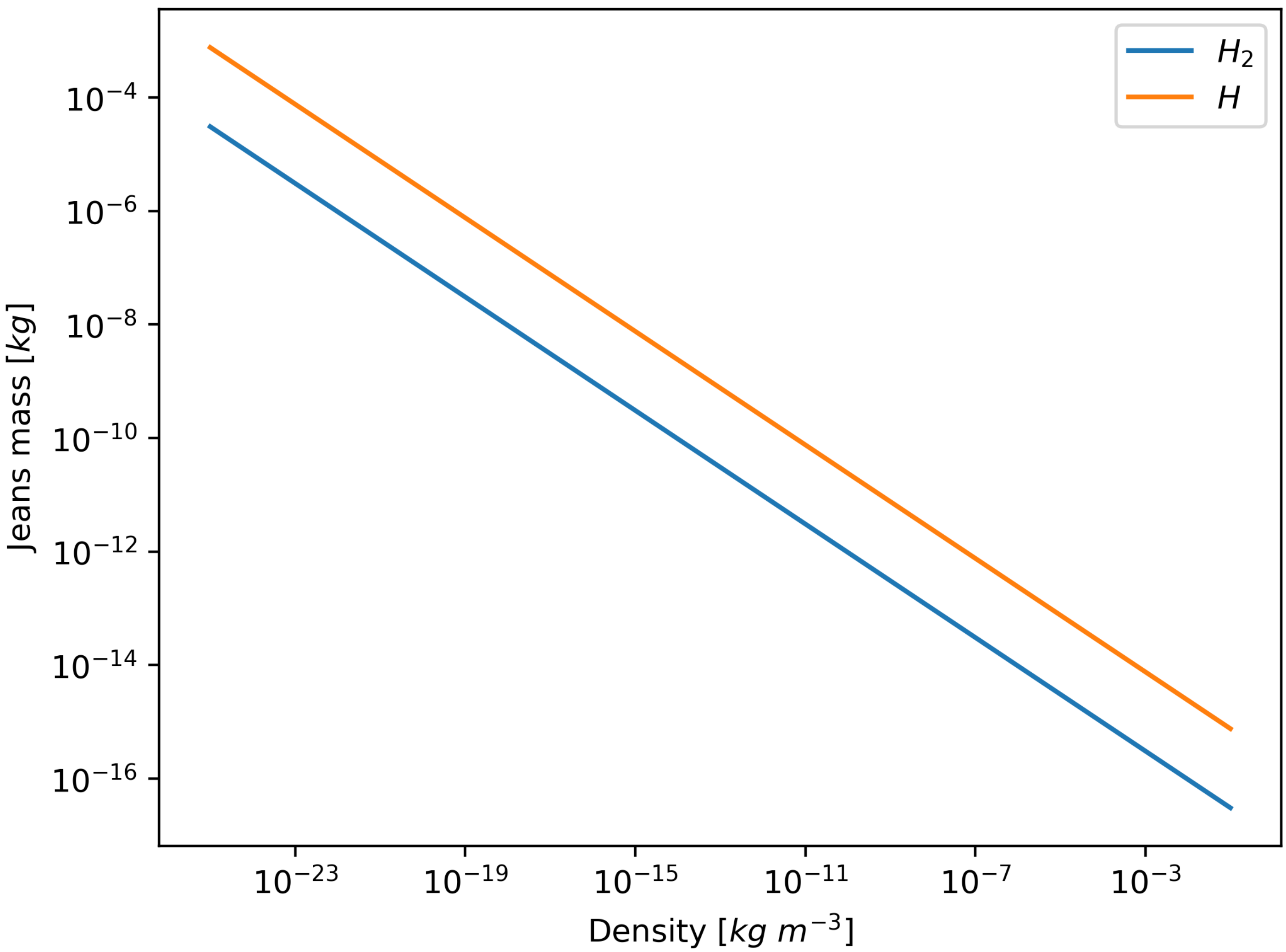

Figure 1: Jeans mass as a function of density for the molecular Hydrogen cloud (H_2) and the neutral atomic Hydrogen cloud (H).

