LEVEL 1: PORTFOLIO MANAGEMENT

Reading 49 (2nd out of 8): RISK & RETURN

Difficulty: **medium**

Benchmark Study Time: **5h**

2022

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INTENTIONALLY EMPTY

THIS E-BOOK:

- ❖ is a selective summary of the corresponding Reading in your CFA® Program Curriculum,
- ❖ provides place for your own notes,
- ❖ helps you structure your study and revision time!

How to use this e-book to maximize your knowledge retention:

1. **Print** the e-book in duplex and bind it to keep **all** important info for this Reading **in one place.**
2. **Read** this e-book, best twice, to grasp the idea of what this Reading is about.
3. **Study** the Reading from your curriculum. **Here add** your notes, examples, formulas, definitions, etc.
4. **Review** the Reading using this e-book, e.g. write your summary of key concepts or revise the formulas at the end of this e-book (if applicable).
5. **Done?** Go to your study plan and change the Reading's status to green:
   (it will make your Chance-to-Pass-Score™ grow ☺).
6. **Come back** to this e-book from time to time to **regularly review for knowledge retention!**

NOTE: While studying or reviewing this Reading, you can use the tables at the end of this e-book and mark your study/review sessions to hold yourself accountable.

RETURN AND RISK

Rate of return and its measures

Sources of return:

- income (e.g. dividends),
- capital gain (increase in the price).

Holding period return (HPR)

HPR measures return over a single period.

$$\mathbf{HPR} = \frac{\mathbf{P_1 + D_1}}{\mathbf{P_0}} - \mathbf{1}$$

Where:

- HPR – holding period return,
- P_1 – value of the investment at the end of the period,
- D_1 – value of dividend,
- P_0 – value of the investment at the beginning of the period.

Arithmetic return (mean return)

$$\overline{\mathbf{R}}_{\mathbf{A}} = \frac{\mathbf{1}}{\mathbf{T}}\sum_{\mathbf{t=1}}^{\mathbf{T}} \mathbf{R_t}$$

Where:

- $\overline{R}_A$ – arithmetic return,
- T – number of holding periods during the lifetime of the investment,
- R_t – rate of return over a single period t.

Geometric mean return

$$\overline{\mathbf{R}}_{\mathbf{G}} = \sqrt[\mathbf{T}]{\prod_{\mathbf{t=1}}^{\mathbf{T}} (\mathbf{1} + \mathbf{R_t})} - \mathbf{1}$$

Where:

- $\overline{R}_G$ – geometric mean return,
- T – number of holding periods during the lifetime of the investment,
- R_t – rate of return over a single period t.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Arithmetic mean return VS Geometric mean return

Arithmetic mean return:

- initial value of the investment is the same for each holding period.

Geometric mean return:

- more commonly used for investments,
- initial value of the investment may be different for each holding period.

rates of return **are equal** for all periods

↓

geometric mean **=** arithmetic mean

rates of return **are not equal** for all periods

↓

geometric mean **<** arithmetic mean

Example

Period	Rate of return (%)
Q1 2019	-3
Q2 2019	1
Q3 2019	4
Q4 2019	8

$$\overline{R}_A = \frac{1}{T}\sum_{t=1}^{T} R_t = \frac{R_1 + R_2 + R_3 + R_4}{T} = \frac{-3+1+4+8}{4} = \frac{10}{4} = 2.5 = 2.5\%$$

$$\overline{R}_G = \sqrt[T]{\prod_{t=1}^{T}(1+R_t)} - 1 = \sqrt[4]{(1+R_1)\times(1+R_2)\times(1+R_3)\times(1+R_4)} - 1 = \sqrt[4]{1.1004} == 1.0242 - 1$$
$$= 0.242 = 2.42\%$$

Money-weighted return

To measure the return on investment over multiple periods, we can also use the money-weighted return. The money-weighted return is an internal rate of return. We use it when the investment amount changes from period to period.

$$\boldsymbol{\sum_{t=0}^{T}\frac{CF_t}{(1+IRR)^t} = 0}$$

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Where:

- CF_t – cash flow in period t,
- IRR – money-weighted return,
- T – number of periods during the lifetime of the investment.

