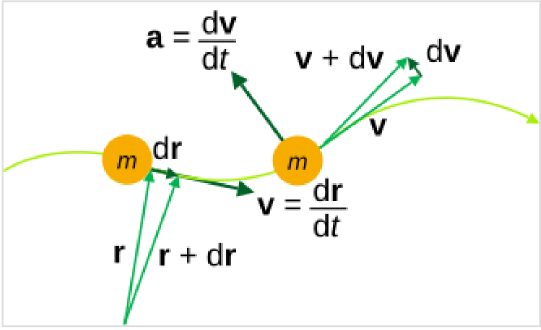
List of equations in classical mechanics

This article lists equations from Newtonian mechanics, see analytical mechanics for the more general formulation of classical mechanics (which includes Lagrangian and Hamiltonian mechanics).

Mass and inertia

Quantity (common name/s)	(Common) symbol/s	Defining equation	SI units	Dimension
Linear, surface, volumetric mass density	λ or μ (especially in acoustics, see below) for Linear, σ for surface, ρ for volume.	$m = \int \lambda \, \mathrm{d}\ell$ $m = \iint \sigma \, \mathrm{d}S$ $m = \iiint \rho \, \mathrm{d}V$	kg m^{-n}, n = 1, 2, 3	M L^{-n}
Moment of mass[5]	$\mathbf{m}$ (No common symbol)	Point mass: $\mathbf{m} = \mathbf{r}m$ Discrete masses about an axis x_i: $\mathbf{m} = \sum_{i=1}^{N} \mathbf{r}_i m_i$ Continuum of mass about an axis x_i: $\mathbf{m} = \int \rho\left(\mathbf{r}\right) x_i \mathrm{d}\mathbf{r}$	kg m	M L
Center of mass	$\mathbf{r}_{\mathrm{com}}$ (Symbols vary)	i-th moment of mass $\mathbf{m}_i = \mathbf{r}_i m_i$ Discrete masses: $\mathbf{r}_{\mathrm{com}} = \frac{1}{M}\sum_i \mathbf{r}_i m_i = \frac{1}{M}\sum_i \mathbf{m}_i$ Mass continuum: $\mathbf{r}_{\mathrm{com}} = \frac{1}{M}\int \mathrm{d}\mathbf{m} = \frac{1}{M}\int \mathbf{r}\,\mathrm{d}m = \frac{1}{M}\int \mathbf{r}\rho\,\mathrm{d}V$	m	L
2-Body reduced mass	m_{12}, μ Pair of masses = m_1 and m_2	$\mu = \frac{m_1 m_2}{m_1 + m_2}$	kg	M
Moment of inertia (MOI)	I	Discrete Masses: $I = \sum_i \mathbf{m}_i \cdot \mathbf{r}_i = \sum_i \left\vert\mathbf{r}_i\right\vert^2 m$ Mass continuum: $I = \int \left\vert\mathbf{r}\right\vert^2 \mathrm{d}m = \int \mathbf{r} \cdot \mathrm{d}\mathbf{m} = \int \left\vert\mathbf{r}\right\vert^2 \rho\,\mathrm{d}V$	kg m^2	M L^2

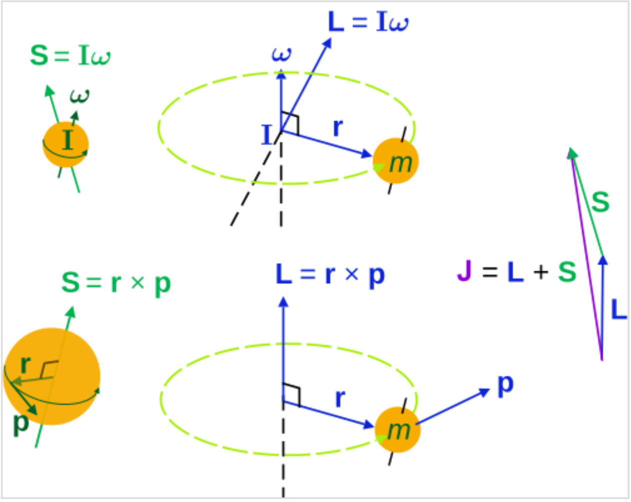
Derived kinematic quantities



Kinematic quantities of a classical particle: mass *m*, position **r**, velocity **v**, acceleration **a**.

Quantity (common name/s)	(Common) symbol/s	Defining equation	SI units	Dimension
Velocity	**v**	$\mathbf{v} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$	m s^{-1}	L T^{-1}
Acceleration	**a**	$\mathbf{a} = \frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\mathbf{r}}{\mathrm{d}t^2}$	m s^{-2}	L T^{-2}
Jerk	**j**	$\mathbf{j} = \frac{\mathrm{d}\mathbf{a}}{\mathrm{d}t} = \frac{\mathrm{d}^3\mathbf{r}}{\mathrm{d}t^3}$	m s^{-3}	L T^{-3}
Jounce	**s**	$\mathbf{s} = \frac{\mathrm{d}\mathbf{j}}{\mathrm{d}t} = \frac{\mathrm{d}^4\mathbf{r}}{\mathrm{d}t^4}$	m s^{-4}	L T^{-4}
Angular velocity	**ω**	$\boldsymbol{\omega} = \hat{\mathbf{n}}\frac{\mathrm{d}\theta}{\mathrm{d}t}$	rad s^{-1}	T^{-1}
Angular Acceleration	**α**	$\boldsymbol{\alpha} = \frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t} = \hat{\mathbf{n}}\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$	rad s^{-2}	T^{-2}
Angular jerk	**ζ**	$\boldsymbol{\zeta} = \frac{\mathrm{d}\boldsymbol{\alpha}}{\mathrm{d}t} = \hat{\mathbf{n}}\frac{\mathrm{d}^3\theta}{\mathrm{d}t^3}$	rad s^{-3}	T^{-3}

Derived dynamic quantities



Angular momenta of a classical object.

Left: intrinsic "spin" angular momentum **S** is really orbital angular momentum of the object at every point,

right: extrinsic orbital angular momentum **L** about an axis,

top: the moment of inertia tensor **I** and angular velocity **ω** (**L** is not always parallel to **ω**)[6]

bottom: momentum **p** and its radial position **r** from the axis.

The total angular momentum (spin + orbital) is **J**.

Quantity (common name/s)	(Common) symbol/s	Defining equation	SI units	Dimension
Momentum	**p**	$\mathbf{p} = m\mathbf{v}$	kg m s^{-1}	M L T^{-1}
Force	**F**	$\mathbf{F} = \mathrm{d}\mathbf{p}/\mathrm{d}t$	N = kg m s^{-2}	M L T^{-2}
Impulse	**J**, Δ**p**, **I**	$\mathbf{J} = \Delta\mathbf{p} = \int_{t_1}^{t_2} \mathbf{F}\,\mathrm{d}t$	kg m s^{-1}	M L T^{-1}
Angular momentum about a position point $\mathbf{r}_0$,	**L**, **J**, **S**	$\mathbf{L} = (\mathbf{r} - \mathbf{r}_0) \times \mathbf{p}$ Most of the time we can set $\mathbf{r}_0 = \mathbf{0}$ if particles are orbiting about axes intersecting at a common point.	kg m^2 s^{-1}	M L^2 T^{-1}
Moment of a force about a position point $\mathbf{r}_0$, Torque	**τ**, **M**	$\boldsymbol{\tau} = (\mathbf{r} - \mathbf{r}_0) \times \mathbf{F} = \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t}$	N m = kg m^2 s^{-2}	M L^2 T^{-2}
Angular impulse	Δ**L** (no common symbol)	$\Delta\mathbf{L} = \int_{t_1}^{t_2} \boldsymbol{\tau}\,\mathrm{d}t$	kg m^2 s^{-1}	M L^2 T^{-1}

General energy definitions

Quantity (common name/s)	(Common) symbol/s	Defining equation	SI units	Dimension
Mechanical work due to a Resultant Force	W	$W = \int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$	J = N m = kg m^2 s^{-2}	M L^2 T^{-2}
Work done ON mechanical system, Work done BY	W_{ON}, W_{BY}	$\Delta W_{\mathrm{ON}} = -\Delta W_{\mathrm{BY}}$	J = N m = kg m^2 s^{-2}	M L^2 T^{-2}
Potential energy	φ, Φ, U, V, E_p	$\Delta W = -\Delta V$	J = N m = kg m^2 s^{-2}	M L^2 T^{-2}
Mechanical power	P	$P = \frac{\mathrm{d}E}{\mathrm{d}t}$	W = J s^{-1}	M L^2 T^{-3}

Every conservative force has a potential energy. By following two principles one can consistently assign a non-relative value to U:

- Wherever the force is zero, its potential energy is defined to be zero as well.
- Whenever the force does work, potential energy is lost.

