soleadea

LEVEL 1: QUANTITATIVE METHODS

Reading 1 (1st out of 7): TIME VALUE OF MONEY

Difficulty: **medium**

Benchmark Study Time: **3.75h**

2022

https://soleadea.org/

INTENTIONALLY EMPTY

THIS E-BOOK:

- is a selective summary of the corresponding Reading in your CFA® Program Curriculum,
- provides place for your own notes,
- helps you structure your study and revision time!

How to use this e-book to maximize your knowledge retention:

1. **Print** the e-book in duplex and bind it to keep **all** important info for this Reading **in one place.**
2. **Read** this e-book, best twice, to grasp the idea of what this Reading is about.
3. **Study** the Reading from your curriculum. **Here add** your notes, examples, formulas, definitions, etc.
4. **Review** the Reading using this e-book, e.g. write your summary of key concepts or revise the formulas at the end of this e-book (if applicable).
5. **Done?** Go to your study plan and change the Reading's status to **green**:
   (it will make your Chance-to-Pass-Score™ grow ☺).
6. **Come back** to this e-book from time to time to **regularly review for knowledge retention!**

NOTE: While studying or reviewing this Reading, you can use the tables at the end of this e-book and mark your study/review sessions to hold yourself accountable.

INTEREST RATE

Definitions

An interest rate is:

- a measure that helps us compare cash flows occurring on different dates,
- the price of money.

Interest rates can be perceived as:

- required rates of return,
- discount rates,
- opportunity costs.

The required rate of return can help us answer the following question:

What are the expected future profits from an investment?

A discount rate can help us answer the following question:

What is the present value of a certain future amount?

An opportunity cost can help us answer the following question:

What future profits do we forego in favor of current consumption?

Components of interest rates

$$\text{interest rate} = (\text{real risk-free interest rate}) + (\text{inflation premium}) +$$
$$+ (\text{default risk premium}) + (\text{liquidity premium}) + (\text{maturity premium})$$

real risk-free interest rate = an interest rate expected for postponing consumption,

inflation premium = a premium connected with expected inflation = a price of the expected inflation rate,

nominal risk-free interest rate = real risk-free interest rate + inflation premium,

default risk premium = a premium for the possibility that the entity concerned may fail to meet its financial obligation,

liquidity premium = a premium for reduced or no liquidity,

maturity premium = a premium connected with the increased price sensitivity of a financial instrument with a longer time to maturity with respect to changes in market interest rates,

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

SINGLE CASH FLOW

Future value of single cash flow (lump sum)

$$FV_N = PV \times (1 + r)^N$$

Where:

- PV – present value of the investment,
- FV_N – future value of the investment N periods from today,
- r – rate of interest per period (periodic interest rate).

Compounding

Compound interest is about adding interest accrued after the end of each compounding period to the principal. Each consecutive period we invest a greater amount and so we receive greater interest.

Remember:

- the more frequent the compounding, the higher the future value,
- the higher the present value, the higher the future value,
- the higher the number of periods, the higher the future value,
- the higher the interest rate, the higher the future value.

Present value of single cash flow (lump sum)

$$PV = \frac{FV_N}{(1 + r)^N}$$

Where:

- PV – present value of the investment,
- FV_N – future value of the investment N periods from today,
- r – rate of interest per period (periodic interest rate).

Remember:

- the more frequent the compounding, the lower the present value,
- the higher the future value, the higher the present value,
- the higher the number of periods, the lower the present value,
- the higher the interest rate, the lower the present value.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Types of interest rates

- Stated annual interest rate (r_s) = quoted interest rate,
- Periodic interest rate ($\frac{r_s}{m}$) = stated annual interest rate divided by the number of compounding periods per year,
- Effective annual rate (EAR).

<u>Effective annual rate (EAR)</u>

$$EAR = \left(1 + \frac{r_s}{m}\right)^m - 1$$

Where:

- r_s – stated annual interest rate,
- m – number of compounding periods in one year,
- $\frac{r_s}{m}$ – periodic interest rate.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

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ANNUITIES

Definitions

An annuity is a series of cash flows of the same value occurring at equal intervals. One type of annuity is **ordinary annuity**, i.e. an annuity in arrears, where the first payment is made **at the end** of the first period. Another type of annuity is **annuity due**. Annuity due is when cash flows occur **at the beginning** of each period of an investment, e.g. at the beginning of each month. The third type of annuity is **perpetuity**, aka. a never-ending sequence of future payments. Perpetuity can be perceived as a perpetual ordinary annuity because payments are made at the end of each period **indefinitely**.

