Theoretical and Computational Star Formation

Exercise 2

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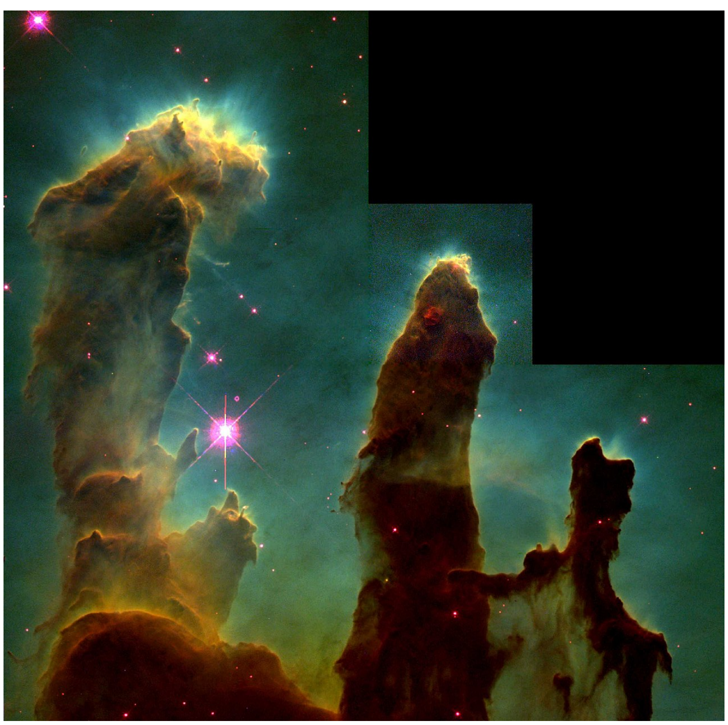

Credit: NASA, Jeff Hester, and Paul Scowen (Arizona State University)
http://hubblesite.org/newscenter/newsdesk/archive/releases/2003/34/image/a

1 Setting up the 1D SPH system

- **Initialization of the particles:** A vector was initialised with n evenly spaced elements between 0 and 1, but not exactly at 0 and 1. This vector contains the positions of the particles. The mass of each particle was set to 1.
- **Determination of the smoothing length:** The smoothing length, h, was determined as $h = \eta MEAN[MIN[r_{ij}]]$, where η is a free parameter of order unity.
- **Determination of particles neighbours:** A function was also defined to determine the positions of the nearest neighbours of a particle in a given position, for a given smoothing length. A particle at position j is defined as a neighbour of a particle at position i if $|r_{ij}| \leq 2h$.
- **Evaluate density at each position:** At each position in our system, we know determine the neighbours of that position for the given smoothing length, and then calculate the density at that position according to equation (1). Where $W(s)$ is given by equations (2) and (3).
- **Output particle positions and densities and plot them:** The computed densities and positions were stored in a vector, which was then written to a .TXT file, and plotted using python.

$$\rho_i = \Sigma_{j=1}^{N} m_j W(r_{ij}, h) \tag{1}$$

$$s = |r_{ij}|/h \tag{2}$$

$$W(s) = \frac{1}{\pi h^3} \begin{cases} 1 - \frac{3}{2}s^2 + \frac{3}{4}s^3 & \text{if } 0 \leq \text{s} \leq 1 \\ \frac{1}{4}(2-s)^3 & \text{if } 1 \leq \text{s} \leq 2 \\ 0 & \text{if s} > 2 \end{cases} \tag{3}$$

