It is fourth and one from the 40 yard line. Should I try a deep 57 yard-field goal? Should I punt or maybe even go for it? Today NFL Coaches are dealt with this type of decision on a game-by-game basis. The conventional wisdom throughout the NFL is to punt on most fourth down situations not in field goal range. However, a coach should wonder if this decision is optimal given the situation. Does my defense really benefit much from just an average of an extra 38 yards (net punt league average) to cover the field? Is that really worth giving up possession of the ball? These are questions that all NFL coaches should think about when making a critical fourth down decision in a game.

This paper investigates a mathematically rigorous way to determine whether a coach should go for it, kick a field goal or punt. The optimal decisions obviously change depending on your yard line, and how many yards you must go to convert the first down. Just these factors however, are clearly not enough to give an accurate representation of whether a team should go for it or not. One also has to consider the offensive and defensive prowess of both the team making the decision and the opposing team. These are the variables considered in our analysis.

The dataset used in our project was rich in nature. It had the play by play information for every game from the 2002 to 2012 seasons. To evaluate our data, we decided to exclude all playoff, pro-bowl, and preseason games. Our project is strictly analyzed from regular season data in the NFL from the 2002-2012 seasons.

The optimal decision is determined from an expected value calculation, given a certain decision was chosen. The average NFL coach or fan might not see the relevance. The purpose of the optimal decision is to win the game, not just increase the expected point margin. But, the graph below might give you pause. This plot shows the very strong, positive linear relationship between expected point margin and winning percentage. These figures are calculated per team, per season over the seasons 2002 – 2012 in the NFL. Expected point margin is an important variable to examine when making a fourth down decision in the early to mid-stages of the game.

Expected point margin is an important factor for determining a decision, but there are situations when this obviously should not be the case. The first oddity and solvable problem are decisions in the fourth quarter. Obviously, it would not be wise to go for a fourth down only because the expected point margin is greater for that particular decision. In the fourth quarter, coaches and management must be playing the score, as well as the clock. Because our model does not account for the clock, all fourth quarter observations were thrown out as well as all observations with less than 4 minutes to go in the first half. If you are interested about decisions in the fourth quarter, you can read about a win percentage model. This model is usually just relevant for fourth quarter situations, because not enough teams go for fourth downs earlier in the game. Our model is purely constructed to give optimal decisions in the first three quarters (excluding the last four minutes before half), where your team most likely has a greater chance of winning if the expected point margin increases.

The expected values are calculated in a complex way, because the observations are strange for usual football analysis. If a team decides to punt, then that team would obtain possession of the ball after the opposing team drives the ball down the field and either scores or does not score. If a team goes for it and converts, they give the ball to the other team after their possession. Obviously, one cannot just calculate what happens in the possession after a decision has been made because they don’t measure the same thing. Depending on the choice, at the end of the observation either the team making the decision or the opposing team would have possession of the ball. Therefore, the observations in our study measure what happens regardless of decision until the team making the decision gets the ball again on offense.

So, if the decision is to punt for example, then the team making that decision would punt the ball and evaluate the expected number of points the other team would score until they get the ball back. The expected number of points the other team would score given you punted from a certain yard line would be the negative expected point margin. So, for example, if the team making the decision decided to punt from their 30-yard line, and given that decision the other team was expected to score 1.7 points after that punt, then the point margin differential would be -1.7.

The expected point margin given that you go for it on fourth down is calculated in a similar fashion as the punt, except it is a little more complex because a greater number of things can happen. If you convert, then you must calculate the expected number of points your team will score after that decision. Then, after this drive you must calculate the expected number of points the other team will score against your defense, in order to create an observation in which you get the ball when the observation is over. If you convert the expected point marginal difference will be estimated by subtracting these two numbers. If you do not convert, the calculation is the same calculation as the punt. You simply calculate the expected number of points the opposing team will score given they take over where you attempted the conversion. Then to calculate the full expected value of going for it, you must combine the probability of conversion and non-conversion, with the point marginal difference of conversion and non-conversion respectively.

