LEVEL 1: FIXED INCOME

Reading 43 (5th out of 6): RISKS ASSOCIATED WITH BONDS

Difficulty: **hard**

Benchmark Study Time: **5.2h**

2022

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INTENTIONALLY EMPTY

THIS E-BOOK:

- is a selective summary of the corresponding Reading in your CFA® Program Curriculum,
- provides place for your own notes,
- helps you structure your study and revision time!

How to use this e-book to maximize your knowledge retention:

1. **Print** the e-book in duplex and bind it to keep all important info for this Reading **in one place.**
2. **Read** this e-book, best twice, to grasp the idea of what this Reading is about.
3. **Study** the Reading from your curriculum. **Here add** your notes, examples, formulas, definitions, etc.
4. **Review** the Reading using this e-book, e.g. write your summary of key concepts or revise the formulas at the end of this e-book (if applicable).
5. **Done?** Go to your study plan and change the Reading's status to **green**:
   (it will make your Chance-to-Pass-Score™ grow ☺).
6. **Come back** to this e-book from time to time to **regularly review for knowledge retention!**

NOTE: While studying or reviewing this Reading, you can use the tables at the end of this e-book and mark your study/review sessions to hold yourself accountable.

DIFFERENT TYPES OF RISK

Risks associated with investing in bonds:

- market price risk,
- reinvestment risk,
- interest rate risk,
- prepayment risk,
- credit risk,
- inflation risk,
- foreign exchange risk,
- liquidity risk, and
- volatility risk.

Market price risk

The risk associated with a fall in bond prices due to an increase in interest rates is called market price risk.

Reinvestment risk

When an investor buys a bond, the realized rate of return depends on:

- coupons,
- a possible difference between the price and the par value,
- the reinvestment rate.

The additional income from reinvestment, sometimes called interest on coupons, depends on both future market rates and the strategy applied by the investor to reinvest coupons.

The risk associated with the reinvestment of coupons and the reinvestment rate is called reinvestment risk.

Interest rate risk

The term 'interest rate risk' is used to describe the risk associated with both increase and decrease in the yield-to-maturity. This is a basic type of risk that investors can suffer from on a debt market.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Prepayment risk

The prepayment risk is associated with a possibility that the issuer will call the issue or partially repay it.

From the bondholder's point of view, there are three major disadvantages of early redemption of the bond by the issuer:

1. It is impossible to say with reasonable certainty what cash flows from the bond will occur in the future.
2. The investor is exposed to the reinvestment risk because the issuer redeems the bonds earlier when the market discount rate decreases.
3. A decline in the market discount rate reduces a potential increase in the price of callable bonds because of the so-called price compression.

Credit risk

The risk that the issuer will not be able to fulfill his obligations arising from issued bonds is called credit risk.

Inflation risk

Inflation risk arises when due to a decline in the purchasing power of money the real value of cash flows from the financial instrument decreases. This type of risk is limited in the case of investing in inflation-indexed bonds.

Foreign exchange risk

The real value of cash flows from bonds denominated in foreign currencies is uncertain because their value depends on the exchange rate used at the time of payment.

Liquidity risk

Liquidity risk is greater for bonds that are illiquid. The bond's liquidity determines whether the investor is able to sell this bond and incur no loss against the current market price. If the bond is illiquid, the investor may not be able to sell the bond at the current market price. This type of risk is less important for investors planning to hold their bonds until maturity.

Volatility risk

The risk that some changes in interest rates volatility will have a negative impact on the price of a bond is called volatility risk. This risk affects mainly bonds with embedded options.

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INTEREST RATE RISK

Duration

Definitions

- **Duration** measures the bond's price sensitivity to changes in interest rates.
- **Yield duration** measures interest rate risk using a change in the bond's YTM.
- **Curve duration** measures interest rate risk using a change in the benchmark yield curve (e.g. government par curve).

Types of duration statistics:

- Macaulay duration,
- approximate Macaulay duration,
- modified duration,
- approximate modified duration,
- money duration,
- price value of a basis point,
- effective duration.

Examples of the yield duration:

- Macaulay duration,
- approximate Macaulay duration,
- modified duration,
- approximate modified duration.

An example of the curve duration is the effective duration.

Macaulay duration

The Macaulay duration can be interpreted as:

- the number of years that an investor has to wait until the investment pays back assuming that the yield doesn't change,
- a measure of the interest rate risk.

