Managing the risks of energy industry
Lecture 2: Electricity markets and derivative products

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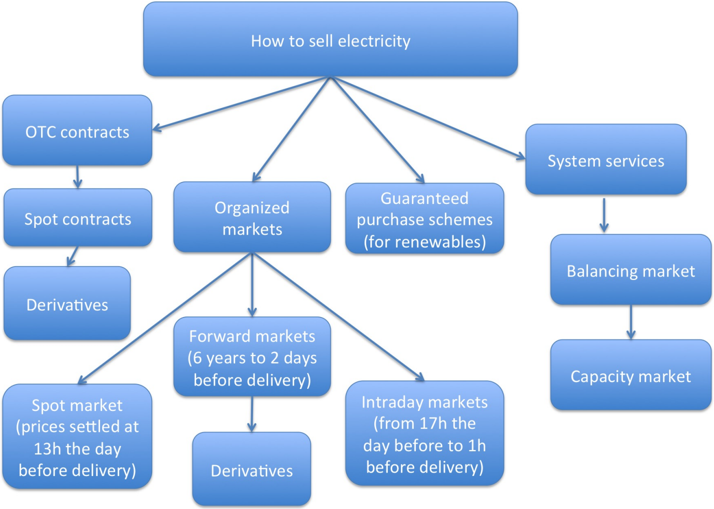
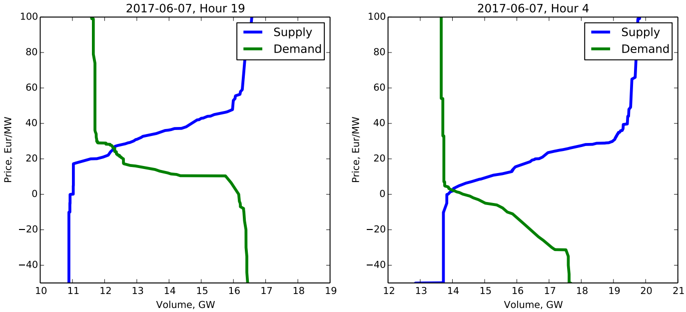
How to sell electricity



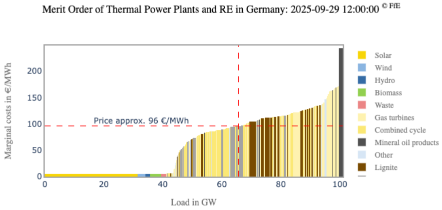
Outline

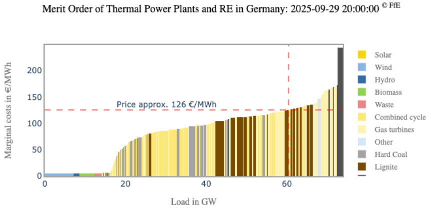
The spot (day-ahead) market

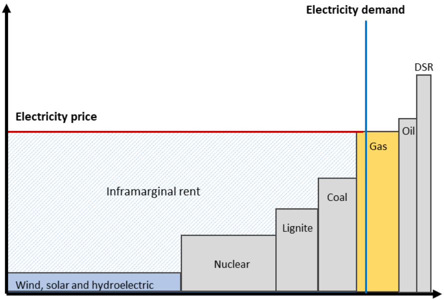
- One of the main trading venues for electricity is the day-ahead market (EPEX Spot in France/Germany).
- In this market trading happens only once: participants submit bids for specific 15-minute intervals or blocks of intervals of the next day until 12:00, then at 12:55 the price is fixed and market clears.



Merit order / generation stack





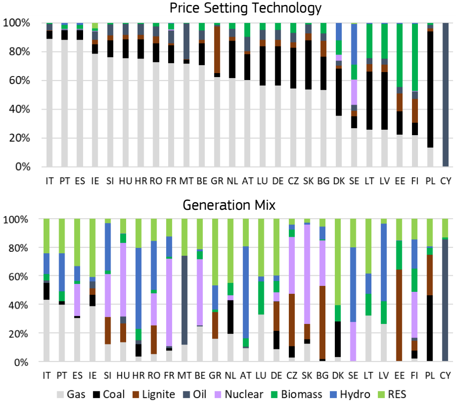


Source: EC JRC

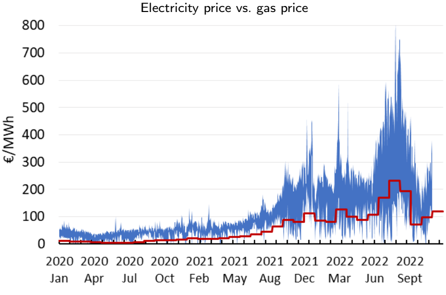
Emergency measures in late 2022 limited inframarginal rents for electricity prices above 180 Eur/MWh

Source for left graph: `ffe.de`. Most other graphs are form “Merit Order and Price-Setting Dynamics in European Electricity Markets”, EC JRC (2023).

Gas is the main price-setting technology



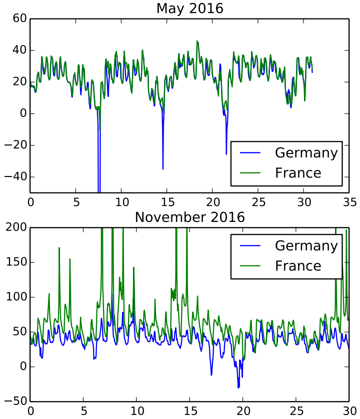
Source: EC JRC



Source: EC JRC

Market coupling mechanism

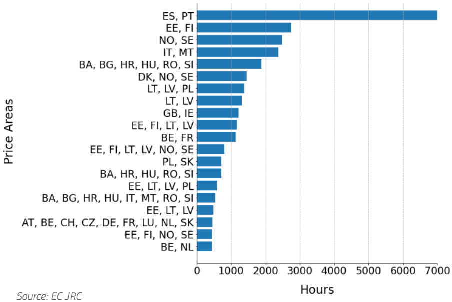
- The European electricity market is split into bidding zones, but due to market coupling prices in different bidding zones coincide in absence of binding transport constraints
- As long as interconnection capacity permits, demand in one market may be matched by supply in any other market
- If the transport constraints become binding, the prices decouple



