Random Walks on Simple Two-Dimensional Manifolds

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Introduction

- Random
- Walk
- Simple
- Two-Dimensional
- Manifolds

Regular Surfaces

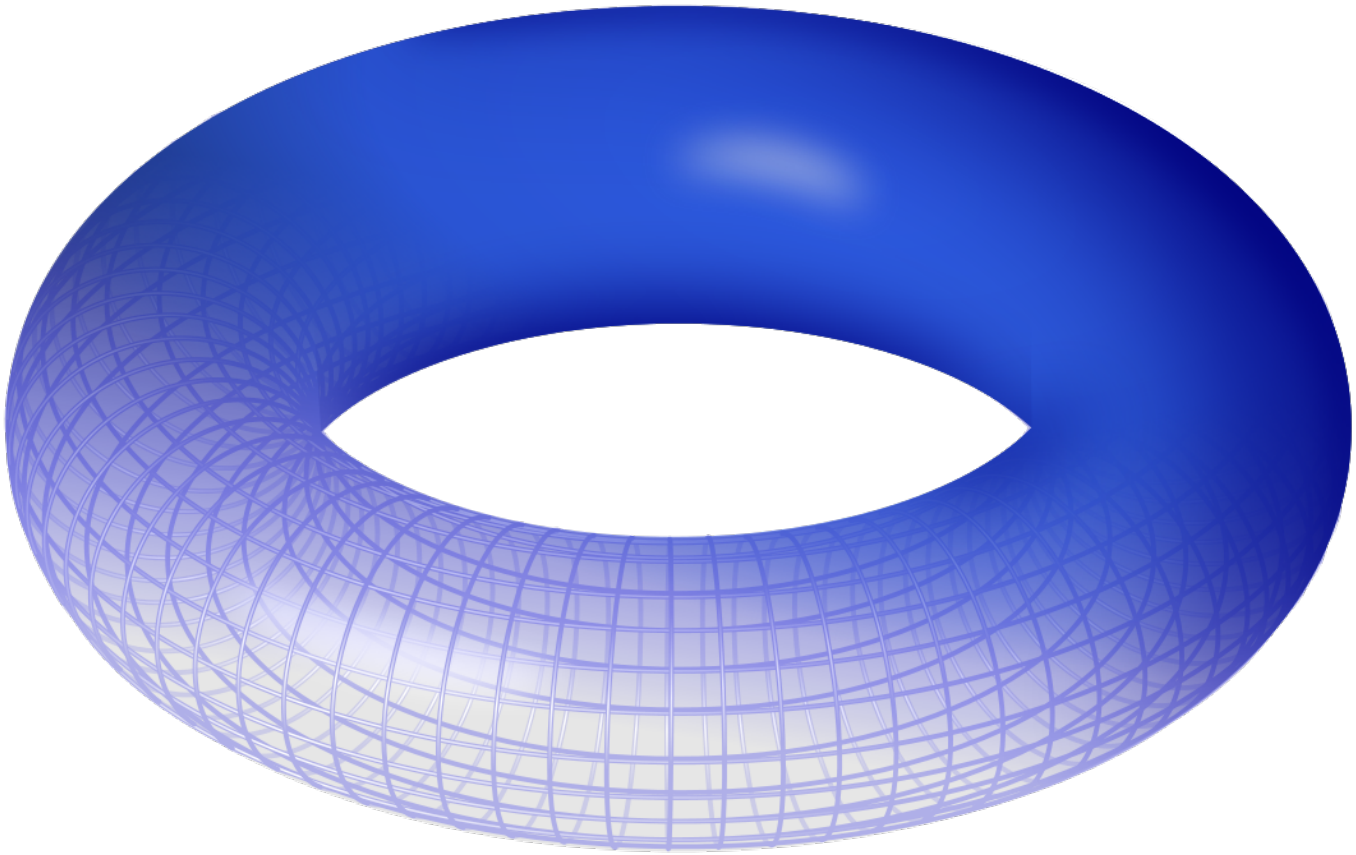


Figure: By Leonid_2 - Own work, CC BY-SA 3.0, https://commons.wikimedia.org/w/index.php?curid=8643414

