Modeling Makes Mathematics Fun and Real -- For Real!!

Sharing by Dr. Brian Winkel, Director SIMIODE, Professor Emeritus
United States Military Academy, West Point NY USA

Faculty Development Program
18 November 2024

www.simiode.org
SIMIODE *A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS*

SVKM S NMIMS
Mukesh Patel School of
Technology Management and Engineering
Mumbai INDIA

18 November 2024

Modeling Makes Mathematics Fun and Real
. . . For Real!!

Thank you DR Mahesh Naik and colleagues
for the invitation.

Personal Career Path

- Undergraduate mathematics degree in 1964
  food service, office work, tutor, organist, Math Club.
- Discovery of Modeling with Mathematics at initial teaching position after PhD in Noetherian Ring Theory, 1971.
- Teaching with modeling at liberal arts colleges, research university, engineering schools, military academies.
- Diversion - Founded (1977) and edited for 30 years *Cryptologia*, devoted to all aspects of cryptology.
- Consolidation - Founded (1991) and edited for 20 years *PRIMUS - Problems, Resources, and Issues in Mathematics undergraduate Studies*.
- Fully on track - Founded (2013) and directs *SIMIODE - Systemic Initiative for Modeling investigations and Opportunities with Differential Equations*.

Emphases - Modeling with Differential Equations

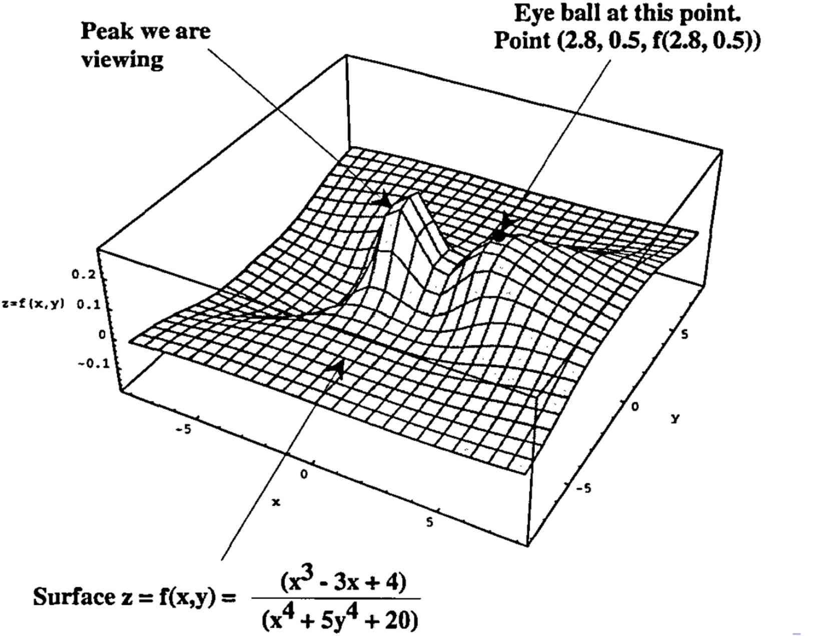
Differential Equations Invented to Study Change

All Models discussed are FREEly available in SIMIODE. Student Version to the public and Teacher Version for teachers at **https://qubeshub.org/community/groups/simiode**. Just Google SIMIODE QUBES. All papers referred to, but not in SIMIODE will be provided for local posting.

Here are the modeling opportunities we will share.

- What Can You See From the Other Side of the Valley?
- Getting Malled
- m&m Modeling of Death and Immigration
- Torricelli's Law for Falling Column of Water
- LSD Drug Model
- Sublimation of Dry Ice
- Chord Path Time
- Spread of Slime
- Eye Surgery Recovery
- Drying Cloth
- Deep Well
- Tuned Mass Dampers
- Sources and Opportunities
- Students as Consultants

What can you see from the other side of the valley?



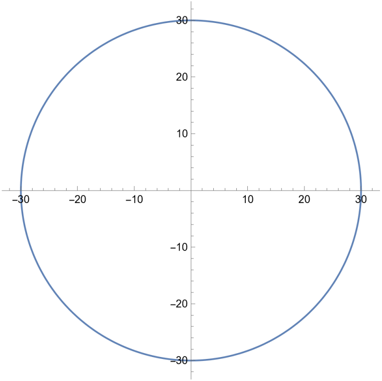
- How would you get started?
- What kinds of mathematical issues might you embrace/invent/need?
- What misconceptions might we have? E.g., do you think we can see the top of the mountain on the other side of the valley?
- Does this problem look like it will permit a "closed form" solution or require some numerical efforts?
- How realistic is this situation and who might be interested in such a problem?

Extensions

- Find the point on the viewing mountain from which we can see the most area on the mountain on the other side of the valley?
- For a set of strategic points on the viewing mountain on the other side of the valley what is the highest (lowest) point from which we can see these points?
- What is the highest point (the lowest point) on the mountain on the other side of the valley we can see - from any point on the viewing mountain?

Getting Malled

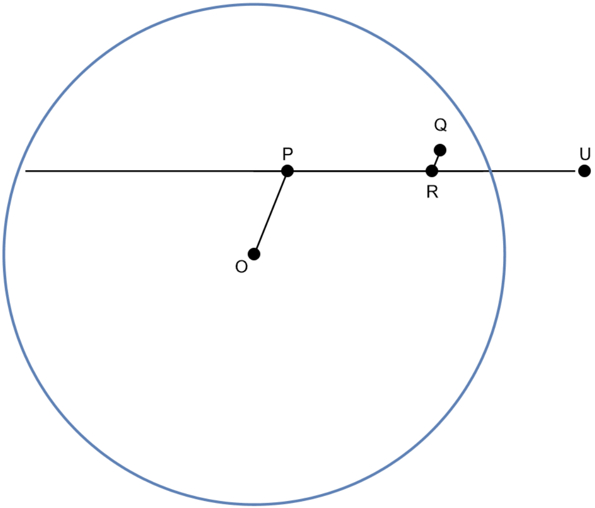
The average driving speed to reach a shopping mall is uniformly 30 km/hr. People are willing to spend no more than 1 hour driving to reach the mall. Suppose a new improved East-West road is built, passing 10 km due North of the shopping mall and that the speed limit on the new road is 55 km/hr. Determine the new neighborhood for the shopping mall.



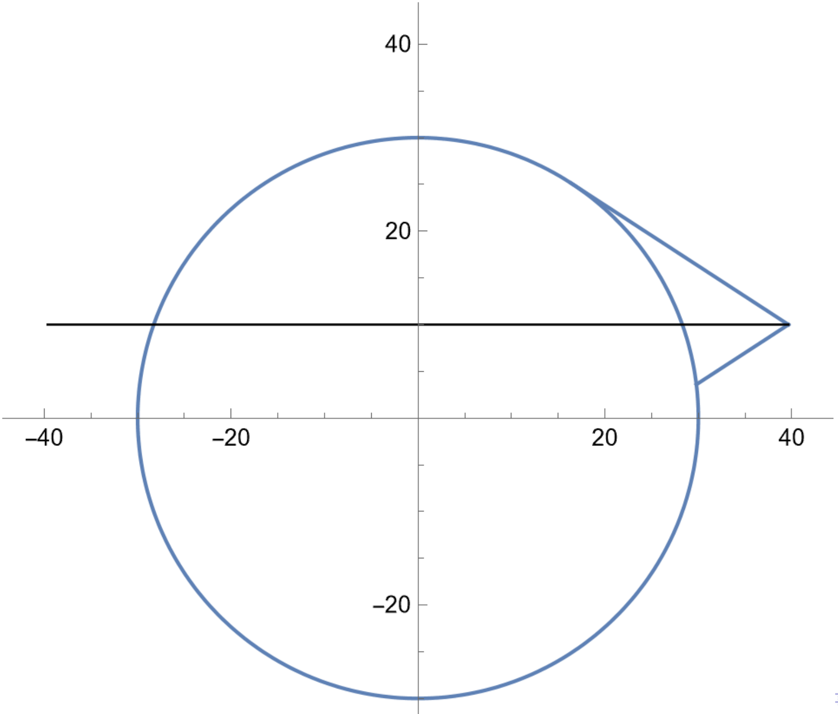
- What reasonable and simplifying assumptions can we make to get started? Why are they so?
- How could we begin?
- Can we identify some points which are in the new neighborhood which were not in the old neighborhood?
- Who would be interested in our result?