Day-ahead prices in France and Germany. In May, prices are coupled almost all the time, except during negative spikes in Germany. In November, prices are decoupled. Data source: `transparency.entsoe.eu`

Bidding zones and price convergence

Source: EC JRC

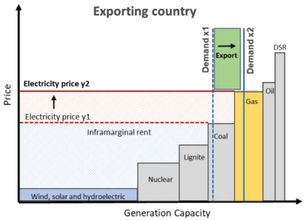


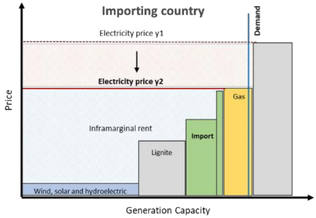
Source: EC JRC

Left: bidding zones are at the heart of market design: trading inside the zones is unrestricted

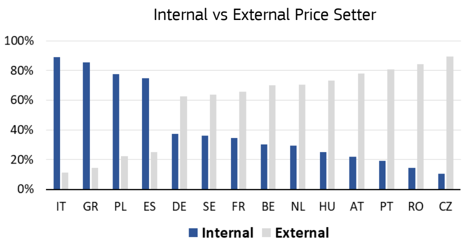
Right: price convergence between bidding zones, total hours in 2022

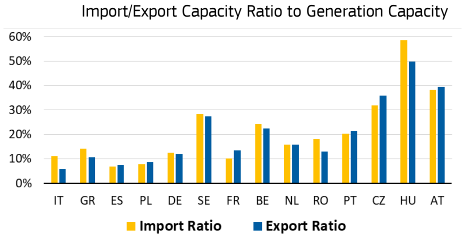
Impact of import/export on market prices





Source: EC JRC

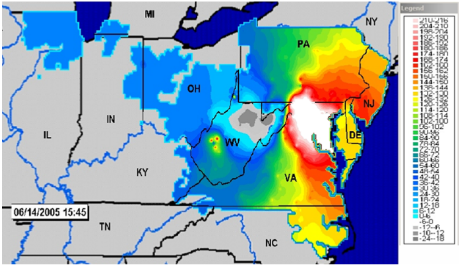




Source: EC JRC

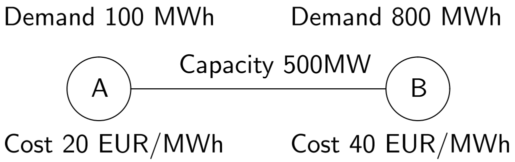
Zonal vs. nodal pricing

- An alternative system implemented in some countries (e.g. PJM market in the US) is the nodal system with locational marginal pricing
- Each node of the network is considered seperately in relation with available transmission capacities
- Optimal dispatch of power plants is calculated based on marginal costs of production



Heat map of locational marginal prices in the PJM network. Source: PJM

Example of LMP calculation



- If there was no transmission limit, the network price would be 20 EUR/MWh and all electricity would be generated at node A
- With limit, marginal price is 20 EUR/MWh at A and 40 EUR/MWh at B, with 600 MWh produced at A and 300 MWh produced at B
- The network operator collects revenues of $40 \times 800 + 20 \times 100 = 34000$ and pays to the generators $600 \times 20 + 300 \times 40 = 24000$
- Difference: congestion revenue, used for infrastructure improvement

Financial Transmission Rights (FTRs)

- Cross-zonal or cross-nodal transmission constraints create *price differences* in electricity markets.
- FTRs are financial instruments that hedge these price differences.
- Two main forms:

$$\begin{aligned} \text{FTR option:} \quad & \max(P_{\text{sink}} - P_{\text{source}}, 0), \\ \text{FTR obligation:} \quad & P_{\text{sink}} - P_{\text{source}}. \end{aligned}$$

- FTRs are *purely financial* — they do not grant physical transmission rights.

Europe: FTRs and Implicit Market Coupling

- European markets used **zonal pricing**
- Cross-zonal capacity is allocated **implicitly** in the **Single Day-Ahead Coupling (SDAC)** using the EUPHEMIA algorithm.
- Long-term transmission rights (LTTRs), usually implemented as FTRs:
  - Allocated in forward auctions (JAO).
  - Settled against **day-ahead zonal price spreads**.
  - Do not interfere with implicit allocation.
- Design objective: maximize physical efficiency and price convergence.

Timeline

1. **Forward (LT):** TSOs auction LTTRs (FTRs) via JAO; capacity remains fully available to SDAC.
2. **Day-Ahead (SDAC):** Implicit coupling sets flows and zonal prices (P_A, P_B).
3. **Settlement:** FTRs pay the DA price spread; physical flows unchanged.

Who Bears the Risk? Congestion Rent as a Natural Hedge

- FTRs are issued by TSOs (jointly on each border).
- When prices diverge, FTR holders receive a payoff.
- The same price divergence generates **congestion rent** in SDAC:

$$\text{Congestion rent} = (P_A - P_B) \times \text{flow}.$$

- Congestion rent is used to fund FTR settlements.
- **Revenue adequacy principle (FCA/HAR):**
  - TSOs are revenue-neutral.
  - Curtailment and compensation rules handle outages or decoupling.