If you decide to kick a field goal, the calculation is similar to the calculation of going for it because you can either convert or not convert the field goal. If you converted the field goal you scored three points, you must subtract this number from the expected number of points the other team will score once they get the ball. If you do not convert the field goal, you must take the negative expected value of the other team’s drive given their new yard line.

One can see that now our observations are all measuring the same thing: the expected point marginal difference given a decision. This is pivotal because it allows us to compare the three decisions directly. It is hard to think about the possibilities of the outcomes of these decisions in an NFL game because they are complex and multi-dimensional, but our model creates a way to make comparisons that are valid and direct.

Because of the nature of our observations a coach must be wary not only for fourth quarter situations, but also decisions towards the end of the first half. If there is not much time left our observations are irrelevant because some of the calculations are based on the other team getting the ball after your possession. This may not happen if there is very little time left in the half. Once again, our model is not without its flaws. If clock management is an important factor in the decision at hand, the fourth down decision should be evaluated by the coach and/or upper management.

After researching and deciding on our implementation method , we needed to try and figure out a way to rank the teams 1 to 32 on offense and defense (1 = best to 32 = worst). In order to rank teams first we calculated the average number of points scored and given up per game per season. The best offensive team for a particular season would be the team that scored the most points per game. The best defensive team for a particular season would be the team that gave up the fewest number of points per game. Then, we plotted the number of points scored and given up for the 32 teams per season. One could see that because our time frame is rather long (11 years), the teams average number of points scored and given up fluctuated greatly from season to season. For example, the San Francisco 49ers went from one of the worst defenses in the league in 2007 to one of the best in 2012.

Because fluctuations were so dramatic for the teams, we chose to rank the teams 1 to 32 per season. So, the number one ranked offensive team for example could change from the Patriots in 2002 to the Packers in 2003. After, we made this correction our plots of ranking and years look much better for both offense and defense.

The graphs below show the top four and bottom two teams are many times outliers in particular seasons. Many times the difference between these outlying teams expected points is greater than the difference for the teams in the middle. Because of this we created dummy variables, one for those teams ranked 31 and 32, and one for those teams ranked 5 – 30 (for both offense and defense). The dummy variable between 5 and 30 are linearly related so we multiply this linear factor by the 5 – 30 dummy. However, teams 31 and 32 are close enough that we simply create a 0-1 dummy variable. Because of these dummies, our models are estimated in comparison to offenses and defenses ranked 1-4.

After evaluating and changing team ranks, a valid estimation of the probabilities of conversion for field goals as well as fourth down conversions must be performed. For each of these probabilities the outcome is binary, either the kick or fourth down attempt was converted or not. Therefore, we must implement a model that is used for a binary response. The types of models that are most commonly performed in these cases are logistic regression models or probit regression models. In our particular case we will use the logistic regression model because our model may not be normal, which is an assumption of the probit regression. The assumptions of the logistic regression model are that the response is binary, and the explanatory variables are independent. Obviously, the response is binary, and we found no reason that in either equation our explanatory variables were mutually related. Therefore, we ran our models and computed 95 percent confidence intervals for given situations.

From the graph below, it is obvious that the probability of a conversion attempt is very close to 1 when it is a short kick (a kick with 0 yards to your goal line, or a 17 yard field goal). This is not surprising at all, because extra point attempts are from 20 yards away and kickers make about 98 percent of extra point attempts. However, as one gets further and further away it may surprise some people that the percentage decrease for very long field goal attempts isn’t more drastic. When you have 50 yards to go (a 67-yard field goal) our model predicts about a 20 percent conversion rate. This obviously is too high; no one has ever converted a field goal of longer than 63 yards. There are three major reasons why these probabilities are clearly inflated. Firstly, only the kickers with huge legs like Sebastian Janikowski kick field goals greater than 60 yards. Secondly, they usually kick these field goals in nice weather, possibly in Denver, where the altitude might give a kicker 5 or so yards of added range. And lastly, 67 yard field goals are not in our data set. The logistic regression model does not accurately predict response values when the explanatory variables are outside of the range in the dataset. For these reasons, deep field goal probability calculations are too high. From about the 40 yard line and in, however, our results are more than believable. But, field goal success rate also depends on your kicker and on the weather. These results average over all kickers and different weather. If you are kicking in snow, or have a terrible or great kicker, the coach should adjust the expected value of kicking a field goal accordingly.