Generalized mechanics

Quantity (common name/s)	(Common) symbol/s	Defining equation	SI units	Dimension
Generalized coordinates	q, Q		varies with choice	varies with choice
Generalized velocities	$\dot{q}, \dot{Q}$	$\dot{q} \equiv \mathrm{d}q/\mathrm{d}t$	varies with choice	varies with choice
Generalized momenta	p, P	$p = \partial L / \partial \dot{q}$	varies with choice	varies with choice
Lagrangian	L	$L(\mathbf{q}, \dot{\mathbf{q}}, t) = T(\dot{\mathbf{q}}) - V(\mathbf{q}, \dot{\mathbf{q}}, t)$ where $\mathbf{q} = \mathbf{q}(t)$ and $\mathbf{p} = \mathbf{p}(t)$ are vectors of the generalized coords and momenta, as functions of time	J	M L^2 T^{-2}
Hamiltonian	H	$H(\mathbf{p}, \mathbf{q}, t) = \mathbf{p} \cdot \dot{\mathbf{q}} - L(\mathbf{q}, \dot{\mathbf{q}}, t)$	J	M L^2 T^{-2}
Action, Hamilton's principal function	S, $\mathcal{S}$	$\mathcal{S} = \int_{t_1}^{t_2} L(\mathbf{q}, \dot{\mathbf{q}}, t) \mathrm{d}t$	J s	M L^2 T^{-1}

Kinematics

In the following rotational definitions, the angle can be any angle about the specified axis of rotation. It is customary to use θ, but this does not have to be the polar angle used in polar coordinate systems. The unit axial vector

$$\hat{\mathbf{n}} = \hat{\mathbf{e}}_r \times \hat{\mathbf{e}}_\theta$$

defines the axis of rotation, $\hat{\mathbf{e}}_r$ = unit vector in direction of $\mathbf{r}$, $\hat{\mathbf{e}}_\theta$ = unit vector tangential to the angle.

	Translation	Rotation
Velocity	Average: $$\mathbf{v}_{\text{average}} = \frac{\Delta\mathbf{r}}{\Delta t}$$ Instantaneous: $$\mathbf{v} = \frac{d\mathbf{r}}{dt}$$	Angular velocity $$\boldsymbol{\omega} = \hat{\mathbf{n}}\frac{\mathrm{d}\theta}{\mathrm{d}t}$$ Rotating rigid body: $$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r}$$
Acceleration	Average: $$\mathbf{a}_{\text{average}} = \frac{\Delta\mathbf{v}}{\Delta t}$$ Instantaneous: $$\mathbf{a} = \frac{d\mathbf{v}}{dt} = \frac{d^2\mathbf{r}}{dt^2}$$	Angular acceleration $$\boldsymbol{\alpha} = \frac{\mathrm{d}\boldsymbol{\omega}}{\mathrm{d}t} = \hat{\mathbf{n}}\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2}$$ Rotating rigid body: $$\mathbf{a} = \boldsymbol{\alpha} \times \mathbf{r} + \boldsymbol{\omega} \times \mathbf{v}$$
Jerk	Average: $$\mathbf{j}_{\text{average}} = \frac{\Delta\mathbf{a}}{\Delta t}$$ Instantaneous: $$\mathbf{j} = \frac{d\mathbf{a}}{dt} = \frac{d^2\mathbf{v}}{dt^2} = \frac{d^3\mathbf{r}}{dt^3}$$	Angular jerk $$\boldsymbol{\zeta} = \frac{\mathrm{d}\boldsymbol{\alpha}}{\mathrm{d}t} = \hat{\mathbf{n}}\frac{\mathrm{d}^2\omega}{\mathrm{d}t^2} = \hat{\mathbf{n}}\frac{\mathrm{d}^3\theta}{\mathrm{d}t^3}$$ Rotating rigid body: $$\mathbf{j} = \boldsymbol{\zeta} \times \mathbf{r} + \boldsymbol{\alpha} \times \mathbf{a}$$

Dynamics

	Translation	Rotation
Momentum	Momentum is the "amount of translation" $\mathbf{p} = m\mathbf{v}$ For a rotating rigid body: $\mathbf{p} = \boldsymbol{\omega} \times \mathbf{m}$	Angular momentum Angular momentum is the "amount of rotation": $\mathbf{L} = \mathbf{r} \times \mathbf{p} = \mathbf{I} \cdot \boldsymbol{\omega}$ and the cross-product is a pseudovector i.e. if **r** and **p** are reversed in direction (negative), **L** is not. In general **I** is an order-2 tensor, see above for its components. The dot · indicates tensor contraction.
Force and Newton's 2nd law	Resultant force acts on a system at the center of mass, equal to the rate of change of momentum: $\mathbf{F} = \frac{d\mathbf{p}}{dt} = \frac{d(m\mathbf{v})}{dt} = m\mathbf{a} + \mathbf{v}\frac{\mathrm{d}m}{\mathrm{d}t}$ For a number of particles, the equation of motion for one particle *i* is:[7] $\frac{\mathrm{d}\mathbf{p}_i}{\mathrm{d}t} = \mathbf{F}_E + \sum_{i \neq j} \mathbf{F}_{ij}$ where $\mathbf{p}_i$ = momentum of particle *i*, $\mathbf{F}_{ij}$ = force ***on*** particle *i* ***by*** particle *j*, and $\mathbf{F}_E$ = resultant external force (due to any agent not part of system). Particle *i* does not exert a force on itself.	Torque Torque $\boldsymbol{\tau}$ is also called moment of a force, because it is the rotational analogue to force:[8] $\boldsymbol{\tau} = \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \mathbf{r} \times \mathbf{F} = \frac{\mathrm{d}(\mathbf{I} \cdot \boldsymbol{\omega})}{\mathrm{d}t}$ For rigid bodies, Newton's 2nd law for rotation takes the same form as for translation: $\boldsymbol{\tau} = \frac{\mathrm{d}\mathbf{L}}{\mathrm{d}t} = \frac{\mathrm{d}(\mathbf{I} \cdot \boldsymbol{\omega})}{\mathrm{d}t} = \frac{\mathrm{d}\mathbf{I}}{\mathrm{d}t} \cdot \boldsymbol{\omega} + \mathbf{I} \cdot \boldsymbol{\alpha}$ Likewise, for a number of particles, the equation of motion for one particle *i* is:[9] $\frac{\mathrm{d}\mathbf{L}_i}{\mathrm{d}t} = \boldsymbol{\tau}_E + \sum_{i \neq j} \boldsymbol{\tau}_{ij}$
Yank	Yank is rate of change of force: $\mathbf{Y} = \frac{d\mathbf{F}}{dt} = \frac{d^2\mathbf{p}}{dt^2} = \frac{d^2(m\mathbf{v})}{dt^2} = m\mathbf{j} + \mathbf{2a}\frac{\mathrm{d}m}{\mathrm{d}t} + \mathbf{v}\frac{\mathrm{d}^2 m}{\mathrm{d}t^2}$ For constant mass, it becomes; $\mathbf{Y} = m\mathbf{j}$	Rotatum Rotatum **P** is also called moment of a Yank, because it is the rotational analogue to yank: $\mathbf{P} = \frac{\mathrm{d}\boldsymbol{\tau}}{\mathrm{d}t} = \mathbf{r} \times \mathbf{Y} = \frac{\mathrm{d}(\mathbf{I} \cdot \boldsymbol{\alpha})}{\mathrm{d}t}$

Impulse	Impulse is the change in momentum: $$\Delta \mathbf{p} = \int \mathbf{F}\, dt$$ For constant force **F**: $$\Delta \mathbf{p} = \mathbf{F}\Delta t$$	Twirl/angular impulse is the change in angular momentum: $$\Delta \mathbf{L} = \int \boldsymbol{\tau}\, dt$$ For constant torque **τ**: $$\Delta \mathbf{L} = \boldsymbol{\tau}\Delta t$$

Precession

The precession angular speed of a spinning top is given by:

$$\boldsymbol{\Omega} = \frac{wr}{I\boldsymbol{\omega}}$$

where w is the weight of the spinning flywheel.