Time-weighted return

For practical reasons, we often apply the time-weighted rate of return. The time-weighted rate of return differs from the money-weighted rate of return as it does not depend on the value of particular cash flows. The time-weighted rate of return is a **geometric return** over the whole investment period.

Money-weighted rate of return vs Time-weighted rate of return

- The money-weighted rate of return gives different weights to different periods, while the time-weighted rate of return gives the same weights to different periods.
- If the portfolio manager has **full control** of the timing and amounts of cash inflows and outflows, the **money-weighted rate of return** should be used.
- When the portfolio manager has **little influence** on the timing and invested amounts, the **time-weighted rate of return** should be applied.

Annualized rate of return

As far as both the computation and the comparison of returns on different investments are concerned, it is important for you to be able to convert rates of return to a single period. The annualized rate of return helps compare returns on different investments.

$$\mathbf{R = (1 + r)^t - 1}$$

Where:

- R – annualized rate of return,
- r – rate of return over the analyzed holding period,
- t – number of holding periods during one year.

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Portfolio return

Portfolio return is the weighted average of the returns of the individual assets in the portfolio.

$$\mathbf{R_p = \sum_{i=1}^{N} w_i \times R_i}$$

Where:

- R_p – rate of return on the portfolio,
- w_i – weight of asset "i" in the portfolio,
- R_i – return on asset "i".

Nominal return VS Real return

Real return is nominal return adjusted for inflation.

$$\mathbf{(1 + R_{real}) = \frac{(1 + R_{nom})}{(1 + R_i)}}$$

Where:

- R_{real} – real return,
- R_{nom} – nominal return,
- R_i – inflation rate.

Asset classes

There are many lines along which we can divide different types of assets. However, the 2 basic characteristics of all financial assets, i.e. return and risk, allow us to identify the major asset classes which include: large company stocks, small company stocks, corporate bonds, government bonds, and Treasury bills. This table shows comprehensive data for the American market:

	2000-2008		1926-2008	
Asset class	Rate of return (%)	Standard deviation (%)	Rate of return (%)	Standard deviation (%)
Large companies	-3.6	15.0	9.60	20.6
Small companies	4.1	24.5	11.7	33.0
Corporate bonds	8.2	11.3	5.90	8.4
Government bonds	10.5	11.7	5.70	9.4
Treasury bills	3.1	0.5	3.90	3.1

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Risk and its measures

Population variance

$$\sigma^2 = \frac{\sum_{i=1}^{T}(R_i - \mu)^2}{T}$$

Where:

- σ^2 – population variance,
- R_i – return "i",
- T – number of periods,
- μ – population mean.

Sample variance

Sample variance is used when there is no full information on the entire population of returns.

$$s^2 = \frac{\sum_{i=1}^{T}(R_i - \overline{R})^2}{T-1}$$

Where:

- s^2 – sample variance,
- R_i – return "i",
- T – number of periods.
- $\overline{R}$ – sample mean.

Standard deviation

Standard deviation is a square root of variance.

$$\sigma = \sqrt{\frac{\sum_{i=1}^{T}(R_i - \mu)^2}{T}}$$

$$s = \sqrt{\frac{\sum_{i=1}^{T}(R_i - \overline{R})^2}{T-1}}$$

Where:

- σ – population standard deviation,
- s – sample standard deviation.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

UTILITY THEORY: INDIFFERENCE CURVE, CLASSIFICATION OF INVESTORS

The utility theory

According to the utility theory, every person should seek to maximize his or her utility (= satisfaction).

The utility theory is based on the utility function which assigns a particular utility value to a given monetary value. The utility function:

- allows us to rank investments from the most to the least beneficial for the investor,
- is different for different persons,
- can change over time.

