Midterm 1 Equation Sheet

$$\frac{\mathrm{d}}{\mathrm{dx}}\sin x = \cos x$$

$$\frac{\mathrm{d}}{\mathrm{dx}}\cos x = -\sin x$$

$$\int \mathrm{d}x\, \frac{1}{1+x^2} = \tan^{-1} x$$

$$\int_a^b \mathrm{d}x u(x)v'(x) = u(x)v(x)|_a^b - \int_a^b \mathrm{d}x u'(x)v(x)$$

$$\int \mathrm{d}x f\big(u(x)\big)u'(x) = \int \mathrm{d}u f(u)$$

$$\int dx\, e^{\alpha x} = \frac{1}{\alpha} e^{\alpha x}$$

$$e^x = \sum_{n=0}^{\infty} \frac{1}{n!} x^n$$

$$\int_{-\infty}^{\infty} dx e^{-x^2} = \sqrt{\pi}$$

$$\cos(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\sin(x) = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\frac{1}{1-x} = \sum_{n=0}^{\infty} x^n \text{, for } |x| \leq 1$$

$$a^n = e^{n \ln(a)}$$

Infinitesimal volume element of a sphere = $r^2 \mathrm{d}r \sin\theta\, \mathrm{d}\theta \mathrm{d}\phi$ = $r^2 \mathrm{d}r \mathrm{d}\cos\theta\, \mathrm{d}\phi$

Infinitesimal volume element of a cylinder = $\rho \mathrm{d}\rho \mathrm{d}\theta \mathrm{dz}$

Midterm 2 Equation Sheet

Infinitesimal volume element of a sphere = $r^2 \mathrm{d}r \sin\theta \, \mathrm{d}\theta \mathrm{d}\phi = r^2 \mathrm{d}r \mathrm{d}\cos\theta \, \mathrm{d}\phi$

Infinitesimal volume element of a cylinder = $\rho \mathrm{d}\rho \mathrm{d}\theta \mathrm{d}z$

Divergence Theorem: $\int_S \vec{F} \cdot \hat{n} \, \mathrm{d}S = \int_V \nabla \cdot \vec{F} \, \mathrm{d}V$

Stokes Theorem: $\int_S \vec{F} \cdot \hat{t} \, \mathrm{d}s = \int_S (\nabla \times \vec{F}) \cdot \hat{n} \, \mathrm{d}S$ $\quad \oint_C \mathbf{F} \cdot \hat{\mathbf{t}} \, ds = \iint_S \hat{\mathbf{n}} \cdot \nabla \times \mathbf{F} \, dS$

Residue Theorem: $\oint f(z)\mathrm{d}z = 2\pi i \sum Res$

	Cartesian Coordinates	Cylindrical Coordinates	Spherical Coordinates
Conversion to Cartesian		$x = r\cos\theta$, $y = r\sin\theta$, $z = z$	$x = r\cos\phi\sin\theta$, $y = r\sin\phi\sin\theta$, $z = r\cos\theta$
Gradient ∇f	$\frac{\partial f}{\partial x}\hat{\imath} + \frac{\partial f}{\partial y}\hat{\jmath} + \frac{\partial f}{\partial z}\hat{k}$	$\frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{\partial f}{\partial z}\hat{z}$	$\frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi}$
Divergence $\nabla \cdot \mathbf{F}$	$\frac{\partial F_x}{\partial x} + \frac{\partial F_y}{\partial y} + \frac{\partial F_z}{\partial z}$	$\frac{1}{r}\frac{\partial (rF_r)}{\partial r} + \frac{1}{r}\frac{\partial F_\theta}{\partial \theta} + \frac{\partial F_z}{\partial z}$	$\frac{1}{r^2}\frac{\partial (r^2 F_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial F_\theta \sin\theta}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial F_\phi}{\partial \phi}$
Curl $\nabla \times \mathbf{F}$	$\left(\frac{\partial F_z}{\partial y} - \frac{\partial F_y}{\partial z}\right)\hat{\imath} + \left(\frac{\partial F_x}{\partial z} - \frac{\partial F_z}{\partial x}\right)\hat{\jmath} + \left(\frac{\partial F_y}{\partial x} - \frac{\partial F_x}{\partial y}\right)\hat{k}$	$\left(\frac{1}{r}\frac{\partial F_z}{\partial \theta} - \frac{\partial F_\theta}{\partial z}\right)\hat{r} + \left(\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right)\hat{\theta} + \left(\frac{1}{r}\frac{\partial (rF_\theta)}{\partial r} - \frac{1}{r}\frac{\partial F_r}{\partial \theta}\right)\hat{z}$	$\left(\frac{1}{r\sin\theta}\frac{\partial (\sin\theta\, F_\phi)}{\partial \theta} - \frac{1}{r\sin\theta}\frac{\partial F_\theta}{\partial \phi}\right)\hat{r} + \left(\frac{1}{r\sin\theta}\frac{\partial F_r}{\partial \phi} - \frac{1}{r}\frac{\partial (F_\phi)}{\partial r}\right)\hat{\theta} + \left(\frac{1}{r}\frac{\partial (rF_\theta)}{\partial r} - \frac{1}{r}\frac{\partial F_r}{\partial \theta}\right)\hat{\phi}$

