Theoretical and Computational Star formation SoSe 2022

Assignment sheet 1

Hand-out: 05.04.2022
Hand-in: 12.04.2022
Discussion: 19.04.2022

Total points: 30

Exercise	1	2	Total
Points			

General Information

All exercises combined over all sheets sum up to a final 100%-score value. In order to pass the course one has to reach a combined score of 50%.

Please note that we can only give points for WORKING code! If your code does not compile or does not run, we will have to give 0 points for the coding part. Please consider this when submitting your exercises.

Handing in your exercises:

- You should hand in groups of 2 - 4 students.
- The assignment sheet has to be handed in directly before the start of the tutorial.
- All sheets will be handed out electronically via Ilias.
- You have to hand in your exercises electronically by sending them to the email adress **lectures@ph1.uni-koeln.de**. In particular your programs should be handed in as text documents with a corresponding suffix (example: *.py for python programs). If required, text has to be scanned in or submitted as a PDF document. Additional output of your programs has to be submitted as well. **All** of your files (code plus compiled executable) should be contained in **one** archive file format (example: *.tar, *.zip).
- You should name all authors in all programs (as a comment) and PDFs.
- Please write 'readable' code. Comment your thoughts and steps.

1 Jeans Analysis [15 Points]

Consider two clouds in the interstellar medium. One cloud consists purely of molecular Hydrogen (H_2), has a temperature of 20 K, and a volume density of 10^{-18} g/cm^3. The other cloud consists purely of neutral Hydrogen (H), has a temperature of 100 K, and a volume density of 10^{-23} g/cm^3.

1. What is the Jeans radius and the corresponding Jeans mass for each cloud?
2. Plot the Jeans mass a function of density for both clouds on a log-log plot. You may use Python or any other programing language to produce this plot.
3. Describe your findings.
4. Now consider that the gas behaviour changes at a critical volume density, $\rho_{\mathrm{CRIT}} = 10^{-13}$ g/cm^3. Above this density the adiabatic index is 7/5. Continue your plot to densities above 10^{-13} g/cm^3 and describe the behaviour of the Jeans mass at these high densities.

   HINT: This can be done in a continuous fashion by using a barotropic equation of state such that $\mathrm{T}(\rho) = \mathrm{T}_0(1 + (\rho/\rho_{\mathrm{CRIT}})^{\gamma-1})$

2 Free-fall time [15 Points]

Derive the free-fall time, i.e. the time a fluid element needs to fall towards the centre of a self-gravitating gaseous sphere. The following steps must be taken:

- Consider the equation of motion for a fluid element in free-fall (no pressure or other terms are taken into account!). The fluid element should initially rest at a radius R and the sphere of gas enclosed by this radius is M. The gas in the sphere should have a uniform density.
- Rewrite the equation of motion such that you get the derivative of the velocity squared towards r on the left-hand side.
- Bring dr to the other side of the equation and integrate both sides.
- Again replace v by dr/dt and choose the solution with $v < 0$ (i.e. collapse).
- Separate dr and dt to be able to integrate once again each side of the equation separately.
- Substitute the dimensionless radius $x = r/R$.
- Integrate the right-hand side from $x = 1$ to $x = 0$ to obtain the free-fall time.

  HINT: By using the substitution $x = \sin^2(\phi)$, the integral can be calculated analytically.

The result should be:

$$\tau_{ff} = \sqrt{\frac{3\pi}{32G\rho_0}} \tag{1}$$

Finally calculate the free-fall times for both clouds in exercise one and briefly discuss the results.