soleadea

LEVEL 1: DERIVATIVE INVESTMENTS

Reading 46 (2nd out of 2): PRICING & VALUATION

Difficulty: **hard**

Benchmark Study Time: **4.75h**

2022

THIS E-BOOK:

- ❖ is a selective summary of the corresponding Reading in your CFA® Program Curriculum,
- ❖ provides place for your own notes,
- ❖ helps you structure your study and revision time!

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6. **Come back** to this e-book from time to time to **regularly review for knowledge retention!**

NOTE: While studying or reviewing this Reading, you can use the tables at the end of this e-book and mark your study/review sessions to hold yourself accountable.

ARBITRAGE, REPLICATION, & RISK NEUTRALITY IN DERIVATIVES PRICING

arbitrage = process through which an investor earns a profit without bearing any risk.

Law of one price:

- says that assets generating identical future cash flows should be priced at the same level,
- is the basis for arbitrage.

risk-neutral pricing = we can price derivatives assuming that discount rates are equal to the risk-free rate (no matter what the investors' aversion to risk is)

arbitrage-free pricing = a process of pricing derivatives using arbitrage and assuming the risk neutrality of investors

Replication:

- long underlying asset + short risk-free asset = long forward contract
- long underlying asset + short forward contract = long risk-free asset
- short forward contract + short risk-free asset = short underlying asset

Where:

- A long position in a risk-free asset is an equivalent of lending the money.
- A short position in a risk-free asset is an equivalent of borrowing the money.

HERE KNOWLEDGE RETENTION HAPPENS | **WRITE: notes, examples, formulas, definitions, relations, etc.**

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

VALUATION VS PRICING

The "**price**" and "**value**" of a contingent claim, like an option, are things that can be directly compared. On the contrary, in the case of forward commitments, like forwards, futures, and swaps, "price" and "value" mean very different things, e.g.:

- Forward price is a fixed value for which the underlying asset will be sold/bought at the forward contract expiration.
- Forward value is the value of the position in a forward contract for the investor in a given moment of time.

FORWARD CONTRACTS – PRICING

Forward price

$$F = S_0 \times (1 + r)^T$$

Where:

- F – forward price,
- r – risk-free interest rate,
- S_0 – underlying price at contract initiation,
- T – time until contract expiration.

Forward price assuming benefits & costs

$$F = \left(S_0 - i + c\right) \times (1 + r)^T$$

Where:

- F – forward price,
- r – risk-free interest rate,
- S_0 – underlying price at contract initiation,
- i – PV of benefits (e.g. dividends, income, interest, convenience yield),
- c – PV of costs (e.g. storage cost),
- T – time until contract expiration.

NOTE: *When calculating the forward price when there's the* ***net cost of carry*** *given (defined as the difference between the benefits of holding the underlying and the cost related to holding the underlying), we decrease the underlying price by this net cost of carry.*

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

FORWARD CONTRACTS – VALUATION

Forward contract value at time t=0

$$V_t = 0$$

Forward contract value at time t (long position)

$$V_t = \frac{S_t \times (1+r)^{T-t} - F}{(1+r)^{T-t}} = S_t - \frac{F}{(1+r)^{T-t}}$$

Where:

- V_t – forward contract value at time t,
- F – forward price established for the contract,
- r – risk-free interest rate,
- S_t – underlying price at time t,
- T – time until contract expiration (set in the forward contract),
- $(T-t)$ – time remaining until the forward contract expiration.

Forward contract value at time t assuming benefits & storage costs (long position)

$$V_t = \frac{\left[S_t - (i-c) \times (1+r)^t\right] \times (1+r)^{T-t} - F}{(1+r)^{T-t}} = S_t - (i-c) \times (1+r)^t - \frac{F}{(1+r)^{T-t}}$$

Where:

- V_t – forward contract value at time t,
- F – forward price established for the contract,
- r – risk-free interest rate,
- S_t – underlying price at time t,
- i – PV of benefits (e.g. dividends, income, interest, convenience yield),
- c – PV of costs (e.g. storage cost),
- T – time until contract expiration (set in the forward contract),
- $(T-t)$ – time remaining until the forward contract expiration.