First off, we must explain fourth downs in our data set. Because all fourth quarter plays were extracted from our data set, and the prevailing conventional wisdom is to not attempt fourth downs unless it is a late game situation, there weren’t enough fourth down plays to make accurate predictions. Therefore, we accounted for this by calculating the probability of third down conversion. This seems odd, but in today’s NFL, the third down is of a similar mind set to fourth down if you went for it. Teams aggressively try and convert the first because they nearly always punt if they do not convert. There are however, some third down situations that are not similar to fourth down situations. For instance, many times on third and long, teams will run the ball and concede punting just to not throw an interception or create more room for their punter. In these situations, if it was fourth down they would obviously be going more aggressively for the first. Because of this, all third down plays of greater than 10 yards which were runs were taken out of the model. We felt these observations would bias our results.

After using these particular plays to model fourth down plays, we know that the probability of a fourth down conversion should change depending on the strength of the offense and opposing teams’ defense, the number of yards to go for a first down, and the field position. From our model, you can see that as the yards to go increases the probability decreases. Also, the better the offense is in comparison to the defense increases the probability of conversion. And the yard line we turned into a categorical variable. This categorical variable indicates whether the fourth down conversion is from the other teams’ 0 – 10, 10 – 30, or 30-50 yard line, or your own 0-10, 10-30, or 30-50 yard line. You can see from these probabilities that the conversion rate from the other teams’ 0-10 is lowest (it is the one left out of the equation) because all the numbers are positive. Then, as one might expect the conversion rate if you have greater than 90 yards to go is lower in relation to the other yard lines as well because teams are nervous about safety possibilities and other factors.

The following graph gives one particular example, of the effect of the number of yards to go on the probability conversion. This graph assumes that the offense and opposing teams defense are both near average (15th and 15th ranked), and the team making the decision is on the 50 yard line. You can clearly see that the probability of conversion is about .7 with one or fewer yards to go, and steadily goes down to about .1 with 20 yards to go. These numbers are pretty consistent with the numbers published in a few papers about league average (fourth and less than 1 is about .68) (according to one paper I read – need to find it!).

If the offensive team’s rank is high (5th) and the defensive rank is low (25th) one can see that the probability increases slightly when comparing across yards to go from the previous plot. Instead of the graph steadily decreasing from about .68 to .1 (0 to 20 yards away), the probabilities range from .72 to .14.

The expected number of points is calculated by multiplying the possible outcomes of a drive by the point values given those outcomes. In each drive there are five possible outcomes: a touchdown or field goal scored by the offense, a safety or touchdown scored by the defense, and no points. These numbers are multiplied by the points of each outcome: 7, 3, -2, -7, and 0 respectively. For the model, we assume every touchdown is 7 points (not 6 or 8) which is true the vast majority of the time. In order to estimate the probabilities of these outcomes we decided to use a multinomial logistic regression model. A multinomial model is used for a categorical response variable (here five categories) when the explanatory variables are independent. Here, the explanatory variables for our model are the offensive rank and the defensive rank of the two respective teams as well as the starting field position. The assumption of independence seems reasonable.

If you notice, the calculations can get complicated if for instance the team who decides to punt gets a defensive touchdown on the other teams next drive. Or, if the opposing team to the one making the decision scores a safety because they get the ball back on offense after this play. These two situations were accounted for and implemented by simply multiplying the probability of the safety or defensive touchdown by the expected values of the next drive. By accounting for these possibilities we ensure that the observations are consistent.