$\vdots$

Getting On and Getting Off Highway Ideas



Sketch with hints.

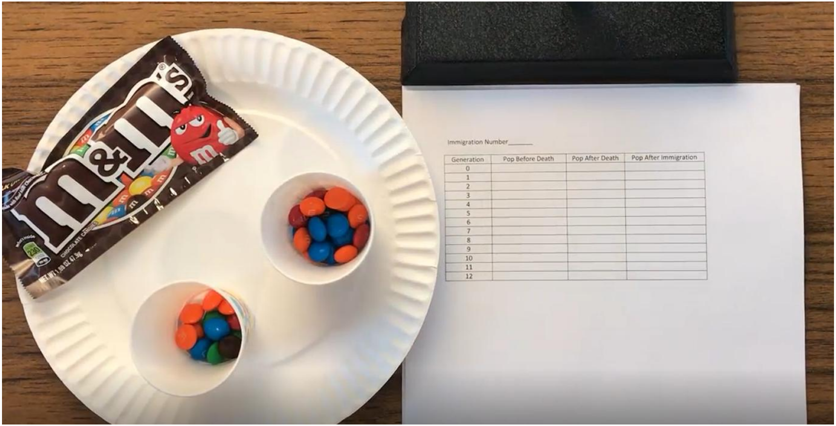


40
20
-20
-40
-20
20
40

Death and immigration

From 1-001-MMDeathImmigration-Modeling Scenario

Here is the picture of the material needed . . .



Equipment:

- 1 small bag regular m&m's
- 1 small cup
- 1 paper plate

Start with initial population, say about 50 m&m's. Keep all others in storage cup. At each generation place "dead" m&m's in cup.

1. Toss the m&m's gently onto the plate.
2. Remove m&m's with the 'm' facing up – they die.
3. Add 10 m&m immigrants from the cup.
4. Count remaining m&m's from that generation. Record data.
5. Go to Step 1 and repeat. Record time (generation number) and number of m&m's each time.

Using Solver to estimate parameters a and b in model M(n+1) = a*M(n) + b, M(0) = 50.

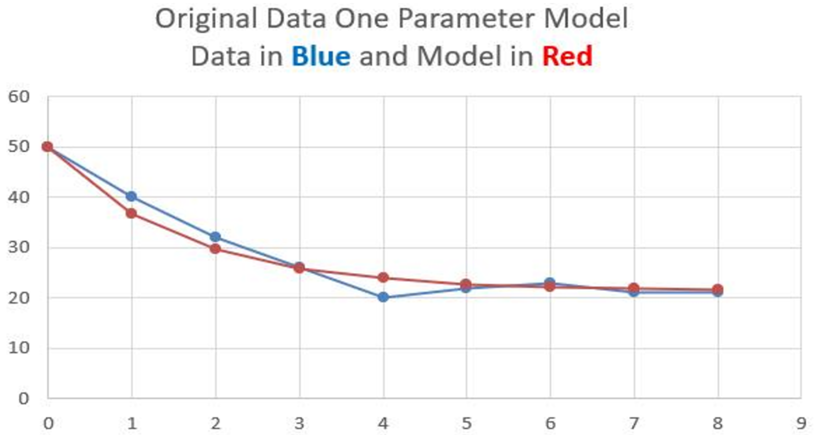
Here we offer results from data collected with initial population of b(0) = 50 and immigration level of b = 10.
You should enter your data and change your <---Fixed immigration level below.
You set your own initial population, say about 20 or 25, not 50. Too tedious if 50.
We estimate first two parameters a and b and then only parameter a with parameter b set to 10 (known).

Perfect model means b(n+1) = 0.5*b(n) + 10.

Two parameter model means b(n+1) = a*b(n) + b.

One parameter model means b(n+1) = a*b(n) + 10.

		Parameters			a =	0.8		a =	0.3	
					b =	9		b =	10	<--Fixed
	Insert Your Data		**Pop And**		**Two**	**Two**		**One**	**One**	
		Perfect	**Perfect**		**Parameter**	**Parameter**		**Parameter**	**Parameter**	
Time	**Pop**	**Model**	**Diff^2**		**Model**	**Diff^2**		**Model**	**Diff^2**	
0	50	50	0		50	0		50	0	
1	40	35	25		49	81		25	225	
2	32	27.5	20.25		48.2	262.44		17.5	210.25	
3	26	23.75	5.0625		47.56	464.8336		15.25	115.5625	
4	20	21.875	3.515625		47.048	731.5943		14.575	29.43063	
5	22	20.9375	1.128906		46.6384	607.0508		14.3725	58.17876	
6	23	20.46875	6.407227		46.31072	543.3897		14.31175	75.48569	
7	21	20.23438	0.586182		46.048576	627.4312		14.29353	44.97681	
8	21	20.11719	0.779358		45.838861	616.969		14.28806	45.05017	
		TotalSSE	62.7298			3934.708			803.9345	


Original Data One Parameter Model
Data in Blue and Model in Red
60
50
40
30
20
10
0
0 1 2 3 4 5 6 7 8 9

We can perform a similar analysis built on a continuous differential equation model:

$$b'(n) = -0.5 \cdot b(n)\,, \quad b(0) = 50\,.$$

Many colleagues use this Modeling Scenario to start class on Day One of their Ordinary Differential Equations course and themes and activities occur on this first day which reoccur throughout the semester, such as:

mathematical model, data,

collecting data, equilibrium,

steady state, long term behavior,

stability, iteration, rate of change,

parameter, data analysis,

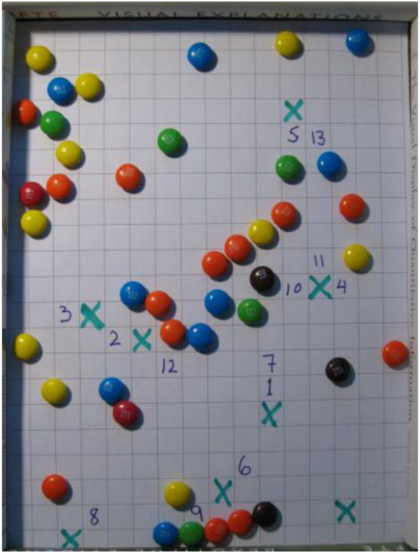
parameter estimation, model validation,

⋮

and more ...

m&m Spread of Disease

From 1-017-DiseaseSpread-Modeling Scenario



	A	B	C	D
1	**Initial Condit**	y(0) =		8
2	**Parameters**		r =	0.473
3			K =	63
4				
5	**Time**	**Infect**	**Model**	**Error^2**
6	0	8	8	0
7	1	14	11.9228	4.3147
8	2	21	17.1688	14.678
9	3	26	23.6539	5.504
10	4	31	30.9353	0.0042
11	5	41	38.2775	7.4118
12	6	47	44.9211	4.322
13	7	50	50.3685	0.1358
14	8	51	54.4856	12.149
15	9	54	57.4097	11.626
16	10	56	59.396	11.533
17	11	60	60.7048	0.4967
18	12	60	61.5499	2.4022
19	13	61	62.0885	1.1848
20	14	61	62.4289	2.0418
21	15	61	62.6429	2.6992
22	16	61	62.777	3.1578
23	17	62	62.8609	0.7411
24	18	62	62.9132	0.834
25	19	63	62.9459	0.0029
26			**SSE =**	**85.24**
27				

y(0) is known initial condition

r is parameter to be estimated for growth rate

K is parameter for carrying capacity which we are presumihg to be 63 = 9 + 55.