Future value of ordinary annuity

$$\mathrm{FV_O} = \mathrm{A} \times (1+\mathrm{r})^{\mathrm{N}-1} + \mathrm{A} \times (1+\mathrm{r})^{\mathrm{N}-2} + .. + \mathrm{A} \times (1+\mathrm{r}) + \mathrm{A}$$

$$\mathrm{FV_O} = \mathrm{A} \times \left(\sum_{i=0}^{\mathrm{N}-1} (1+\mathrm{r})^{i}\right) = \mathrm{A} \times \left(\frac{(1+\mathrm{r})^{\mathrm{N}} - 1}{\mathrm{r}}\right)$$

Where:

- $\mathrm{FV_O}$ – future value of ordinary annuity,
- r – periodic interest rate,
- N – number of periods,
- A – annuity amount (annuity payment),
- $\left(\frac{(1+\mathrm{r})^{\mathrm{N}}-1}{\mathrm{r}}\right)$ – future value annuity factor.

Future value of annuity due

$$\mathrm{FV_D} = \mathrm{A} \times (1+\mathrm{r})^{\mathrm{N}} + \mathrm{A} \times (1+\mathrm{r})^{\mathrm{N}-1} + .. + \mathrm{A} \times (1+\mathrm{r})$$

$$\mathrm{FV_D} = \mathrm{A} \times \left(\sum_{i=1}^{\mathrm{N}} (1+\mathrm{r})^{i}\right) = \mathrm{A} \times \left(\frac{(1+\mathrm{r})^{\mathrm{N}} - 1}{\mathrm{r}}\right) \times (1+\mathrm{r}) = \mathrm{FV_O} \times (1+\mathrm{r})$$

Where:

- $\mathrm{FV_D}$ – future value of annuity due,
- r – periodic interest rate,
- N – number of periods,
- A – annuity amount (annuity payment).

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Present value of ordinary annuity

$$PV_O = \frac{A}{(1+r)} + \frac{A}{(1+r)^2} + .. + \frac{A}{(1+r)^N}$$

$$PV_O = A \times \left(\sum_{i=1}^{N} \frac{1}{(1+r)^i}\right) = A \times \left(\frac{1 - \frac{1}{(1+r)^N}}{r}\right)$$

Where:

- PV_O – present value of ordinary annuity,
- r – periodic interest rate,
- N – number of periods,
- A – annuity amount (annuity payment),
- $\left(\frac{1-\frac{1}{(1+r)^N}}{r}\right)$ – present value annuity factor.

Present value of annuity due

$$PV_D = A + \frac{A}{(1+r)} + \frac{A}{(1+r)^2} + .. + \frac{A}{(1+r)^{N-1}}$$

$$PV_D = A \times \left(\sum_{i=0}^{N-1} \frac{1}{(1+r)^i}\right) = A \times \left(\frac{1 - \frac{1}{(1+r)^N}}{r}\right) \times (1+r) = PV_O \times (1+r)$$

Where:

- PV_D – future value of annuity due,
- r – periodic interest rate,
- N – number of periods,
- A – annuity amount (annuity payment).

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Present value of perpetuity

$$PV_P = \frac{A}{r}$$

Where:

- PV_P – present value of perpetuity,
- A – annuity amount (annuity payment).

Note: When using the above formula, we get the present value of a perpetuity one period before the first annuity payment. For example, if the perpetuity consists of regular annual payments and the first payment is one year from now, the present value we get is for today. But if the perpetuity consists of regular annual payments and the first payment is three years from now, the present value we get is two years from now **OR** if the perpetuity consists of regular annual payments and the first payment is ten years from now, the present value we get is nine years from now. **Remember!** PV of perpetuity is always one period before the first payment!

