LEVEL 1: FIXED INCOME

Reading 41 (3rd out of 6): BOND VALUATION

Difficulty: **medium**

Benchmark Study Time: **5.2h**

2022

INTENTIONALLY EMPTY

THIS E-BOOK:

- is a selective summary of the corresponding Reading in your CFA® Program Curriculum,
- provides place for your own notes,
- helps you structure your study and revision time!

How to use this e-book to maximize your knowledge retention:

1. **Print** the e-book in duplex and bind it to keep **all** important info for this Reading **in one place.**
2. **Read** this e-book, best twice, to grasp the idea of what this Reading is about.
3. **Study** the Reading from your curriculum. **Here add** your notes, examples, formulas, definitions, etc.
4. **Review** the Reading using this e-book, e.g. write your summary of key concepts or revise the formulas at the end of this e-book (if applicable).
5. **Done?** Go to your study plan and change the Reading's status to **green**:
   (it will make your Chance-to-Pass-Score™ grow ☺).
6. **Come back** to this e-book from time to time to **regularly review for knowledge retention!**

NOTE: While studying or reviewing this Reading, you can use the tables at the end of this e-book and mark your study/review sessions to hold yourself accountable.

INTRODUCTION TO BOND VALUATION

Bond value

price of a well-priced bond = present value of the future cash flows from the bond

In order to determine the price of a bond, investors must know:

- the value of the cash flows from the bond, and
- the value of the discount rate used to discount the cash flows.

A discount rate depends on the risk associated with the bond's cash flows.

Methods of bond valuation

An investor can value a bond using:

- a market discount rate,
- spot rates and forward rates,
- binomial interest rate tree,
- matrix pricing.

Market discount rate method

The market discount rate method assumes using only one discount rate for the entire period from today to the maturity date.

market discount rate = required yield = required rate of return

Example

Valuation of a 2-year bond using market discount rate:

$$\mathbf{P = \frac{C}{1+r} + \frac{C+FV}{(1+r)^2}}$$

Where:

- P – bond's value,
- C – coupon payment,
- FV – par value,
- r – market discount rate.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Spot rates or Forward rates method

The spot rates or forward rates method assumes the use of a set of spot rates or forward rates, respectively.

spot rate = yield-to-maturity on a zero-coupon bond (the interest rate applicable to the period from today to some point in time in the future).

spot curve = set of yields-to-maturity on zero-coupon bonds (spot rates) with different maturities.

forward rate = the interest rate applicable to two periods in the future.

forward curve = set of forward rates for equal periods at different points in time.

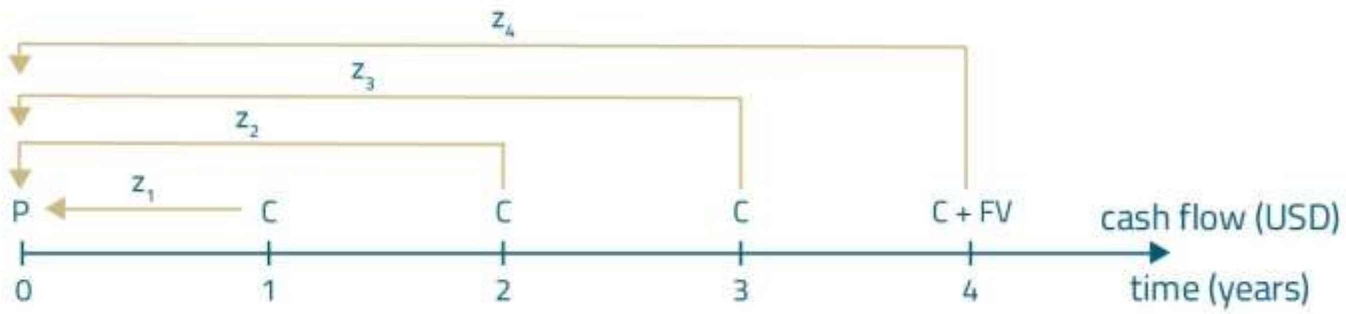
Example

Valuation of a 4-year bond using spot rates:

$$\mathbf{P = \frac{C}{1+z_1} + \frac{C}{(1+z_2)^2} + \frac{C}{(1+z_3)^3} + \frac{C+FV}{(1+z_4)^4}}$$

Where:

- P – bond's value,
- C – coupon payment,
- FV – par value,
- z_1, z_2, z_3, z_4 – spot rates.



HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Example

Valuation of a 3-year bond using forward rates:

$$\mathbf{P = \frac{C}{1 + f_{0,1}} + \frac{C}{(1 + f_{0,1}) \times (1 + f_{1,2})} + \frac{C + FV}{(1 + f_{0,1}) \times (1 + f_{1,2}) \times (1 + f_{2,3})}}$$

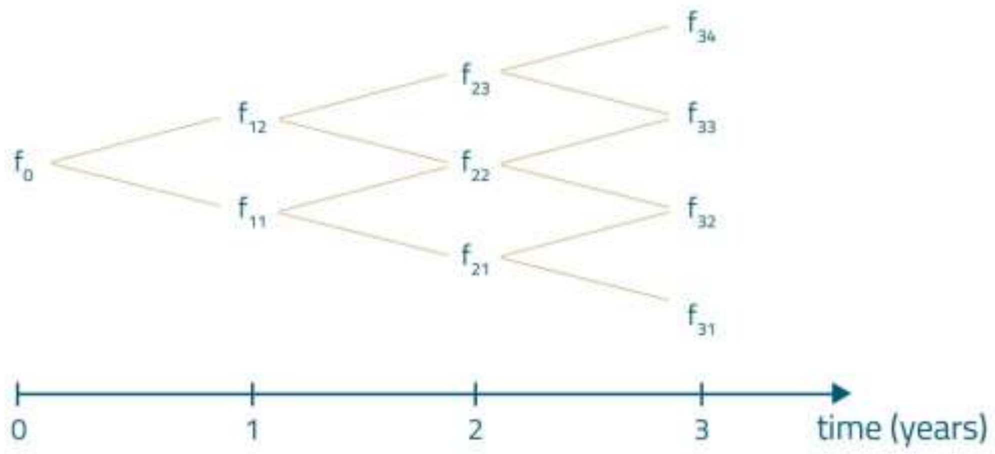
Where:

- P – bond's value,
- C – coupon payment,
- FV – par value,
- $f_{0,1}, f_{1,2}, f_{2,3}$ – forward rates.

Binomial interest rate trees method

The binomial interest rate trees method:

- assumes that interest rates are volatile → in a given period the discount rate can take different values → the further the period, the more different the values of the discount rate,
- is used to value bonds with embedded options, because it takes into account possible changes in the cash flows of the bond.



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Matrix pricing

Matrix pricing is an estimation process that is used when the market discount rate for a particular bond is unknown.

Matrix pricing:

- is used when the bond is illiquid or when investors intend to value the bond that is to be issued in the near future,
- assumes that investors do not know the discount rate at the beginning.

To compute the discount rate, investors use:

- the linear interpolation,
- yields to maturity of similar bonds.

Implied spot rates vs Implied forward rates

Implied forward rate

If we know two different spot rates, we are able to compute the implied forward rate:

[NOTE: The formula below holds true if we assume an annual bond basis. In the case of the periodicity of 2 or more, we should adjust the formula.]

$$\mathbf{(1+z_n)^n \times (1+f_{n,n+k})^k = (1+z_{n+k})^{n+k}}$$

$$\mathbf{f_{n,n+k} = \left(\frac{(1+z_{n+k})^{n+k}}{(1+z_n)^n}\right)^{\frac{1}{k}} - 1}$$

Where:

- $\mathbf{z_n}$ – yield-to-maturity on a zero-coupon bond maturing in n years,
- $\mathbf{z_{n+k}}$ – yield-to-maturity on a zero-coupon bond maturing in (n+k) years,
- $\mathbf{f_{n,n+k}}$ – implied k-year forward yield n years into the future (n-year into k-year rate; nyky; n's, k's).

