soleadea

LEVEL 1: PORTFOLIO MANAGEMENT

Reading 50 (3rd out of 8): MODERN PORTFOLIO THEORY

Difficulty: **medium**

Benchmark Study Time: **4h**

2022

https://soleadea.org/

INTENTIONALLY EMPTY

THIS E-BOOK:

- is a selective summary of the corresponding Reading in your CFA® Program Curriculum,
- provides place for your own notes,
- helps you structure your study and revision time!

How to use this e-book to maximize your knowledge retention:

1. **Print** the e-book in duplex and bind it to keep **all** important info for this Reading **in one place.**
2. **Read** this e-book, best twice, to grasp the idea of what this Reading is about.
3. **Study** the Reading from your curriculum. **Here add** your notes, examples, formulas, definitions, etc.
4. **Review** the Reading using this e-book, e.g. write your summary of key concepts or revise the formulas at the end of this e-book (if applicable).
5. **Done?** Go to your study plan and change the Reading's status to green:
   (it will make your Chance-to-Pass-Score™ grow ☺).
6. **Come back** to this e-book from time to time to **regularly review for knowledge retention!**

NOTE: While studying or reviewing this Reading, you can use the tables at the end of this e-book and mark your study/review sessions to hold yourself accountable.

CAL, CML & MARKET PORTFOLIO

Homogeneous expectations

Capital allocation line:

- connects risky assets with the risk-free asset,
- the risky assets in the portfolio mostly determine the expected risk and return of the portfolio.

Every investor has a different efficient frontier and CAL because:

- not every investor has access to all kinds of assets,
- investors have different expectations.

Modern Portfolio Theory applies the concept of homogeneous expectations.

The concept of homogeneous expectations means that all investors come up with the same effective frontier, the same CAL, and the same optimal risky portfolio.

Market portfolio

Because of the homogeneity of expectations, all investors invest in the same portfolio of risky assets.

This portfolio:

- is called the market portfolio,
- will include all assets available in the market,
- the weights of individual assets are equal to the percentage share of those assets in the total market value of all assets.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

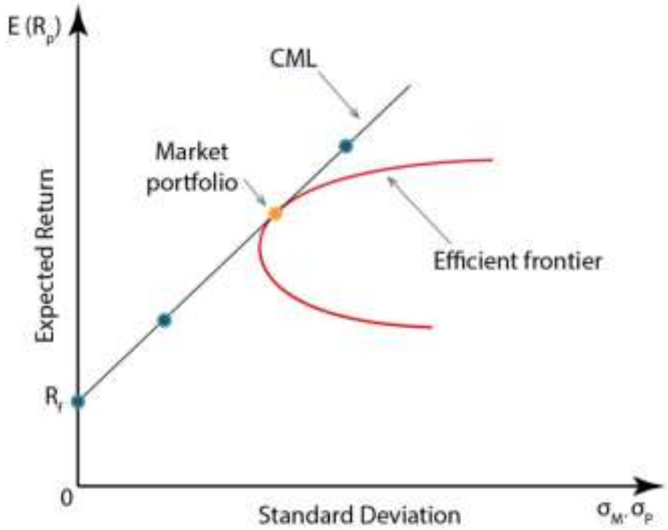
Capital market line (CML)

CML = CAL (where: optimal risky portfolio = market portfolio)

$$\mathbf{E\left(R_p\right) = R_f + \frac{[E(R_M) - R_f]}{\sigma_M} \times \sigma_P}$$

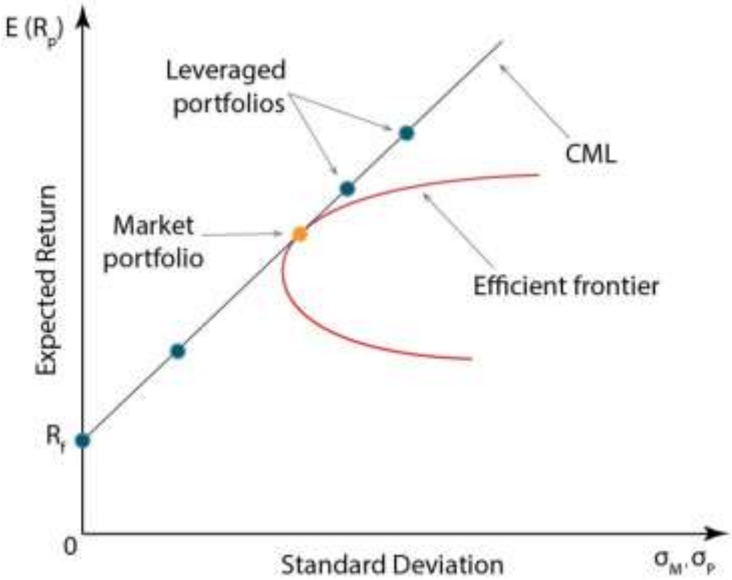
Where:

- $E(R_p)$ – expected return on the portfolio,
- R_f – risk-free interest rate,
- $E(R_M)$ – market expected return,
- $[E(R_M) - R_f]$ – excess return of the market (market risk premium),
- σ_M – market's standard deviation,
- σ_P – portfolio's standard deviation,
- $\frac{E(R_M)-R_f}{\sigma_M}$ – slope.



If: $\boldsymbol{\sigma_P = 0}$ then $\mathbf{E\left(R_p\right) = R_f}$

Leveraged portfolios



When an investor constructs a portfolio that includes risk-free assets and the market portfolio, both the rate of return and the risk will be proportionate to the shares of the risk-free assets and the market portfolio. The bigger the share of the market portfolio as compared to the share of risk-free assets, the higher the expected risk and return of the selected portfolio. In an extreme case, an investor with a high risk tolerance may also borrow money and invest additional capital in a market portfolio. Such portfolios are often called **leveraged portfolios**.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Informationally efficient markets

If markets were informationally efficient, then:

- the prices of assets would adequately reflect their real value,
- investors would have equal access to information,
- it would be impossible for any investor to use any piece of information in order to beat the market.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

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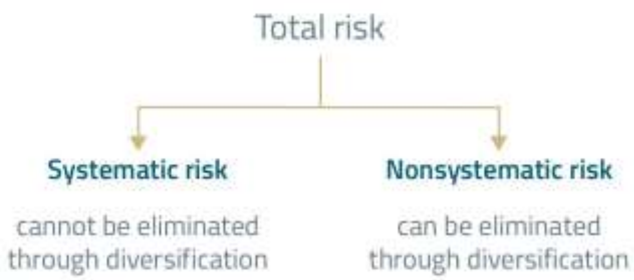
SYSTEMATIC RISK VS NONSYSTEMATIC RISK

Portfolio risk

When analyzing a portfolio, it is important to distinguish between systematic and nonsystematic risk. The lower the correlation between selected assets, the lower the portfolio risk. Note, however, that diversification usually eliminates only a part of the total risk of a portfolio. This is because certain factors affect only one company in the portfolio, while others can affect the entire portfolio or even the whole market. The risk associated with factors affecting only one company in the portfolio (nonsystematic risk) can be effectively eliminated through diversification of the portfolio. However, the risk resulting from factors that affect the whole market (systematic risk) cannot be eliminated through diversification.



Systematic risk

systematic risk = market risk = nondiversifiable risk

Systematic risk:

- is inherent in the market,
- cannot be avoided,
- affects all assets in the market.

Nonsystematic risk

nonsystematic risk = unique risk = firm-specific risk = diversifiable risk

Nonsystematic risk:

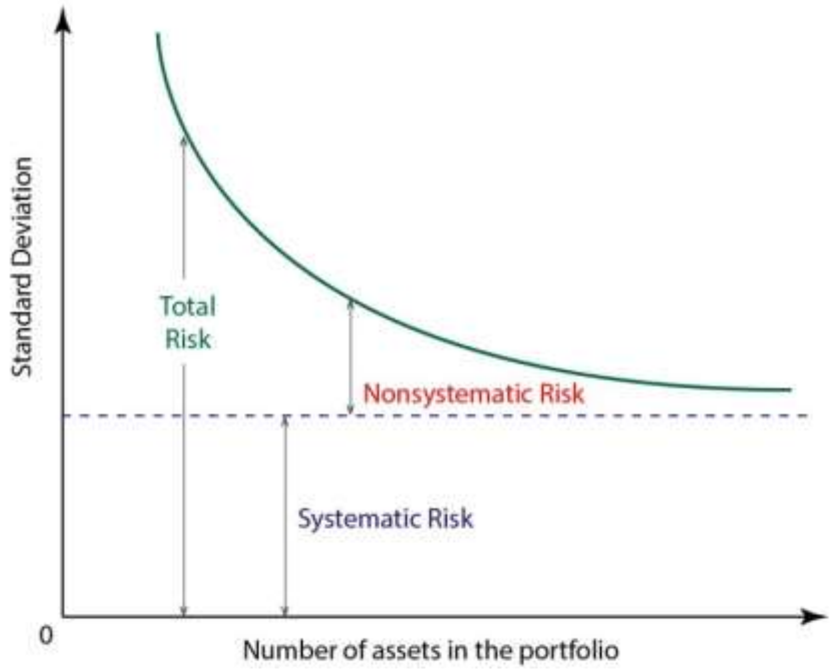
- is limited to a particular asset or asset classes,
- can be avoided through diversification,
- of the market portfolio equals zero.

The more assets there are in the portfolio and the less correlated they are, the lower the nonsystematic risk.

A reasonable investor shouldn't require a higher rate of return for nonsystematic risk as it can be avoided through portfolio diversification.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Total risk



The graph shows the relation between the total risk, nonsystematic risk, systematic risk and the number of assets in the portfolio. **The first important thing** is that the total risk is equal to the sum of systematic risk and nonsystematic risk. **Secondly**, the more assets we have in the portfolio, the lower its total risk gets. **Thirdly**, with every asset added to the portfolio, its total risk decreases more slowly. So, at some point, the manager should consider the benefits of adding a new asset (further diversification) – whether it outweighs the costs of operation, analysis, and management, as well as transaction costs. **The next point** is that for a well-diversified portfolio, the nonsystematic risk is virtually equal to zero. Also, the graph shows that there is a certain level of risk that we cannot go below. This level of risk is determined by the systematic risk. Such risk can only be eliminated through the use of derivatives.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

BETA & RETURN-GENERATING MODELS

Return-generating models

Return-generating models provide an estimate of the portfolio expected return given certain factors.

Main categories of factors:

- macroeconomic factors,
- fundamental factors,
- statistical factors.

Macroeconomic factors include:

- GDP growth,
- the rate of inflation,
- the level of consumption.

Fundamental factors include:

- sales growth,
- P/B ratio,
- size of the company,
- earnings.

Statistical factors identify statistical correlations between certain variables.

Multi-factor model

$$\mathbf{E(R_P) - R_f = \sum_{i=1}^{n} \beta_{Pi} \times E(F_i)}$$

Where:

- $E(R_P)$ – portfolio expected rate of return,
- R_f – risk-free interest rate,
- $E(F_i)$ – 'i' factor,
- β_{Pi} – 'i' factor weight.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Single-index model

The simplest model that allows return estimation is a single-index model. It is a single-factor linear model. Most often, the single factor is the rate of return on the market portfolio over the risk-free rate.

<u>Portfolio expected return</u>

$$\mathbf{E(R_P) - R_f = \beta_P \times [E(R_M) - R_f]}$$

OR

$$\mathbf{E(R_P) - R_f = \frac{\sigma_P}{\sigma_M} \times [E(R_M) - R_f]}$$

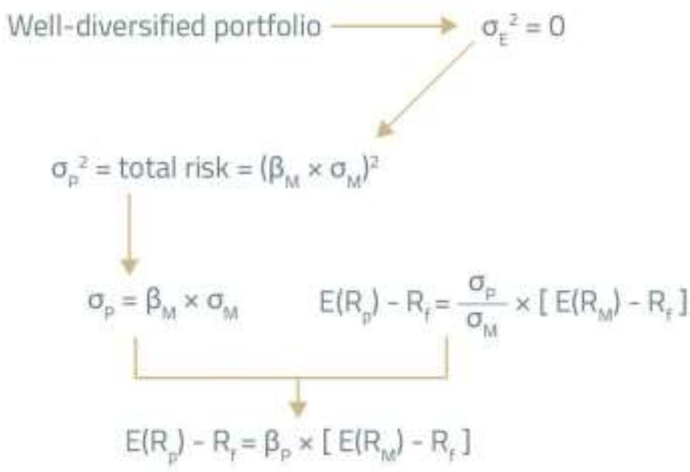
<u>Portfolio risk</u>

$$\textbf{total risk} = \textbf{systematic risk} + \textbf{nonsystematic risk}$$

$$\mathbf{\sigma_P^2 = \textbf{total risk} = (\beta_M \times \sigma_M)^2 + \sigma_E^2}$$