EU vs US FTR Markets

Europe (Zonal)

- Zonal prices
- FTRs hedge *zonal* price spreads
- Primary allocation via JAO auctions
- No organized secondary market
- Design focus: physical efficiency and market coupling

United States (Nodal)

- Nodal pricing (LMPs)
- FTRs hedge *nodal congestion*
- ISO-run auctions with netting
- Active reconfiguration auctions
- Design focus: financial completeness

Key message: Europe treats FTRs as a supporting hedge; U.S. markets treat them as a core financial asset.

US FTRs: Thousands of Nodes, Yet Liquidity

- U.S. markets have **thousands of pricing nodes**.
- Liquidity does *not* come from trading every node-to-node path.

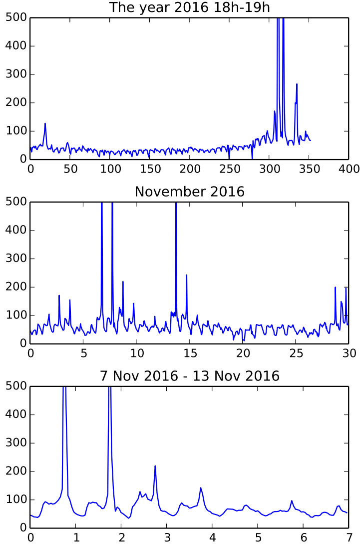
Why liquidity exists:

1. **Aggregation**
   - Trading hubs (e.g. PJM Western Hub, CAISO NP15).
   - Zones, interfaces, and constraints.
2. **Centralized ISO auctions**
   - Annual, monthly, and reconfiguration auctions.
   - Positions are netted system-wide.
   - ISO acts as central counterparty.
3. **Approximate hedging**
   - Market participants hedge node exposure with hub/zone FTRs.
   - Residual basis risk is accepted.

Features of spot electricity prices

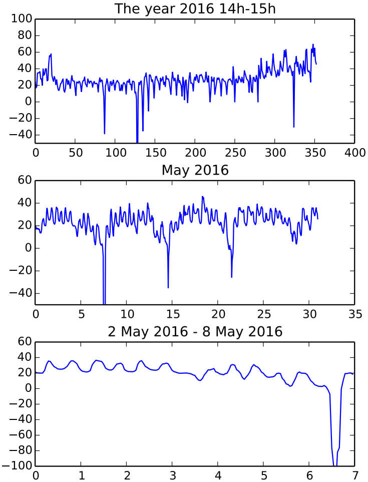
- Spot electricity prices possess daily, weekly and annual seasonality
- Prices are highly correlated with consumption and in countries where electricity is used for heating / air conditioning, with the temperature
- Due to non-storability, prices exhibit spikes which occur, e.g., in case of plant outage, especially in winter

Day-ahead prices in France. Data source: `transparency.entsoe.eu`



Features of spot electricity prices

- Negative prices: since it is costly to shut down coal-fired and nuclear plants, producers are ready to pay to keep the plant running
- This phenomenon is particularly important in Germany due to the large-scale production from renewable sources (at zero marginal cost)



Day-ahead prices in Germany. Data source: `transparency.entsoe.eu`

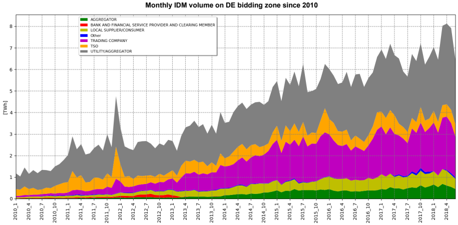
Outline

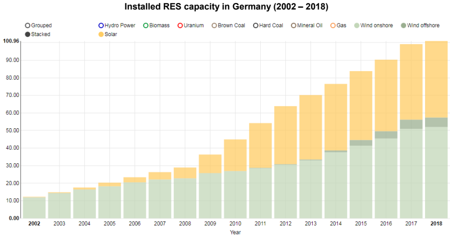
The intraday maket

The intraday market opens at 15h and allows continuous trading for each hour/quarter-hour of the next day, up to 5–30 minutes before delivery.

Every delivery hour of every day corresponds to a different product: the life time of a single product is from 9 to 32 hours.

The development of intraday market is largely driven by increasing penetration of renewables.

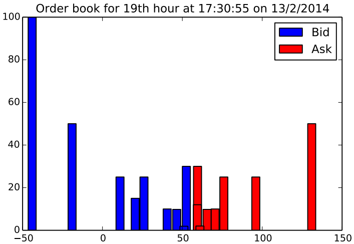




Source: EPEX Spot / P. Vassilopoulos

The intraday market

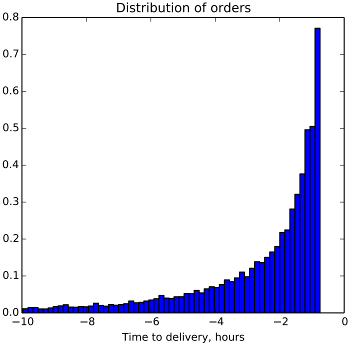
- Trading in intraday markets is order book-based, with a separate order book for each delivery hour.
- Each country has a separate intraday market, but the markets are coupled: if transmission capacity exists, traders in any market see the orders from other markets in their order books.

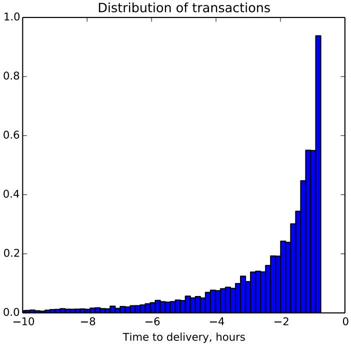


Intraday electricity markets are gradually acquiring the characteristics of other high-frequency markets with automated trading, optimal execution algorithms, presence of arbitrageurs, price manipulation attempts etc.