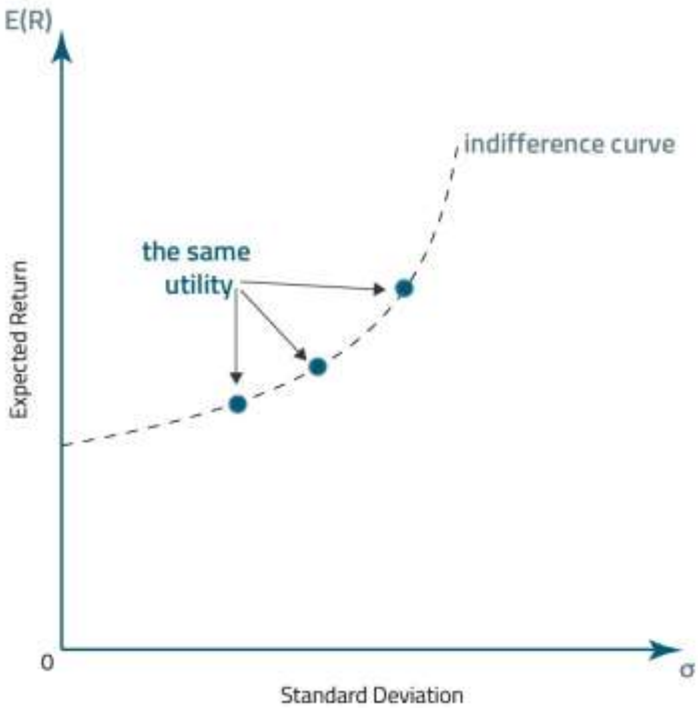
$$\mathcal{U} = \mathbf{E(R)} - \frac{\mathbf{1}}{\mathbf{2}} \times \mathbf{A} \times \boldsymbol{\sigma}^{\mathbf{2}}$$

Where:

- U – utility of the investment,
- $\mathrm{E(R)}$ – expected return,
- A – risk aversion coefficient,
- σ^2 – variance.

Indifference curve

A graph representing the utility function is called an indifference curve. At each point of the indifference curve, the investor's utility is the same.

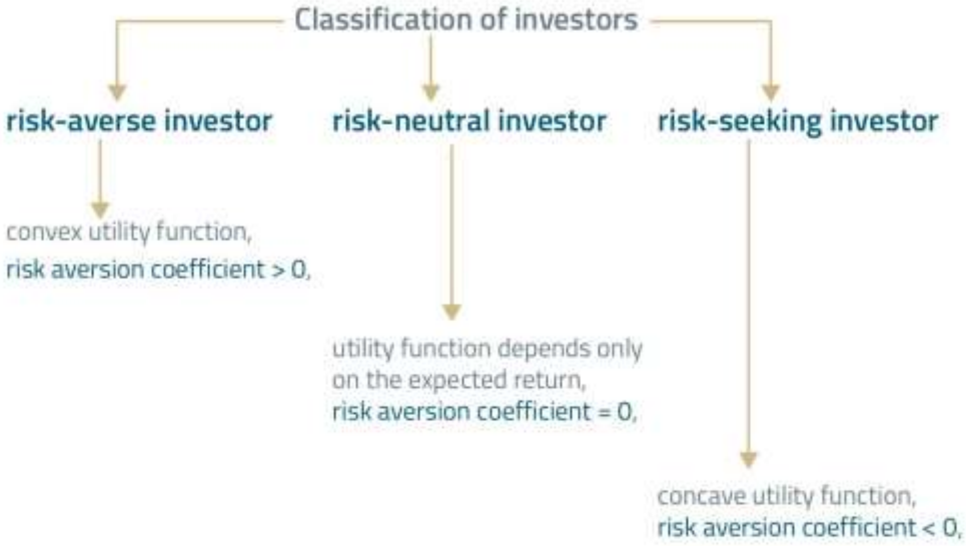


HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Classification of investors

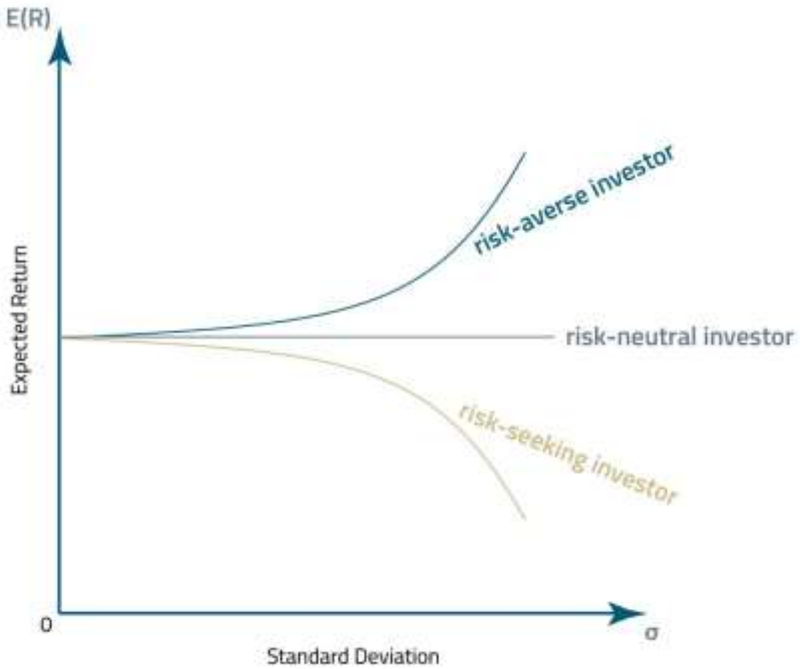
Based on an investor's attitude to risk, we distinguish 3 types of investors:

- risk-averse investors,
- risk-neutral investors,
- risk-seeking investors.



The lower the risk aversion, the higher the risk tolerance.

Indifference curves for different types of investors



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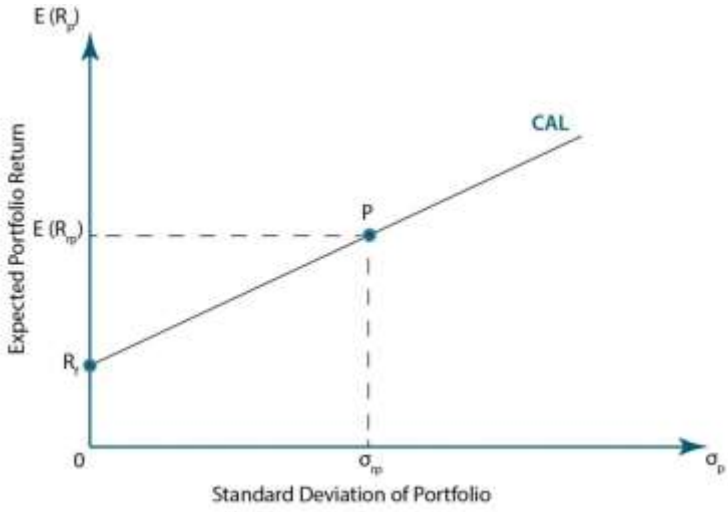
Portfolio selection

The capital allocation line (CAL) reflects the return of an investor's portfolio depending on the risk measured with standard deviation. The capital allocation line is a line connecting a risk-free asset with the portfolio of risky assets.

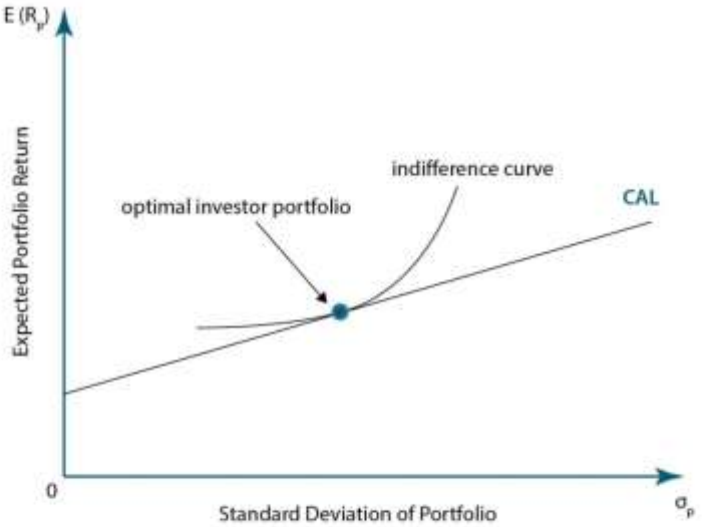


$$\mathbf{E}\left(\mathbf{R_p}\right) = \mathbf{R_f} + \frac{\mathbf{E(R_{rp}) - R_f}}{\boldsymbol{\sigma}_{\mathbf{rp}}} \times \boldsymbol{\sigma}_{\mathbf{P}}$$

Where:

- $E(R_P)$ – expected return of the portfolio,
- R_f – risk-free rate,
- $E(R_{rp})$ – expected return of risky assets,
- σ_{rp} – standard deviation of risky assets,
- σ_P – standard deviation of the portfolio.