Midterm 3 Equation Sheet

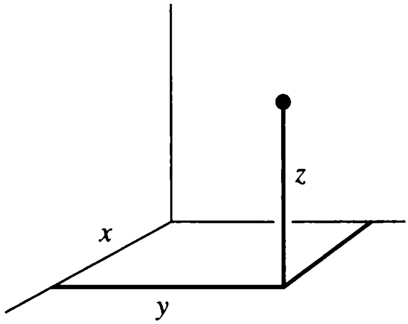
Angle between two vectors **a** and **b**:

$$\theta = \cos^{-1}\left(\frac{(\mathbf{a} \cdot \mathbf{b})}{\sqrt{(\mathbf{a} \cdot \mathbf{a})(\mathbf{b} \cdot \mathbf{b})}}\right)$$

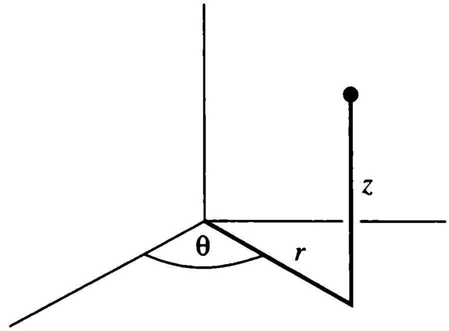
First order linear differential equation solution:

$$y(t) = y_0 e^{-\int_{t_0}^{t} q(t')dt'} + e^{-\int_{t_0}^{t} q(t')dt'}\left(\int_{t_0}^{t} r(t')dt' \, e^{\int_{t_0}^{t'} q(t'')dt''}\right)$$

CARTESIAN

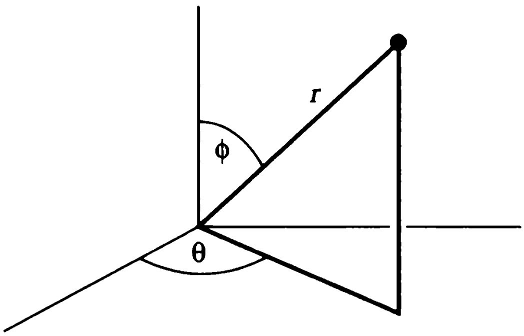


CYLINDRICAL



DIVERGENCE div **F** $\nabla \cdot \mathbf{F}$	$\dfrac{\partial F_x}{\partial x} + \dfrac{\partial F_y}{\partial y} + \dfrac{\partial F_z}{\partial z}$	$\dfrac{1}{r}\dfrac{\partial}{\partial r}(rF_r) + \dfrac{1}{r}\dfrac{\partial F_\theta}{\partial \theta} + \dfrac{\partial F_z}{\partial z}$
GRADIENT grad f ∇f	$(\nabla f)_x = \dfrac{\partial f}{\partial x}$	$(\nabla f)_r = \dfrac{\partial f}{\partial r}$
	$(\nabla f)_y = \dfrac{\partial f}{\partial y}$	$(\nabla f)_\theta = \dfrac{1}{r}\dfrac{\partial f}{\partial \theta}$
	$(\nabla f)_z = \dfrac{\partial f}{\partial z}$	$(\nabla f)_z = \dfrac{\partial f}{\partial z}$
CURL curl **F** $\nabla \times \mathbf{F}$	$(\nabla \times \mathbf{F})_x = \dfrac{\partial F_z}{\partial y} - \dfrac{\partial F_y}{\partial z}$	$(\nabla \times \mathbf{F})_r = \dfrac{1}{r}\dfrac{\partial F_z}{\partial \theta} - \dfrac{\partial F_\theta}{\partial z}$
	$(\nabla \times \mathbf{F})_y = \dfrac{\partial F_x}{\partial z} - \dfrac{\partial F_z}{\partial x}$	$(\nabla \times \mathbf{F})_\theta = \dfrac{\partial F_r}{\partial z} - \dfrac{\partial F_z}{\partial r}$
	$(\nabla \times \mathbf{F})_z = \dfrac{\partial F_y}{\partial x} - \dfrac{\partial F_x}{\partial y}$	$(\nabla \times \mathbf{F})_z = \dfrac{1}{r}\dfrac{\partial}{\partial r}(rF_\theta) - \dfrac{1}{r}\dfrac{\partial F_r}{\partial \theta}$
LAPLACIAN $\nabla^2 f$	$\dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2}$	$\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial f}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2 f}{\partial \theta^2} + \dfrac{\partial^2 f}{\partial z^2}$

SPHERICAL



$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2F_r) + \frac{1}{r\sin\phi}\frac{\partial}{\partial\phi}(\sin\phi\, F_\phi) + \frac{1}{r\sin\phi}\frac{\partial F_\theta}{\partial\theta}.$$

$$(\nabla f)_r = \frac{\partial f}{\partial r}$$

$$(\nabla f)_\phi = \frac{1}{r}\frac{\partial f}{\partial\phi}$$

$$(\nabla f)_\theta = \frac{1}{r\sin\phi}\frac{\partial f}{\partial\theta}$$

$$(\nabla\times\mathbf{F})_r = \frac{1}{r\sin\phi}\frac{\partial}{\partial\phi}(\sin\phi\, F_\theta) - \frac{1}{r\sin\phi}\frac{\partial F_\phi}{\partial\theta}$$

$$(\nabla\times\mathbf{F})_\phi = \frac{1}{r\sin\phi}\frac{\partial F_r}{\partial\theta} - \frac{1}{r}\frac{\partial}{\partial r}(rF_\theta)$$

$$(\nabla\times\mathbf{F})_\theta = \frac{1}{r}\frac{\partial}{\partial r}(rF_\phi) - \frac{1}{r}\frac{\partial F_r}{\partial\phi}$$

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\phi}\frac{\partial}{\partial\phi}\left(\sin\phi\frac{\partial f}{\partial\phi}\right) + \frac{1}{r^2\sin^2\phi}\frac{\partial^2 f}{\partial\theta^2}$$