Intraday market liquidity patterns

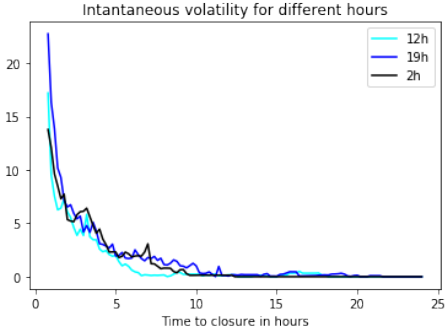
Liquidity only appears a few hours before delivery.

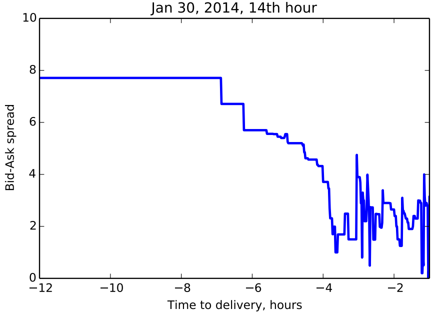




Distribution of orders/transactions as function of time to delivery for all contracts expiring in February 2015.

Bid-ask spread and volatility

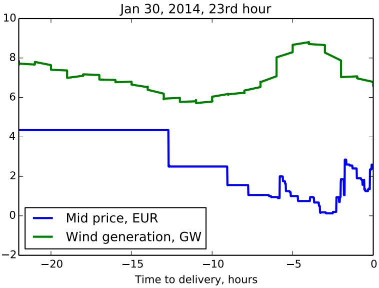


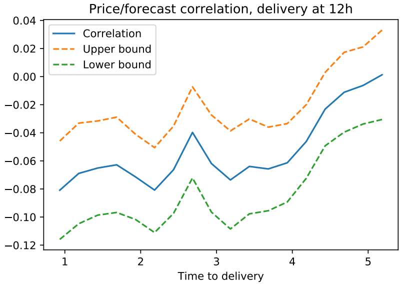


Left: (Normal) volatility averaged over all days of February 2014 (kernel estimator, source: L. Tinsi). Right: bid-ask spread evolution on a typical day.

The intraday market: effect of renewables

The development of intraday markets has been fueled by the expansion of intermittent renewables: prices are negatively correlated with renewable production forecasts.





Left: dynamics of intraday price vs. wind production in Germany. Right: empirical correlation between price and wind power production forecast in the Germany delivery zone. Data source: EPEX spot

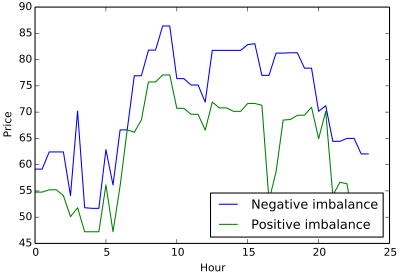
Outline

The Balance Responsible Entity system

- Balance Responsible Entities (BRE, responsables d'équilibre) are basic agents of the French electricity markets
- A BRE, declares to RTE its balance perimeter: portfolio of activities such as
  - Physical sites consuming or generating powe
  - Purchases and/or sales on the power exchanges operating in France;
  - Purchases and/or sales of electricity from/to counterparts;
  - Energy exports and/or imports;
  - Sales of energy to RTE to compensate losses.
- All energy production / consumption must be affected to a balance perimeter of a BRE
- All imbalances within the balance perimeter are compensated to RTE using the imbalance price.

Imbalance prices

- In case of system imbalance, the network operator compensates over-producing agents and applies a penalty to under-producing agents.
- The compensation price and penalty are fixed to enable the network operator to recover the cost of using additional generation.

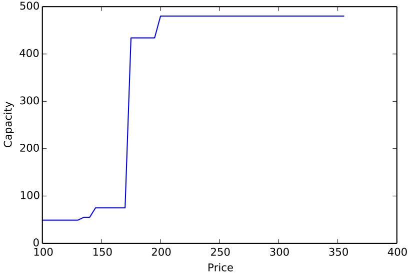


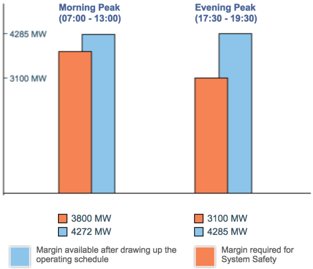
	Balancing trend > 0	**Trend** < 0
Imbalance > 0: actor is paid	$VWAP_u(1-k)$	$VWAP_d(1-k)$
Imbalance < 0: actor pays	$VWAP_u(1+k)$	$VWAP_d(1+k)$

Here $VWAP$ is the volume weighted average price of the balancing and $k = 0.05$.

The balancing mechanism

- The balancing mechanism (adjustment market) allows the network operator (RTE) to ensure precise overall balance between generation and consumption for the entire system (reconstitute primary and secondary reserve).
- Market players submit bids for increasing production (or reducing consumption).

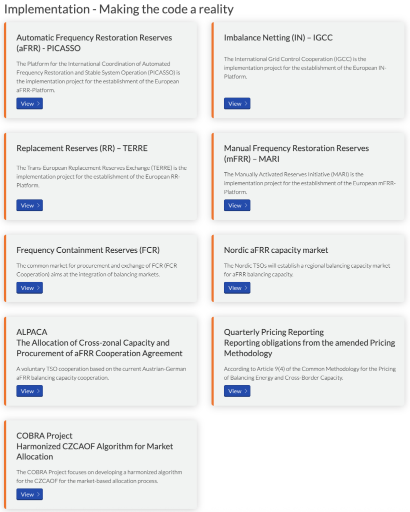




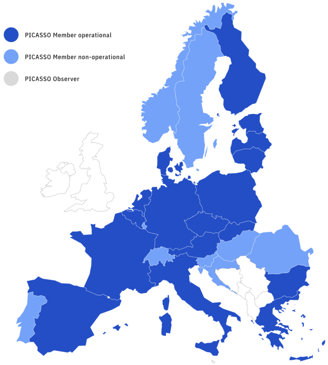
Capacity / price curve (left) and available margins on 15/12/2016. Data: RTE