Where:

- $E(R_P)$ – portfolio expected rate of return,
- R_f – risk-free interest rate,
- $E(R_M)$ – market portfolio expected rate of return,
- β_P – beta of the portfolio,
- σ_M – market portfolio standard deviation,
- σ_P – portfolio standard deviation.



Market model

Market model is:

- the simplest implementation of the single-index model,
- used for beta estimation and the computation of abnormal returns.

$$\mathbf{R_i = \alpha_i + \beta_i \times R_M + e_i}$$

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Where:

- R_i – return on the asset,
- α_i – intercept,
- R_M – return on the market,
- β_i – asset beta,
- e_i – asset specific return,
- $\alpha_i = R_f \times (1 - \beta)$.

Beta

Beta is the measure of systematic risk (the market risk) because it shows how sensitive the return on a stock is compared to the return on the market portfolio. Beta shows how much the rate of return on a particular asset will change if the rate of return on the market changes by 1 percentage point.

$$\boldsymbol{\beta_i = \frac{Cov(R_i, R_M)}{\sigma_M^2} = \frac{\rho_{i,M} \times \sigma_i}{\sigma_M}}$$

Where:

- β_i – asset beta,
- $Cov(R_i, R_M)$ – covariance of the asset and the market portfolio,
- σ_M^2 – market variance,
- $\rho_{i,M}$ – correlation coefficient between the asset and the market,
- σ_i – standard deviation of the asset,
- σ_M – market standard deviation.

Beta of the market equals 1

Beta = 1	the rate of return on the asset moves in the same direction as the market, the asset has **the same systematic risk** as the market,
Beta > 1	the rate of return on the asset moves in the same direction as the market, the asset is **more risky** than the market,
0 < Beta < 1	the rate of return on the asset moves in the same direction as the market, the asset is **less risky** than the market,
Beta = 0	the rate of return on the asset **doesn't depend** on the market,
Beta < 0	the rate of return on the asset moves in the opposite direction than the market.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

CAPITAL ASSET PRICING MODEL (CAPM)

CAPM assumptions

The Capital Asset Pricing Model (CAPM) shows the relationship between the risk and the expected return and is particularly appreciated in today's finance and used by analysts from across the world. What are the key characteristics of the model? On the one hand, it is simple and intuitive. On the other, it involves many assumptions that often do not stand up to reality. It was introduced independently by Treynor, Sharpe, Lintner and Mossin who drew on Harry Markowitz's earlier work.

The CAPM assumptions:

- Investors are risk-averse, utility-maximizing, rational individuals.
- Markets are frictionless, including no transaction costs and no taxes.
- Investors plan for the same single holding period.
- All investors have homogeneous expectations.
- All investments are infinitely divisible.
- Investors cannot influence prices.

The CAPM is based on 2 equations:

- the capital market line (**CML**),
- the security market line (**SML**).

<u>The capital market line (CML)</u>

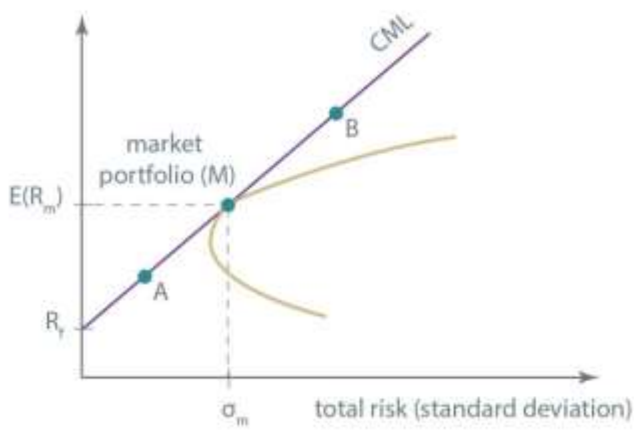
The formula is true only for efficient portfolios.