Use Solver to Minimize the sum of square errors between model

Estimating r only.

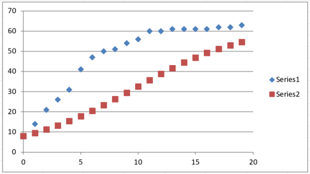
$$y(t) = \frac{8\, e^{r\,t}\, K}{-8 + 8\, e^{r\,t} + K}$$

and observed data in Infecteds Column.

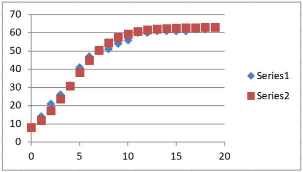
In Solver minimize on Cell E32 by changing variable cell E8.

If you were to alter your initial condition you would change 8 to y0 in the above formal equation.

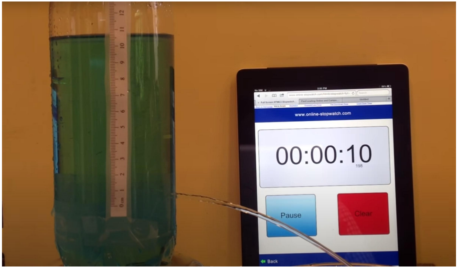
Original data witih r = 0.2



Best fit model with r = 0.473



Modeling a Falling Column of Water - Torricelli's Law



We use data taken from video at **SIMIODE YouTube Channel**. Cylindrical column (radius $= 4.17$ cm) of water empties through a bore hole (diameter $= 11/16" = 0.218281$ cm) in bottom of column. Exit hole at bottom of column - height is 0 cm.

We seek to model $h(t)$, the height of the column of water.

What Students Can Accomplish

Outline of modeling process

- seeing and collecting data,
- conjecturing empirical models,
- building an analytical model from scientific first principles,
- creating a differential equation model,
- solving of the differential equation,
- estimating parameter,
- comparing model with the actual data.

IDEA - have students collect data from different configurations, i.e. different bore hole sizes at bottom of column of water and compare models and parameters.

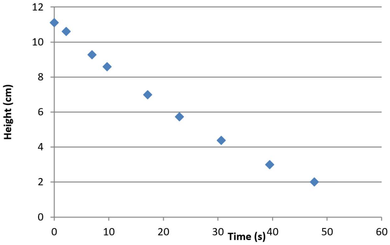
Here is data we collected. What do you see or notice?

Make some observations now.

What about the shape of the data plot?

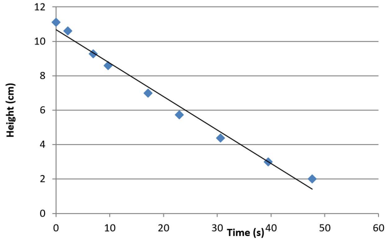
Does this fit your reality?

Time (s)	Height (cm)
0.0	11.1
2.187	10.6
6.933	9.3
9.717	8.6
17.102	7.0
22.968	5.75
30.603	4.4
39.503	3.0
47.663	2.0



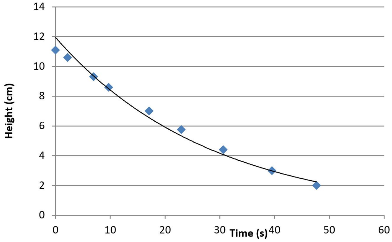
Linear Fit? Easy to do with Excel's TrendLine capabilities.

Time (s)	Height (cm)
0.0	11.1
2.187	10.6
6.933	9.3
9.717	8.6
17.102	7.0
22.968	5.75
30.603	4.4
39.503	3.0
47.663	2.0



Exponential Decay Fit? Also, easy to do with Excel's TrendLine capabilities.

Time (s)	Height (cm)
0.0	11.1
2.187	10.6
6.933	9.3
9.717	8.6
17.102	7.0
22.968	5.75
30.603	4.4
39.503	3.0
47.663	2.0



All are empirical fits with no understanding.

They just fit a function to data.

Neither line nor exponential are good.

Can we articulate why neither is that great?

What happens to height $h(t)$?

How fast is column of water falling? Early and later?

For large $h(t)$ the column of water falls faster.

For small $h(t)$ falls slower.

Time (s)	Height (cm)
0.0	11.1
2.187	10.6
6.933	9.3
9.717	8.6
17.102	7.0
22.968	5.75
30.603	4.4
39.503	3.0
47.663	2.0

Ideas about $\dfrac{dh(t)}{dt}$ - rate of change in height $h(t)$?

Time (s)	Height (cm)
0.0	11.1
2.187	10.6
6.933	9.3
9.717	8.6
17.102	7.0
22.968	5.75
30.603	4.4
39.503	3.0
47.663	2.0

Check out the average rate of falling of the height of the column of water in several intervals, say, $[0, 2.187]$,

$$\frac{10.6 - 11.1}{2.187 - 0} = -0.2286\,,$$

or in the interval $[39.503, 47.663]$,

$$\frac{2.0 - 3.0}{47.663 - 39.503} = -0.122549\,.$$

What do you see? What can you say about $h'(t)$?

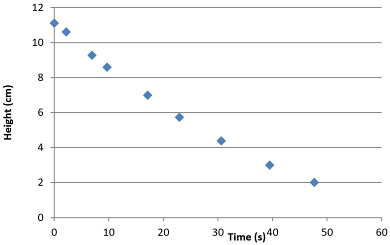
Let's find a model from some first principles.

This would be an analytic model.

NOT just fit a function to data.

NOT just "it looks like it falls faster or slower."

Time (s)	Height (cm)
0.0	11.1
2.187	10.6
6.933	9.3
9.717	8.6
17.102	7.0
22.968	5.75
30.603	4.4
39.503	3.0
47.663	2.0



Enter Evangelista Torricelli 1608–1647, an Italian physicist and mathematician, and a student of Galileo. Best known for his invention of the barometer. Obviously, also known for his wicked mustache!

Torricelli's Law to the rescue!

$$\frac{dh(t)}{dt} = -b\sqrt{g \cdot h(t)}, \quad h(0) = h_0 \quad b > 0.$$

Say it out loud in sentence form.

Explain to yourself what it means, what it implies.

Does Torricelli's Law agree with observations?

For large $h(t)$ the column of water DOES fall faster.

For small $h(t)$ the column of water DOES fall slower.

We build the model that IS Torricelli's Law from First Principles.

This will be an analytic model.

Basically, The Law of Conservation of Energy says that

Total Energy is conserved

.

We will apply it to a slab of water, first at the surface of the column of water and then at the bottom of the column ($h = 0$)

Total Energy is the the sum of the **potential energy** and the **kinetic energy** of a particle of mass m and this sum is constant at each instance in time, t.

Now by The Law of Conservation of Energy - Initial Total Energy equals Final Total Energy.

$$TE_i = \frac{1}{2}mv_i^2 + mgh = \frac{1}{2}mv_f^2 + mg \cdot 0 = \frac{1}{2}mv_f^2 = TE_f \, ,$$

Divide both sides by m and multiply by 2 - to solve for v_f:

$$v_f = \sqrt{2gh + v_i^2} \, .$$

Since $v_i = 0$ we have one classical form of Torricelli's Law

$$v_f = \sqrt{2gh} \, ,$$

where v_f is the speed of the water as it leaves the exit or bore hole.

