Theoretical and Computational Star Formation SoSe 2022

Assignment sheet 9

Hand-out	14.06.2022
Hand-in	21.06.2022
Discussion	28.06.2022

Total points: 30 Points

General Information

All exercises combined over all sheets sum up to a final 100%-score value. In order to pass the course one has to reach a combined score of 50%.

Please note that we can only give points for WORKING code! If your code does not compile or does not run, we will have to give 0 points for the coding part. Please consider this when submitting your exercises.

Handing in your exercises:

- You should submit in groups of 2 - 4 students.
- The assignment sheet has to be handed in directly before the start of the tutorial.
- All sheets will be handed out electronically via Ilias.
- You have to hand in your exercises electronically by sending them to the email address **lectures@ph1.uni-koeln.de**. In particular your programs should be handed in as text documents with a corresponding suffix (example: *.py for Python programs). If required, text has to be scanned in or submitted as a PDF document. Additional output of your programs has to be submitted as well. **All** of your files should be contained in **one** archive file format (example.: *.tar, *zip).
- You should name all authors in all programs (as a comment) and PDFs.
- Please write 'readable' code. Comment your thoughts and steps.

1 Sedov blast wave [10 Points]

The Sedov blast wave solution describes the expansion of a blast wave, e.g. triggered by a supernova explosion into the surrounding medium, which occurs in an adiabatic fashion. For this we consider a medium with an initially constant density ρ in which we insert at one point a large amount of thermal energy E. This energy will lead to an expansion of a spherically symmetric bubble, the so-called Sedov blast wave.
The solution, i.e. the position of the shock front R_s is self similar and only depends on the initial energy of the explosion E (which remains constant over time), the density of the uniform, ambient medium ρ and the time t.
Explain in detail the steps you make in the following.

1. Derive the basic dependence of R_s on E, ρ, and t by means of a dimensional analysis.
2. Calculate the velocity of the shock front $v_s = \frac{\partial R_s}{\partial t}$.
3. Calculate the mass which is swept up by the blast wave over time.
4. Explain (phenomenologically) what phases in the evolution of a supernova follow after this initial Sedov-Taylor phase and which assumption(s) made in the Sedov-Taylor phase brake down at later stages.

2 Finite volume method in one dimension - the implementation [10 Points]

After having coded a working Riemann solver in the previous assignment sheet, you are well prepared for finalizing your 1D hydrodynamics code.
Task: Implement the one-dimensional finite volume method

$$\vec{q}_i^{\,n+1} = \vec{q}^{\,n} - \frac{\Delta t}{\Delta x}(\vec{f}^{n}_{x,i+1/2} - \vec{f}^{n}_{x,i-1/2}). \tag{1}$$

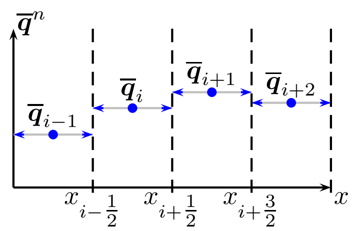

Figure 1: Discretization of the computational domain $\Omega = [a, b] \subset \mathbb{R}$ divided into $N \in \mathbb{N}$ uniform, equidistant Cartesian cells indexed by i. Each cell has the same size $\Delta x = \frac{b-a}{N}$

to solve the three-dimensional compressible Euler equations

$$\frac{\partial}{\partial t}\begin{bmatrix}\rho\\ \rho v_x\\ \rho v_y\\ \rho v_z\\ E\end{bmatrix}+\frac{\partial}{\partial x}\begin{bmatrix}\rho v_x\\ \rho v_x^2+p\\ \rho v_x v_y\\ \rho v_x v_z\\ v_x(\frac{1}{2}\rho\|\vec{v}\|^2+\frac{\gamma p}{\gamma-1})\end{bmatrix}=\begin{bmatrix}0\\0\\0\\0\\0\end{bmatrix}. \tag{2}$$

in one spatial dimension.