Electricity balancing in the EU

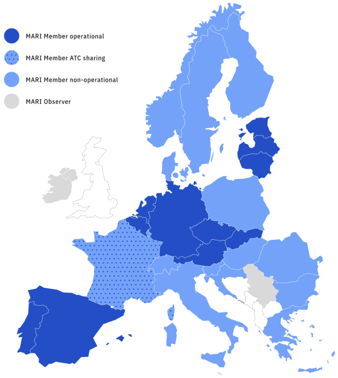
- The European regulation "Electricity balancing" aiming to create a Europe-wide adjustment market was approved by the European Commission in March 2017 and is progressively implemented in a series of projects.
- The goal is to create a market-oriented framework automatic frequency restoration and primary and secondary reserves at the European scale.



Frequency restoration reserve pooling



PICASSO implementation project (as of July 2025)



MARI implementation project (as of December 2025)

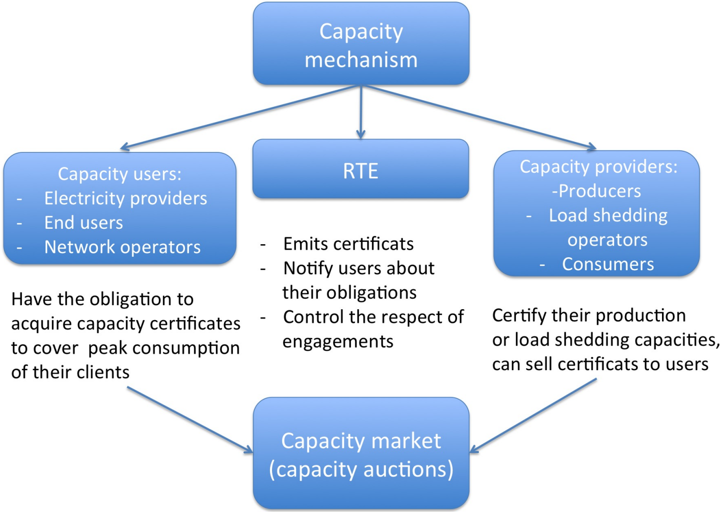
PICASSO (Platform for the International Coordination of Automated Frequency Restoration and Stable System Operation) and MARI (Manually Activated Reserves Initiative) are fully operational and active. See `www.entsoe.eu/network_codes/eb/` for status of other projects.

Outline

Effect of renewables on the market evolution

- The cost of renewable energy has declined dramatically in the last decade (by 70% for onshore wind and 90% for PV)
- Renewable generators, producing at zero marginal prices, are driving baseload generators out of the market (39 GW of coal generation was retired in the US between 2017 and 2019)
- To handle the intermittency of VRE, system operators need to activate costly flexible resources to meet demand
- This can raise total system costs even as renewable costs decline
- Financing flexibility in an energy-only market may lead to very high peak prices, which are not acceptable politically
- As a result, alternative market design are introduced in various countries

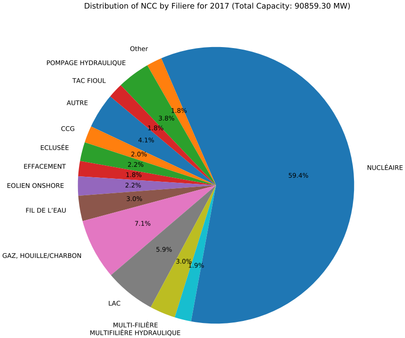
The capacity mechanism/market

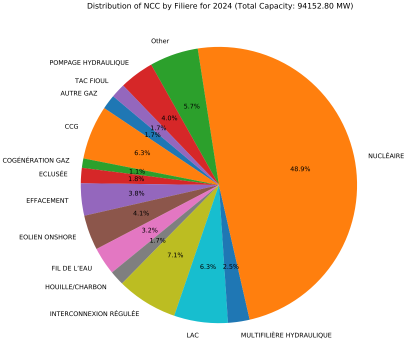


The capacity mechanism in France

- Introduced in 2017 until 2026
- Annual certification up to 4 years before the delivery year
- Via the 'certification perimieter responsible entity', financially responsible for capacity violations
- Two certification methods:
  - Standard method: based on realized values observed during peak periods
  - Normative method (for renewable energies): based on historical production multiplied by a coefficient
  - Capacity market in France is characterized by high volatility and relatively low prices
  - Additional income for producers but not sufficient to finance new investment

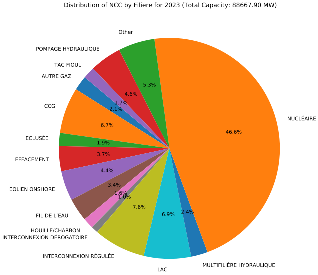
The capacity mechanism in France

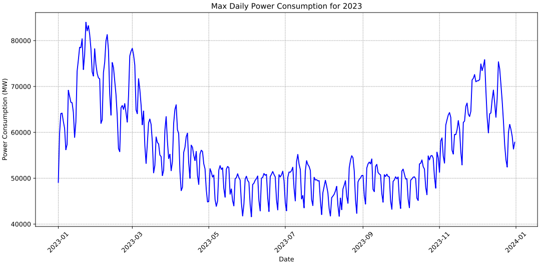




Left: certified capacity in 2017. Right: certified capacity in 2024.