You are helping students realize notions from the "S" of STEM

. . . while doing the "M" of STEM.

So, for a cylinder of constant cross sectional area we have an analytic model (differential equation!) for $h(t)$.

$$\frac{dh(t)}{dt} = -b\sqrt{g \cdot h(t)}\,, \quad h(0) = h_0\,.$$

We solve this differential equation for $h(t)$ to realize a model.

What strategy/technique can we employ? What technology?

We use this solution and our data to estimate parameter b and validate our model by comparing model predictions to data.

$$\frac{dh(t)}{dt} = -b\sqrt{g \cdot h(t)} = -b\sqrt{g} \cdot (h(t))^{1/2}.$$

KEY POINT: In traditional courses the differential equation is presented and students work on techniques with no knowledge of where the differential equation comes from or what its purpose is.

SIMIODE motivates with modeling.

Separate the variables (Done to introduce technique or practice.)

$$(h(t))^{-1/2} \cdot \frac{dh(t)}{dt} = -b\sqrt{g}.$$

OR

$$(h(t))^{-1/2} \cdot dh = -b\sqrt{g} \cdot dt.$$

Integrate both sides. (What is C?)

$$\int (h(t))^{-1/2} \cdot \frac{dh(t)}{dt}\, dt = \int -b\sqrt{g}\, dt + C\,,$$

$$2(h(t))^{1/2} = -b\sqrt{g} \cdot t + C.$$

Now to find C using Initial Conditions:

$$2(h(t))^{1/2} = -b\sqrt{g} \cdot t + C.$$

$$2(h(0))^{1/2} = -b\sqrt{g} \cdot 0 + C = C.$$

Thus we have

$$2(h(t))^{1/2} = -b\sqrt{g} \cdot t + 2(h(0))^{1/2}.$$

Divide both sides by 2 and then square both sides yields:

$$h(t) = \left(-\frac{b\sqrt{g}}{2} \cdot t + (h(0))^{1/2}\right)^2. \tag{1}$$

This is model for height of the column of water, $h(t)$, at time t.

What do we know and what do we need to estimate b in (1)?

$h(0) = 11.1$ cm and $g = 980$ cm/s^2

Thus from $h(0) = 11.1$ cm and $g = 980$ cm/s^2

$$h(t) = \left(-\frac{b\sqrt{g}}{2} \cdot t + (h(0))^{1/2}\right)^2$$

becomes

$$h(t) = \left(-\frac{b\sqrt{980}}{2} \cdot t + (11.1)^{1/2}\right)^2 ,$$

and expanded in decimals we have

$$h(t) = (-15.6525 \cdot b \cdot t + 3.33166)^2 . \tag{2}$$

We have arrived at our model and now we seek to determine b and validate our model and predict our data.

How might we do this?

We turn to our Excel spreadsheet and seek to determine the parameter b which minimizes the sum of the squared errors between our data (h_i) and our model $(h(t_i))$ over our data points.

$$SSE(b) = \sum_{i=1}^{9} (h_i - h(t_i))^2 \ .$$

Minimize as a function of the parameter b:

$$SSE(b) = \sum_{i=1}^{9} (h_i - h(t_i))^2 \ .$$

where

- t_i is the i^{th} time observation,
- h_i is the observed height at time t_i,
- $h(t_i)$ is our model's prediction of the height at time t_i, and
- $n = 9$ is the number of data points we have.

Model Analysis in Excel Using Solver

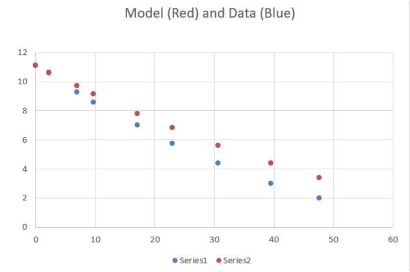
Data collected Friday, 5 August 2016 by Brian Winkel

SOURCE for Data
https://www.youtube.com/watch?v=NBr4DOj4OTE
Radius of hole 11/64" = 0.218281 cm and radius of cylinder 4.17 cm

Model h'(t) = -b Sqrt(g h(t))

Model h(t) = (-b Sqrt(g)/2 + h(0)^(1/2))^2

				b=	0.002
	Zeroed	Actual	Model	SSE	
Time (s)	Time	Height (cm)			
8.679	0	11.1	11.09995836	1.73426E-09	
10.866	2.187	10.6	10.64844791	0.0023472	
15.612	6.933	9.3	9.700872913	0.160699092	
18.396	9.717	8.6	9.165570453	0.319869938	
25.781	17.102	7	7.819192408	0.671076202	
31.647	22.968	5.75	6.825923093	1.157610501	
39.282	30.603	4.4	5.634134022	1.523086785	
48.182	39.503	3	4.389102871	1.929606788	
56.342	47.663	2	3.384016994	1.915503039	
			Total SEE	7.679799546	



Torricelli Modeling Scenario 1-015-Torricell-ModelingScenarioi.

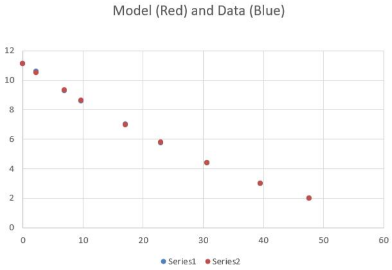
Solver can minimize the TOTAL SEE or SSE which is currently 7.679799546 with parameter $b = 0.002$ by asking Solver to minimize SEE or SSE as a function of b cell.

Parameter Estimation with Excel Solver - Results

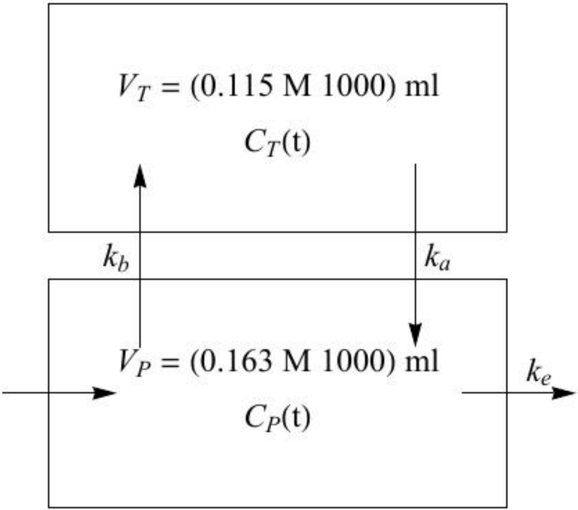
Model h'(t) = -b Sqrt(g h(t))

Model h(t) = (-b Sqrt(g)/2 + h(0)^(1/2))^2

				b=	0.002581
	Zeroed	Actual	Model	SSE	
Time (s)	**Time**	**Height (cm)**			
8.679	0	11.1	11.09995836	1.73426E-09	
10.866	2.187	10.6	10.51908638	0.006547014	
15.612	6.933	9.3	9.312232219	0.000149627	
18.396	9.717	8.6	8.638501405	0.001482358	
25.781	17.102	7	6.973871293	0.000682709	
31.647	22.968	5.75	5.778477118	0.000810946	
39.282	30.603	4.4	4.390799244	8.46539E-05	
48.182	39.503	3	3.013347567	0.000178158	
56.342	47.663	2	1.977592125	0.000502113	
			Total SEE	0.010437581	



We offer a model for LSD flow between plasma and tissue compartments in the human body.