2 Results

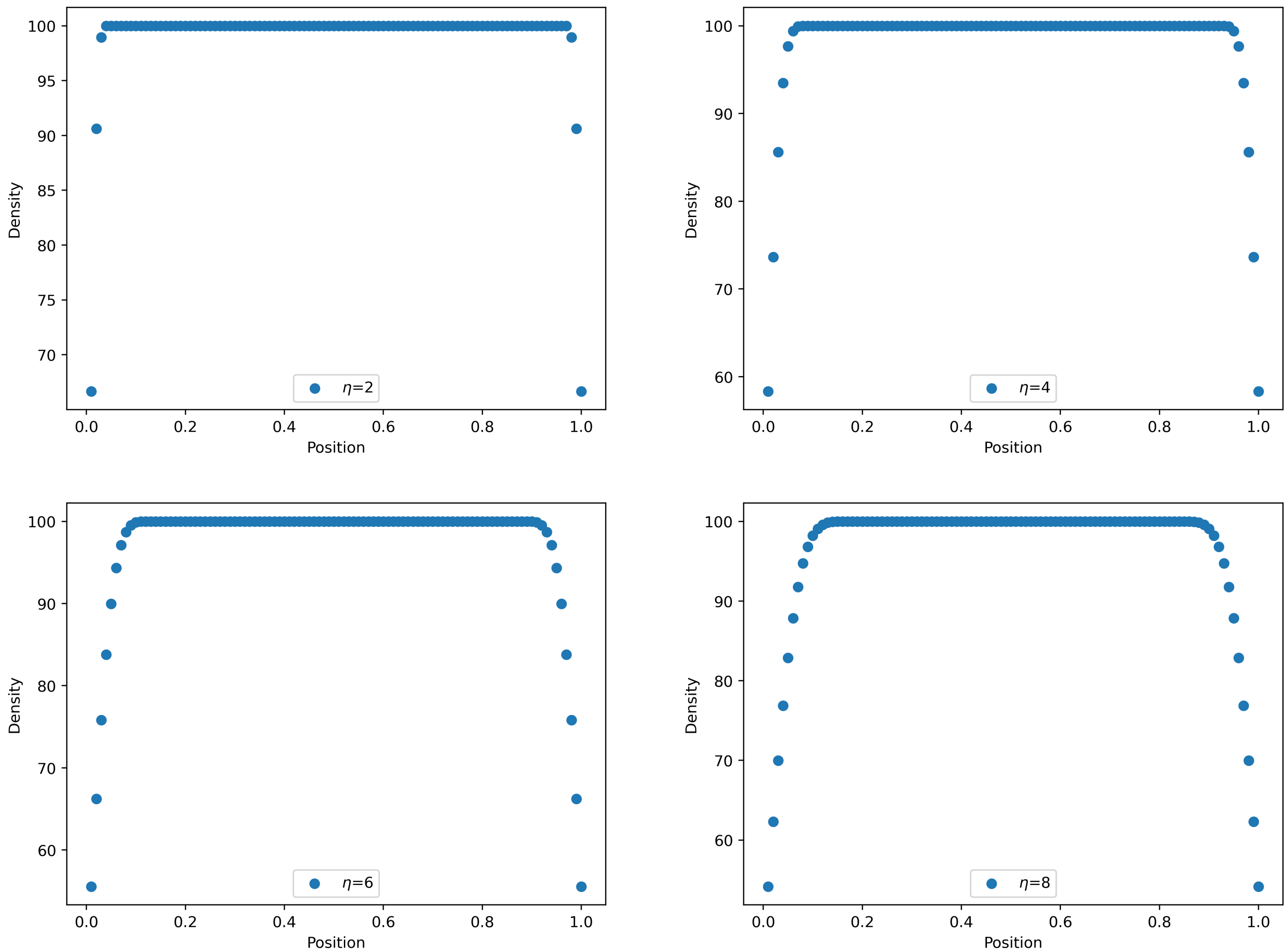

Figure 1: Particle densities as a function of position for n=100 particles and a range of values for η.

The density of particles should be equal to the number of particles used. Why is this not the case for the particles near x=0 and x=1? Figure 1 shows the particle density as a function of position for n=100 particles, with 4 different values of η. The density of particles at each point is equal to the number of particles, in this case n=100. However, at the edge positions, near 0 and 1, the density decreases. This is because the density for a position i is computed by summing the mass of the nearest neighbours by the M4 spline kernel which has a value dependent on the distance from i to the neighbouring position. For particles close to 0 and 1, they have no neighbours beyond 0 and 1, so less contributions to the density.

What does varying the parameter η do to the density and why? Figure 1 also shows the effect of the value chosen for η. Increasing the value of eta increases the smoothing length, which in turn increases the number of neighbours contributing to the density at position i. This increases the number of particles near 0 and 1 which experience the edge effects previously discussed. The value of η does not however affect the value of the densities of most particles as the M4 spline kernel takes into consideration the smoothing length through the normalisation factor $(2/3h)$ and the value of $s = |r_{ij}|/h$.