Forward contract value at expiration (T) assuming benefits & storage costs (long position)

$$V_t = S_T - (i-c) \times (1+r)^T - F$$

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Monetary & Non-Monetary Benefits & Costs

Dividends and a **risk-free rate earned on a foreign currency** are monetary (usually) benefits of holding an underlying asset.

Cost of storage is an example of a monetary (usually) cost.

Convenience yield is a non-monetary benefit of holding an asset and is observed mainly when a commodity:

- cannot be sold short, or
- is in a short supply.

The convenience yield can be observed in the case of commodities derivatives like oil futures, but not in the case of financial instruments derivatives like S&P 500 futures.

PRICES OF FORWARDS VS PRICES OF FUTURES

Forwards and futures are characterized by different cash flow patterns, which may lead to different prices. In the case of a futures, we can expect small cash flows during the life of the contract, whereas in the case of a forward – there is one bigger cash flow at the contract expiration. This different cash flow pattern may produce different prices of forwards and futures when we take the time value of money concept into account.

When interest rates are constant, there will be no differences between futures and forward prices.

On the other hand, BOTH positive correlation between futures prices and interest rates AND negative correlation between futures prices and interest rates lead to differences between futures and forward prices:

- If there is a positive correlation between futures prices and interest rates, the long will prefer futures over forwards (he can reinvest cash from daily settlements at higher interest rates).
- If there is a negative correlation between futures prices and interest rates, the short will prefer futures over forwards (he can reinvest cash from daily settlements at higher interest rates).

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

FORWARD RATE AGREEMENTS (FRAs)

Basics

A forward rate agreement is an agreement made to fix an interest rate at a specified level at a specified future time.

With an FRA, it is possible to hedge against the risk of future interest rate changes.

In an FRA, a party can take:

- a long position,
- a short position.

The party that goes long is the one that seeks to take out a loan in the future at a specified interest rate and wants to hedge against an increase in interest rates.

The party that goes short is the one that lends money or makes a deposit in the future and wants to hedge against a drop in interest rates.

FRA payoff formula

FRA payoff formula for the long:

$$\text{notional principal} \times$$
$$\times \frac{(\text{underlying rate at expiration} - \text{forward contract rate}) \times \left(\frac{\text{days in underlying rate}}{360}\right)}{1 + (\text{underlying rate at expiration}) \times \left(\frac{\text{days in underlying rate}}{360}\right)}$$

Where:

- underlying rate at expiration = LIBOR or EURIBOR value on the contract expiration day
- days in underlying rate = the number of days left to maturity of the loan

The numerator is the buyer's payoff amount and it is:

- positive if the underlying rate is higher than the forward contract rate,
- negative if the underlying rate is lower than the forward contract rate.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

LIBOR

LIBOR = London Interbank Offered Rate

FRAs are often based on LIBOR.

LIBOR is calculated each day by Thomson Reuters, to whom major banks submit their cost of borrowing unsecured funds for 15 periods of time ranging from overnight to 12 months in 10 currencies.

150 different LIBOR rates are published every day.

- LIBOR is quoted as an annualized rate based on a 360-day year.
- Exception: in the case of British pounds, the year is assumed to have 365 days.

FRA notation examples

3 × 6 FRA:

- the contract expires in 90 days (3 times 30 days),
- the interest on the loan is paid in another 3 months, i.e. in 180 days (6 times 30 days) from the initiation of the contract,
- the contract is based on 90-day LIBOR.

12 × 24 FRA:

- the contract expires in 360 days (12 times 30 days),
- the interest on the loan is paid in another 12 months, that is in 720 days (24 times 30 days) from the initiation of the contract,
- the contract is based on 360-day LIBOR.