$$V_P = \underbrace{(0.163M)}_{\text{kg of plasma}} \cdot \underbrace{(1)}_{\text{liter/kg}} \cdot \underbrace{(1000)}_{\text{ml/liter}} \cdot = \underbrace{(163M)}_{\text{ml of plasma}} \tag{3}$$

Using the notion of "simple change in something," in this case amount of LSD in each compartment we can produce the system of differential equations in (2). We discourage them from building rate of change models of just concentration as they can be difficult with units. The last term in the equation for $C_T'(t)$ in (2) reflects the exponential decay of the LSD in the tissue compartment due to excretion.

$$\begin{aligned} V_P C_P'(t) &= k_a V_T C_T(t) - k_b V_P C_P(t) - k_e V_P C_P(t) \\ V_T C_T'(t) &= k_b V_P C_P(t) - k_a V_T C_T(t)\,. \end{aligned} \tag{4}$$

We seek the model built from parameter (k_a, k_b, and k_e) estimates using the solution of the system of differential equations and minimization of the sum of square error function

$$SSE(k_a, k_b, k_e) = \sum_{i=1}^{7} (C_P(t_i) - O_i)^2 \,.$$

Now with *Mathematica*'s powerful `FindMinimum` command we can determine the values of the parameters k_a, k_b, and k_e which minimize this $SSE(k_a, k_b, k_e)$ function,

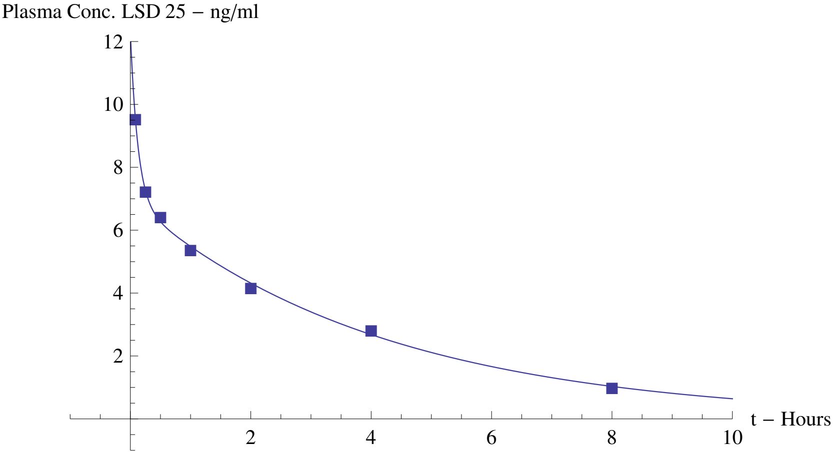
```
FindMinimum[SSE[ka, kb, ke], {ka, 1/3}, {kb, 1/4}, {ke, 1/4}]
```

- Indeed, we obtain the minimum sum of square errors to be 0.080945, when $k_a = 4.63679$, $k_b = 3.18659$, and $k_e = 0.41128$.
- We encourage students to use different initial guesses in the `FindMinimum` command for each of the three parameters, k_a, k_b, and k_e, to give them some idea of the robustness of the command itself and confidence that they have a true minimum sum of square errors.

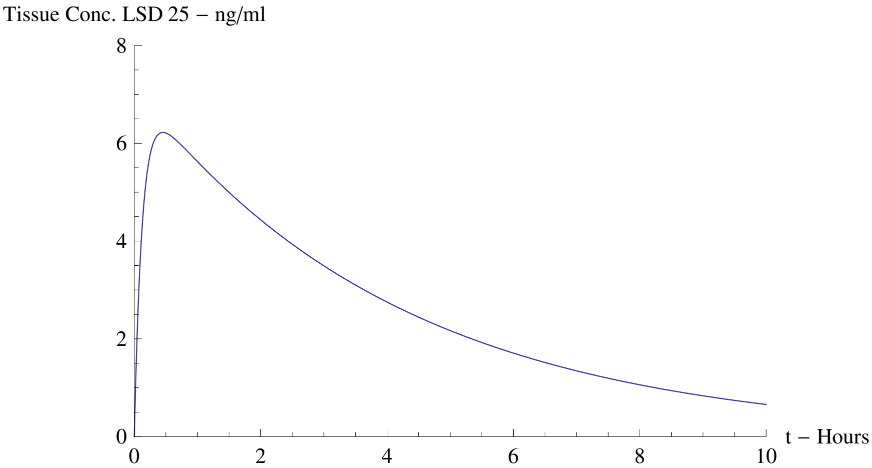
- This gives us a final model expression for $C_P(t)$

$$C_P(t) = 0.128905\left(41.2194e^{-7.99617t} + 53.9669e^{-0.238492t}\right) .$$

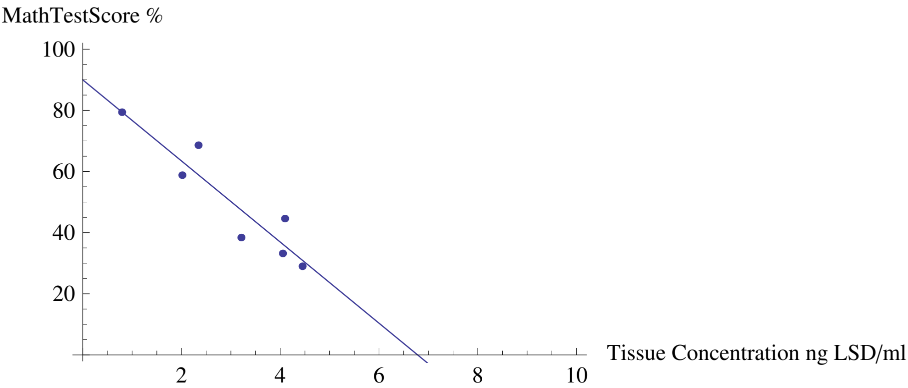
Plot of the observed values of the average concentration of LSD (ng/ml) (squares) and the model built from parameter (k_a, k_b, and k_e) estimates using the solution of the system of differential equations and minimization of the sum of square error function



Plot of LSD (drug) in tissue obtain from data on LSD in plasma, all because of our modeling efforts.

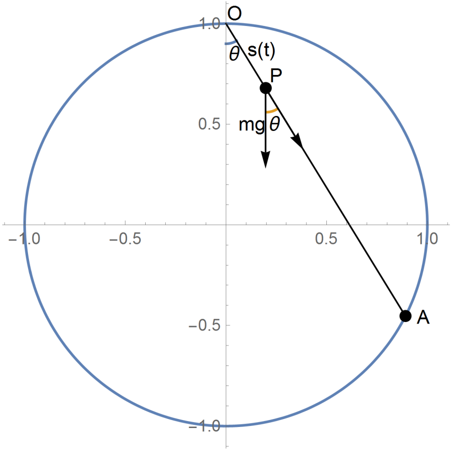


Strong correlation between amount of LSD in tissue and performance on simple arithmetic questions.



Chord Path Time

A mass particle starts from rest at point, $O = (0, 1)$, of a vertical circle and slides along a chord to a point A on the circle.

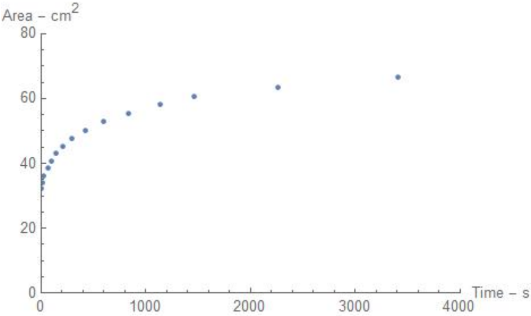


What is the time(s) it takes for the mass to hit the point A in terms of a changing angle θ?

Spread of Slime

Model the spread of slime. That's it!