Energy

The mechanical work done by an external agent on a system is equal to the change in kinetic energy of the system:

General work-energy theorem (translation and rotation)

The work done W by an external agent which exerts a force **F** (at **r**) and torque **τ** on an object along a curved path C is:

$$W = \Delta T = \int_C \left(\mathbf{F} \cdot \mathrm{d}\mathbf{r} + \boldsymbol{\tau} \cdot \mathbf{n}\, \mathrm{d}\theta \right)$$

where θ is the angle of rotation about an axis defined by a unit vector **n**.

Kinetic energy

The change in kinetic energy for an object initially traveling at speed v_0 and later at speed v is:

$$\Delta E_k = W = \frac{1}{2}m(v^2 - {v_0}^2)$$

Elastic potential energy

For a stretched spring fixed at one end obeying Hooke's law, the elastic potential energy is

$$\Delta E_p = \frac{1}{2}k(r_2 - r_1)^2$$

where r_2 and r_1 are collinear coordinates of the free end of the spring, in the direction of the extension/compression, and k is the spring constant.

Euler's equations for rigid body dynamics

Euler also worked out analogous laws of motion to those of Newton, see Euler's laws of motion. These extend the scope of Newton's laws to rigid bodies, but are essentially the same as above. A new equation Euler formulated is:[10]

$$\mathbf{I} \cdot \boldsymbol{\alpha} + \boldsymbol{\omega} \times (\mathbf{I} \cdot \boldsymbol{\omega}) = \boldsymbol{\tau}$$

where **I** is the moment of inertia tensor.

General planar motion

The previous equations for planar motion can be used here: corollaries of momentum, angular momentum etc. can immediately follow by applying the above definitions. For any object moving in any path in a plane,

$$\mathbf{r} = \mathbf{r}(t) = r\hat{\mathbf{r}}$$

the following general results apply to the particle.

Kinematics	Dynamics
Position $$\mathbf{r} = \mathbf{r}\left(r, \theta, t\right) = r\hat{\mathbf{r}}$$	
Velocity $$\mathbf{v} = \hat{\mathbf{r}}\frac{\mathrm{d}r}{\mathrm{d}t} + r\omega\hat{\boldsymbol{\theta}}$$	Momentum $$\mathbf{p} = m\left(\hat{\mathbf{r}}\frac{\mathrm{d}r}{\mathrm{d}t} + r\omega\hat{\boldsymbol{\theta}}\right)$$ Angular momenta $$\mathbf{L} = m\mathbf{r} \times \left(\hat{\mathbf{r}}\frac{\mathrm{d}r}{\mathrm{d}t} + r\omega\hat{\boldsymbol{\theta}}\right)$$
Acceleration $$\mathbf{a} = \left(\frac{\mathrm{d}^2 r}{\mathrm{d}t^2} - r\omega^2\right)\hat{\mathbf{r}} + \left(r\alpha + 2\omega\frac{\mathrm{d}r}{\mathrm{d}t}\right)\hat{\boldsymbol{\theta}}$$	The centripetal force is $$\mathbf{F}_\perp = -m\omega^2 R\hat{\mathbf{r}} = -\omega^2\mathbf{m}$$ where again **m** is the mass moment, and the Coriolis force is $$\mathbf{F}_c = 2\omega m\frac{\mathrm{d}r}{\mathrm{d}t}\hat{\boldsymbol{\theta}} = 2\omega m v\hat{\boldsymbol{\theta}}$$ The Coriolis acceleration and force can also be written: $$\mathbf{F}_c = m\mathbf{a}_c = -2m\boldsymbol{\omega} \times \boldsymbol{v}$$

Central force motion

For a massive body moving in a central potential due to another object, which depends only on the radial separation between the centers of masses of the two objects, the equation of motion is:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{\mathbf{r}}\right)+\frac{1}{\mathbf{r}}=-\frac{\mu\mathbf{r}^2}{\mathbf{l}^2}\mathbf{F}(\mathbf{r})$$

Equations of motion (constant acceleration)

These equations can be used only when acceleration is constant. If acceleration is not constant then the general calculus equations above must be used, found by integrating the definitions of position, velocity and acceleration (see above).

Linear motion	Angular motion
$\mathbf{v}-\mathbf{v_0}=\mathbf{a}t$	$\boldsymbol{\omega}-\boldsymbol{\omega_0}=\boldsymbol{\alpha}t$
$\mathbf{x}-\mathbf{x_0}=\frac{1}{2}(\mathbf{v_0}+\mathbf{v})t$	$\boldsymbol{\theta}-\boldsymbol{\theta_0}=\frac{1}{2}(\boldsymbol{\omega_0}+\boldsymbol{\omega})t$
$\mathbf{x}-\mathbf{x_0}=\mathbf{v}_0t+\frac{1}{2}\mathbf{a}t^2$	$\boldsymbol{\theta}-\boldsymbol{\theta_0}=\boldsymbol{\omega}_0t+\frac{1}{2}\boldsymbol{\alpha}t^2$
$\mathbf{x}_{n^{th}}=\mathbf{v}_0+\mathbf{a}(n-\frac{1}{2})$	$\boldsymbol{\theta}_{n^{th}}=\boldsymbol{\omega}_0+\boldsymbol{\alpha}(n-\frac{1}{2})$
$v^2-v_0^2=2\mathbf{a}(\mathbf{x}-\mathbf{x_0})$	$\omega^2-\omega_0^2=2\boldsymbol{\alpha}(\boldsymbol{\theta}-\boldsymbol{\theta_0})$

Galilean frame transforms

For classical (Galileo-Newtonian) mechanics, the transformation law from one inertial or accelerating (including rotation) frame (reference frame traveling at constant velocity - including zero) to another is the Galilean transform.

Unprimed quantities refer to position, velocity and acceleration in one frame F; primed quantities refer to position, velocity and acceleration in another frame F' moving at translational velocity **V** or angular velocity **Ω** relative to F. Conversely F moves at velocity (—**V** or —**Ω**) relative to F'. The situation is similar for relative accelerations.

<table>
<tr><th>Motion of entities</th><th>Inertial frames</th><th>Accelerating frames</th></tr>
<tr><td>Translation
V = Constant relative velocity between two inertial frames F and F'.
A = (Variable) relative acceleration between two accelerating frames F and F'.</td><td>Relative position
$$\mathbf{r}' = \mathbf{r} + \mathbf{V}t$$
Relative velocity
$$\mathbf{v}' = \mathbf{v} + \mathbf{V}$$
Equivalent accelerations
$$\mathbf{a}' = \mathbf{a}$$</td><td>Relative accelerations
$$\mathbf{a}' = \mathbf{a} + \mathbf{A}$$
Apparent/fictitious forces
$$\mathbf{F}' = \mathbf{F} - \mathbf{F}_{\text{app}}$$</td></tr>
<tr><td rowspan="2">Rotation
Ω = Constant relative angular velocity between two frames F and F'.
Λ = (Variable) relative angular acceleration between two accelerating frames F and F'.</td><td>Relative angular position
$$\theta' = \theta + \Omega t$$
Relative velocity
$$\boldsymbol{\omega}' = \boldsymbol{\omega} + \boldsymbol{\Omega}$$
Equivalent accelerations
$$\boldsymbol{\alpha}' = \boldsymbol{\alpha}$$</td><td>Relative accelerations
$$\boldsymbol{\alpha}' = \boldsymbol{\alpha} + \boldsymbol{\Lambda}$$
Apparent/fictitious torques
$$\boldsymbol{\tau}' = \boldsymbol{\tau} - \boldsymbol{\tau}_{\text{app}}$$</td></tr>
<tr><td colspan="2">Transformation of any vector T to a rotating frame
$$\frac{\mathrm{d}\mathbf{T}'}{\mathrm{d}t} = \frac{\mathrm{d}\mathbf{T}}{\mathrm{d}t} - \boldsymbol{\Omega} \times \mathbf{T}$$</td></tr>
</table>

Mechanical oscillators

SHM, DHM, SHO, and DHO refer to simple harmonic motion, damped harmonic motion, simple harmonic oscillator and damped harmonic oscillator respectively.