A properly defined utility function can help us select the right portfolio for an investor. The **optimal investor portfolio** is located at the tangent of CAL and indifference curve.



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PORTFOLIO RISK

Covariance and correlation coefficient

Covariance

Covariance is a measure of **linear association** between two variables.

If both variables deviate from the mean simultaneously up or simultaneously down, the covariance has a positive value.

If the values of two variables deviate in opposite directions, the covariance has a negative value.

A covariance equal or close to zero means that there is no linear association between two variables.

The absence of linear association does not mean that the variables are not associated!

Correlation coefficient

The correlation coefficient:

- is easier to interpret than the covariance,
- its value ranges from -1 to 1.

$$\boldsymbol{\rho_{12} = \frac{Cov(R_1, R_2)}{\sigma_1 \times \sigma_2}}$$

Where:

- ρ_{12} – correlation coefficient between asset 1 and asset 2,
- σ_1, σ_2 – standard deviations,
- $\mathrm{Cov}(R_1, R_2)$ – covariance between asset 1 and asset 2.

The correlation coefficient shows how strong the linear association is between the returns on portfolio assets.

- $\rho_{12} = +1$ (perfect positive correlation),
- $\rho_{12} = 0$ (returns are not correlated at all),
- $\rho_{12} = -1$ (perfect negative correlation).

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Standard deviation of a portfolio

$$\sigma = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} w_i \times w_j \times Cov\left(R_i, R_j\right)} = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} w_i \times w_j \times \rho_{ij} \times \sigma_i \times \sigma_j}$$

Where:

- σ – standard deviation of the portfolio,
- w – weight of an individual asset in the portfolio.

the covariance of A and A = the variance of A

<u>Standard deviation of a portfolio comprised of two assets</u>

$$\sigma_p = \sqrt{w_A^2 \times \sigma_A^2 + w_B^2 \times \sigma_B^2 + 2 \times w_A \times w_B \times Cov(R_A R_B)} =$$
$$= \sqrt{w_A^2 \times \sigma_A^2 + w_B^2 \times \sigma_B^2 + 2 \times w_A \times w_B \times \rho_{AB} \times \sigma_A \times \sigma_B}$$

The lower the covariance between the assets:

- the lower the portfolio risk,
- the stronger the effect of portfolio diversification.

When there's a **perfect positive correlation**, the portfolio risk is equal to the weighted average risk of individual assets in the portfolio and the **diversification** of the portfolio **doesn't reduce the portfolio's risk**:

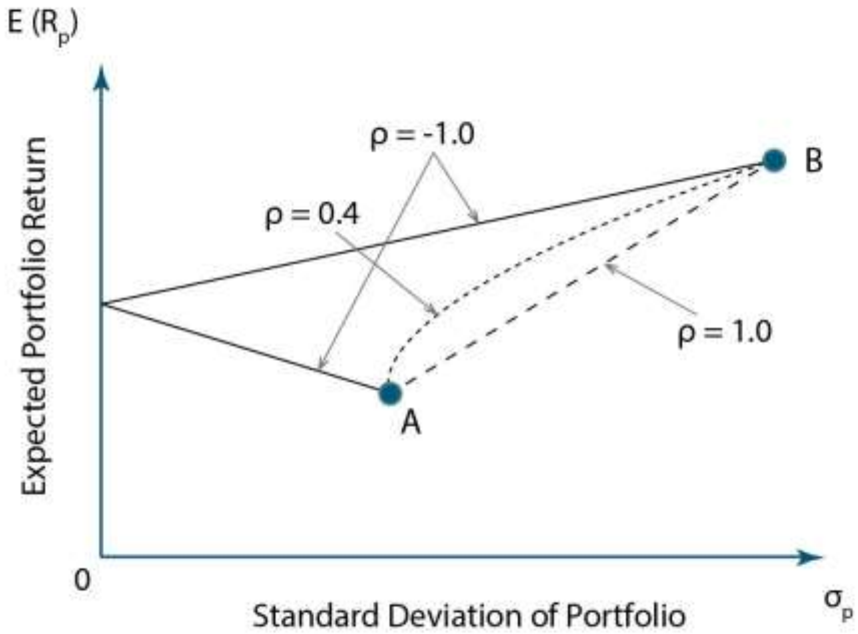
$$\sigma_p = w_A \times \sigma_A + w_B \times \sigma_B$$

If the correlation coefficient **is lower than +1**, the portfolio standard deviation is lower than the weighted average of the standard deviations of individual assets in the portfolio:

$$\sigma_p < w_A \times \sigma_A + w_B \times \sigma_B$$

Therefore, the **diversification** of the portfolio **reduces the portfolio's risk**.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.



HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

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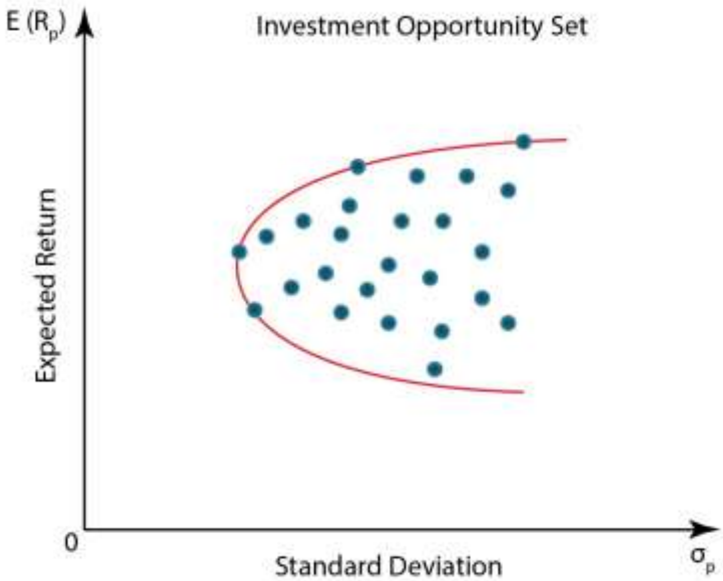
EFFICIENT FRONTIER & INVESTOR'S OPTIMAL PORTFOLIO

Opportunity set and Efficient frontier

<u>Opportunity set</u>

An opportunity set includes all risky portfolios available to an investor.



<u>Efficient frontier</u>

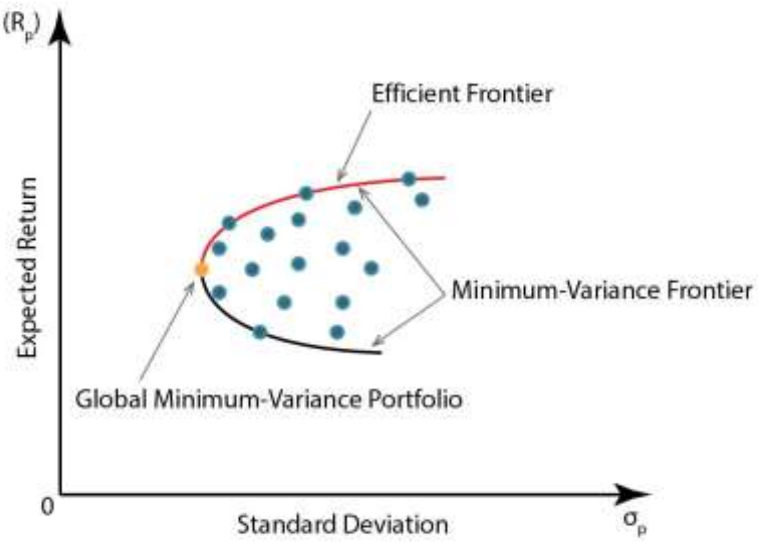
The efficient frontier is:

- a set of portfolios that a rational investor will choose,
- a part of the minimum-variance frontier,
- also called Markowitz efficient frontier or efficient set.

The global minimum-variance portfolio is:

- the portfolio characterized by the lowest risk of all possible portfolios that the investor can build,
- the beginning of the efficient frontier.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.



Two-fund separation theorem

If we are dealing with an investment that includes a risk-free asset and a portfolio consisting of risky assets, we may assume that the portfolio consists of two elements – a risk-free element and a risky element. In theory, it is often referred to as the two-fund separation theorem.