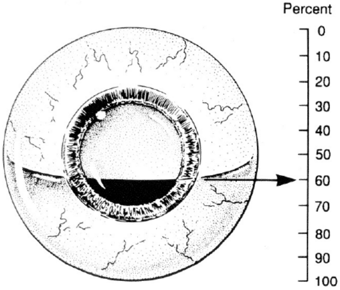

00:07:45902
Pause


Area – cm^2
80
60
40
20
0
0
1000
2000
3000
4000
Time – s

Modeling Intraocular Gas Bubbles in Retinal Surgery Patients

- Following some retinal surgery inert gas is injected into the intraocular region to serve as a tampinade for the wound.
- This gas must be replaced slowly, but eventually, with normal body fluids for proper functioning and healing of the retina.
- It is important not to risk change in air pressures (e.g., loss of cabin pressure in flight) else the eye might explode!

- But how does the gas get out and get replaced by body fluids?



60% Bubble

- The meniscus height of an intraocular gas bubble is estimated clinically as a percentage of the vertical diameter of the eye seen through the dilated pupil with the plane of the cornea perpendicular to the ground.
- Ophthalmologists dilate your pupil, examine, and measure height of inverted bubble to check that it is getting smaller.

126	I	C	0.1	**1**	0.90	**4.5850**	1.5228	0.2
126	I	C	0.1	**2**	0.90	**4.5850**	1.5228	0.2
126	I	C	0.1	**3**	0.73	**3.8377**	1.3449	0.54
126	I	C	0.1	**6**	0.45	**2.0266**	0.7064	1.1
126	I	C	0.1	**13**	0.33	**1.2460**	0.2199	1.34
126	I	C	0.1	**16**	0.25	**0.7830**	-0.2446	1.5
126	I	C	0.1	**22**	0.10	**0.1460**	-1.9241	1.8
126	I	C	0.1	**34**	0.02	**0.0062**	-5.0832	1.96

Patient 126 has an Intraocular lens (I); gas used Perfluoropropane gas (C_3F_8) (C); concentration of gas 10% (0.1); **times of observation in days were 1, 2, 3, 6, 13, 16, 22, and 34**; the percentage (decimal) of meniscus height of the bubble (beginning with 0.90); the **computed bubble volume ($\mathbf{cm^3}$) from meniscus height** (beginning with 4.5850); natural logarithm of bubble volume (beginning with 1.5228); and actual height of the bubble in cm (beginning with 0.2).

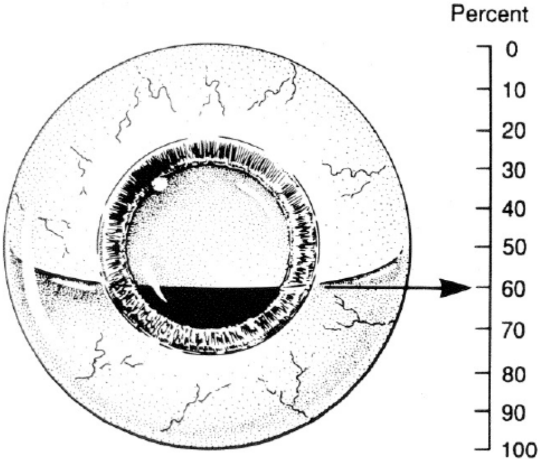
- But how does the gas get out and get replaced by body fluids?
- This is basis for mathematical model of change in the volume of the bubble of gas - a differential equation!
- The medical literature suggests that the volume of the intraocular gas bubble in the vitreous cavity, $V = V(t)$ in cm^3 at time t in days can be modeled by the following exponential decay differential equation model.

$$\frac{dV}{dt} = -kV \, , V(0) = \frac{4}{3}\pi(1^3) \, . \tag{3}$$

NB: The radius of the typical eye is presumed to be 1 cm.

What might $\frac{dV}{dt}$ depend upon?

Think about how the gas escapes.



60% Bubble

Three other models one could consider. What do these say and what is the basis for each? But, which is best?

$$\frac{dV(t)}{dt} = -kS(t)\,, \tag{4}$$

$$\frac{dV}{dt} = -kS^{3/2}\,, V(0) = \frac{4}{3}\pi(1^3)\,, S(0) = 4\pi(1^2). \tag{5}$$

$$\frac{dV}{dt} = -k_1 S(t)^{3/2} - k_2 M(t)^{3/2}\,, V(0) = V_0\,, S(0) = S_0\,, M(0) = M_0\,. \tag{6}$$

And again,

$$\frac{dV}{dt} = -kV\,, \quad V(0) = \frac{4}{3}\pi(1^3)\,.$$

After analysis and comparison of models it turns out that the first model, exponential decay, works just fine:

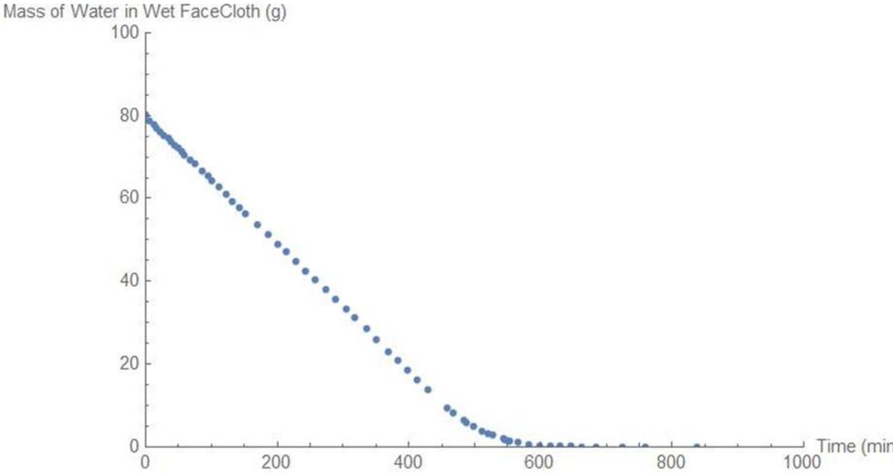
$$\frac{dV}{dt} = -kV\,, \quad V(0) = \frac{4}{3}\pi(1^3)\,.$$

And so one can use half-life for the size of the bubble in terms of the individual choice of gas used and predict the time at which it is safe to risk change in air pressures, e.g., is it OK to fly?

Wet Cloth Drying

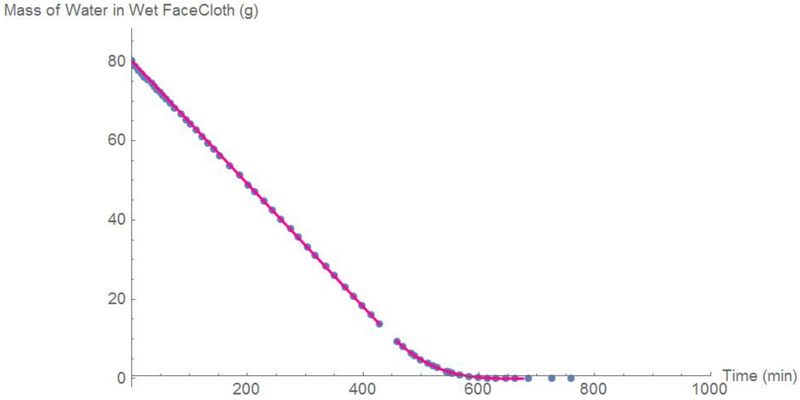
We build a mathematical model for the rate at which drying takes place in a cloth wet with water while hanging in air. A model can be based on underlying physical principles. Such a model is called an *analytic* model.

Plot of data collected.



Notice the ribbed trim of the cloth.

We form a composite model for when the “interior” of the cloth is completely dry and then when the “ribbed trim” is drying and plot that over the data.