Equations of motion

Physical situation	Nomenclature	Translational equations	Angular equations
SHM	x = Transverse displacement θ = Angular displacement A = Transverse amplitude Θ = Angular amplitude	$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} = -\omega^2 x$ Solution: $x = A\sin(\omega t + \phi)$	$\frac{\mathrm{d}^2 \theta}{\mathrm{d}t^2} = -\omega^2 \theta$ Solution: $\theta = \Theta\sin(\omega t + \phi)$
Unforced DHM	b = damping constant κ = torsion constant	$\frac{\mathrm{d}^2 x}{\mathrm{d}t^2} + b\frac{\mathrm{d}x}{\mathrm{d}t} + \omega^2 x = 0$ Solution (see below for ω'): $x = Ae^{-bt/2m}\cos(\omega')$ Resonant frequency: $\omega_{\mathrm{res}} = \sqrt{\omega^2 - \left(\frac{b}{4m}\right)^2}$ Damping rate: $\gamma = b/m$ Expected lifetime of excitation: $\tau = 1/\gamma$	$\frac{\mathrm{d}^2 \theta}{\mathrm{d}t^2} + b\frac{\mathrm{d}\theta}{\mathrm{d}t} + \omega^2 \theta = 0$ Solution: $\theta = \Theta e^{-\kappa t/2m}\cos(\omega)$ Resonant frequency: $\omega_{\mathrm{res}} = \sqrt{\omega^2 - \left(\frac{\kappa}{4m}\right)^2}$ Damping rate: $\gamma = \kappa/m$ Expected lifetime of excitation: $\tau = 1/\gamma$

Angular frequencies

Physical situation	Nomenclature	Equations
Linear undamped unforced SHO	k = spring constant m = mass of oscillating bob	$\omega = \sqrt{\frac{k}{m}}$
Linear unforced DHO	k = spring constant b = Damping coefficient	$\omega' = \sqrt{\frac{k}{m} - \left(\frac{b}{2m}\right)^2}$
Low amplitude angular SHO	I = Moment of inertia about oscillating axis κ = torsion constant	$\omega = \sqrt{\frac{\kappa}{I}}$
Low amplitude simple pendulum	L = Length of pendulum g = Gravitational acceleration Θ = Angular amplitude	Approximate value $\omega = \sqrt{\frac{g}{L}}$ Exact value can be shown to be: $\omega = \sqrt{\frac{g}{L}}\left[1 + \sum_{k=1}^{\infty} \frac{\prod_{n=1}^{k}(2n-1)}{\prod_{n=1}^{m}(2n)}\sin^{2n}\Theta\right]$

Physical situation	Nomenclature	Equations
SHM energy	T = kinetic energy U = potential energy E = total energy	Potential energy $$U = \frac{m}{2}(x)^2 = \frac{m(\omega A)^2}{2}\cos^2(\omega t + \phi)$$ Maximum value at $x = A$: $$U_{\max} = \frac{m}{2}(\omega A)^2$$ Kinetic energy $$T = \frac{\omega^2 m}{2}\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 = \frac{m(\omega A)^2}{2}\sin^2(\omega t + \phi)$$ Total energy $$E = T + U$$
DHM energy		$E = \frac{m(\omega A)^2}{2} e^{-bt/m}$

Energy in mechanical oscillations

See also

- List of physics formulae
- Defining equation (physical chemistry)
- Constitutive equation
- Mechanics
- Optics
- Electromagnetism
- Thermodynamics
- Acoustics
- Isaac Newton
- List of equations in wave theory
- List of relativistic equations
- List of equations in fluid mechanics
- List of equations in gravitation
- List of electromagnetism equations
- List of photonics equations
- List of equations in quantum mechanics
- List of equations in nuclear and particle physics

Notes

1. Mayer, Sussman & Wisdom 2001, p. xiii
2. Berkshire & Kibble 2004, p. 1
3. Berkshire & Kibble 2004, p. 2
4. Arnold 1989, p. v
5. "*Section: Moments and center of mass*" (http://www.ltcconline.net/greenl/courses/202/multipleIntegration/MassMoments.htm).

List of equations in quantum mechanics

This article summarizes equations in the theory of quantum mechanics.

Wavefunctions

A fundamental physical constant occurring in quantum mechanics is the **Planck constant**, h. A common abbreviation is $\hbar = h/2\pi$, also known as the *reduced Planck constant* or *Dirac constant*.

Quantity (common name/s)	(Common) symbol/s	Defining equation	SI unit	Dimension
Wavefunction	ψ, Ψ	To solve from the Schrödinger equation	varies with situation and number of particles	
Wavefunction probability density	ρ	$\rho = \left\|\Psi\right\|^2 = \Psi^* \Psi$	m^{-3}	$[L]^{-3}$
Wavefunction probability current	$\mathbf{j}$	Non-relativistic, no external field: $\begin{aligned}\mathbf{j} &= \frac{-i\hbar}{2m}\left(\Psi^*\nabla\Psi - \Psi\nabla\Psi^*\right) \\ &= \frac{\hbar}{m}\operatorname{Im}(\Psi^*\nabla\Psi) = \operatorname{Re}\left(\Psi^*\frac{\hbar}{im}\nabla\Psi\right)\end{aligned}$ star * is complex conjugate	$m^{-2} \cdot s^{-1}$	$[T]^{-1}$ $[L]^{-2}$

The general form of wavefunction for a system of particles, each with position $\mathbf{r}_i$ and z-component of spin $s_{z\,i}$. Sums are over the discrete variable s_z, integrals over continuous positions $\mathbf{r}$.

For clarity and brevity, the coordinates are collected into tuples, the indices label the particles (which cannot be done physically, but is mathematically necessary). Following are general mathematical results, used in calculations.

Property or effect	Nomenclature	Equation
Wavefunction for *N* particles in 3d	$\mathbf{r} = (\mathbf{r}_1, \mathbf{r}_2 \ldots \mathbf{r}_N)$ $\mathbf{s_z} = (s_{z\,1}, s_{z\,2}, \ldots, s_{z\,N})$	In function notation: $\Psi = \Psi\left(\mathbf{r}, \mathbf{s_z}, t\right)$ in bra–ket notation: $\lvert\Psi\rangle = \sum_{s_{z1}} \sum_{s_{z2}} \cdots \sum_{s_{zN}} \int_{V_1} \int_{V_2} \cdots \int_{V_N} \mathrm{d}\mathbf{r}_1 \mathrm{d}\mathbf{r}_2 \cdots \mathrm{d}\mathbf{r}_N \Psi \lvert \mathbf{r}, \mathbf{s_z}\rangle$ for non-interacting particles: $\Psi = \prod_{n=1}^{N} \Psi\left(\mathbf{r}_n, s_{zn}, t\right)$
Position-momentum Fourier transform (1 particle in 3d)	Φ = momentum–space wavefunction Ψ = position–space wavefunction	$\Phi(\mathbf{p}, s_z, t) = \frac{1}{\sqrt{2\pi\hbar}^3} \int\limits_{\text{all space}} e^{-i\mathbf{p}\cdot\mathbf{r}/\hbar} \Psi(\mathbf{r}, s_z, t) \mathrm{d}^3\mathbf{r}$ $\upharpoonleft\downharpoonright$ $\Psi(\mathbf{r}, s_z, t) = \frac{1}{\sqrt{2\pi\hbar}^3} \int\limits_{\text{all space}} e^{+i\mathbf{p}\cdot\mathbf{r}/\hbar} \Phi(\mathbf{p}, s_z, t) \mathrm{d}^3\mathbf{p}_n$
General probability distribution	V_j = volume (3d region) particle may occupy, P = Probability that particle 1 has position $\mathbf{r}_1$ in volume V_1 with spin s_{z1} **and** particle 2 has position $\mathbf{r}_2$ in volume V_2 with spin s_{z2}, etc.	$P = \sum_{s_{zN}} \cdots \sum_{s_{z2}} \sum_{s_{z1}} \int_{V_N} \cdots \int_{V_2} \int_{V_1} \lvert\Psi\rvert^2 \mathrm{d}^3\mathbf{r}_1 \mathrm{d}^3\mathbf{r}_2 \cdots \mathrm{d}^3\mathbf{r}_N$
General normalization condition		$P = \sum_{s_{zN}} \cdots \sum_{s_{z2}} \sum_{s_{z1}} \int\limits_{\text{all space}} \cdots \int\limits_{\text{all space}} \int\limits_{\text{all space}} \lvert\Psi\rvert^2 \mathrm{d}^3\mathbf{r}_1 \mathrm{d}^3\mathbf{r}_2 \cdots \mathrm{d}^3\mathbf{r}_N = 1$