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If the bond has just paid its coupon:

$$\mathbf{MacD = \left(\frac{C}{1+r} + \frac{2 \times C}{(1+r)^2} + \cdots + \frac{(n-1) \times C}{(1+r)^{n-1}} + \frac{n \times C}{(1+r)^n} + \frac{n \times FV}{(1+r)^n}\right) \times \frac{1}{P}}$$

$$\mathbf{MacD = 1 \times \frac{\frac{C}{1+r}}{P} + 2 \times \frac{\frac{C}{(1+r)^2}}{P} + \cdots + (n-1) \times \frac{\frac{C}{(1+r)^{n-1}}}{P} + n \times \frac{\frac{C+FV}{(1+r)^n}}{P}}$$

Where:

- P – bond price,
- C – coupon payment,
- FV – par value,
- n – number of periods until maturity,
- MacD – Macaulay duration,
- r – yield-to-maturity.

If the bond is between coupon payments, we can use the following approximation for Macaulay duration:

$$\mathbf{MacD = \frac{1+r}{r} - \frac{1+r+n \times (c-r)}{c \times [(1+r)^n - 1] + r} - \frac{t}{T}}$$

Where:

- t – number of days from the last coupon date,
- c – coupon rate,
- T – number of days in the coupon period,
- n – number of periods until maturity,
- MacD – Macaulay duration,
- r – yield-to-maturity.

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Modified duration

$$\mathbf{ModD} = \frac{\mathbf{MacD}}{\mathbf{1+r}}$$

Where:

- ModD – modified duration,
- MacD – Macaulay duration,
- r – yield-to-maturity.

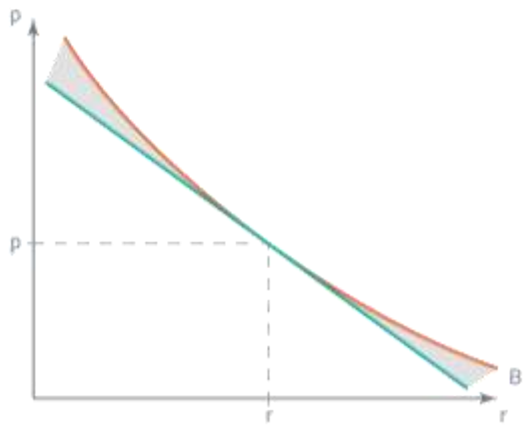
Duration as the measure of interest rate risk

The modified duration can be used to compute the percentage price change for a bond assuming a change in the yield.

$$\mathbf{Price\ change}\ (\%) = -\mathbf{ModD} \times (\mathbf{yield\ change})$$

Drawbacks of the formula:

- it gives only approximate results,
- according to the formula, when the yield increases or decreases by the same value, the bond price changes by exactly the same amount but in the opposite direction.



the greater the modified duration → the greater the percentage price change

the lower the modified duration → the lower the percentage price change

the greater the yield change (yield volatility) → the greater the percentage price change

the lower the yield change (yield volatility) → the lower the percentage price change

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Characteristics of Macaulay and modified durations

1. The higher the duration, the more the bond price changes due to changes in the yield. This means that the higher the duration, the higher the bond interest rate risk.
2. The duration differs for bonds with different coupons.
3. For all bonds without embedded options, the Macaulay duration is lower than time to maturity except for zero-coupon bonds.
4. A zero-coupon bond has the Macaulay duration equal to its time to maturity.
5. Since the modified duration is always lower than the Macaulay duration, all bonds without embedded options have modified durations lower than their times to maturity.
6. The lower the ratio of the coupon to the par value, the greater the duration and the greater the interest rate risk. So, a zero-coupon bond has the greatest interest rate risk from all bonds with the same maturity and the same credit rating.
7. The modified duration of a consol bond is always equal to 1 divided by yield-to-maturity.
8. For floaters, duration does not exceed the period to the next coupon payment. So, the interest rate risk associated with these bonds is low.
9. The duration changes in time.
10. The duration of all bonds approaches 0 when the maturity date gets near. The only exception are long-term discount bonds. For this type of bonds, if the term to maturity is long enough, the duration increases as the bond approaches the maturity date.
11. Usually, between coupon payments, the duration decreases in a linear manner. After the coupon is paid, the duration increases suddenly and then starts to decrease once again. And so on.
12. Modified duration depends on the slope of the line tangent to the price-yield curve. The lower the slope, the lower the modified duration. The greater the slope, the greater the modified duration.
13. If the interest rate increases, the new line tangent to the price-yield curve is flatter, which means that the modified duration is lower, and the interest rate risk is also reduced.
14. If the interest rate decreases, the slope of the new line tangent to the price-yield curve is greater, which means that the modified duration is greater, and the interest rate risk is also greater.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Duration gap

$$\text{duration gap} = \text{MacD} - \text{investment horizon}$$

Assuming a one-time, immediate, parallel shift in a yield curve:

- If a **duration gap is negative**, then an investor is **afraid of a decrease in interest rates**.
- If a **duration gap is positive**, then an investor is **afraid of an increase in interest rates**.
- If a **duration gap is equal to 0**, then an investor is **hedged against changes in interest rates**.