3 × 9 FRA:

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Real FRA vs Synthetic FRA

<u>Example:</u>

Real FRA	Underlying	Synthetic FRA consists of:
3 × 9 FRA (long position)	6-month LIBOR	- short position in 3-month Eurodollar time deposit, **and** - long position in 9-month Eurodollar time deposit
3 × 12 FRA (short position)	9-month LIBOR	- long position in 3-month Eurodollar time deposit, **and** - short position in 12-month Eurodollar time deposit

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

SWAPS

Valuation

- At initiation, a swap usually has a zero value.
- However, during its life, a swap can have a zero, negative, or positive value, depending on the changes in market values.

Example:

A plain vanilla swap is a basic interest rate swap for which we agree to exchange some known fixed rate for unknown floating rates. For each payment date, the value of the floating rate is derived from the market. If, for example, market interest rates go up, the party that pays the fixed rate (the long) and receives the floating rates will expect to receive higher values in the future. Because at initiation the value of the swap was 0 (it's a forward commitment), the increase of interest rates will lead to a higher (positive) value of the swap to the party that goes long. Of course, then the value for the short will be negative.

Swaps Replication Using Off-Market Forwards

Both forwards and swaps usually have a zero value at initiation. However, there might be exceptions, e.g. off-market forwards are defined as forwards with nonzero values at initiation. This kind of forward is rather uncommon but is sometimes created.

We can replicate a swap contract with a series of forward contracts with different expiration dates. However, generally forwards expiring on different dates will have different forward prices and the swap price is the same for each date. So, to be able to replicate the swap with forwards, we need to use off-market forwards – each of them will have the same forward price (equal to the swap price) and some of them will have a negative and some of them – a positive value at initiation. However, the sum of their values at initiation will be 0, the same as of the swap. So, to make a long story short:

A swap can be perceived as a series of off-market forwards with the same forward price.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

OPTIONS – MONEYNESS

Based on the moneyness criterion, we distinguish between options that are either profitable or unprofitable to exercise at a given moment.

There are three types of such options:

- in-the-money options,
- out-of-the-money options,
- at-the-money options.

If the exercise of an option produces a positive payoff → in-the-money option

If the exercise of an option produces a negative payoff → out-of-the-money option

If payoff = 0 → at-the-money option

Whether an option is in-the-money, at-the-money or out-of-the-money depends on:

- the price of the underlying stock, and
- the strike price of the option.

$X < S$ → a call option is in-the-money

$X > S$ → a put option is in-the-money

$X > S$ → a call option is out-of-the-money

$X < S$ → a put option is out-of-the-money

$X = S$ → both a call option and a put option are at-the-money

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

OPTION VALUE

option value

intrinsic value + time value

Intrinsic value

The intrinsic value of:

call option = $MAX[(S_T - X),0]$

put option = $MAX[0,(X - S_T)]$

Intrinsic value is also called exercise value.

Time value

The value related to the fact that an option may be more profitable in the future than today is called time value.

$$\text{time value} = \text{option price} - \text{intrinsic value}$$

Time value represents the amount that an investor is ready to pay out of hope that before expiration the value of an option will increase as a result of a beneficial change in the price of the underlying.

Time value vs time:

- the more time left to expiration, usually the greater the time value of an option
- at expiration, the time value of an option is always equal to 0

NOTE: Both intrinsic value and time value are greater than or equal to 0, hence option value is always greater than or equal to 0.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

FACTORS AFFECTING THE PRICE OF AN OPTION

Factors affecting the price of an option:

- price of the underlying asset (S),
- income or costs on the underlying asset within the lifetime of the option (D),
- exercise price (X),
- time to expiration (T),
- volatility of the underlying asset's price (σ),
- risk-free interest rate (r).

	Call option premium	Put option premium	Option price sensitivity measure
Higher price of the underlying asset (S)	higher	lower	Delta
Higher expected income from the underlying (e.g. dividend in the case of stocks) (D)	lower	higher	-
Higher exercise price (X)	lower	higher	-
Longer time to expiration (T)	higher	higher (except for some European-style puts)	Theta
Greater volatility (σ)	higher	higher	Vega
Higher interest rate (r)	higher	lower	Rho

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

PUT-CALL PARITY

Assumptions

We have an underlying asset, e.g. a stock, and two options on the underlying:

- a call option, and
- a put option.