Geodesic Equations

1. Extend definition of line to other surfaces
2. Assume a path is a geodesic contained in a coordinate patch
3. Derive geodesic equations for coordinate functions of path

Geodesic Equations

$$u'' + \frac{\mu_{uu} \cdot \mu_u}{\mu_u \cdot \mu_u}(u')^2 + \frac{\mu_{vv} \cdot \mu_u}{\mu_u \cdot \mu_u}(v')^2 + 2\frac{\mu_{uv} \cdot \mu_u}{\mu_u \cdot \mu_u}u'v' = 0$$
$$v'' + \frac{\mu_{uu} \cdot \mu_v}{\mu_v \cdot \mu_v}(u')^2 + \frac{\mu_{vv} \cdot \mu_v}{\mu_v \cdot \mu_v}(v')^2 + 2\frac{\mu_{uv} \cdot \mu_v}{\mu_v \cdot \mu_v}u'v' = 0$$

Christoffel Symbols

$$\Gamma^i_{jk} = \frac{\mu_{jk} \cdot \mu_i}{\mu_i \cdot \mu_i} \quad \text{where } i, j, k \in \{u, v\}$$

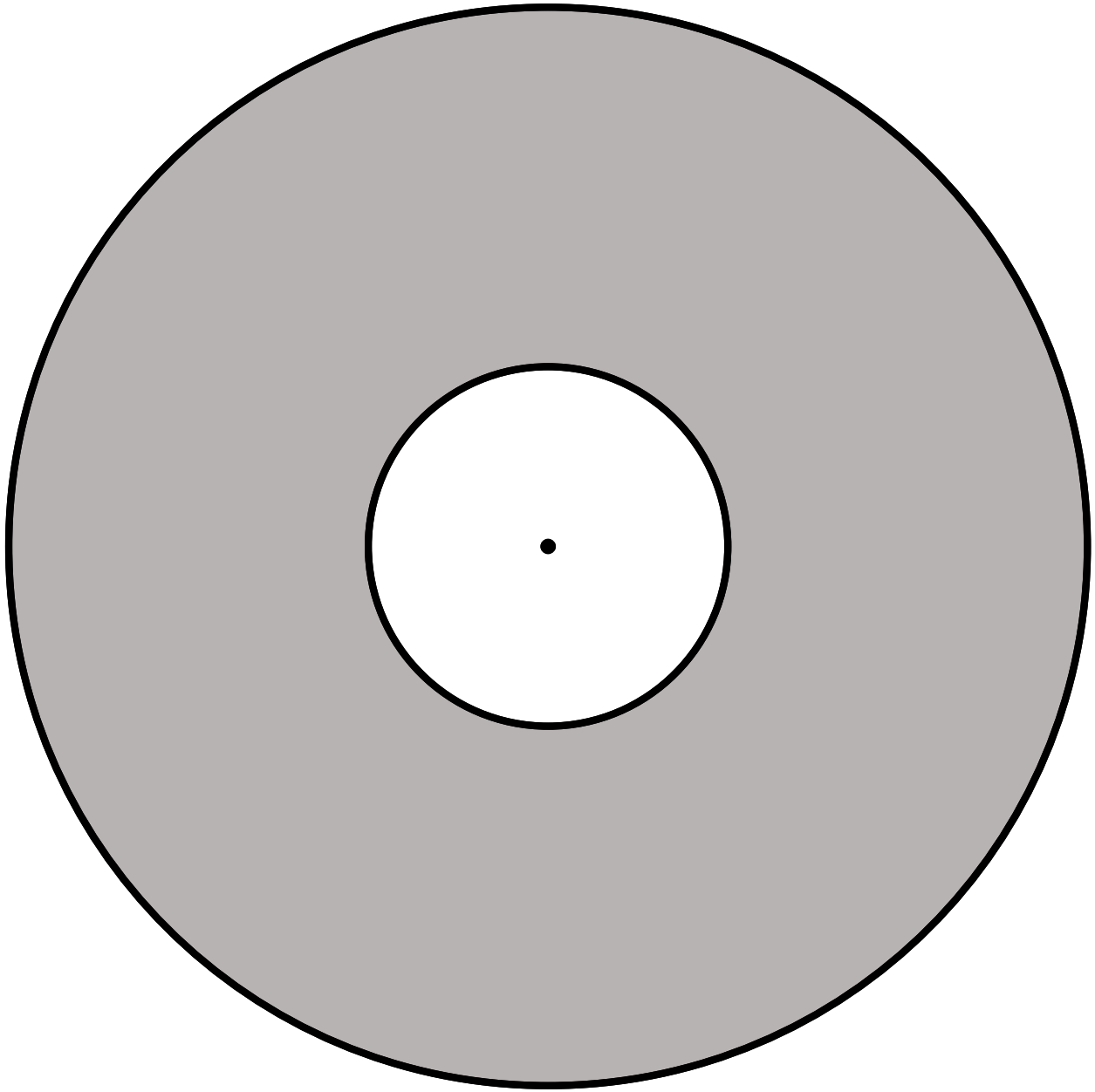
$$\Downarrow \text{abuse of symbols}$$

$$\frac{d^2 x^i}{dt^2} + \sum_{j,k \in \{1,2\}} \Gamma^i_{jk} \frac{dx^j}{dt} \frac{dx^k}{dt} = 0$$