Key elements of your grid implementation should be:

- Discretization of the computational grid into N equidistant cells as shown in Figure 1.
- Isolated domain Ω, i.e., the value in the 0-th and $(N+1)$-th cell (which do not physically exist) are assumed to be identical to the 1-th and N-th cells, respectively

Hints:

- Define one solution array like `double q[i][k];` holding the data for each i-th cell and k-th conserved variable, where
  $i = [1, N]$ (N = number of cells), and
  $k = [1, 5]$ ($1 = \rho$, $2 = \rho v_x$, $3 = \rho v_y$, $4 = \rho v_z$, $5 = E$)
- Define various separate functions to be able to test them more easily.
  Pseudocode:

  init(a, b, N, q) Initialize 1D grid with initial conditions

  finterface = getRiemannFluxX(qL, qR) Computes the fluxes from the conserved quantities

  lambda = getMaxEigenvalueX(qL, qR) Computes the maximum eigenvalue from the conserved quantities

  dt = getTimestep(lambda) Computes the next time step from the previously computed eigenvalues

  q = updateSolution(q, finterface, dt) Update solution at time t^n to the next step, t^{n+1}, using the computed fluxes and the time step

- Your code structure should look like this:
  1. Choose number of grid cells N and size of computational domain (a, b)
  2. Compute Δx, set $t = 0$.
  3. Initialize the solution array `q` with initial conditions of a physical problem (see next page for examples, start with the Sod problem)
  4. Loop: compute in *every* sweep:

(a) The maximum wave speed at time level t^n.
(b) Compute new time step (all cells should use the same time step)
(c) Use boundary (see above) for edge cells ($i = 0, N + 1$).
(d) Compute the numerical fluxes at each interface
(e) Update the new solution with the computed time step and the

3 Numerical test: The Sod shock tube [10 points]

The Sod shock tube is probably the most popular shock-tube test. For the Sod shock tube, an analytic solution exists and is known. The initial conditions and runtime parameters are listed in Table 1.

	$x < x_{\text{shock}}$	$x > x_{\text{shock}}$
Density	1	0.125
Pressure	1	0.1
Momentum	$\vec{0}$	$\vec{0}$
Domain size	[0,1]	
Shock location	$x_{\text{shock}} = 0.5$	
Simulation end time	$t_{\text{max}} = 0.2$	
Adiabatic index	$\gamma = 1.4$	

Table 1: Initial conditions of the Sod shock-tube problem.

1. Make yourself familiar with the Sod shock tube problem.
2. Use your new 1D Hydro code to compute the solution of the Sod shock tube for various choices of N.
3. Plot the results, in particular the density, the pressure, and the velocity and compare it with the Figures 2, 3, and 4. Plot the solution for different times.
4. Show that your solution converges with increasing N at the time $t = 0.2$.
5. Based on your results, discuss problems and deviations from an ideal solution.

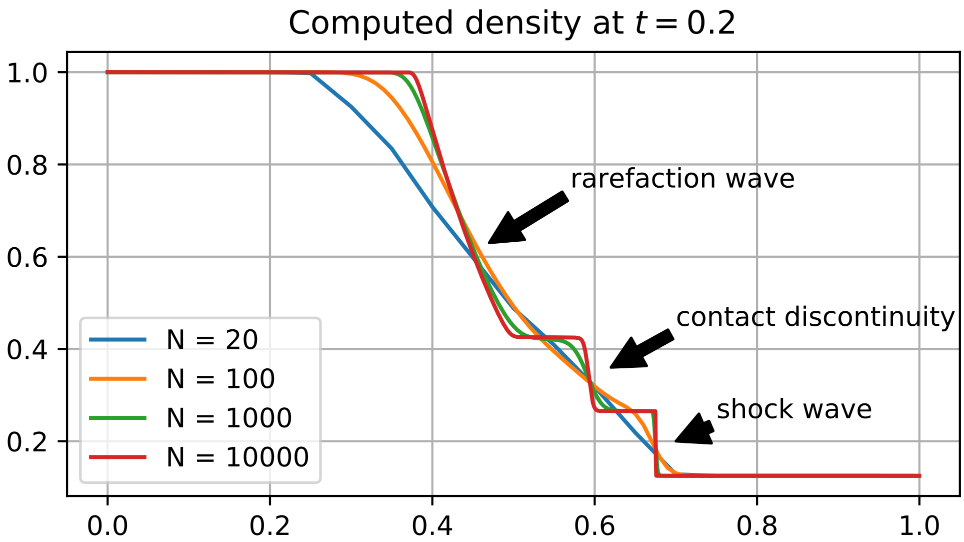

Figure 2: Computed density for a few different choices of N. We highlight the individual waves that emerge from the single initial jump.