Both options:

- are European-style options,
- have the same expiration date,
- have the same exercise price, and
- cover the same quantity of the underlying.

$$c_0 + \frac{X}{(1 + r)^T} = S_0 + p_0$$

Where:

- c_0 – current price of a call option,
- X – exercise price,
- r – risk-free interest rate,
- S_0 – current price of the underlying asset less the PV of any future benefits & plus the PV of any future costs expected to be earned or incurred on the underlying before the expiration of the option,
- p_0 – current price of a put option.

left-hand side of the equation ←→ fiduciary call

right-hand side of the equation ←→ protective put

fiduciary call = call option + zero-coupon bond (bond par value = X)

protective put = underlying asset + put option

On expiration day, both the fiduciary call and the protective put pay off the same amounts.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Put–call parity:

- allows us to determine the price of one option if we know the price of the other,
- is useful for determining the minimum price of both a call and a put,
- thanks to relationships involved in the put–call parity, allows us to create synthetic instruments.

Synthetic long call

$$c = S + p - \frac{X}{(1 + r)^T}$$

A synthetic long call consists of:

- a long position in an underlying asset,
- a long put,
- a short position in a bond.

Synthetic short call

$$-c = -S - p + \frac{X}{(1 + r)^T}$$

A synthetic short call consists of:

- a short position in an underlying asset,
- a short put,
- a long position in a bond.

Synthetic long put

$$p = c + \frac{X}{(1 + r)^T} - S$$

A synthetic long put consists of:

- a long call,
- a long position in a bond,
- a short position in an underlying asset.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Synthetic short put

$$-p = -c - \frac{X}{(1 + r)^{T}} + S$$

A synthetic short put consists of:

- a short call,
- a short position in a bond,
- a long position in an underlying asset.

PUT-CALL-FORWARD PARITY

Assumptions

We have an underlying asset, forward contract on the underlying asset, and two options on the underlying asset:

- a call option, and
- a put option.

Both options:

- are European-style options,
- have the same expiration date,
- have the same exercise price, and
- cover the same quantity of the underlying.

$$p_0 - c_0 = \frac{X - F}{(1 + r)^{T}}$$

Where:

- c_0 – current price of a call option,
- p_0 – current price of a put option,
- X – exercise price,
- F – forward price,
- r – risk-free interest rate.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

LOWER BOUNDS & MAXIMUM VALUES OF OPTIONS

	Minimum value	Maximum value
American-style call option	$\mathrm{MAX}\left[S_0 - \left(\frac{X}{(1+r)^T}\right), 0\right]$	S_0
European-style call option	$\mathrm{MAX}\left[S_0 - \left(\frac{X}{(1+r)^T}\right), 0\right]$	S_0
American-style put option	$\mathrm{MAX}[(X - S_0), 0]$	X
European-style put option	$\mathrm{MAX}\left[\left(\frac{X}{(1+r)^T} - S_0\right), 0\right]$	$\frac{X}{(1+r)^T}$

Where:

- X – exercise price,
- r – risk-free interest rate,
- S_0 – current price of the underlying asset less the PV of any future benefits & plus the PV of any future costs expected to be earned or incurred on the underlying before the expiration of the option,
- T – time to expiration of an option.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

BINOMIAL VALUATION OF OPTIONS

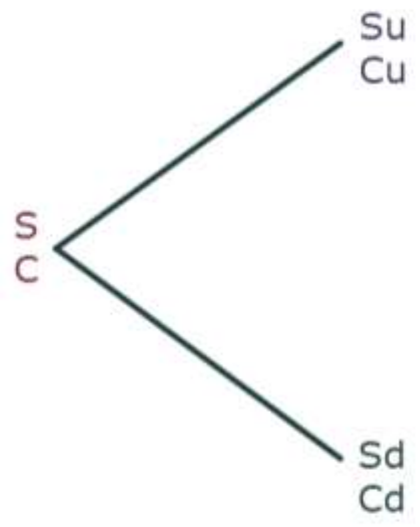
Assumptions:

- we have a call option on a stock,
- the value of the call option today is C,
- the price of the stock is S,
- the stock doesn't pay dividends,
- we assume only one period,
- there are only two scenarios available for the period: the price of the stock can either increase or decrease.