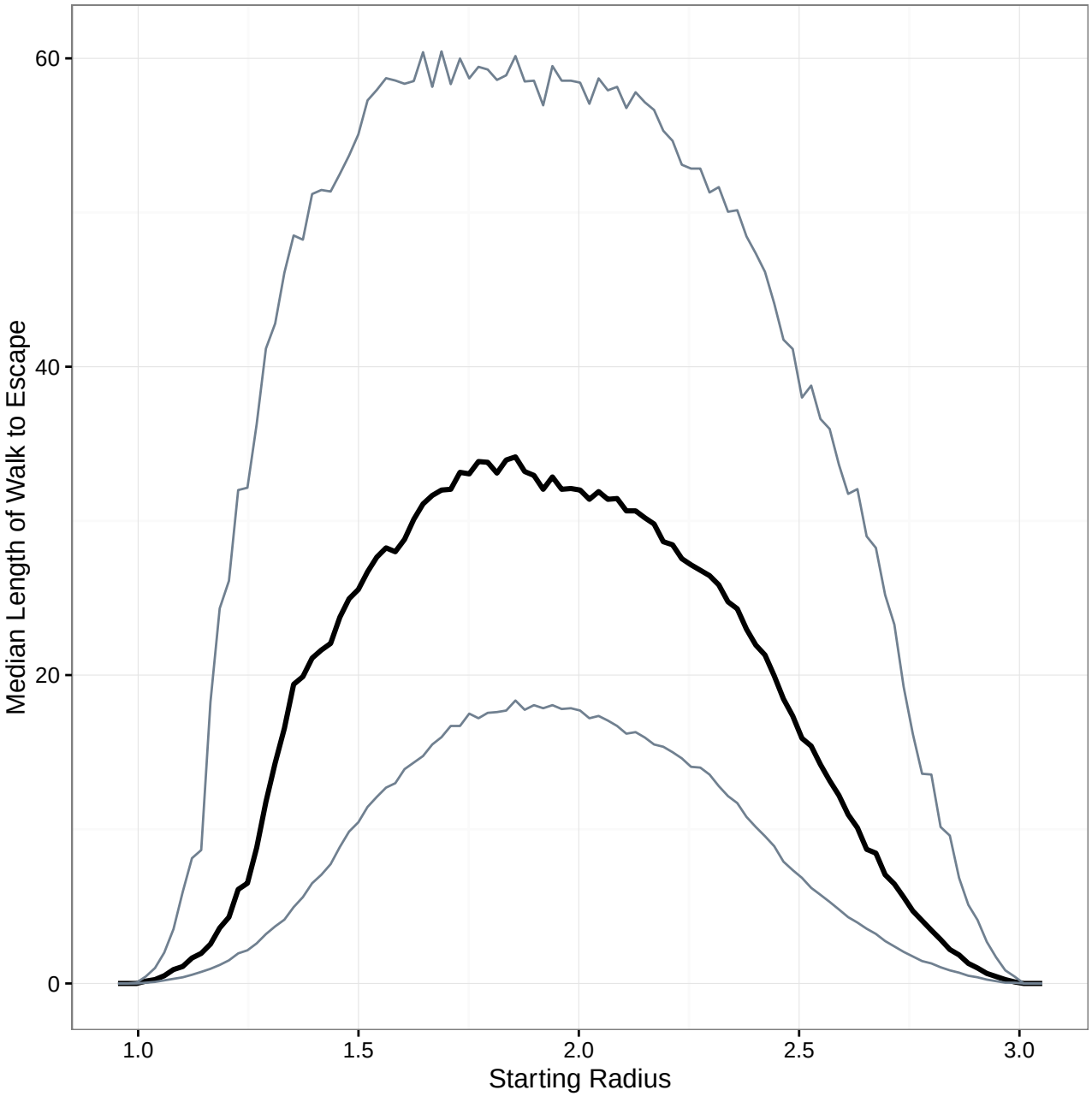
C++ Implementation

- Runge-Kutta 4th Order Method
- Stack Linked List