1.2 Change in gas behaviour

Figure 2 also shows the Jeans mass as a function of density for the molecular Hydrogen cloud (H_2) and the neutral atomic Hydrogen cloud (H), however, we now consider a scenario where the gas behaviour changes at a critical volume density $\rho_{CRIT} = 10^{-13}\ g/cm^3$. Above this density, the adiabatic index is 7/5. The temperature is then calculated using a barotropic equation of state, $T(\rho) = T_0(1 + (\rho/\rho_{CRIT})^{\gamma-1})$. In the regime above the critical density, the Jeans mass reaches a minimum, after which the Jeans mass begins to increase again with increasing densities.

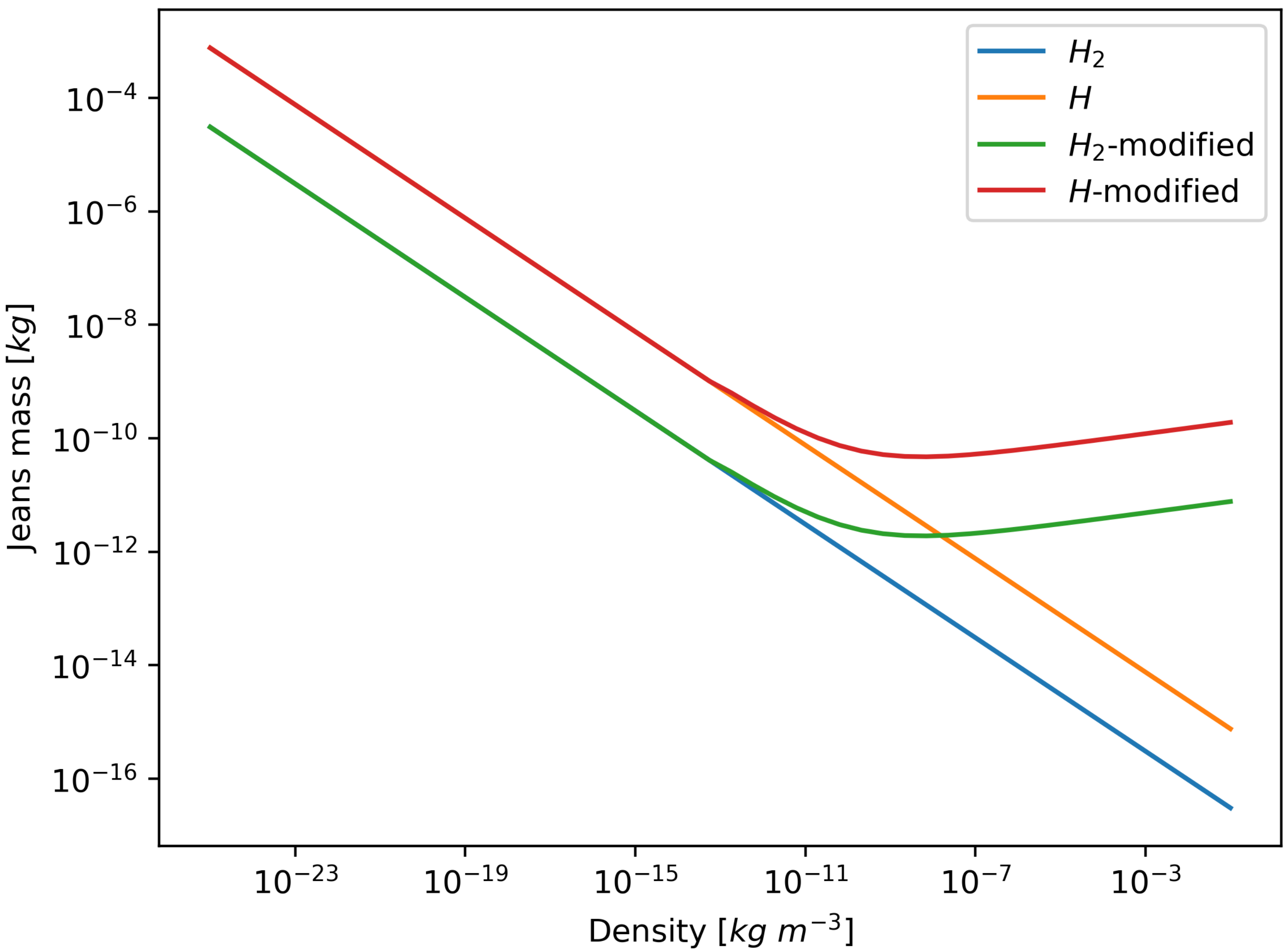

Figure 2: Jeans mass as a function of density for the molecular Hydrogen cloud (H_2) and the neutral atomic Hydrogen cloud (H). The lines denoted "modified" represent the clouds behaviour when we consider the change in gas behaviour above the critical density $\rho_{CRIT} = 10^{-13}\ g/cm^3$.