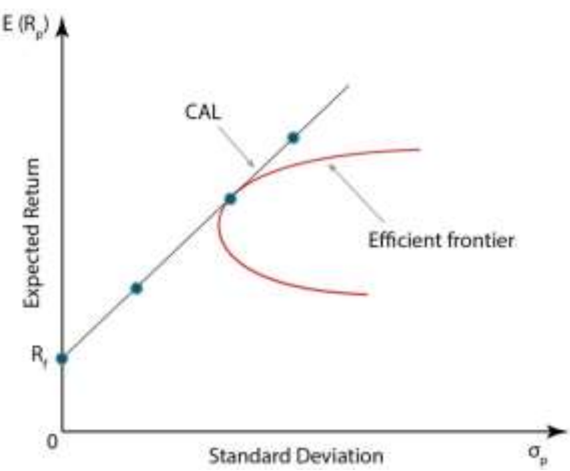
The standard deviation of a risk-free asset is 0.

According to the two-fund separation theorem, every investor invests in a portfolio consisting of:

- a risk-free asset, and
- a portfolio of risky assets.

<u>Capital allocation line (CAL)</u>

The two-fund separation theorem states that all investors regardless of risk preferences will aim at having a portfolio located on the capital allocation line that is tangent to the efficient frontier. The portfolios on the capital allocation line tangent to the efficient frontier are efficient, which means that for every portfolio lying on the CAL there is no other portfolio that would have a higher rate of return for a given level of risk or lower risk for a given level of expected return.

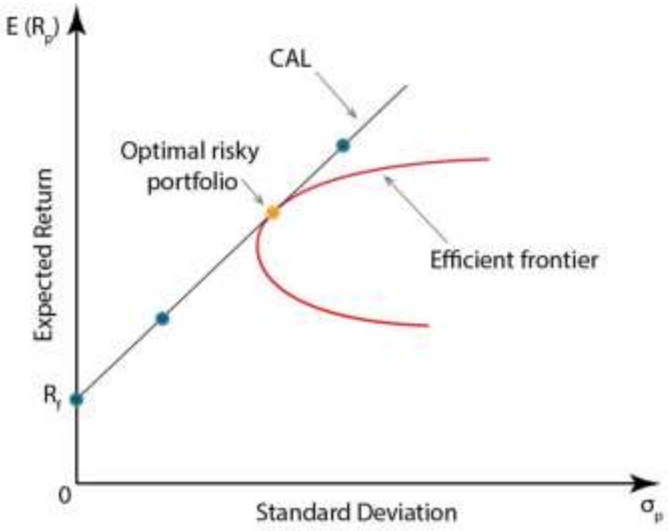


HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

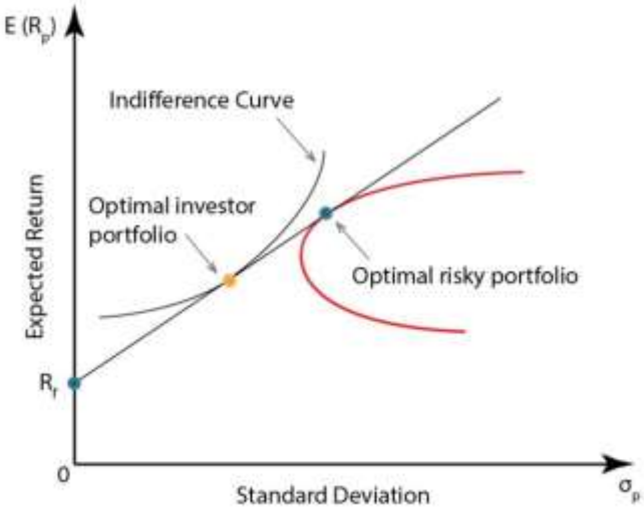
<u>Optimal risky portfolio</u>

An optimal risky portfolio:

- consists only of risky assets,
- offers the best ratio of expected return to risk,
- doesn't depend on the investor's individual preferences.