- Function Pointers

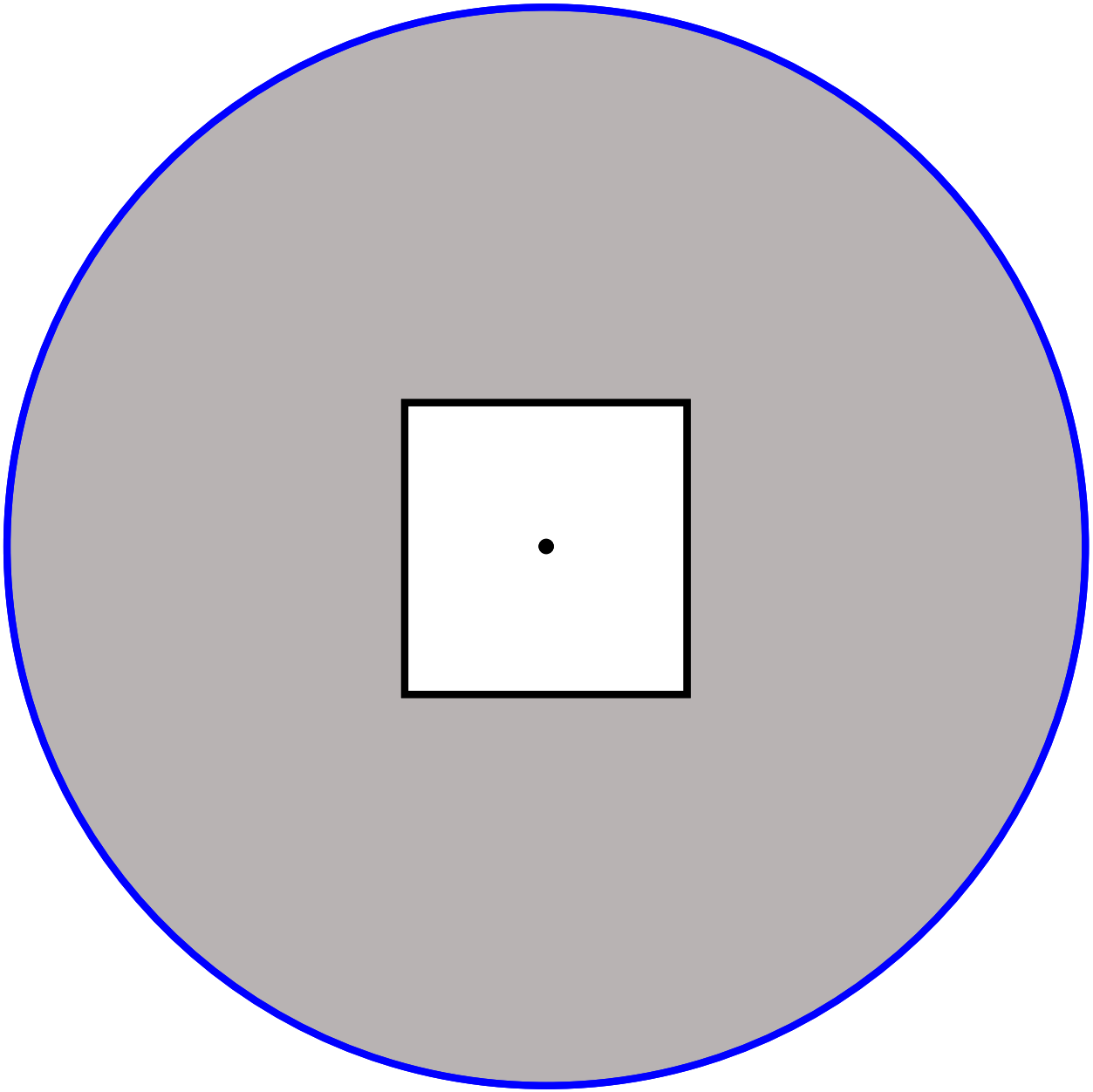
Plane



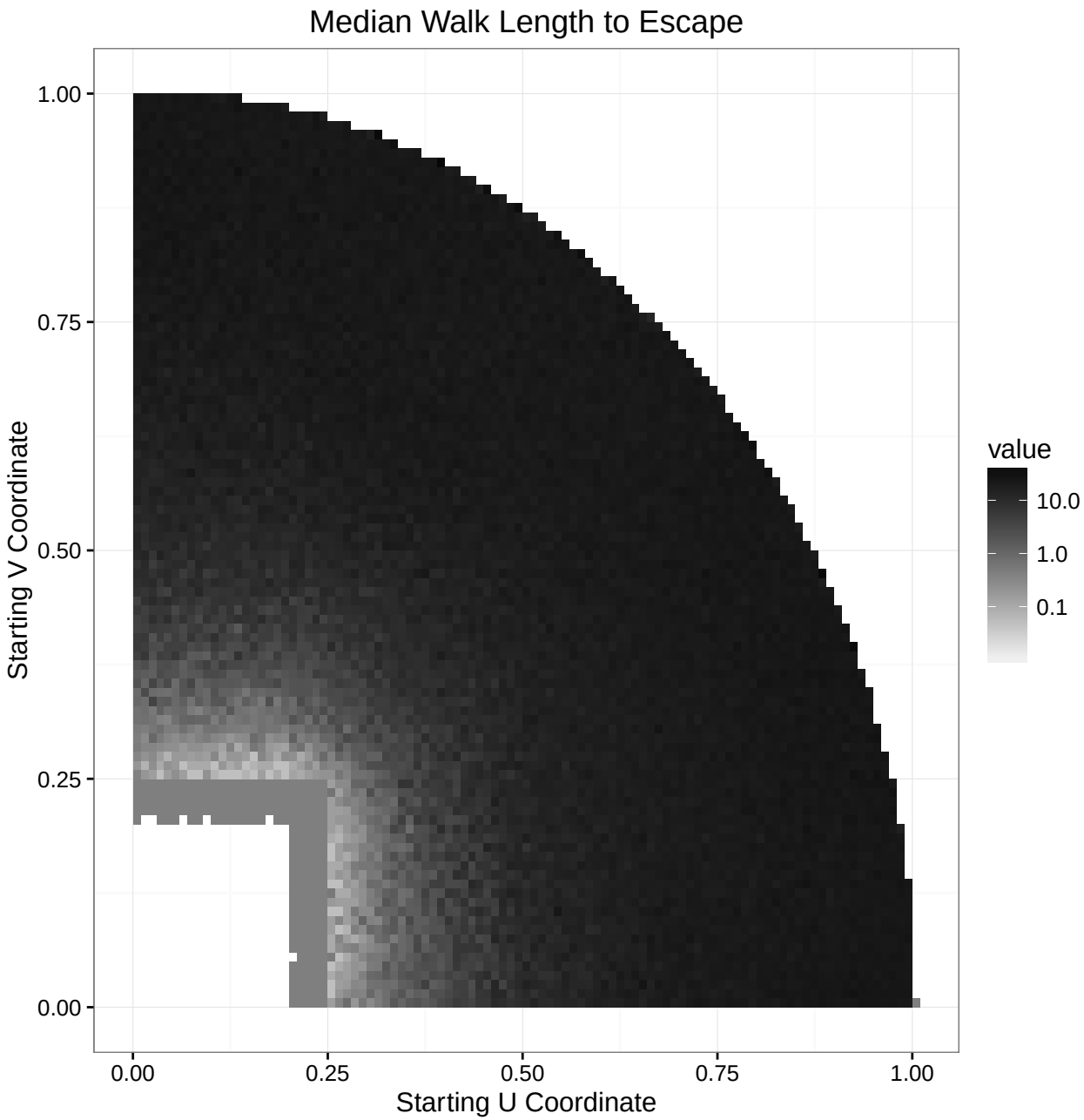
Plane


60
40
20
0