So, if you are asked to compute the present value of perpetuity on T=0 (now) and the perpetuity consists of regular annual payments and the first payment is three years from now, you'll use the following adjusted formula (adjustment = discount the PV of perpetuity $\frac{A}{r}$ two years back):

$$PV_{(T=0)} = \frac{\frac{A}{r}}{(1+r)^2}$$

Algorithm for solving TVM problems

After reading a question:

- determine what you need to calculate,
- establish whether you are dealing with a single payment or an annuity or a series of unequal cash flows,
- in the case of annuity, check if it is an ordinary annuity or annuity due (payment at the end or at the beginning of periods) or perpetuity,
- **put the cash flows on a time line to visualize the problem,**
- apply the formulas and – using a calculator – solve the problem (use either direct formulas or TVM or CF+NPV+IRR worksheets),
- the combination of CF+NPV+IRR worksheets is especially useful when dealing with a series of unequal cash flows → e.g. there are cash flows in year 1, year 3, and year 6 → then: for years 1, 3, and 6 enter the cash flows given and for years 2, 4, and 5 input CF=0 → next, use the NPV worksheet to compute the present value (NPV) or the future value (NFV).
- interpret the results when necessary.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Summarizing key concepts:

☐ Interest rates: 3 interpretations

My summary:

☐ Components of interest rates

My summary:

☐ Future value & present value of single cash flow (lump sum)

My summary:

☐ Compounding

My summary:

☐ Types of interest rates

My summary:

☐ Future value & present value of ordinary annuity

My summary:

☐ Future value & present value of annuity due

My summary:

☐ Present value of perpetuity

My summary:

Reviewing formulas:

$$FV_N = PV \times (1 + r)^N$$

Write down the formula:

$$PV = \frac{FV_N}{(1 + r)^N}$$

Write down the formula:

$$EAR = \left(1 + \frac{r_s}{m}\right)^m - 1$$

Write down the formula:

$$FV_O = A \times (1 + r)^{N-1} + A \times (1 + r)^{N-2} + .. + A \times (1 + r) + A$$

$$FV_O = A \times \left(\sum_{i=0}^{N-1} (1 + r)^i\right) = A \times \left(\frac{(1 + r)^N - 1}{r}\right)$$

Write down the formula:

$$FV_D = A \times (1+r)^N + A \times (1+r)^{N-1} +..+ A \times (1+r)$$

$$FV_D = A \times \left(\sum_{i=1}^{N} (1+r)^i \right) = A \times \left(\frac{(1+r)^N - 1}{r} \right) \times (1+r) = FV_O \times (1+r)$$

Write down the formula:

$$PV_O = \frac{A}{(1+r)} + \frac{A}{(1+r)^2} +..+ \frac{A}{(1+r)^N}$$

$$PV_O = A \times \left(\sum_{i=1}^{N} \frac{1}{(1+r)^i} \right) = A \times \left(\frac{1 - \frac{1}{(1+r)^N}}{r} \right)$$

Write down the formula:

$$PV_D = A + \frac{A}{(1+r)} + \frac{A}{(1+r)^2} + .. + \frac{A}{(1+r)^{N-1}}$$

$$PV_D = A \times \left(\sum_{i=0}^{N-1} \frac{1}{(1+r)^i} \right) = A \times \left(\frac{1 - \frac{1}{(1+r)^N}}{r} \right) \times (1+r) = PV_O \times (1+r)$$

Write down the formula:

$$PV_P = \frac{A}{r}$$

Write down the formula:

Keeping myself accountable:

TABLE 1 | STUDY

When you sit down to study, you may want to **try the Pomodoro Technique** to handle your study sessions: study for 25 minutes, then take a 5-minute break. Repeat this 25+5 study-break sequence all throughout your daily study session.

✔ Tick off as you proceed.

POMODORO TIMETABLE: study-break sequences (25′ + 5′)													
date		date		date		date		date		date		date	
25′		25′		25′		25′		25′		25′		25′	
5′		5′		5′		5′		5′		5′		5′	
25′		25′		25′		25′		25′		25′		25′	
5′		5′		5′		5′		5′		5′		5′	
25′		25′		25′		25′		25′		25′		25′	
5′		5′		5′		5′		5′		5′		5′	
25′		25′		25′		25′		25′		25′		25′	
5′		5′		5′		5′		5′		5′		5′	

TABLE 2 | REVIEW

Never ever neglect revision! Though it's not the most popular thing among CFA candidates, regular revision is what makes the difference. If you want to pass your exam, **schedule & do your review sessions.**

REVIEW TIMETABLE: When did I review this Reading?													
date		date		date		date		date		date		date	
date		date		date		date		date		date		date	