The capacity mechanism in France

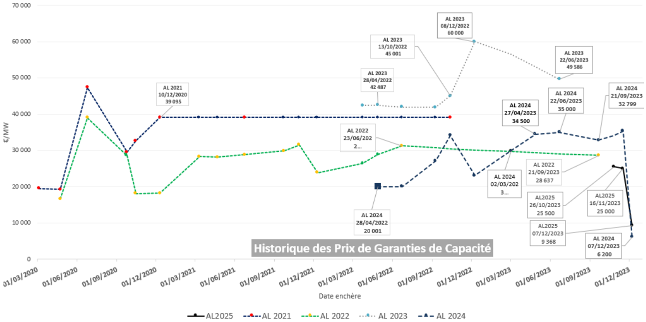




Left: certified capacity in 2023. Right: maximum daily consumption in 2023.

Capacity market in France

Delivery	Auction date	07/12/2023	25/04/2024	20/06/2024	19/09/2024	24/10/2024	05/12/2024
2026	**Price**	-	15,538.1	-	6,124.7	3,540.2	2,521.8
	Quantity	-	2,850.9	-	4,358.6	2,965.2	4,203.6
2025	**Price**	9,368.3	19,999.6	14,999.9	10,800.9	6,191.6	0.00
	Quantity	6,572.4	4,165.3	4,181.1	3,760.4	4,092.4	10,277.7



Top: capacity auction results in 2024. Source: EPEX Spot. Bottom: historical results. Source: Haya Energy Solutions.

Capacity mechanisms in the EU

- **Strategic reserve:** (Belgium, Germany, Poland and Sweden) A central agency contracts capacity through a competitive tender. The contracted plants cannot participate in the market and are only activated in case of shortfalls.
- **Capacity auction:** (UK) The total required capacity is centrally procured in an auction. The new capacity participates in the energy-only market.
- **Capacity obligation:** (France) An obligation for large consumers or electricity suppliers to contract an amount of capacity linked to their self-assessed future consumption or supply, plus a reserve margin.
- **Reliability options:** (planned in Italy) A capacity provider enters into an option contract with a counterparty. The contract offers the counterparty the option to procure electricity at a predetermined strike price.
- **Capacity payments:** (Italy, Poland, Portugal, Spain, Ireland) Pre-determined fees are set by the regulator and paid to capacity providers. The plants receiving capacity payments continue to participate in the energy-only market.

Outline

Ancillary services markets

Ancillary services include capabilities of

- Frequency regulation (FCR/FRR)
- Reactive power generation (reactive power: no net power flow but losses and capacity limits in lines)
- Black start capabilities

Outline

Commodity futures

- For financial futures, cash and carry arbitrage yields

$$F_t(T) = e^{r(T-t)} S_t.$$

- In presence of storage cost c per unit of time and underlying,

$$F_t(T) = e^{(r+c)(T-t)} S_t,$$

- and for underlyings which cannot be sold short,

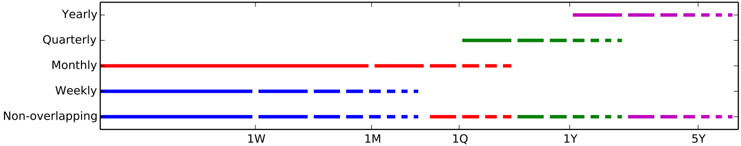
$$F_t(T) = e^{(r+c-y)(T-t)} S_t,$$

  where $y \geq 0$ is the "convenience yield" per unit of time and underlying.

Electricity cannot be stored at large scale at reasonable cost, so this relationship breaks down and one needs to model jointly the spot and future prices.

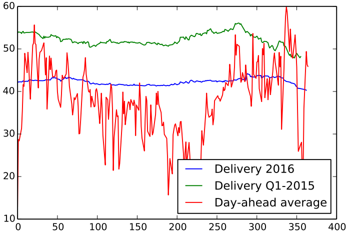
Electricity future contracts

- An electricity future contract (swap) specifies a delivery period
- A future with delivery between T_1 and T_2 settles financially against the average day-ahead price of this period



The forward market

- Futures prices are much less volatile than spot prices, especially for longer delivery periods.
- Due to non-storability of energy, futures prices are not correlated with spot prices and one cannot speak of convergence of futures prices to spot prices.
- Futures for winter delivery are more expensive.



2014 yearly and quarterly base-load futures prices compared to daily average of day-ahead prices. Data source: EDF.

Electricity future contracts: modeling approaches

- Introduce and model a fictitions instantaneous delivery contract

$$F_t(T_1, T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} f_t(T) dT;$$

$\Rightarrow$complicated dynamics for swap prices which are the underlying of options;

- Model directly the swap prices

$\Rightarrow$ complicated constraints on the volatility structure;

- Model the spot price and compute forward prices as risk-neutral expectations of spot price

$\Rightarrow$ calibration to the initial forward curve difficult

Modeling instantaneous-delivery contracts

Assume a multifactor log-normal dynamics for $f_t(T)$ under a risk-neutral probability measure $\mathbb{Q}$:

$$\frac{df_t(T)}{f_t(T)} = \sum_{i=1}^{n} \sigma_i(t,T)dW_t^i = \sigma^T(t,T)dW_t,$$

where W^i are independent Brownian motions (risk factors) and σ_i are risk factor volatilities.