Equations

Wave–particle duality and time evolution

Property or effect	Nomenclature	Equation
Planck–Einstein equation and de Broglie wavelength relations	**P** = (E/c, **p**) is the four-momentum, **K** = (ω/c, **k**) is the four-wavevector, E = energy of particle $\omega = 2\pi f$ is the angular frequency and frequency of the particle $\hbar = h/2\pi$ are the Planck constants c = speed of light	$\mathbf{P} = (E/c, \mathbf{p}) = \hbar(\omega/c, \mathbf{k}) = \hbar\mathbf{K}$
Schrödinger equation	Ψ = wavefunction of the system $\hat{H}$ = Hamiltonian operator, E = energy eigenvalue of system i is the imaginary unit t = time	General time-dependent case: $i\hbar\frac{\partial}{\partial t}\Psi = \hat{H}\Psi$ Time-independent case: $\hat{H}\Psi = E\Psi$
Heisenberg equation	$\hat{A}$ = operator of an observable property [ ] is the commutator $\langle\,\rangle$ denotes the average	$\frac{d}{dt}\hat{A}(t) = \frac{i}{\hbar}[\hat{H}, \hat{A}(t)] + \frac{\partial \hat{A}(t)}{\partial t}$
Time evolution in Heisenberg picture (Ehrenfest theorem)	m = mass, V = potential energy, **r** = position, **p** = momentum, of a particle.	$\frac{d}{dt}\langle\hat{A}\rangle = \frac{1}{i\hbar}\langle[\hat{A}, \hat{H}]\rangle + \left\langle\frac{\partial \hat{A}}{\partial t}\right\rangle$ For momentum and position; $m\frac{d}{dt}\langle\mathbf{r}\rangle = \langle\mathbf{p}\rangle$ $\frac{d}{dt}\langle\mathbf{p}\rangle = -\langle\nabla V\rangle$

Non-relativistic time-independent Schrödinger equation

Summarized below are the various forms the Hamiltonian takes, with the corresponding Schrödinger equations and forms of wavefunction solutions. Notice in the case of one spatial dimension, for one particle, the partial derivative reduces to an ordinary derivative.

	One particle	***N* particles**
One dimension	$\hat{H} = \frac{\hat{p}^2}{2m} + V(x) = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)$	$\begin{aligned}\hat{H} &= \sum_{n=1}^{N}\frac{\hat{p}_n^2}{2m_n} + V(x_1, x_2, \cdots x_N)\\ &= -\frac{\hbar^2}{2}\sum_{n=1}^{N}\frac{1}{m_n}\frac{\partial^2}{\partial x_n^2} + V(x_1, x_2, \cdots x_N)\end{aligned}$ where the position of particle *n* is x_n.
	$E\Psi = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2}\Psi + V\Psi$	$E\Psi = -\frac{\hbar^2}{2}\sum_{n=1}^{N}\frac{1}{m_n}\frac{\partial^2}{\partial x_n^2}\Psi + V\Psi\,.$
	$\Psi(x,t) = \psi(x)e^{-iEt/\hbar}\,.$ There is a further restriction — the solution must not grow at infinity, so that it has either a finite ***L***$\mathbf{L^2}$**-norm** (if it is a bound state) or a slowly diverging norm (if it is part of a continuum):[1] $\Vert\psi\Vert^2 = \int \vert\psi(x)\vert^2\, dx.$	$\Psi = e^{-iEt/\hbar}\psi(x_1, x_2 \cdots x_N)$ for non-interacting particles $\Psi = e^{-iEt/\hbar}\prod_{n=1}^{N}\psi(x_n)\,,\quad V(x_1, x_2, \cdots x_N) = \sum_{n=1}^{N}V(x_n)\,.$
Three dimensions	$\begin{aligned}\hat{H} &= \frac{\hat{\mathbf{p}}\cdot\hat{\mathbf{p}}}{2m} + V(\mathbf{r})\\ &= -\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r})\end{aligned}$ where the position of the particle is **r** = (*x*, *y*, *z*).	$\begin{aligned}\hat{H} &= \sum_{n=1}^{N}\frac{\hat{\mathbf{p}}_n\cdot\hat{\mathbf{p}}_n}{2m_n} + V(\mathbf{r}_1, \mathbf{r}_2, \cdots \mathbf{r}_N)\\ &= -\frac{\hbar^2}{2}\sum_{n=1}^{N}\frac{1}{m_n}\nabla_n^2 + V(\mathbf{r}_1, \mathbf{r}_2, \cdots \mathbf{r}_N)\end{aligned}$ where the position of particle *n* is $\mathbf{r}_n = (x_n, y_n, z_n)$, and the Laplacian for particle *n* using the corresponding position coordinates is $\nabla_n^2 = \frac{\partial^2}{{\partial x_n}^2} + \frac{\partial^2}{{\partial y_n}^2} + \frac{\partial^2}{{\partial z_n}^2}$
	$E\Psi = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi$	$E\Psi = -\frac{\hbar^2}{2}\sum_{n=1}^{N}\frac{1}{m_n}\nabla_n^2\Psi + V\Psi$
	$\Psi = \psi(\mathbf{r})e^{-iEt/\hbar}$	$\Psi = e^{-iEt/\hbar}\psi(\mathbf{r}_1, \mathbf{r}_2 \cdots \mathbf{r}_N)$ for non-interacting particles $\Psi = e^{-iEt/\hbar}\prod_{n=1}^{N}\psi(\mathbf{r}_n)\,,\quad V(\mathbf{r}_1, \mathbf{r}_2, \cdots \mathbf{r}_N) = \sum_{n=1}^{N}V(\mathbf{r}_n)$

Non-relativistic time-dependent Schrödinger equation

Again, summarized below are the various forms the Hamiltonian takes, with the corresponding Schrödinger equations and forms of solutions.

	One particle	*N* particles
One dimension	$\hat{H} = \frac{\hat{p}^2}{2m} + V(x,t) = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x,t)$	$\hat{H} = \sum_{n=1}^{N} \frac{\hat{p}_n^2}{2m_n} + V(x_1, x_2, \cdots x_N, t)$ $= -\frac{\hbar^2}{2}\sum_{n=1}^{N} \frac{1}{m_n}\frac{\partial^2}{\partial x_n^2} + V(x_1, x_2, \cdots x_N, t)$ where the position of particle *n* is x_n.
	$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2}\Psi + V\Psi$	$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2}\sum_{n=1}^{N}\frac{1}{m_n}\frac{\partial^2}{\partial x_n^2}\Psi + V\Psi\,.$
	$\Psi = \Psi(x,t)$	$\Psi = \Psi(x_1, x_2 \cdots x_N, t)$
Three dimensions	$\hat{H} = \frac{\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}}{2m} + V(\mathbf{r},t)$ $= -\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{r},t)$	$\hat{H} = \sum_{n=1}^{N} \frac{\hat{\mathbf{p}}_n \cdot \hat{\mathbf{p}}_n}{2m_n} + V(\mathbf{r}_1, \mathbf{r}_2, \cdots \mathbf{r}_N, t)$ $= -\frac{\hbar^2}{2}\sum_{n=1}^{N}\frac{1}{m_n}\nabla_n^2 + V(\mathbf{r}_1, \mathbf{r}_2, \cdots \mathbf{r}_N, t)$
	$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2m}\nabla^2\Psi + V\Psi$	$i\hbar\frac{\partial}{\partial t}\Psi = -\frac{\hbar^2}{2}\sum_{n=1}^{N}\frac{1}{m_n}\nabla_n^2\Psi + V\Psi$ This last equation is in a very high dimension,[2] so the solutions are not easy to visualize.
	$\Psi = \Psi(\mathbf{r},t)$	$\Psi = \Psi(\mathbf{r}_1, \mathbf{r}_2, \cdots \mathbf{r}_N, t)$

Photoemission

Property/Effect	Nomenclature	Equation
Photoelectric equation	K_{max} = Maximum kinetic energy of ejected electron (J) *h* = Planck constant *f* = frequency of incident photons (Hz = s^{-1}) *φ*, Φ = Work function of the material the photons are incident on (J)	$K_{\mathrm{max}} = hf - \Phi$
Threshold frequency and Work function	*φ*, Φ = Work function of the material the photons are incident on (J) f_0, ν_0 = Threshold frequency (Hz = s^{-1})	Can only be found by experiment. The De Broglie relations give the relation between them: $\phi = hf_0$
Photon momentum	*p* = momentum of photon (kg m s^{-1}) *f* = frequency of photon (Hz = s^{-1}) *λ* = wavelength of photon (m)	The De Broglie relations give: $p = hf/c = h/\lambda$

