Intermediate Econometrics M1-TSE
Homework 1 - Fall 2023

Upload your work as a pdf file (of size <10 Mo) on Moodle before Thursday 5th of October at 8:00 p.m.. Late submissions will not be considered for grading and will receive a grade of 0. This homework must NOT be done alone. You have to be in a group of two or three members - the group members can be from different TD groups. You are not authorized to share/copy other groups' work.

Groups must be registered on Moodle and the document needs to be uploaded by one member of the group only. Please indicate the names of all group members in the file.

Handwriting is allowed but preferably use Office Word or LateX for both equations and written text. If your handwritten answer to a question is unreadable you will get 0 for that question.

Explain each step taken to find the solution including the theorems involved and the required assumptions unless it is a short answer (SA) question.

We advise you to spend no more than four hours on this homework. A good understanding of the course should make you able to complete the homework in less than this amount of time.

Total of points: 20

Exercise 1 (12.5 points)

We consider the following model:

$$y_i = x_i\beta + u_i \qquad E(u_i|\, x_i) = 0 \qquad Var(u_i|\, x_i) = \sigma^2$$

with $(y_i, x_i), i = 1, \dots, n$, an i.i.d. sample where x_i is a single random variable. All variables have a bounded moment of order 4: $E(x^4) < \infty$ and $E(y^4) < \infty$. We also assume $E(x^2) \neq 0$, $E(x^3) \neq 0$, $E(x^4) \neq 0$.

Consider the following estimators:

$$\tilde{\beta} = \frac{\sum_{i=1}^n {x_i}^2 y_i}{\sum_{i=1}^n {x_i}^3}$$

$$\breve{\beta} = \frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n {x_i}^2} + \frac{1}{n}$$

$$\tilde{\tilde{\beta}} = \frac{1}{2}\left(\frac{\sum_{i=1}^n x_i y_i}{\sum_{i=1}^n {x_i}^2} + \tilde{\beta}\right)$$

Question 1. Are these estimators consistent for β ?

Question 2. Find the asymptotic distribution of $\tilde{\beta}$ and $\breve{\beta}$.

Exercise 2 (7.5 points)

We have a sample of 2,324 working men aged 25 to 55 who receive an hourly wage rate whose logarithm is denoted y_i. We observe the following variables:

- Education level: a dummy variable bac equal to 1 if the level of education is a general baccalaureate or a higher education diploma, 0 otherwise.
- Age: a dummy variable $senior$ equal to 1 if the age is over 40, 0 otherwise.

We consider the following linear model

$$y_i = \beta_0 + \beta_E bac_i + \beta_A senior_i + u_i$$

with $x_i = (bac_i, senior_i)$ and $E(u_i|\, x_i) = 0, Var(u_i|\, x_i) = \sigma^2$.

The following estimation results are obtained using ordinary least squares

$$\hat{\beta}_E = 0.258\ (0.013), \hat{\beta}_A = 0.132\ (0.012)$$

with standard errors in parenthesis.

Question 1. We want to test H_0: $\beta_E \leq 0.20$ against H_1: $\beta_E > 0.20$ at the 5% level. Give the test statistic and its behavior under H_0. Give the formal decision rule, compute the test statistic and conclude. What economic conclusion do you draw from this?

Question 2. We want to test H_0: $\beta_E = \beta_A$ against H_1: $\beta_E \neq \beta_A$ at a level of 5%. Describe a test procedure by explaining the formula for the test statistic. Give its behavior under H_0. Give the formal decision rule. What information is missing in the results to calculate this test statistic?

Question 3. What is a confidence interval for β_A at confidence level 95%? Give the formula and the result. Interpret this confidence interval.