- Log-normal modeling may be used for long-dated forwards which are not as volatile as the spot
- The number of factors in electricity markets is quite high since forwards of different maturities are loosely coupled
- This modeling is not compatible with lognormal swap price dynamics, often assumed by the market

Implied spot price dynamics

The spot price may be recovered as $S_t = \lim_{T\to t} f_t(T)$. Itô formula yields:

$$\frac{dS_t}{S_t} = \left(\partial \ln f_0(t) - \frac{1}{2}\sigma^2(t,t) + \int_0^t \sigma^T(s,t)\partial_2\sigma(s,t)ds + \int_0^t \partial_2\sigma(s,t)^T dW_s\right)dt + \sigma^T(t,t)dW_t.$$

- The spot price is not martingale under $\mathbb{Q}$ because it is not traded;
- The spot price dynamics may not be Markovian

Example

Assume an exponential volatility structure: $\sigma_i(t,T) = \sigma_i e^{-\lambda_i(T-t)}$. Then,

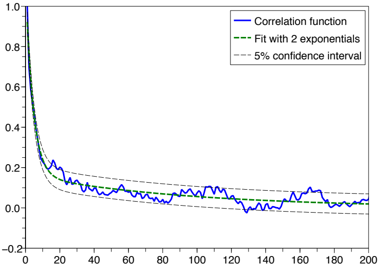
$$S_t = f_0(t)\exp\left(-\frac{1}{2}\int_0^t \|\sigma(s,t)\|^2 ds + \sum_{i=1}^n X_t^i\right)$$

where

$$X_t^i = \sigma_i e^{-\lambda_i t}\int_0^t e^{\lambda_i s} dW_s^i \quad \Rightarrow \quad dX_t^i = -\lambda_i X_t^i + \sigma_i dW_t^i$$

$\Rightarrow$ the spot price is the exponential of a sum of Ornstein-Uhlenbeck processes (may be used to model price spikes)

The graph shows the autocovariance function in the of spot price in Germany fitted with sum of 2 exponentials

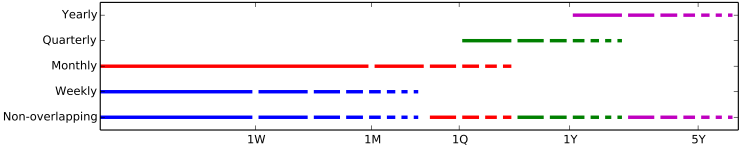


Modeling swap contracts

- An alternative is to model directly the dynamics of traded swap contracts.
- In the presence of overlaps, dynamics is constrained by

$$F_t(T_1, T_2) = \frac{1}{T_2 - T_1} \int_{T_1}^{T_2} f_t(T)dT.$$

- We choose to model the non-overlapping contracts, discarding some information.



A similar log-normal model $dF_t(T_1, T_2) = F_t(T_1, T_2)\Sigma^T(t, T_1, T_2)dW_t$ may be used.

Reduced-form price models

In reduced-form spot price models, one models the day-ahead electricity price directly with a Markov process, and the forward price is deduced by risk-neutral expectation

Usually one models the daily average since intraday structure is complex and irrelevant for forwards

Reduced-form spot price models must respect the following "stylized features":

- Seasonality;
- Mean reversion;
- Spikes and non-Gaussian behavior.

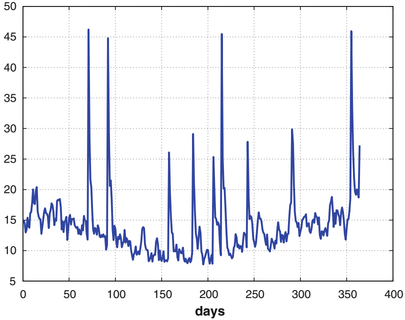
Cartea and Figueroa (2005) model

$$\ln S_t = g(t) + Y_t$$

$$dY_t = -\alpha Y_t dt + \sigma(t) dW_t + J \cdot dq_t$$

where

- $g(t)$ is a deterministic seasonality;
- J is a log-normal proportional jump size;
- q is a Poisson process of jump times.



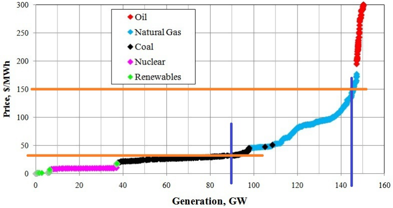
Price trajectory in the Cartea and Figueroa model. Source: Aïd (2015).

In this model, forward prices can be computed explicitly under deterministic market price of risk assumption.

Structural spot price models

- Unlike stock price process which are hardly predictable, electricity prices are related to a multitude of observable factors: consumption, fuel prices, plant outages etc.
- Structural models focus on the price formation mechanism and aim to predict day-ahead prices based on the available information.

- In demand-based models the spot price is obtained by matching a constant supply function with a random inelastic demand.
- In stack-curve models, the supply function is constructed from unit costs and capacities of different generation technologies.



Example of generation stack

Source: B. Posner / psu.edu

Demand-based models

- The demand for electricity is described by a stochastic process:

$$D_t = \overline{D_t} + X_t,$$
$$dX_t = (\mu - \lambda X_t)dt + \sigma dW_t,$$

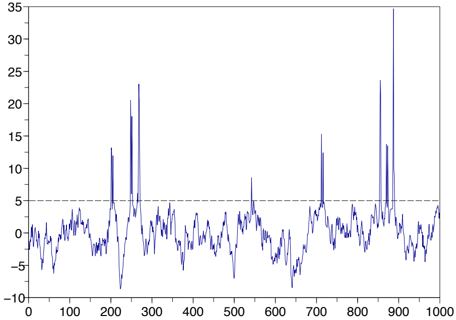
  where $\overline{D_t}$ is the seasonal component and X_t is the stationary stochastic part.
- The price is obtained by matching the demand level with a deterministic supply function which must be nonlinear to account for spikes.
- Barlow (2002) proposes

$$P_t = \left(\frac{a_0 - D_t}{b_0}\right)^{1/\alpha}$$

  for some $\alpha > 0$.
- Kanamura and Ohashi (2004) suggest a "hockey stick" profile

$$P_t = (a_1 + b_1 D_t)1_{D_t \le D_0} + (a_1 + b_1 D_t)1_{D_t > D_0}.$$

Demand-based models



Spot price trajectory in the demand-based model by Kanamura and Ohashi (2004).