Quantum uncertainty

Property or effect	Nomenclature	Equation
Heisenberg's uncertainty principles	n = number of photons φ = wave phase [,] = commutator	Position–momentum $\sigma(x)\sigma(p) \geq \frac{\hbar}{2}$ Energy-time $\sigma(E)\sigma(t) \geq \frac{\hbar}{2}$ Number-phase $\sigma(n)\sigma(\phi) \geq \frac{\hbar}{2}$
Dispersion of observable	A = observables (eigenvalues of operator)	$\begin{aligned}\sigma(A)^2 &= \langle (A - \langle A \rangle)^2 \rangle \\ &= \langle A^2 \rangle - \langle A \rangle^2\end{aligned}$
General uncertainty relation	A, B = observables (eigenvalues of operator)	$\sigma(A)\sigma(B) \geq \frac{1}{2} \langle i[\hat{A}, \hat{B}] \rangle$

Probability Distributions

Property or effect	Equation
Density of states	$N(E) = 8\sqrt{2}\pi m^{3/2} E^{1/2} / h^3$
Fermi–Dirac distribution (fermions)	$P(E_i) = \frac{g(E_i)}{e^{(E-\mu)/kT} + 1}$ where $P(E_i)$ = probability of energy E_i $g(E_i)$ = degeneracy of energy E_i (no of states with same energy) μ = chemical potential
Bose–Einstein distribution (bosons)	$P(E_i) = \frac{g(E_i)}{e^{(E_i-\mu)/kT} - 1}$

Angular momentum

Property or effect	Nomenclature	Equation
Angular momentum quantum numbers	s = spin quantum number m_s = spin magnetic quantum number ℓ = Azimuthal quantum number m_ℓ = azimuthal magnetic quantum number j = total angular momentum quantum number m_j = total angular momentum magnetic quantum number	Spin: $\|\mathbf{s}\| = \sqrt{s\,(s+1)}\,\hbar$ $m_s \in \{-s, -s+1 \cdots s-1, s\}$ Orbital: $\ell \in \{0 \cdots n-1\}$ $m_\ell \in \{-\ell, -\ell+1 \cdots \ell-1, \ell\}$ Total: $j = \ell + s$ $m_j \in \{\lvert\ell-s\rvert, \lvert\ell-s\rvert+1 \cdots \lvert\ell+s\rvert-1, \lvert\ell+s\rvert\}$
Angular momentum magnitudes	angular momementa: S = Spin, L = orbital, J = total	Spin magnitude: $\lvert\mathbf{S}\rvert = \hbar\sqrt{s(s+1)}$ Orbital magnitude: $\lvert\mathbf{L}\rvert = \hbar\sqrt{\ell(\ell+1)}$ Total magnitude: $\mathbf{J} = \mathbf{L} + \mathbf{S}$ $\lvert\mathbf{J}\rvert = \hbar\sqrt{j(j+1)}$
Angular momentum components		Spin: $S_z = m_s\hbar$ Orbital: $L_z = m_\ell\hbar$

Magnetic moments

In what follows, **B** is an applied external magnetic field and the quantum numbers above are used.

Property or effect	Nomenclature	Equation
orbital magnetic dipole moment	e = electron charge m_e = electron rest mass **L** = electron orbital angular momentum g_ℓ = orbital Landé g-factor μ_B = Bohr magneton	$\boldsymbol{\mu}_\ell = -e\mathbf{L}/2m_e = g_\ell \frac{\mu_B}{\hbar}\mathbf{L}$ z-component: $\mu_{\ell,z} = -m_\ell\mu_B$
spin magnetic dipole moment	**S** = electron spin angular momentum g_s = spin Landé g-factor	$\boldsymbol{\mu}_s = -e\mathbf{S}/m_e = g_s \frac{\mu_B}{\hbar}\mathbf{S}$ z-component: $\mu_{s,z} = -eS_z/m_e = g_s eS_z/2m_e$
dipole moment potential	U = potential energy of dipole in field	$U = -\boldsymbol{\mu} \cdot \mathbf{B} = -\mu_z B$

Hydrogen atom

Property or effect	Nomenclature	Equation
Energy level	E_n = energy eigenvalue n = principal quantum number e = electron charge m_e = electron rest mass ε_0 = permittivity of free space h = Planck constant	$E_n = -me^4/8\varepsilon_0^2 h^2 n^2 = -13.61\,\mathrm{eV}/n^2$
Spectrum	λ = wavelength of emitted photon, during electronic transition from E_i to E_j	$\frac{1}{\lambda} = R\left(\frac{1}{n_j^2} - \frac{1}{n_i^2}\right),\, n_j < n_i$

See also

- Defining equation (physical chemistry)
- List of electromagnetism equations
- List of equations in classical mechanics
- List of equations in fluid mechanics
- List of equations in gravitation
- List of equations in nuclear and particle physics
- List of equations in wave theory
- List of photonics equations
- List of relativistic equations

Footnotes

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List of quantum field theories

This is a list of quantum field theories. The first few sections are organized according to their matter content, that is, the types of fields appearing in the theory. This is just one of many ways to organize quantum field theories, but reflects the way the subject is taught pedagogically.

Scalar field theory

Theories whose matter content consists of only scalar fields

- Klein-Gordon: free scalar field theory
- φ^4 theory
- Sine-Gordon
- Toda field theory

Spinor field theory

Theories whose matter content consists only of spinor fields

- Dirac theory: free spinor field theory
- Thirring model
- Nambu–Jona-Lasinio model
- Gross–Neveu model

Gauge field theory

Theories whose matter content consists only of gauge fields

- Yang–Mills theory
- Proca theory
- Chern–Simons theory

Interacting theories

- Spinor and scalar
  - Yukawa model
- Scalar and gauge
  - Scalar electrodynamics
  - Scalar chromodynamics
  - Yang–Mills–Higgs
- Spinor and gauge

    - Quantum electrodynamics (QED)
        - Schwinger model (1+1D case of QED)
    - Quantum chromodynamics (QCD)
- Scalar, spinor and gauge
    - Standard Model

Sigma models

- Chiral model
- Non-linear sigma model
- Wess–Zumino–Witten model

Supersymmetric quantum field theories

- Wess–Zumino model
- Supersymmetric Yang–Mills
- 4D N = 1 global supersymmetry
- Seiberg–Witten theory
- Super QCD (sQCD)

Superconformal quantum field theories

- N = 4 supersymmetric Yang–Mills theory
- ABJM superconformal field theory
- 6D (2,0) superconformal field theory

Supergravity quantum field theories

- Pure 4D N = 1 supergravity
- 4D N = 1 supergravity
- Type I supergravity
- Type IIA supergravity
- Type IIB supergravity
- Eleven-dimensional supergravity

String theories

Theories studied in the branch of quantum field theory known as string theory. These theories are without supersymmetry.

- Polyakov action
- Nambu-Goto action
- Bosonic string theory

Other quantum field theories

- Kondo model (s-d model)
- Minimal model (Virasoro minimal model)

Branches of quantum field theory

- String theory
- Conformal field theory
- Supersymmetry
- Topological quantum field theory
- Noncommutative quantum field theory
- Local quantum field theory (also known as Algebraic quantum field theory or AQFT)

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List of relativistic equations

Following is a list of the frequently occurring equations in the theory of special relativity.

Postulates of Special Relativity

To derive the equations of special relativity, one must start with two other

1. The laws of physics are invariant under transformations between inertial frames. In other words, the laws of physics will be the same whether you are testing them in a frame 'at rest', or a frame moving with a constant velocity relative to the 'rest' frame.
2. The speed of light in a perfect classical vacuum (c_0) is measured to be the same by all observers in inertial frames and is, moreover, finite but nonzero. This speed acts as a supremum for the speed of local transmission of information in the universe.

In this context, "speed of light" really refers to the speed supremum of information transmission or of the movement of ordinary (nonnegative mass) matter, locally, as in a classical vacuum. Thus, a more accurate description would refer to c_0 rather than the speed of light per se. However, light and other massless particles do theoretically travel at c_0 under vacuum conditions and experiment has nonfalsified this notion with fairly high precision. Regardless of whether light itself does travel at c_0, though c_0 does act as such a supremum, and that is the assumption which matters for Relativity.

From these two postulates, all of special relativity follows.

In the following, the relative velocity *v* between two inertial frames is restricted fully to the *x*-direction, of a Cartesian coordinate system.

Kinematics

Lorentz transformation

The following notations are used very often in special relativity:

Lorentz factor

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

where $\beta = \frac{v}{c}$ and *v* is the relative velocity between two inertial frames.