Our goal is to find out the value of the call option.

Have a look at the graph presenting the two scenarios:

- **Scenario 1** assumes that the stock price will increase in the period.
- **Scenario 2** assumes that the stock price will decrease in the period.



Where:

- S – current price of stock,
- C – current value of call option,
- u – represents an up move in the stock price,
- d – represents a down move in the stock price,
- Su – stock price assuming scenario 1,
- Sd – stock price assuming scenario 2,
- Cu – call option value assuming scenario 1,
- Cd – call option value assuming scenario 2.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Create a risk-free portfolio

To find out what the price of a call option is, we create a risk-free portfolio consisting of call options and stocks. How do we achieve this?

We know that the prices of a call option and a stock are directly related, namely when the stock price increases, the call option premium also increases and vice versa – when the stock price decreases, the call option premium also decreases. Therefore, in the risk-free portfolio we should have call options and stocks in different positions. Thanks to this, changes in the price of one instrument will be offset by changes in the price of the other instrument. There is an infinite number of such risk-free portfolios, and we choose one of them.

We assume that our risk-free portfolio (**portfolio F**) consists of:

- 1 short call, and
- a number of long positions in a stock. We will refer to the number of long positions in a stock as **Δ**.

$$\text{current value of portfolio F} = \Delta \times \text{S} - \text{C}$$

If the price of the stock increases (**scenario 1**), then the value of portfolio F will be equal to:

$$\Delta \times \text{Su} - \text{Cu}$$

If the price of the stock decreases (**scenario 2**), then the value of portfolio F will be equal to:

$$\Delta \times \text{Sd} - \text{Cd}$$

If portfolio F is risk-free, then the value of the portfolio in the upper and the lower node should be equal. Why? This follows from the definition of a risk-free investment. Regardless of the circumstances and assumed scenarios, the value of the risk-free investment should be the same at the end of a given period.

Therefore:

$$\Delta \times \text{Su} - \text{Cu} = \Delta \times \text{Sd} - \text{Cd}$$

$$\Delta = \frac{\text{Cu} - \text{Cd}}{\text{Su} - \text{Sd}}$$

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

if the number of long positions in the stock is $\frac{Cu-Cd}{Su-Sd}$ → portfolio F is a risk-free portfolio

Because the rate of return on a risk-free investment is equal to the risk-free interest rate, then:

$$[\Delta \times S - C] \times (1 + r) = \Delta \times Su - Cu = \Delta \times Sd - Cd$$

Where:

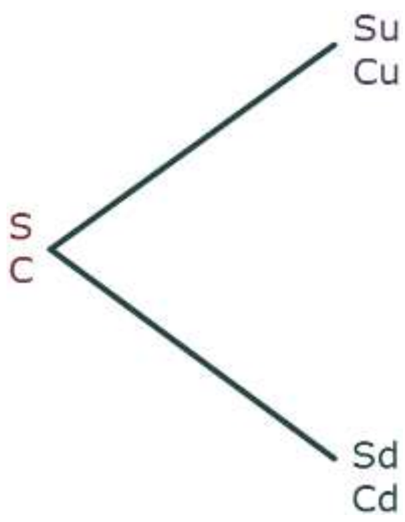
- r – risk-free interest rate over one period.