<u>Optimal investor portfolio</u>



Portfolio selection:

- depends on the personal preferences of an investor and his or her utility function,
- the highest indifference curve tangent to the CAL indicates the optimal investor portfolio.

optimal investor portfolio = portfolio with the highest utility for the investor

optimal investor portfolio ≠ optimal risky portfolio

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Summarizing key concepts:

☐ Rate of return and its measures

My summary:

☐ Money-weighted return vs Time-weighted return

My summary:

☐ Asset classes

My summary:

☐ Risk and its measures

My summary:

☐ The utility theory: indifference curve, classification of investors, risk aversion

My summary:

☐ Portfolio selection

My summary:

☐ Covariance and correlation coefficient – portfolio implications

My summary:

☐ Portfolio risk & return

My summary:

☐ Opportunity set & Efficient frontier

My summary:

☐ Minimum-variance frontier & Global minimum-variance portfolio

My summary:

Reviewing formulas:

$$\mathbf{HPR = \frac{P_1 + D_1}{P_0} - 1}$$

Write down the formula:

$$\mathbf{\bar{R}_A = \frac{1}{T}\sum_{t=1}^{T} R_t}$$

Write down the formula:

$$\mathbf{\bar{R}_G = \sqrt[T]{\prod_{t=1}^{T}(1 + R_t)} - 1}$$

Write down the formula:

$$\mathbf{\sum_{t=0}^{T}\frac{CF_t}{(1 + IRR)^t} = 0}$$

Write down the formula:

$$\mathbf{R = (1 + r)^t - 1}$$

Write down the formula:

$$\mathbf{R_p = \sum_{i=1}^{N} w_i \times R_i}$$

Write down the formula:

$$\mathbf{(1 + R_{real}) = \frac{(1 + R_{nom})}{(1 + R_i)}}$$

Write down the formula:

$$\mathbf{\sigma^2 = \frac{\sum_{i=1}^{T}(R_i - \mu)^2}{T}}$$

Write down the formula:

$$\mathbf{s^2 = \frac{\sum_{i=1}^{T}(R_i - \overline{R})^2}{T - 1}}$$

Write down the formula:

$$\mathbf{\sigma = \sqrt{\frac{\sum_{i=1}^{T}(R_i - \mu)^2}{T}}}$$

$$\mathbf{s = \sqrt{\frac{\sum_{i=1}^{T}(R_i - \overline{R})^2}{T - 1}}}$$

Write down the formula:

$$\mathcal{U} = E(R) - \frac{1}{2} \times A \times \sigma^2$$

Write down the formula:

$$E\left(R_p\right) = R_f + \frac{E(R_{rp}) - R_f}{\sigma_{rp}} \times \sigma_P$$

Write down the formula:

$$\rho_{12} = \frac{Cov\left(R_1, R_2\right)}{\sigma_1 \times \sigma_2}$$

Write down the formula:

$$\sigma = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} w_i \times w_j \times Cov\left(R_i, R_j\right)} = \sqrt{\sum_{i=1}^{N}\sum_{j=1}^{N} w_i \times w_j \times \rho_{ij} \times \sigma_i \times \sigma_j}$$

Write down the formula:

$$\sigma_p = \sqrt{w_A^2 \times \sigma_A^2 + w_B^2 \times \sigma_B^2 + 2 \times w_A \times w_B \times Cov\left(R_A R_B\right)} =$$

$$= \sqrt{w_A^2 \times \sigma_A^2 + w_B^2 \times \sigma_B^2 + 2 \times w_A \times w_B \times \rho_{AB} \times \sigma_A \times \sigma_B}$$

Write down the formula:

Keeping myself accountable:

TABLE 1 | STUDY

When you sit down to study, you may want to **try the Pomodoro Technique** to handle your study sessions: study for 25 minutes, then take a 5-minute break. Repeat this 25+5 study-break sequence all throughout your daily study session.

✔ Tick off as you proceed.

POMODORO TIMETABLE: study-break sequences (25′ + 5′)													
date		date		date		date		date		date		date	
25′		25′		25′		25′		25′		25′		25′	
5′		5′		5′		5′		5′		5′		5′	
25′		25′		25′		25′		25′		25′		25′	
5′		5′		5′		5′		5′		5′		5′	
25′		25′		25′		25′		25′		25′		25′	
5′		5′		5′		5′		5′		5′		5′	
25′		25′		25′		25′		25′		25′		25′	
5′		5′		5′		5′		5′		5′		5′	

TABLE 2 | REVIEW

Never ever neglect revision! Though it's not the most popular thing among CFA candidates, regular revision is what makes the difference. If you want to pass your exam, **schedule & do your review sessions.**

REVIEW TIMETABLE: When did I review this Reading?													
date		date		date		date		date		date		date	
date		date		date		date		date		date		date	