For two frames at rest, γ = 1, and increases with relative velocity between the two inertial frames. As the relative velocity approaches the speed of light, γ → ∞.

Time dilation (different times *t* and *t′* at the same position *x* in same inertial frame)

$$t' = \gamma t$$

Derivation of time dilation

Applying the above postulates, consider the inside of any vehicle (usually exemplified by a train) moving with a velocity *v* with respect to someone standing on the ground as the vehicle passes. Inside, a light is shone upwards to a mirror on the ceiling, where the light reflects back down. If the height of the mirror is *h*, and the speed of light *c*, then the time it takes for the light to go up and come back down is:

$$t = \frac{2h}{c}$$

However, to the observer on the ground, the situation is very different. Since the train is moving by the observer on the ground, the light beam appears to move diagonally instead of straight up and down. To visualize this, picture the light being emitted at one point, then having the vehicle move until the light hits the mirror at the top of the vehicle, and then having the train move still more until the light beam returns to the bottom of the vehicle. The light beam will have appeared to have moved diagonally upward with the train, and then diagonally downward. This path will help form two-right sided triangles, with the height as one of the sides, and the two straight parts of the path being the respective hypotenuses:

$$c^2\left(\frac{t'}{2}\right)^2 = h^2 + v^2\left(\frac{t'}{2}\right)^2$$

Rearranging to get t':

$$\left(\frac{t'}{2}\right)^2 = \frac{h^2}{c^2 - v^2}$$
$$\frac{t'}{2} = \frac{h}{\sqrt{c^2 - v^2}}$$
$$t' = \frac{2h}{\sqrt{c^2 - v^2}}$$

Taking out a factor of *c*, and then plugging in for *t*, one finds:

$$t' = \frac{2h}{c}\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} = \frac{t}{\sqrt{1 - \frac{v^2}{c^2}}}$$

This is the formula for time dilation:

$$t' = \gamma t$$

In this example the time measured in the frame on the vehicle, *t*, is known as the proper time. The proper time between two events - such as the event of light being emitted on the vehicle and the event of light being received on the vehicle - is the time between the two events in a frame where the events occur at the same location. So, above, the emission and reception of the light both took place in the vehicle's frame, making the time that an observer in the vehicle's frame would measure the proper time.

Length contraction (different positions *x* and *x′* at the same instant *t* in the same inertial frame)

$$\ell' = \frac{\ell}{\gamma}$$

Derivation of length contraction

Consider a long train, moving with velocity *v* with respect to the ground, and one observer on the train and one on the ground, standing next to a post. The observer on the train sees the front of the train pass the post, and then, some time *t′* later, sees the end of the train pass the same post. He then calculates the train's length as follows:

$$\ell = vt'$$

However, the observer on the ground, making the same measurement, comes to a different conclusion. This observer finds that time *t* passed between the front of the train passing the post, and the back of the train passing the post. Because the two events - the passing of each end of the train by the post - occurred in the same place in the ground observer's frame, the time this observer measured is the proper time. So:

$$\ell' = vt = v\left(\frac{t'}{\gamma}\right) = \frac{\ell}{\gamma}$$

This is the formula for length contraction. As there existed a proper time for time dilation, there exists a proper length for length contraction, which in this case is ℓ. The proper length of an object is the length of the object in the frame in which the object is at rest. Also, this contraction only affects the dimensions of the object which are parallel to the relative velocity between the object and observer. Thus, lengths perpendicular to the direction of motion are unaffected by length contraction.

Lorentz transformation

$$x' = \gamma\left(x - vt\right)$$
$$y' = y$$
$$z' = z$$
$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

Derivation of Lorentz transformation using time dilation and length contraction

Now substituting the length contraction result into the Galilean transformation (i.e. $x = \ell$), we have:

$$\frac{x'}{\gamma} = x - vt$$

that is:

$$x' = \gamma\left(x - vt\right)$$

and going from the primed frame to the unprimed frame:

$$x = \gamma\left(x' + vt'\right)$$

Going from the primed frame to the unprimed frame was accomplished by making v in the first equation negative, and then exchanging primed variables for unprimed ones, and vice versa. Also, as length contraction does not affect the perpendicular dimensions of an object, the following remain the same as in the Galilean transformation:

$$y' = y$$
$$z' = z$$

Finally, to determine how t and t' transform, substituting the $x \leftrightarrow x'$ transformation into its inverse:

$$x = \gamma\left(\gamma\left(x - vt\right) + vt'\right)$$
$$x = \gamma\left(\gamma x - \gamma vt + vt'\right)$$
$$x = \gamma^2 x - \gamma^2 vt + \gamma vt'$$
$$\gamma vt' = \gamma^2 vt - \gamma^2 x + x$$
$$\gamma vt' = \gamma^2 vt + x\left(1 - \gamma^2\right)$$

Plugging in the value for γ:

$$\begin{aligned}
\gamma vt' &= \gamma^2 vt + x\left(1 - \frac{1}{1-\beta^2}\right)\\
\gamma vt' &= \gamma^2 vt + x\left(\frac{1-\beta^2}{1-\beta^2} - \frac{1}{1-\beta^2}\right)\\
\gamma vt' &= \gamma^2 vt - x\left(\frac{\beta^2}{1-\beta^2}\right)\\
\gamma vt' &= \gamma^2 vt - \gamma^2 \beta^2 x
\end{aligned}$$

Finally, dividing through by γ*v*:

$$t' = \gamma\left(t - \beta\frac{x}{c}\right)$$

Or more commonly:

$$t' = \gamma\left(t - \frac{vx}{c^2}\right)$$

And the converse can again be gotten by changing the sign of *v*, and exchanging the unprimed variables for their primed variables, and vice versa. These transformations together are the Lorentz transformation:

$$\begin{aligned}
x' &= \gamma\left(x - vt\right)\\
y' &= y\\
z' &= z\\
t' &= \gamma\left(t - \frac{vx}{c^2}\right)
\end{aligned}$$

Velocity addition

$$V'_x = \frac{V_x - v}{1 - \frac{V_x v}{c^2}}$$

$$V'_y = \frac{V_y}{\gamma\left(1 - \frac{V_x v}{c^2}\right)}$$

$$V'_z = \frac{V_z}{\gamma\left(1 - \frac{V_x v}{c^2}\right)}$$

Derivation of velocity addition

The Lorentz transformations also apply to differentials, so:

$$\begin{aligned}
dx' &= \gamma\left(dx - vdt\right)\\
dy' &= dy\\
dz' &= dz\\
dt' &= \gamma\left(dt - \frac{vdx}{c^2}\right)
\end{aligned}$$

The velocity is *dx/dt*, so

$$\frac{dx'}{dt'} = \frac{\gamma\left(dx - vdt\right)}{\gamma\left(dt - \frac{vdx}{c^2}\right)}$$

$$\frac{dx'}{dt'} = \frac{dx - vdt}{dt - \frac{vdx}{c^2}}$$

$$\frac{dx'}{dt'} = \frac{\frac{dx}{dt} - v}{1 - \frac{dx}{dt}\frac{v}{c^2}}$$

Now substituting:

$$V_x = \frac{dx}{dt} \quad V'_x = \frac{dx'}{dt'}$$

gives the velocity addition (actually below is subtraction, addition is just reversing the signs of V_x, V_y, and V_z around):

$$V'_x = \frac{V_x - v}{1 - \frac{V_x v}{c^2}}$$

$$V'_y = \frac{V_y}{\gamma\left(1 - \frac{V_x v}{c^2}\right)}$$

$$V'_z = \frac{V_z}{\gamma\left(1 - \frac{V_x v}{c^2}\right)}$$

Also, the velocities in the directions perpendicular to the frame changes are affected, as shown above. This is due to time dilation, as encapsulated in the *dt*/*dt′* transformation. The V'_y and V'_z equations were both derived by dividing the appropriate space differential (e.g. *dy′* or *dz′*) by the time differential.

The metric and four-vectors

In what follows, bold sans serif is used for 4-vectors while normal bold roman is used for ordinary 3-vectors.