Median Length of Walk to Escape
1.0
1.5
2.0
2.5
3.0
Starting Radius

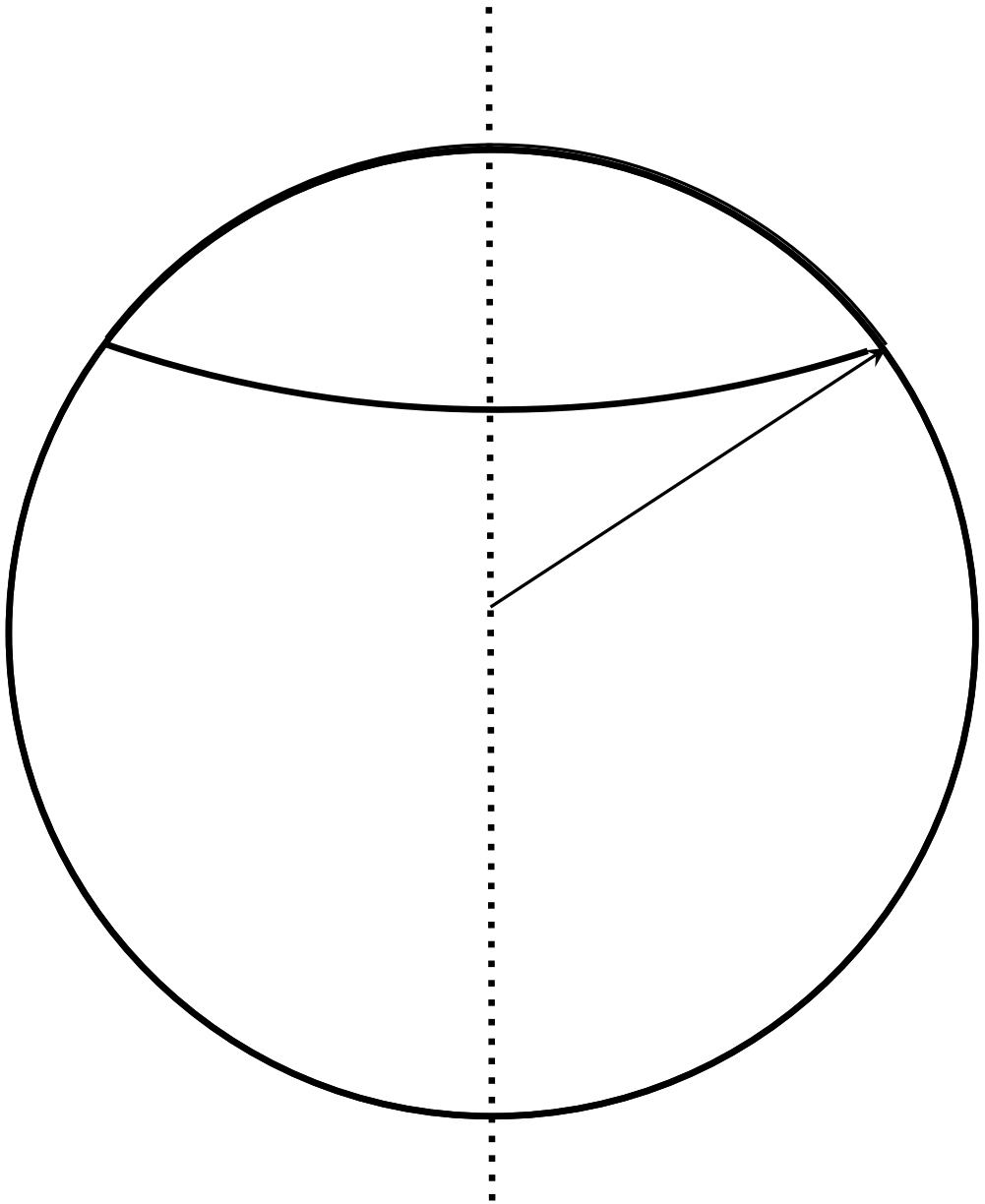
Plane



Plane


Median Walk Length to Escape
Starting V Coordinate
Starting U Coordinate
0.00
0.25
0.50
0.75
1.00
value
10.0
1.0
0.1