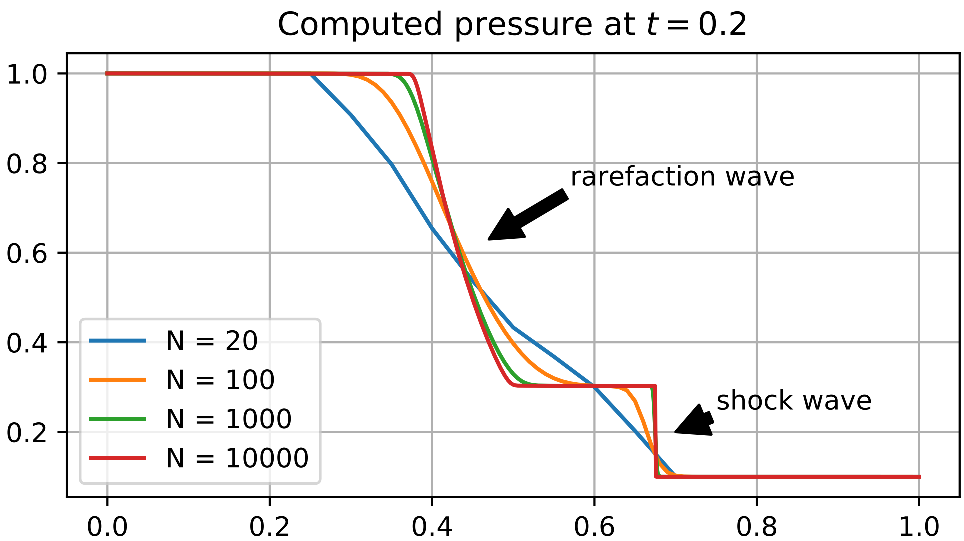

Figure 3: Computed pressure for a few different choices of N. Note that the contact discontinuity is absent in the pressure plot. Contact discontinuities are only present in density and temperature, but not in pressure (or magnetic fields).

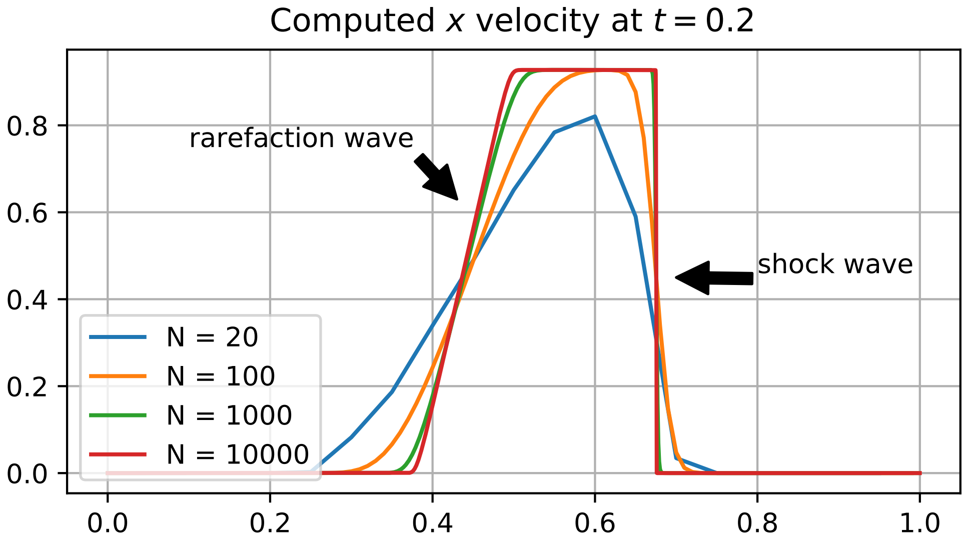

Figure 4: Computed velocity for a few different choices of N.