$$\mathbf{E(R_i) = R_f + \frac{\sigma_i}{\sigma_M} \times [E(R_M) - R_f]}$$

$$\textbf{rate of return on an efficient portfolio} = \textbf{time price} + (\textbf{total risk}) \times (\textbf{risk price})$$

Where:

- $E(R_i)$ – expected rate of return on the efficient portfolio,
- R_f – risk-free rate,
- $E(R_M)$ – expected market portfolio return,
- σ_i – standard deviation of the efficient portfolio,
- σ_M – standard deviation of the market portfolio.



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<u>The security market line (SML)</u>

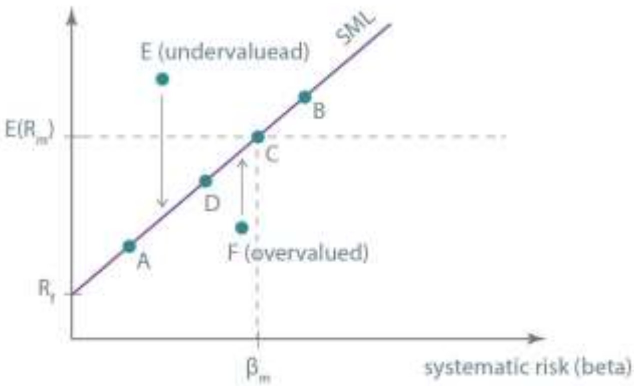
The formula applies to all portfolios, including portfolios comprised of a single security.

$$\mathbf{E(R_i) = R_f + \beta_i \times [E(R_M) - R_f]}$$

$$\textbf{rate of return} = (\textbf{risk-free rate}) + (\textbf{systematic risk}) \times (\textbf{market risk premium})$$

Where:

- $E(R_i)$ – expected rate of return on the asset or portfolio i,
- R_f – risk-free rate,
- $E(R_M)$ – expected market portfolio return,
- β_i – beta of the asset or portfolio i.



Beta vs Rate of return

$$\beta_i = 1 \longrightarrow E(R_i) = E(R_m)$$

$$\beta_i = 0 \longrightarrow E(R_i) = E(R_f)$$

$$0 < \beta_i < 1 \longrightarrow E(R_f) < E(R_i) < E(R_m)$$

$$\beta_i > 1 \longrightarrow E(R_i) > E(R_m)$$

CAPM application

The CAPM is used for:

- selecting undervalued securities,
- selecting overvalued securities,
- estimating rates of return,
- evaluating the performance of investments,
- evaluating the fund manager's performance.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Evaluation of the fund manager's performance – ratios & tools:

Ratios and tools used for evaluating investment performance:

- Sharpe ratio,
- Treynor ratio,
- Jensen's Alpha,
- M-Squared.

Sharpe ratio

The Sharpe ratio:

- uses the total risk measured with the standard deviation of the portfolio,
- the higher the Sharpe ratio, the better the portfolio performance (higher return per unit of total risk).

$$\mathbf{S = \frac{R_i - R_f}{\sigma_i}}$$

Where:

- R_i – rate of return on the portfolio,
- R_f – risk-free rate,
- σ_i – standard deviation of the portfolio.

Treynor ratio

The Treynor ratio:

- uses the systematic risk measured with the beta of the portfolio,
- the higher the Treynor ratio, the better the portfolio performance (higher return per unit of systematic risk),
- can only be applied to well-diversified portfolios (portfolios with systematic risk only).

$$\mathbf{T = \frac{R_i - R_f}{\beta_i}}$$

Where:

- R_i – rate of return on the portfolio,
- R_f – risk-free rate,
- β_i – beta of the portfolio.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Jensen's alpha

Jensen's alpha:

- is expressed as a percentage,
- is an absolute measure,
- if positive, it means that the portfolio is managed efficiently,
- if negative, it means that the portfolio is managed inefficiently.

$$\boldsymbol{\alpha = R_i - (R_f + \beta_i \times [R_M - R_f])}$$

Where:

- R_i – rate of return on the portfolio,
- R_f – risk-free rate,
- R_M – expected market portfolio return,
- β_i – beta of the portfolio.