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Implied spot rate

If we know forward rates, we are able to compute the implied spot rate:

[NOTE: The formula below holds true if we assume an annual bond basis. In the case of the periodicity of 2 or more, we should adjust the formula.]

$$\mathbf{(1+f_{0,1}) \times (1+f_{1,2}) \times (1+f_{2,3}) \times \ldots \times (1+f_{n-2,n-1}) \times (1+f_{n-1,n}) = (1+z_n)^n}$$

Where:

- $\mathbf{z_n}$ – implied yield-to-maturity on a zero-coupon bond (spot rate) maturing in n years,
- $\mathbf{f_{i,i+j}}$ – j-year forward yield i years into the future (i-year into j-year rate; iyjy; i's, j's).

Types of yield curves (maturity structure/term structure of interest rates)

yield curve = set of yields-to-maturity on coupon bonds with similar credit ratings and different maturities.

spot curve = set of yields-to-maturity on zero-coupon bonds (spot rates) with similar credit ratings and different maturities.

par curve = set of yields-to-maturity on coupon bonds priced at par with similar credit ratings and different maturities.

forward curve = set of forward rates for equal periods at different points in time.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

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MARKET DISCOUNT RATE METHOD

Zero-coupon bond

$$\mathbf{P} = \frac{\mathbf{FV}}{(\mathbf{1} + \mathbf{YTM})^{\mathbf{n}}}$$

Where:

- P – bond's value,
- FV – par value,
- n – number of periods to maturity,
- YTM – market discount rate.

Coupon bond

$$\mathbf{P} = \sum_{\mathbf{i}=\mathbf{1}}^{\mathbf{n}} \frac{\mathbf{C}}{(\mathbf{1} + \mathbf{YTM})^{\mathbf{i}}} + \frac{\mathbf{FV}}{(\mathbf{1} + \mathbf{YTM})^{\mathbf{n}}}$$

Where:

- P – bond's value,
- FV – par value,
- C – coupon payment,
- n – number of periods to maturity,
- YTM – market discount rate.

Accrued interest

Accrued interest is the interest earned but not yet paid.

Flat price (clean price) is the price of a bond without accrued interest.

Full price (dirty price) is the price that is actually paid for the bond.

$$\mathbf{AI} = \frac{\mathbf{t}}{\mathbf{T}} \times \mathbf{C}$$

Where:

- AI – accrued interest,
- t – number of days from the last coupon date,
- T – number of days in the coupon period,
- C – coupon payment.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Main day count conventions:

- Act/Act,
- 30/360

The Act/Act day count convention means that we take the actual number of days in a year and the actual number of days in a month.

The 30/360 day count convention assumes that there are 360 days in one year and 30 days in every month.

Bond price vs Coupons

The price of a bond may be:

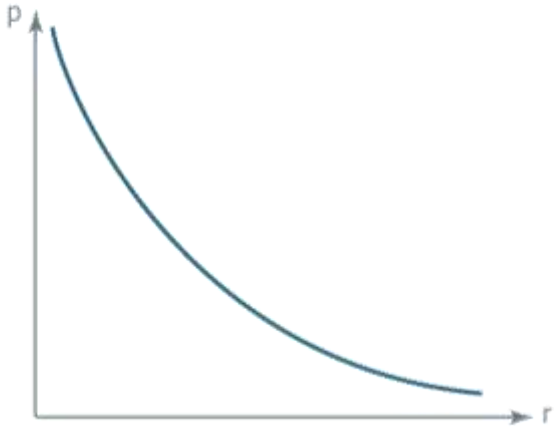
- higher than the par value (the bond sells **at a premium**).
- lower than the par value (the bond sells **at a discount**).
- equal to the par value (the bond sells **at par**).

The price of the bond with fixed coupons:

- equals the par value if the coupon rate is equal to the market discount rate.
- is lower than the par value if the coupon rate is lower than the market discount rate.
- is greater than the par value if the coupon rate is greater than the market discount rate.

Bond price vs Market discount rate

- The greater the market discount rate, the lower the bond price.
- The lower the market discount rate, the greater the bond price.
- The relationship is not linear and the curve is convex.