Approximate modified duration

We calculate the approximate modified duration if we don't know the Macaulay duration and we want to compute the modified duration:

$$\text{approximate modified duration} = \frac{P_- - P_+}{2 \times P \times \Delta r}$$

Where:

- P – current bond price,
- P_+ – price of the bond if we assume that r increases,
- P_- – price of the bond if we assume that r decreases,
- r – bond's own YTM,
- Δr – yield change.

the assumed change in the yield should be the same for both $\mathbf{P_+}$ and $\mathbf{P_-}$

The algorithm for calculating the approximate modified duration (if we know P & r):

Assuming that r decreases by 1 basis point ($\Delta r = -0.0001$) →

→ calculate the price of the bond (P_-) → [STO] 1

Assuming that r increases by 1 basis point ($\Delta r = +0.0001$) →

→ calculate the price of the bond (P_+) → [STO] 2

Calculate the denominator of the formula: $2 \times P \times 0.0001$ → [STO] 3

Calculate the approximate modified duration using the formula above:

[RCL]1 [–] [RCL]2 [÷] [RCL]3 [=]

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Approximate Macaulay duration

$$\mathbf{MacD = ModD \times (1 + r)}$$

Where:

- ModD – approximate modified duration,
- MacD – approximate Macaulay duration,
- r – yield-to-maturity.

Effective duration

$$\mathbf{effective\ duration = \frac{P_{-} - P_{+}}{2 \times P \times \Delta r_B}}$$

Where:

- P – current bond price,
- $\mathrm{P_+}$ – price of the bond if we assume that $\mathrm{r_B}$ increases,
- $\mathrm{P_-}$ – price of the bond if we assume that $\mathrm{r_B}$ decreases,
- $\mathrm{r_B}$ – benchmark yield curve,
- $\Delta \mathrm{r_B}$ – yield change.

the assumed change in the yield should be the same for both $\mathbf{P_+}$ and $\mathbf{P_-}$

The algorithm for calculating the effective duration is the same as in the case of the approximate modified duration

Generally, we use the effective duration for bonds with embedded options:

- putable bonds,
- callable bonds,
- mortgage-backed bonds.

Money duration

The money duration equals the annual modified duration multiplied by the current full price of the bond. So, if we use money duration to measure the interest rate risk, the change in the price will be given in dollars or some other currency, and not as a percentage.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Price value of a basis point

$$\textbf{price value of a basis point} = \frac{\mathbf{P_{-} - P_{+}}}{\mathbf{2}}$$

Where:

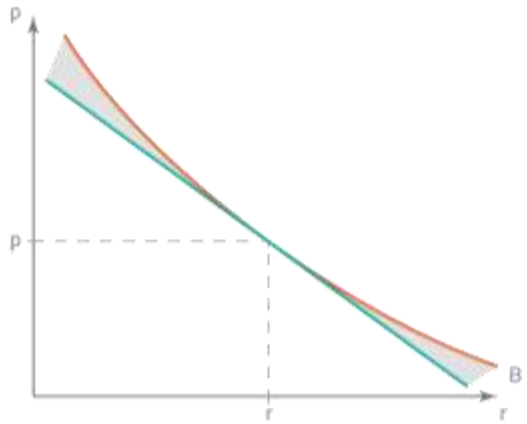
- P_+ – price of the bond if we assume that r increases by 1 basis point,
- P_- – price of the bond if we assume that r decreases by 1 basis point,
- r – bond's own YTM.

We can interpret the price value of a basis point as the money duration for the bond assuming that the change in the YTM is equal to 1 basis point.

Convexity

The convexity is another measure of interest rate risk.

It makes our calculations more precise.



The relationship between the bond price and the yield-to-maturity is not linear and the curve is convex. There are **two consequences** of this:

1. If the yield-to-maturity changes only a bit, namely it either increases or decreases a bit, the absolute value of the percentage change in the bond price will be the same for both increase and decrease in the yield-to-maturity.

2. If the yield-to-maturity changes a lot, the absolute value of the percentage change in the bond price will be lower if the yield-to-maturity increases than if the yield-to-maturity decreases. It's a consequence of the convexity of the curve. It means that if the yield-to-maturity changes a lot, the bond price will increase more as a result of a decrease in the required rate of return than it will decrease as a result of an increase in the yield-to-maturity.