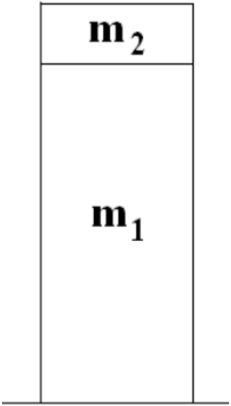


How Deep is the Well?

Drop a stone and listen for the sound of the splash. From total time elapsed over the two time periods, (1) down - falling under gravity with no resistance and (2) up - speed of sound, we can compute the depth.

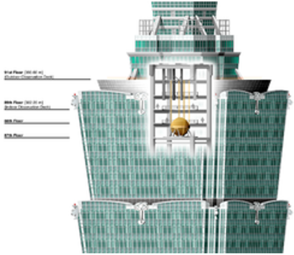
Tuned Mass Dampers (TMD)

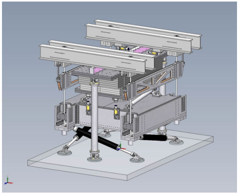
Used at the top of tall structures to mitigate/stop swaying of structure when under influence of earthquake or wind shear.

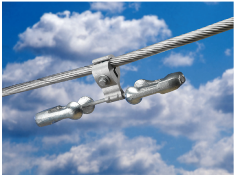


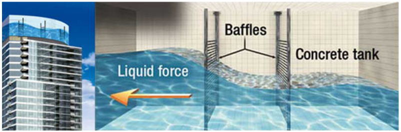
Building of Mass m_1 with Tuned Mass Damper of mass m_2 atop the building.

Examples of Tuned Mass Dampers

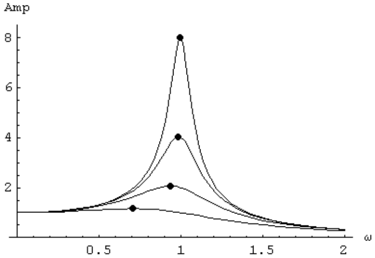




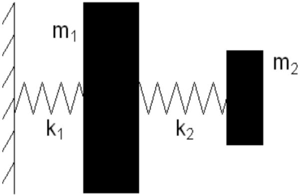




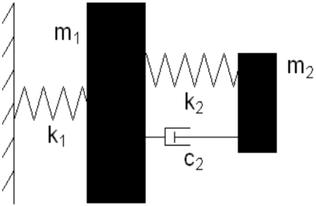
$$m \cdot y''(t) + c \cdot y'(t) + k \cdot y(t) = p_0 \cos(\omega t), \quad y(0) = y_0, \quad y'(0) = v_0 .$$

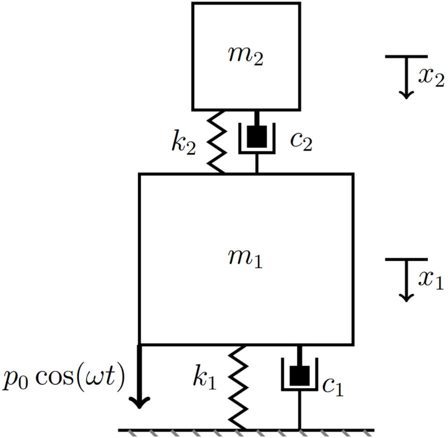


Plot of peak frequency response, both input frequency, ω, and amplitude, of steady state solution for $m = 1$, $k = 1$, $\omega_0 = 1$, $p_0 = 1$, for $c = 1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}$ where peak is highest for lowest value of c, i.e. $\omega_{\max} \to \omega_0$ as c decreases.



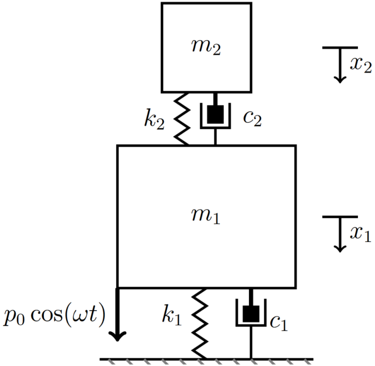
Horizontal depiction of two mass spring system (no damping on either mass or damping on Tuned Mass Damper) with smaller mass m_2 serving as Tuned Mass Damper.



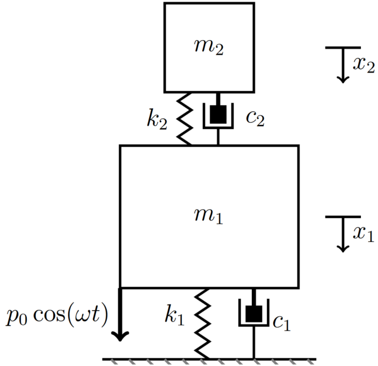


m_2
x_2
k_2
c_2
m_1
x_1
$p_0 \cos(\omega t)$
k_1
c_1

Using Newton's Second Law and our Free Body Diagram we can deduce the governing equations for the motions of the structure and the damper.



Hint: For consistency all downward forces, positions, velocities, and accelerations are positive.



$$
\begin{aligned}
m_1 x_1''(t) &= -k_1 x_1(t) - c_1 x_1'(t) - k_2(x_1(t) - x_2(t)) \\
&\qquad - c_2(x_1'(t) - x_2'(t)) + p_0 \cos(\omega t) \\
m_2 x_2''(t) &= -k_2(x_2(t) - x_1(t)) - c_2(x_2'(t) - x_1'(t))
\end{aligned}
$$

SIMIODE has over 400 Project Ideas and Resources

Hundreds of resources, mainly published articles and links to articles, at Project Ideas and Resources can serve as resources for faculty to create class projects or author and publish Modeling Scenarios in SIMIODE.

They can also serve students who are seeking projects for their course work or own edification.

Resources are listed as Year, Author(s), and Title (or descriptive name) with a description, often the abstract, and some narrative.

All have lots of tag words or keywords so the topic areas can be found in the search capability.

All Models discussed are FREEly available in SIMIODE. Student Version to the public and Teacher Version for teachers - all found at

https://qubeshub.org/community/groups/simiode.

OR just Google SIMIODE QUBES.

Students' cognate courses, e.g., chemical kinetics, life science Petri dish growth, economics modeling phenomena, sociology, linguistics.

Get hold of their textbook or article in the field and bring it into your class.

SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

TEACHER VERSION
KINETICS - RATE OF CHEMICAL REACTION

Brian Winkel
Director SIMIODE
Cornwall NY 12518 USA
BrianWinkel@simiode.org

Abstract: We help students see the connection between college level chemistry course work and their differential equations coursework. We do this through modeling kinetics, or rates of chemical reaction. We study zeroth, first, and second order reactions and offer many opportunities to model these chemical reactions with data, some of which comes from traditional introductory chemistry textbooks. We ask students to verify their model through parameter estimation. We use Excel's Trendline addition to graphs/charts to select the models for the data and transformed data to take advantage of Trendline's set function choices and we also use Mathematica's direct nonlinear fitting capabilities.

Keywords: chemical reaction, kinetics, rate of reaction, order, Law of Mass Action

Tags: first order, differential equation, data, modeling, fitting, parameter estimation, zeroth order, first order, second order, chemistry textbook

Decomposition of H_2O_2

5) Consider the following data (Table 5) for the reaction describing the decomposition of H_2O_2, hydrogen peroxide.

$$2H_2O_2 \longrightarrow 2H_2O + O_2 \tag{7}$$

Time t in s	$[H_2O_2]$ mol/L
0	1
120	0.91
300	0.78
600	0.59
1200	0.37
1800	0.22
2400	0.13
3000	0.082
3600	0.05

Table 5. Data [8, p. 682] for the decomposition of H_2O_2 is given.

a) Plot the data.

b) From the plot make a conjecture as to the order ($m = 0, 1, 2$) of the reaction.

c) Conduct a complete analysis, determining the order and the parameters. Plot the data and the model, being sure to defend what the order is and what the order is not vis-á-vis $m = 0, 1, 2$ orders.

SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

STUDENT VERSION
MODELING EVICTIONS

Mary Vanderschoot
Department of Mathematics and Computer Science
Wheaton College
Wheaton IL USA

STATEMENT

According to the National Law Center on Homelessness and Poverty, unaffordable rents and a lack of legal protections for renters have created a national "eviction epidemic" [4]. Matthew Desmond, author of *Evicted: Poverty and Profit in the American City* and director of the Eviction Lab at Princeton University, estimates that 2.3 million evictions were filed in the U.S. in 2016 (four evictions per minute). Desmond writes, "Eviction is a direct cause of homelessness, but it also is a cause of residential instability, school instability [and] community instability" [1]. In this project you will develop and analyze two mathematical models to study eviction trends in a city using an actual eviction rate.

www.simiode.org
SIMIODE A SYSTEMIC INITIATIVE FOR MODELING INVESTIGATIONS & OPPORTUNITIES WITH DIFFERENTIAL EQUATIONS

STUDENT VERSION
PROPAGATION OF THE WORD JUMBO

Rachel L. Bayless
Department of Mathematics
Agnes Scott College
Decatur GA USA

Rachelle C. DeCoste
Department of Mathematics
Wheaton College
Norton MA USA

STATEMENT

In 1861, the elephant who became known as Jumbo was captured in Ethiopia and purchased by animal collector Johan Schmidt. First known as the "Children's Giant Pet," the largest elephant known at the time was renamed Jumbo, a word believed to have been created from the combination of the Swahili words *jumbe* and *jambo*, meaning "chief" and "hello", respectively. [1]

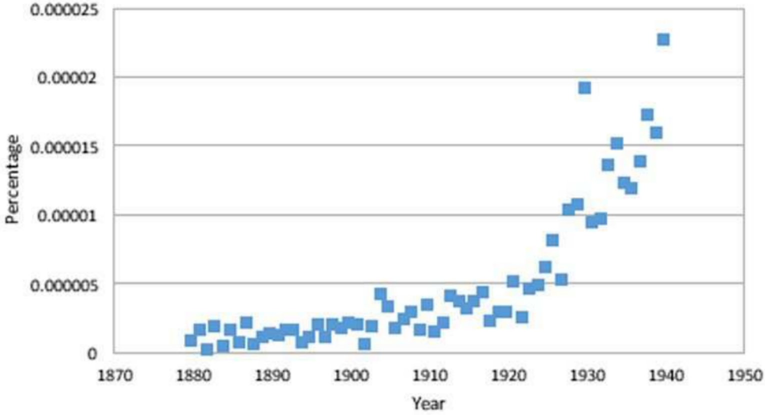


Figure 1. A scatter plot of percentage of printed words that are the word jumbo versus time for the years 1880 to 1940.

Set students free to find their own projects.

WARNING: Make sure it is worthy of differential equation structure, not just data fitting with curve.

PRIMUS, 23(3): 274–290, 2013

ISSN: 1051-1970 print / 1935-4053 online
DOI: 10.1080/10511970.2012.753966

Taylor & Francis
Taylor & Francis Group

Browsing Your Way to Better Teaching

Brian Winkel

Abstract: We describe the use of browsing and searching (in libraries, online, inside sources, at meetings, in abstracts, etc.) as a way to stimulate the teacher of undergraduate mathematics, specifically in differential equations. The approach works in all other areas of mathematics. Browsing can help build new and refreshing teaching materials based on how mathematics is used and explored in places other than mathematics. These "other" places are where almost all of our students will be going after they study with us and we should: (i) know about their journey and arrival points; and (ii) understand the disciplinary approaches for those areas which sent these students to us in the first place for their mathematics studies. We describe a personal browsing experience that spanned almost 40 years and proved to be very worthwhile in finding applications of differential equations to modeling Lysergic Acid Diethylamide in the human body.

Examples of browsing

- Collegial Conversation
- Conference and Meeting Presentations
- Invited Speakers
- Friends and Colleagues Who Know Your Own Interests
- Cognate area textbooks and journals
- The Projects or Starred Exercises in the Text
- New Issues of Journals in Your Institution's Library
- Back Issues of Journals on Dusty Library Shelves
- The Internet - Google Search - Doh!
- Older Books in Your Library and On-Line
- On-Line Course Descriptions and Courses

More examples of browsing

- Newspapers and Magazines
- SIMIODE - Doh!
- Professional Society Websites
- Blogs
- Funded sites - government and foundations
- COMAP - Consortium for Mathematics and its Application
- General Online Support Materials
- YouTube Videos
- Student Assistants
- On a walk through campus
- Online data sets - Census, Covid, Malaria, etc.

Work modeling into your classes all of them.

Take your time and . . .

try not to do too much all at once.

Give students something challenging, interesting, real, and with a view to more mathematics to do even better in modeling, e.g., consulting article and writing for audience - client and senior consultant - is one framework.

INT. J. MATH. EDUC. SCI. TECHNOL., 1990, VOL. 21, NO. 3, 363–368

First year calculus students as in-class consultants

by BRIAN J. WINKEL

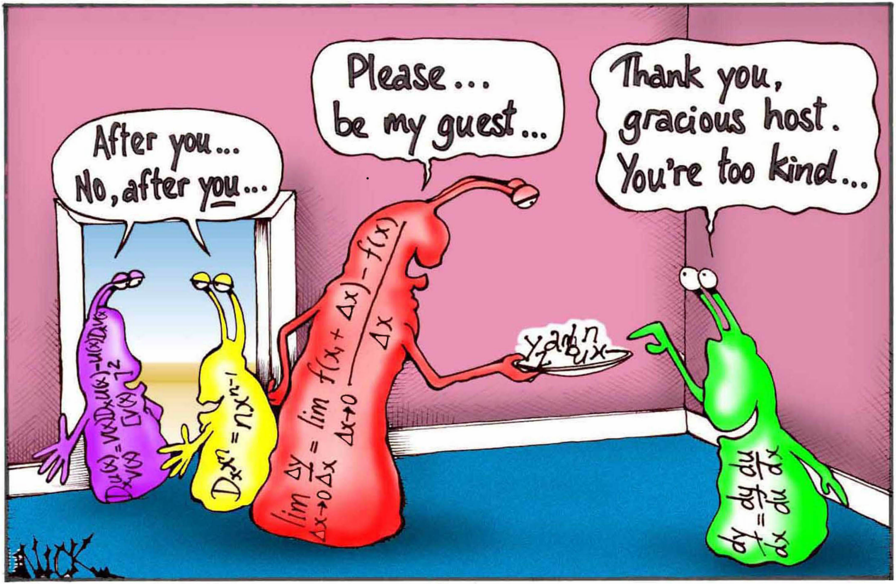
Department of Mathematics, Rose-Hulman Institute of Technology, Terre Haute, Indiana 47803, U.S.A.

(*Received 7 June 1988*)

We demonstrate how we use traditional text book calculus problems to involve first year calculus students as consultants. Students are asked to solve a 'real world' problem after interviewing 'guest' client—a printer. Students go through the initial steps of interviewing client, extracting information, ascertaining the problem, formulating the problem, solving the problem, and writing up two 'solutions'—one is a step by step solution for the client and the second is a technical support document for the 'senior consultant'. This description can serve as a model for further student involvement with fresh approaches to applications of the calculus.

Thank you for visiting with me.

Questions, Comments, Discussion . . .



Deferential equations.