M-squared

The M-squared:

- uses the total risk measured with the standard deviation of the portfolio,
- gives rankings identical to those of the Sharpe ratio,
- is expressed as a percentage,
- is an absolute measure.

$$\boldsymbol{M^2 = (R_i - R_f) \times \frac{\sigma_M}{\sigma_i} - (R_M - R_f)}$$

Where:

- R_i – rate of return on the portfolio,
- R_f – risk-free rate,
- R_M – expected market portfolio return,
- σ_i – standard deviation of the portfolio,
- σ_M – standard deviation of the market portfolio.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Limitations of CAPM

Limitations of the CAPM:

- it is a single-index model (it states the return on the asset only by means of the market risk premium),
- it is a single-period model,
- the market portfolio should include all available assets,
- beta is changeable and difficult to estimate,
- the assumption about the homogeneous expectations of investors is doubtful,
- empirical tests have shown that the CAPM is a poor predictor of future prices.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

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Summarizing key concepts:

☐ CAL, CML & Market portfolio

My summary:

☐ Systematic risk vs Nonsystematic risk

My summary:

☐ Return-generating models

My summary:

☐ Single-index model and Market model

My summary:

☐ Beta

My summary:

☐ CAPM: assumptions

My summary:

☐ CAPM: capital market line (CML), security market line (SML)

My summary:

☐ CAPM: application

My summary:

☐ CAPM: limitations

My summary:

☐ Evaluation of fund manager's performance: Sharpe ratio, Treynor ratio, Jensen's alpha, M-squared

My summary:

Reviewing formulas:

$$\mathbf{E\left(R_p\right) = R_f + \frac{[E(R_M) - R_f]}{\sigma_M} \times \sigma_P}$$

Write down the formula:

$$\mathbf{E(R_P) - R_f = \sum_{i=1}^{n} \beta_{Pi} \times E(F_i)}$$

Write down the formula:

$$\mathbf{E(R_P) - R_f = \beta_P \times [E(R_M) - R_f]}$$

Write down the formula:

$$\mathbf{\sigma_P^2 = total\ risk = (\beta_M \times \sigma_M)^2 + \sigma_E^2}$$

Write down the formula:

$$\mathbf{R_i = \alpha_i + \beta_i \times R_M + e_i}$$

Write down the formula:

$$\mathbf{\beta_i = \frac{Cov(R_i, R_M)}{\sigma_M^2} = \frac{\rho_{i,M} \times \sigma_i}{\sigma_M}}$$

Write down the formula:

$$\mathbf{E(R_i) = R_f + \frac{\sigma_i}{\sigma_M} \times [E(R_M) - R_f]}$$

Write down the formula:

$$\mathbf{E(R_i) = R_f + \beta_i \times [E(R_M) - R_f]}$$

Write down the formula:

$$\mathbf{S = \frac{R_i - R_f}{\sigma_i}}$$

Write down the formula:

$$\mathbf{T = \frac{R_i - R_f}{\beta_i}}$$

Write down the formula:

$$\mathbf{\alpha = R_{\mathit{i}} - \left(R_f + \beta_i \times \left[R_M - R_f\right]\right)}$$

Write down the formula:

$$\mathbf{M^2 = \left(R_i - R_f\right) \times \frac{\sigma_M}{\sigma_i} - \left(R_M - R_f\right)}$$

Write down the formula:

Keeping myself accountable:

TABLE 1 | STUDY

When you sit down to study, you may want to **try the Pomodoro Technique** to handle your study sessions: study for 25 minutes, then take a 5-minute break. Repeat this 25+5 study-break sequence all throughout your daily study session.

✔ Tick off as you proceed.

POMODORO TIMETABLE: study-break sequences (25′ + 5′)													
date		date		date		date		date		date		date	
25′		25′		25′		25′		25′		25′		25′	
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TABLE 2 | REVIEW

Never ever neglect revision! Though it's not the most popular thing among CFA candidates, regular revision is what makes the difference. If you want to pass your exam, **schedule & do your review sessions.**

REVIEW TIMETABLE: When did I review this Reading?													
date		date		date		date		date		date		date	
date		date		date		date		date		date		date	