Inner product (i.e. notion of length)

$$\mathbf{a} \cdot \mathbf{b} = \eta(\mathbf{a}, \mathbf{b})$$

where $\boldsymbol{\eta}$ is known as the metric tensor. In special relativity, the metric tensor is the Minkowski metric:

$$\eta = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Space-time interval

$$ds^2 = dx^2 + dy^2 + dz^2 - c^2dt^2 = \begin{pmatrix} cdt & dx & dy & dz \end{pmatrix} \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} cdt \\ dx \\ dy \\ dz \end{pmatrix}$$

In the above, ds^2 is known as the spacetime interval. This inner product is invariant under the Lorentz transformation, that is,

$$\eta(\mathbf{a}', \mathbf{b}') = \eta\left(\Lambda\mathbf{a}, \Lambda\mathbf{b}\right) = \eta(\mathbf{a}, \mathbf{b})$$

The sign of the metric and the placement of the *ct*, *ct'*, *cdt*, and *cdt'* time-based terms can vary depending on the author's choice. For instance, many times the time-based terms are placed first in the four-vectors, with the spatial terms following. Also, sometimes η is replaced with $-\eta$, making the spatial terms produce negative contributions to the dot product or spacetime interval, while the time term makes a positive contribution. These differences can be used in any combination, so long as the choice of standards is followed completely throughout the computations performed.

Lorentz transforms

It is possible to express the above coordinate transformation via a matrix. To simplify things, it can be best to replace *t*, *t'*, *dt*, and *dt'* with *ct*, *ct'*, *cdt*, and *cdt'*, which has the dimensions of distance. So:

$$\begin{aligned}
x' &= \gamma x - \gamma\beta ct \\
y' &= y \\
z' &= z \\
ct' &= \gamma ct - \gamma\beta x
\end{aligned}$$

then in matrix form:

$$\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$$

The vectors in the above transformation equation are known as four-vectors, in this case they are specifically the position four-vectors. In general, in special relativity, four-vectors can be transformed from one reference frame to another as follows:

$$\mathbf{a}' = \Lambda \mathbf{a}$$

In the above, $\mathbf{a}'$ and $\mathbf{a}$ are the four-vector and the transformed four-vector, respectively, and Λ is the transformation matrix, which, for a given transformation is the same for all four-vectors one might want to transform. So $\mathbf{a}'$ can be a four-vector representing position, velocity, or momentum, and the same Λ can be used when transforming between the same two frames. The most general Lorentz transformation includes boosts and rotations; the components are complicated and the transformation requires spinors.

4-vectors and frame-invariant results

Invariance and unification of physical quantities both arise from four-vectors.[1] The inner product of a 4-vector with itself is equal to a scalar (by definition of the inner product), and since the 4-vectors are physical quantities their magnitudes correspond to physical quantities also.

Property/effect	3-vector	4-vector	Invariant result
Space-time events	3-position: $\mathbf{r} = (x_1, x_2, x_3)$ $\mathbf{r}\cdot\mathbf{r} \equiv r^2 \equiv x_1^2 + x_2^2 + x_3^2$	4-position: $\mathbf{X} = (ct, x_1, x_2, x_3)$	$\mathbf{X}\cdot\mathbf{X} = (c\tau)^2$ $(ct)^2 - \left(x_1^2 + x_2^2 + x_3^2\right)$ $= (ct)^2 - r^2$ $= -\chi^2 = (c\tau)^2$ τ = proper time χ = proper distance
Momentum-energy invariance	$\mathbf{p} = \gamma m \mathbf{u}$ 3-momentum: $\mathbf{p} = (p_1, p_2, p_3)$ $\mathbf{p}\cdot\mathbf{p} \equiv p^2 \equiv p_1^2 + p_2^2 + p_3^2$	4-momentum: $\mathbf{P} = (E/c, p_1, p_2, p_3)$ $\mathbf{P} = m\mathbf{U}$	$\mathbf{P}\cdot\mathbf{P} = (mc)^2$ $\left(\frac{E}{c}\right)^2 - \left(p_1^2 + p_2^2 + p_3^2\right)$ $= \left(\frac{E}{c}\right)^2 - p^2$ $= (mc)^2$ which leads to: $E^2 = (pc)^2 + \left(mc^2\right)^2$ E = total energy m = invariant mass
Velocity	3-velocity: $\mathbf{u} = (u_1, u_2, u_3)$ $\mathbf{u} = \frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}$	4-velocity: $\mathbf{U} = (U_0, U_1, U_2, U_3)$ $\mathbf{U} = \frac{\mathrm{d}\mathbf{X}}{\mathrm{d}\tau} = \gamma\left(c, \mathbf{u}\right)$	$\mathbf{U}\cdot\mathbf{U} = c^2$
Acceleration	3-acceleration: $\mathbf{a} = (a_1, a_2, a_3)$ $\mathbf{a} = \frac{\mathrm{d}\mathbf{u}}{\mathrm{d}t}$	4-acceleration: $\mathbf{A} = (A_0, A_1, A_2, A_3)$ $\mathbf{A} = \frac{\mathrm{d}\mathbf{U}}{\mathrm{d}\tau} = \gamma\left(c\frac{\mathrm{d}\gamma}{\mathrm{d}t}, \frac{\mathrm{d}\gamma}{\mathrm{d}t}\mathbf{u} + \gamma\mathbf{a}\right)$	$\mathbf{A}\cdot\mathbf{U} = 0$
Force	3-force: $\mathbf{f} = (f_1, f_2, f_3)$ $\mathbf{f} = \frac{\mathrm{d}\mathbf{p}}{\mathrm{d}t}$	4-force: $\mathbf{F} = (F_0, F_1, F_2, F_3)$ $\mathbf{F} = \frac{\mathrm{d}\mathbf{P}}{\mathrm{d}\tau} = \gamma m\left(c\frac{\mathrm{d}\gamma}{\mathrm{d}t}, \frac{\mathrm{d}\gamma}{\mathrm{d}t}\mathbf{u} + \gamma\mathbf{a}\right)$	$\mathbf{F}\cdot\mathbf{U} = 0$

Doppler shift

General doppler shift:

$$\nu' = \gamma\nu\left(1 - \beta\cos\theta\right)$$

Doppler shift for emitter and observer moving right towards each other (or directly away):

$$\nu' = \nu\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}}$$

Doppler shift for emitter and observer moving in a direction perpendicular to the line connecting them:

$$\nu' = \gamma\nu$$

Derivation of the relativistic Doppler shift

If an object emits a beam of light or radiation, the frequency, wavelength, and energy of that light or radiation will look different to a moving observer than to one at rest with respect to the emitter. If one assumes that the observer is moving with respect to the emitter along the x-axis, then the standard Lorentz transformation of the four-momentum, which includes energy, becomes:

$$\begin{pmatrix} \frac{E'}{c} \\ p'_x \\ p'_y \\ p'_z \end{pmatrix} = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \frac{E}{c} \\ p_x \\ p_y \\ p_z \end{pmatrix}$$

$$\frac{E'}{c} = \gamma\frac{E}{c} - \gamma\beta p_x$$

Now, if

$$p_x = \|p\| \cos\theta$$

where θ is the angle between p_x and $\vec{p}$, and plugging in the formulas for frequency's relation to momentum and energy:

$$\frac{h\nu'}{c} = \gamma\frac{h\nu}{c} - \gamma\beta \|p\| \cos\theta = \gamma\frac{h\nu}{c} - \gamma\beta\frac{h\nu}{c}\cos\theta$$

$$\nu' = \gamma\nu - \gamma\beta\nu\cos\theta = \gamma\nu\left(1 - \beta\cos\theta\right)$$

This is the formula for the relativistic doppler shift where the difference in velocity between the emitter and observer is not on the x-axis. There are two special cases of this equation. The first is the case where the velocity between the emitter and observer is along the x-axis. In that case θ = 0, and cos θ = 1, which gives:

$$\begin{aligned} \nu' &= \gamma\nu\left(1-\beta\right) \\ &= \nu\frac{1}{\sqrt{1-\beta^2}}\left(1-\beta\right) \\ &= \nu\frac{1}{\sqrt{\left(1-\beta\right)\left(1+\beta\right)}}\left(1-\beta\right) \\ &= \nu\frac{\sqrt{1-\beta}}{\sqrt{1+\beta}} \end{aligned}$$

This is the equation for doppler shift in the case where the velocity between the emitter and observer is along the x-axis. The second special case is that where the relative velocity is perpendicular to the x-axis, and thus θ = π/2, and cos θ = 0, which gives:

$$\nu' = \gamma\nu$$

This is actually completely analogous to time dilation, as frequency is the reciprocal of time. So, doppler shift for emitters and observers moving perpendicular to the line connecting them is completely due to the effects of time dilation.

See also

- Theory of relativity
- Special relativity