2 Free-fall time

2.1 Derivation of free-fall time

The mass of the collapsing cloud is given by the initial density and volume of the cloud. $M = \frac{4}{3}\pi R^3 \rho_0$. The fluid element has acceleration due to gravitational collapse $\vec{g}$ given by $\vec{g} = -\frac{GM}{r^2}$. Thus we have;

$$\begin{aligned}
\frac{d^2 r}{dt^2} &= -\frac{4\pi G R^3 \rho_0}{3r^2} \\
\frac{d}{dt}\frac{dr}{dt} &= -\frac{4\pi G R^3 \rho_0}{3r^2} \\
\frac{dr}{dt}\frac{d}{dr}\nu &= -\frac{4\pi G R^3 \rho_0}{3r^2} \\
\nu \frac{d\nu}{dr} &= -\frac{4\pi G R^3 \rho_0}{3r^2} \\
\nu d\nu &= -\frac{4\pi G R^3 \rho_0}{3r^2} dr \\
\int \nu d\nu &= -\int \frac{4\pi G R^3 \rho_0}{3r^2} dr \\
\frac{1}{2}\nu^2 &= \frac{4\pi G R^3 \rho_0}{3r} + C
\end{aligned}$$

With initial conditions r=R and ν=0, we have $C = \frac{4}{3}\pi G R^2 \rho_0$.

$$\begin{aligned}
\frac{1}{2}\nu^2 &= \frac{4}{3}\pi G R^2 \rho_0 \left(\frac{R}{r} - 1\right) \\
\left(\frac{dr}{dt}\right)^2 &= \frac{8}{3}\pi G R^2 \rho_0 \left(\frac{R}{r} - 1\right) \\
\frac{dr}{dt} &= \left(\frac{8}{3}\pi G R^2 \rho_0 \left(\frac{R}{r} - 1\right)\right)^{\frac{1}{2}} \\
\int dt &= \sqrt{\frac{3}{8\pi G R^2 \rho_0}} \int_0^R \frac{dr}{\sqrt{\frac{R}{r} - 1}}
\end{aligned}$$

Using x=r/R:

$$\int dt = \sqrt{\frac{3}{8\pi G \rho_0}} \int_0^1 \frac{dx}{\sqrt{\frac{1}{x} - 1}}$$

Let $x = sin^2\theta$:

$$\begin{aligned}
t_{ff} &= \sqrt{\frac{3}{8\pi G \rho_0}} \int_0^{\frac{\pi}{2}} \frac{2sin\theta cos\theta d\theta}{\sqrt{\frac{1}{sin^2\theta} - 1}} \\
t_{ff} &= \sqrt{\frac{3}{8\pi G \rho_0}} \int_0^{\frac{\pi}{2}} \frac{2sin^2\theta cos\theta d\theta}{cos\theta} \\
t_{ff} &= \sqrt{\frac{3}{8\pi G \rho_0}} \int_0^{\frac{\pi}{2}} 2sin^2\theta d\theta \\
t_{ff} &= \sqrt{\frac{3}{8\pi G \rho_0}} \left[\theta - \frac{1}{2}sin(2\theta)\right]_0^{\frac{\pi}{2}} \\
t_{ff} &= \sqrt{\frac{3}{8\pi G \rho_0}} \left[\frac{\pi}{2}\right] \\
t_{ff} &= \sqrt{\frac{3\pi}{32 G \rho_0}}
\end{aligned} \tag{1}$$

2.2 Free-fall time of the atomic and molecular clouds

Equation (1) to calculate the free-fall time of the molecular Hydrogen cloud (H_2) and the neutral atomic Hydrogen cloud (H). Since the free-fall time is inversely proportional to $\rho^{\frac{1}{2}}$, the free-fall time is longer for the neutral atomic Hydrogen cloud (H).

Cloud	Free fall time [s]
H_2	$2.100 \cdot 10^{12}$
H	$6.643 \cdot 10^{14}$

Table 2: The free-fall time of both clouds analysed in Section 1 is calculated using equation (1)