From the multinomial and logistic calculations, we can calculate the expected point values given each decision. Then a confidence interval is computed for each of the three scenarios by taking each of the lower bounds and upper bounds of the combined intervals and adding them together. Because of the complexities in this calculation the estimated line is not in the middle of the confidence interval. Also, this interval is technically greater than 95 percent because we are combining other 95 percent confidence intervals.

The next plots illustrate how the yard-line affects the optimal decision, with given offensive and defensive ranks of both the team making the decision and the opposing team (all ranks around 15). In the first plot the optimal decision is to kick a field goal regardless of the number of yards to go. However, with less than one yard to go the confidence intervals overlap which means it may be optimal to go for the first down. The coach should make the final decision if the intervals overlap. With 50 yards to go, and the same rankings, the optimal decision changes in a pretty major way. Now, the graph indicates that a team should go for the first down on fourth and less than 6, between 6 and 12 yards either punt or go for it, and past 12 yards punt. From a team’s own 20 yard line (80 yards to the goal line), a team’s optimal decision is to go for it on fourth and less than 3, either punt or go for it between 3 and 9 yards to go, and punt the ball if there are more than 9 yards to go for a first.

This progression is common to all offensive and defensive ranks. The 3 charts below paint the picture for the optimal decision for every position on the field and any fourth down situation less than 20 yards to go for certain rankings of the offensive and defensive units of both teams. If the team making the decision has a high ranking offense and defense, and the opposing team has a low ranking offense and defense, the optimal decision in many circumstances is to go for it. Our calculations project that it is optimal to attempt the first down anywhere on the field which is fourth and less than 2. The optimal decision map also indicates that you should go for it from about fourth and 15 from the 50, as well as fourth and 7 from your own 3. These numbers are obviously very surprising, but they are partially high because of the unusually large difference between the teams’ rankings.

If the teams are ranked evenly for both offense and defense then our map become more conservative. Now, the map indicates that a team should attempt the first down conversion from fourth and 8 with 50 yards to go and fourth and 4 with 96 yards to go to the goal line.

The optimal decision map becomes even more conservative when the team making the decision is weak on both offense and defense, and the opposing team is strong offensively and defensively. Now, the decision map’s optimal values are to go for the first down on fourth and four from the 50, and go for it on fourth and one with 91 yards to go to the goal line.

Obviously, the ranking of both teams’ offense and defense have an enormous impact on the optimal decision. For a high ranking offense, a coach should go for it much more often given other rankings remain constant. The dramatic changes in these plots purely due to teams’ offensive and defensive ranks are interesting, but how does the overall trend compare to current NFL fourth down decision making may be a better question? The plot below shows the decisions coaches and upper management have made over these 11 seasons. When comparing the maps, there are stark differences from our previous three optimal decision maps. The first major difference is that teams rarely go for it. In fact anywhere past the 45 yard line the decision most often made in every scenario (combination of yard line and yards to go) is to punt. One other noticeable feature is that our earlier plots had kickers going for a field goal attempt from much larger distances. Once again, this value is heavily inflated because of the higher than should be conversion rate of deep attempts.

Our analysis should not convince you that the results we obtained are perfect. The data used in our analysis is observational, and there is no way to prove causality. Therefore, coaches’ deep level of knowledge of the game should still play a major role in determining the fourth down decision. But, their football expertise along with an understanding of the estimated expected values could help guide their decision under uncertainty. The stark difference between our logical model and the current NFL’s usual decision is obvious. We still would not recommend going for a first down on fourth and seven from your own 3 yard line. However, our process makes intuitive and mathematical sense. And the optimal decision is most likely somewhere in the middle. Obviously, giving up possession of the ball by punting has a much greater negative effect than teams currently realize.

The model could be improved, and the assumptions fortified with the help of other statistician gurus. But, this is a good start in showing that something is off. The way current NFL coaches and upper management evaluate fourth down decisions is seriously flawed. In the next decade, statistics will probably play a more integral role in fourth down decision making because the first few teams that makes sense of the data will have a distinct advantage.