Compute the value of a call option

After we substitute Δ by $\frac{Cu-Cd}{Su-Sd}$ and transform the above equation, we get

the formula for the value of a call option today:

$$C = \frac{p \times Cu + (1 - p) \times Cd}{1 + r}$$



Where:

- C – call option current value,
- S –stock current price,
- u – represents an up move in the stock price,
- d – represents a down move in the stock price,
- Su – stock price assuming an up move in the stock price,
- Sd – stock price assuming a down move in the stock price,
- $p = \frac{1+r-d}{u-d}$,
- $p; (1 - p)$ – risk-neutral probabilities (the probabilities that would exist if we assumed that investors are risk-neutral),
- r – risk-free interest rate over one period,
- Cu – call option value assuming an up move in the stock price,
- Cd – call option value assuming a down move in the stock price.

The same formula can be applied to put option valuation.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Summarizing key concepts:

☐ Arbitrage, replication & risk neutrality in derivatives pricing

My summary:

☐ Valuation vs Pricing

My summary:

☐ Forward contracts – pricing

My summary:

☐ Forward contracts – valuation

My summary:

☐ Prices of forwards vs Prices of futures

My summary:

☐ Forward Rate Agreements (FRAs)

My summary:

☐ Swaps valuation & replication

My summary:

☐ Options – moneyness, value

My summary:

☐ Factors affecting the price of an option

My summary:

☐ Put-call parity

My summary:

☐ Put-call-forward parity

My summary:

☐ Lower bounds & Maximum values of options

My summary:

☐ Binomial valuation of options

My summary:

Reviewing formulas:

$$F = S_0 \times (1 + r)^T$$

Write down the formula:

$$F = \left(S_0 - i + c\right) \times (1 + r)^T$$

Write down the formula:

$$V_t = \frac{S_t \times (1 + r)^{T-t} - F}{(1 + r)^{T-t}} = S_t - \frac{F}{(1 + r)^{T-t}}$$

Write down the formula:

$$V_t = \frac{\left[S_t - (i - c) \times (1 + r)^t\right] \times (1 + r)^{T-t} - F}{(1 + r)^{T-t}} = S_t - (i - c) \times (1 + r)^t - \frac{F}{(1 + r)^{T-t}}$$

Write down the formula:

$$V_t = S_T - (i - c) \times (1 + r)^T - F$$

Write down the formula:

$$c_0 + \frac{X}{(1 + r)^T} = S_0 + p_0$$

Write down the formula:

$$p_0 - c_0 = \frac{X - F}{(1 + r)^T}$$

Write down the formula:

$$\mathrm{MAX}\left[S_0 - \left(\frac{X}{(1 + r)^T}\right), 0\right]$$

Write down the formula:

$$\mathrm{MAX}[(X - S_0), 0]$$

Write down the formula:

$$\mathrm{MAX}\left[\left(\frac{X}{(1+r)^T} - S_0\right), 0\right]$$

Write down the formula:

$$\Delta = \frac{Cu - Cd}{Su - Sd}$$

$$C = \frac{p \times Cu + (1-p) \times Cd}{1+r}$$

$$p = \frac{1+r-d}{u-d}$$

Write down the formula:

Keeping myself accountable:

TABLE 1 | STUDY

When you sit down to study, you may want to **try the Pomodoro Technique** to handle your study sessions: study for 25 minutes, then take a 5-minute break. Repeat this 25+5 study-break sequence all throughout your daily study session.

✔ Tick off as you proceed.

POMODORO TIMETABLE: study-break sequences (25′ + 5′)													
date		date		date		date		date		date		date	
25′		25′		25′		25′		25′		25′		25′	
5′		5′		5′		5′		5′		5′		5′	
25′		25′		25′		25′		25′		25′		25′	
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25′		25′		25′		25′		25′		25′		25′	
5′		5′		5′		5′		5′		5′		5′	
25′		25′		25′		25′		25′		25′		25′	
5′		5′		5′		5′		5′		5′		5′	

TABLE 2 | REVIEW

Never ever neglect revision! Though it's not the most popular thing among CFA candidates, regular revision is what makes the difference. If you want to pass your exam, **schedule & do your review sessions.**

REVIEW TIMETABLE: When did I review this Reading?													
date		date		date		date		date		date		date	
date		date		date		date		date		date		date	