Sphere



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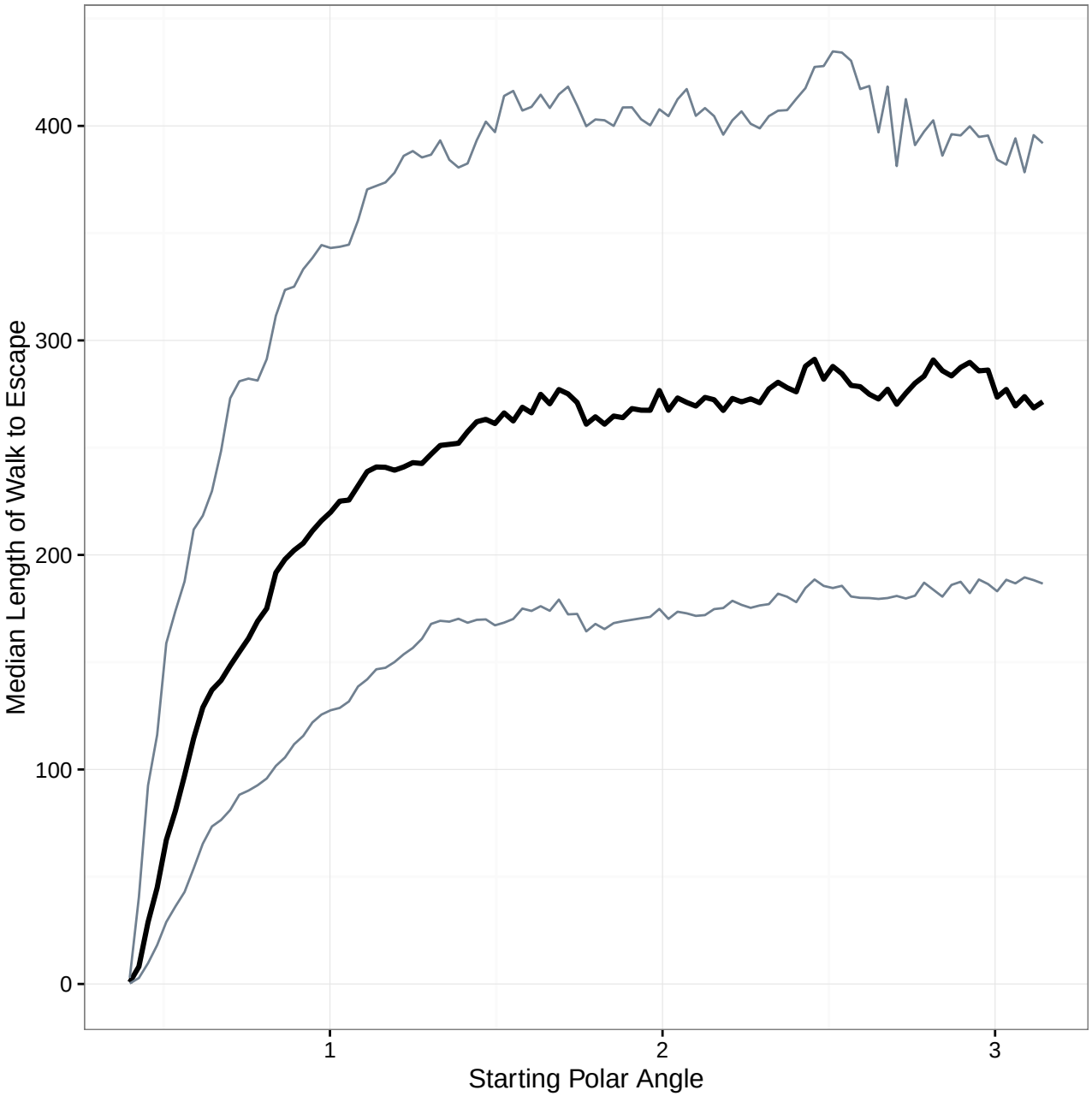
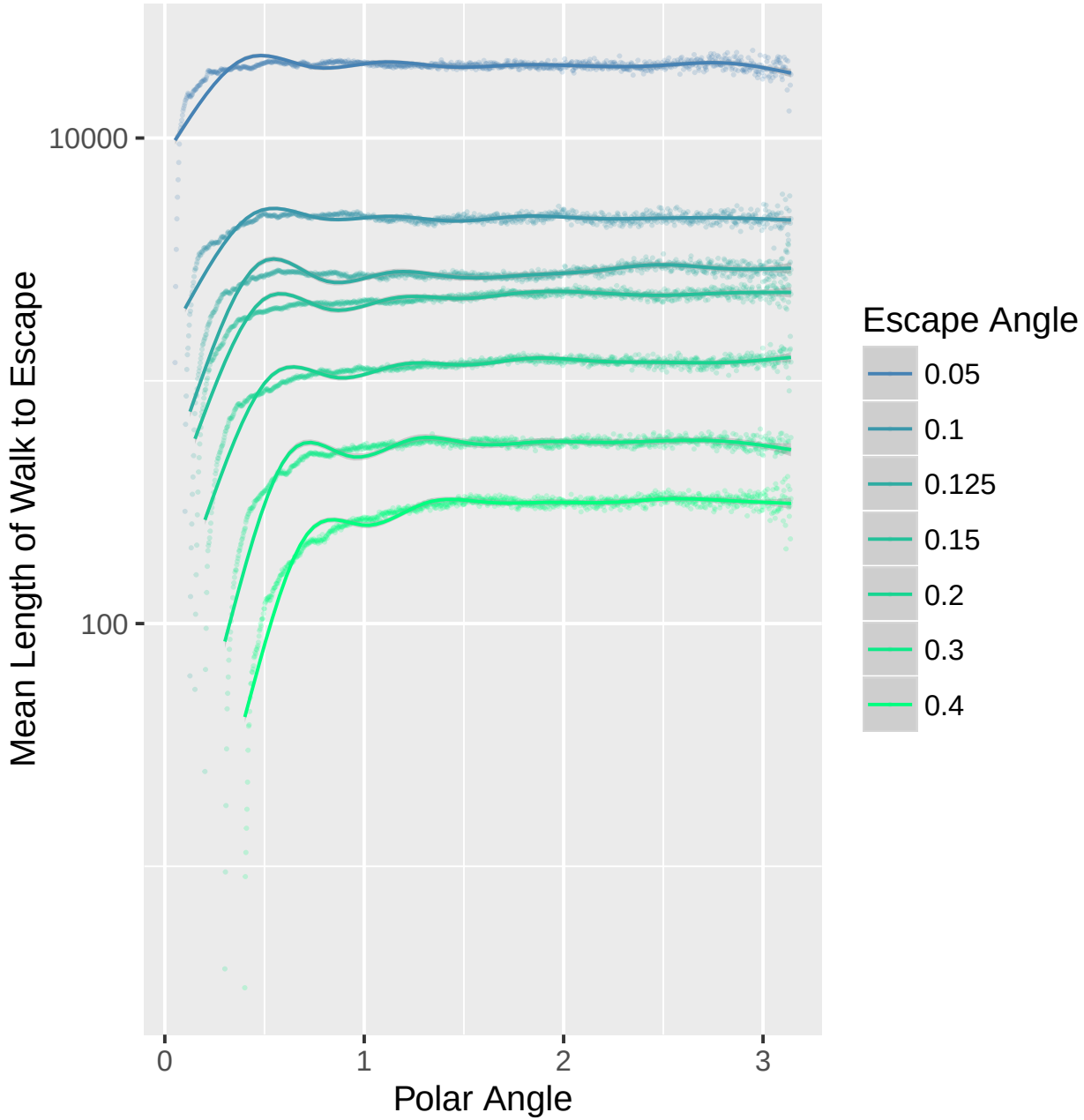

Median Length of Walk to Escape
400
300
200
100
0
1
2
3
Starting Polar Angle

Sphere


10000
100
Mean Length of Walk to Escape
0
1
2
3
Polar Angle
Escape Angle
0.05
0.1
0.125
0.15
0.2
0.3
0.4

Questions?