In these models, spikes can only be caused by surges in demand, while in electricity markets spikes can also be due to sudden changes in supply, such as plant outages.

Stack curve model of Aïd (2009)

- The electricity demand D_t can be satisfied with n different technologies;
- Each technology has available capacity C_t^i and fuel cost $h_i S_t^i$, where S^i is the fuel price and h_i is the heat rate;
- The marginal fuel cost is

$$\widehat{P}_t = \sum_{i=1}^{n} h_i S_t^i \mathbf{1}_{D_t \in I_t^i}, \qquad I_t^i = \left(\sum_{k=1}^{i-1} C_t^k, \sum_{k=1}^{i} C_t^k \right]$$

- The spot price depends on the marginal fuel cost and the reserve margin:

$$P_t = g(R_t) \times \widehat{P}_t, \qquad R_t = \sum_{i=1}^{n} C_t^i - D_t$$

where g is the *scarcity function*:

$$g(x) = \min\left(\frac{\gamma}{x^\nu}, M\right) \mathbf{1}_{x>0} + M \mathbf{1}_{x \le 0}.$$

Electricity derivatives: financial

Standard Calls/Puts on electricity futures are traded in power exchanges such as EEX. They can be valued as standard financial options since the underlying is liquidly traded.

Call option gives the right to enter a long futures position at the expiry date of the option, at the specified price (strike price)

Put option gives the right to enter a short futures position at the expiry date of the option, at the specified price

Electricity derivatives: physical

Physical options are embedded in portfolios of energy assets:

- Fuel spreads mimick the profit of a power plant at a given moment in time: the clean fuel spread option pays

$$\left(S_T^e - hS_T^f - gS_T^c\right)^+,$$

  where

  - S^e is the spot price of electricity;
  - S^f is the spot price of fuel (e.g., gas or coal);
  - S^c is the price of carbon emission allowances;
  - h is the heat rate of the plant;
  - g is the emission rate of the plant.

Electricity derivatives

- Cross-border transmission rights are spread options on the price differential of two neighboring countries (e.g., France vs. Germany). Their pricing is complexified by market coupling (when markets are coupled the spread is zero).
- A tolling agreement mimicks the operation of a power plant over time: it pays

$$\int_0^T \left(S_t^e - hS_t^f - gS_t^c\right)^+ dt.$$

- A swing option is a flexible delivery contract which mimicks a hydroelectric reservoir: the buyer has the right to receive energy (at most $\bar{q}$) on a certain number of days N during a period of time T subject to the constraint that the total consumed power is between $\underline{Q}$ and $\overline{Q}$.

Call and put options

- In the EEX market, call and put options on 6 monthly swaps, 6 quarterly swaps and 3 yearly swaps (base-load) are traded
- For every swap, a single maturity, shortly before the swap expiry is offered, with a variety of strikes
- The underlying of each option is therefore traded throughout the lifetime of the option
- Risk-neutral pricing approaches are therefore justified

Consider the pricing of a call option on an electricity swap contract with pay-off

$$(F_T(T_1, T_2) - K)^+.$$

Its price at t is given by

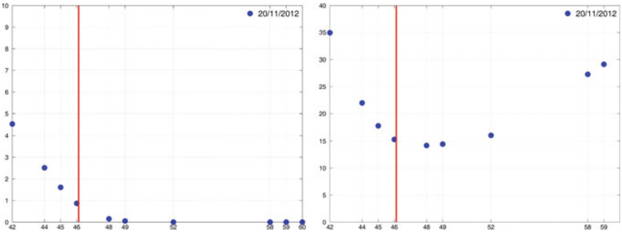
$$V_t = e^{-r(T-t)}\mathbb{E}^{\mathbb{Q}}[(F_T(T_1, T_2) - K)^+|\mathcal{F}_t]$$

Call and put options

- In a log-normal model for the forward $f_t(T)$: pricing by Monte-Carlo.
- In a log-normal model for the swap price $F_t(T_1, T_2)$: Black's formula

$$V_t = e^{-r(T-t)} F_t(T_1, T_2) N(d_1) - e^{-r(T-t)} K N(d_2),$$

$$d_{1,2} = \frac{\log \frac{F_t(T_1,T_2)}{K} \pm \frac{1}{2} v_{t,T}}{\sqrt{v_{t,T}}}, \quad v_{t,T} = \int_t^T \|\Sigma(s, T_1, T_2)\|^2 ds.$$



Prices and implied volatilities of year-ahead base-load futures options with expiry on Jan 2013, as of Nov 20, 2012. Source: R. Aïd (2015).

Spread options on forward contracts

- Most commodity spread options are written on forward contracts
- The underlyings are liquidly traded and risk-neutral valuation may be used

Consider two forward contracts with risk-neutral dynamics

$$dF_t^1 = F_t^1 \sigma_1(t) dW_t, \qquad dF_t^1 = F_t^2 \sigma_2(t)(\rho dW_t + \sqrt{1-\rho^2} dW_t'),$$

where σ_1 and σ_2 are deterministic volatility functions and W and W' are independent standard BMs

Magrabe's formula gives the price of the zero-strike option on $F_T^1 - F_T^2$:

$$e^{-r(T-t)} \mathbb{E}^{\mathbb{Q}}[(F_T^1 - F_T^2)^+ | \mathcal{F}_t] = e^{-r(T-t)}(F_t^1 N(d_1) - F_t^2 N(d_2)),$$

$$d_{12} = \frac{\log \frac{F_t^1}{F_t^2} \pm \frac{1}{2} v_{t,T}}{\sqrt{v_{t,T}}}, \quad v_{t,T} = \int_t^T (\sigma_1^2(t) + \sigma_2^2(t) - 2\rho\sigma_1(t)\sigma_2(t)) ds$$