The consequences of the relationship:

1. If the required yield changes only a bit (it either increases or decreases a bit), the absolute value of the percentage change in the bond price will be the same for both an increase and a decrease of the market discount rate.
2. If the required yield changes a lot, the absolute value of the percentage change in the bond price will be lower if the market discount rate increases than if the discount rate decreases.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

The sensitivity of the bond price to changes in interest rates is affected by:

- the value of coupons,
- the frequency of coupon payments,
- the time to maturity.

the greater the coupon rate → the lower the sensitivity of the bond's price to changes in the market discount rate

the more frequently coupons are paid → the lower the sensitivity of the bond's price to changes in the market discount rate

the longer the time to maturity → the greater the sensitivity of the bond's price to changes in the market discount rate

FRN pricing (assuming annual coupon payment)

$$\mathbf{P = \sum_{i=1}^{n} \frac{(LIBOR + QM) \times FV}{(1 + LIBOR + DM)^i} + \frac{FV}{(1 + LIBOR + DM)^n}}$$

Where:

- P – bond's value,
- FV – par value,
- n – number of years to maturity,
- LIBOR – 12-month LIBOR,
- QM – quoted margin (stated on an annual basis; as a %),
- DM – discount margin aka. required margin (stated on an annual basis; as a %).

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Yields & Yield Spreads

Yield to maturity

Yield to maturity is:

- the annual rate that the bondholder will earn if she decides to hold the bond until maturity, assuming the issuer will pay all his obligations in time,
- the internal rate of return (IRR).

$$\mathbf{P} = \sum_{i=1}^{n} \frac{\mathbf{C}}{(1+\mathbf{YTM})^{i}} + \frac{\mathbf{FV}}{(1+\mathbf{YTM})^{n}}$$

- P – bond's price,
- FV – par value,
- C – coupon payment,
- n – number of periods to maturity,
- YTM – yield to maturity.

Yield measures in money market

Discount rate basis

$$\mathbf{PV} = \mathbf{FV} \times \left(1 - \frac{\mathbf{days}}{\mathbf{year}} \times \mathbf{d}\right)$$

$$\mathbf{d} = \left(\frac{\mathbf{FV} - \mathbf{PV}}{\mathbf{FV}}\right) \times \frac{\mathbf{year}}{\mathbf{days}}$$

- PV – price of the money market instrument,
- FV – face value of the money market instrument,
- d – discount rate,
- year – number of days in the year,
- days – number of days until maturity.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Add-on rate basis

$$\mathbf{FV = PV \times \left(1 + \frac{days}{year} \times a\right)}$$

$$\mathbf{a = \left(\frac{FV - PV}{PV}\right) \times \frac{year}{days}}$$

- PV – price of the money market instrument,
- FV – face value of the money market instrument,
- a – add-on rate,
- year – number of days in the year,
- days – number of days until maturity.

Bond equivalent yield (BEY; investment yield)

BEY is a money-market rate stated assuming:

- **365 days** in a year,
- **add-on** rate basis.

YIELD SPREADS

$$\mathbf{YTM = benchmark\ yield + spread}$$

$$\mathbf{spread = YTM - benchmark\ yield}$$

G-spread = spread over actual or interpolated government bond yield.

I-spread = interpolated spread = spread over the standard swap rate assuming the same currency and tenor as for the bond.

Z-spread = zero-volatility spread = static spread = constant yield spread over government spot curve or interest rate swap spot curve.

Option-adjusted spread (OAS) = Z-spread **–** call option value (in basis points per year) **OR**

Option-adjusted spread (OAS) = Z-spread **+** put option value (in basis points per year)

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Summarizing key concepts:

☐ Methods of bond valuation: Market discount rate method, Spot rates or Forward rates method, Binomial interest rate trees method, Matrix pricing

My summary:

☐ Implied spot rates vs Implied forward rates

My summary:

☐ Types of yield curves

My summary:

☐ Bond price vs Coupons, Bond price vs Market discount rate

My summary:

☐ Zero-coupon bond pricing, Fixed-coupon bond pricing, FRN pricing

My summary:

☐ Yield to maturity

My summary:

☐ Yield measures in money market

My summary:

☐ Yield spreads

My summary:

Reviewing formulas:

$$\mathbf{P} = \frac{\mathbf{C}}{\mathbf{1 + z_1}} + \frac{\mathbf{C}}{(\mathbf{1 + z_2})^{\mathbf{2}}} + \frac{\mathbf{C}}{(\mathbf{1 + z_3})^{\mathbf{3}}} + \frac{\mathbf{C + FV}}{(\mathbf{1 + z_4})^{\mathbf{4}}}$$

Write down the formula:

$$\mathbf{P} = \frac{\mathbf{C}}{\mathbf{1 + f_{0,1}}} + \frac{\mathbf{C}}{(\mathbf{1 + f_{0,1}}) \times (\mathbf{1 + f_{1,2}})} + \frac{\mathbf{C + FV}}{(\mathbf{1 + f_{0,1}}) \times (\mathbf{1 + f_{1,2}}) \times (\mathbf{1 + f_{2,3}})}$$

Write down the formula:

$$(\mathbf{1 + z_n})^{\mathbf{n}} \times (\mathbf{1 + f_{n,n+k}})^{\mathbf{k}} = (\mathbf{1 + z_{n+k}})^{\mathbf{n+k}}$$

$$\mathbf{f_{n,n+k}} = \left(\frac{(\mathbf{1 + z_{n+k}})^{\mathbf{n+k}}}{(\mathbf{1 + z_n})^{\mathbf{n}}} \right)^{\frac{\mathbf{1}}{\mathbf{k}}} - \mathbf{1}$$

Write down the formula:

$$\left(1+f_{0,1}\right)\times\left(1+f_{1,2}\right)\times\left(1+f_{2,3}\right)\times\ldots\times\left(1+f_{n-2,n-1}\right)\times\left(1+f_{n-1,n}\right)=\left(1+z_n\right)^n$$

Write down the formula:

$$P=\sum_{i=1}^{n}\frac{C}{(1+YTM)^i}+\frac{FV}{(1+YTM)^n}$$

Write down the formula:

$$AI=\frac{t}{T}\times C$$

Write down the formula:

$$P=\sum_{i=1}^{n}\frac{(LIBOR+QM)\times FV}{(1+LIBOR+DM)^i}+\frac{FV}{(1+LIBOR+DM)^n}$$

Write down the formula:

$$\mathbf{P = \sum_{i=1}^{n} \frac{C}{(1+YTM)^i} + \frac{FV}{(1+YTM)^n}}$$

Write down the formula:

$$\mathbf{PV = FV \times \left(1 - \frac{days}{year} \times d\right)}$$

$$\mathbf{d = \left(\frac{FV - PV}{FV}\right) \times \frac{year}{days}}$$

Write down the formula:

$$\mathbf{FV = PV \times \left(1 + \frac{days}{year} \times a\right)}$$

$$\mathbf{a = \left(\frac{FV - PV}{PV}\right) \times \frac{year}{days}}$$

Write down the formula:

Keeping myself accountable:

TABLE 1 | STUDY

When you sit down to study, you may want to **try the Pomodoro Technique** to handle your study sessions: study for 25 minutes, then take a 5-minute break. Repeat this 25+5 study-break sequence all throughout your daily study session.

✔ Tick off as you proceed.

POMODORO TIMETABLE: study-break sequences (25′ + 5′)													
date		date		date		date		date		date		date	
25′		25′		25′		25′		25′		25′		25′	
5′		5′		5′		5′		5′		5′		5′	
25′		25′		25′		25′		25′		25′		25′	
5′		5′		5′		5′		5′		5′		5′	
25′		25′		25′		25′		25′		25′		25′	
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25′		25′		25′		25′		25′		25′		25′	
5′		5′		5′		5′		5′		5′		5′	

TABLE 2 | REVIEW

Never ever neglect revision! Though it's not the most popular thing among CFA candidates, regular revision is what makes the difference. If you want to pass your exam, **schedule & do your review sessions.**

REVIEW TIMETABLE: When did I review this Reading?													
date		date		date		date		date		date		date	
date		date		date		date		date		date		date	