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These consequences have a great impact on the interest rate risk measurement. If we assume that the yield-to-maturity will change only a bit, we can use duration alone to measure the interest rate risk. However, if we assume that the yield-to-maturity will change a lot, we have to use both duration and convexity and adjust the results obtained with the aid of the former measure using the so-called convexity adjustment.

Types of convexity statistics:

- approximate convexity,
- effective convexity,
- money convexity.

Approximate convexity

$$\mathbf{approximate\ convexity} = \frac{\mathbf{P_{-} + P_{+} - 2 \times P}}{\mathbf{P \times (\Delta r)^2}}$$

Where:

- P – current bond price,
- $\mathrm{P_{+}}$ – price of the bond if we assume that r increases,
- $\mathrm{P_{-}}$ – price of the bond if we assume that r decreases,
- r – bond's own YTM,
- $\Delta\mathrm{r}$ – yield change.

the assumed change in the yield should be the same for both $\mathbf{P_{+}}$ and $\mathbf{P_{-}}$

Effective convexity

$$\mathbf{effective\ convexity} = \frac{\mathbf{P_{-} + P_{+} - 2 \times P}}{\mathbf{P \times \left(\Delta r_B\right)^2}}$$

Where:

- P – current bond price,
- $\mathrm{P_{+}}$ – price of the bond if we assume that $\mathrm{r_B}$ increases,
- $\mathrm{P_{-}}$ – price of the bond if we assume that $\mathrm{r_B}$ decreases,
- $\mathrm{r_B}$ – benchmark yield curve,
- $\Delta\mathrm{r_B}$ – yield change.

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Similarly to the effective duration, the effective convexity is used to measure interest rate risk for bonds with embedded options:

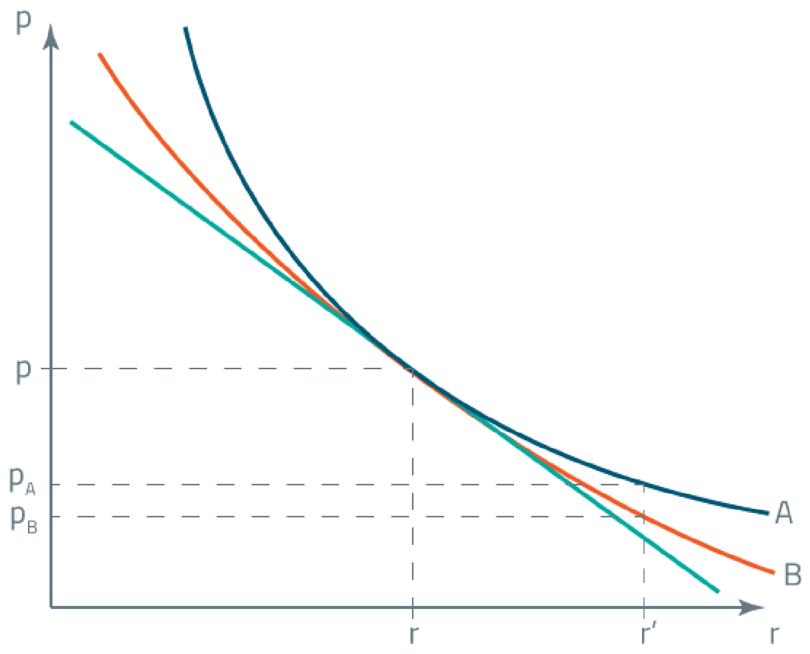
- putable bonds,
- callable bonds,
- mortgage-backed bonds.

Money convexity

Money convexity is equal to annual convexity multiplied by the bond full price.

Duration and convexity as measures of interest rate risk

$$\textbf{Price change (\%)} = -\textbf{ModD} \times (\textbf{yield change}) + \mathbf{0.5} \times \textbf{Convexity} \times (\textbf{yield change})^{\mathbf{2}}$$



Bond A has greater convexity. If the yields for both bonds change by the same amount, then:

- if the yields increase, the price of Bond A will decrease less than the price of Bond B,
- if the yields decrease, the price of Bond A will increase more than the price of Bond B.

Therefore, in both cases, Bond A is a better investment choice.

For <u>bonds without embedded options</u>, convexity is always positive, so the <u>impact of convexity</u> adjustments on this type of bonds <u>is always positive</u>. However, for bonds with embedded call options, which include callable bonds or mortgage-backed securities, the convexity can take a negative value if the yield is low enough. Because the convexity adjustment is added, the effect of convexity adjustment for this type of debt instruments will be then negative.

HERE KNOWLEDGE RETENTION HAPPENS | WRITE: notes, examples, formulas, definitions, relations, etc.

Summarizing key concepts:

☐ Different types of risk

My summary:

☐ Interest rate risk

My summary:

☐ Duration: yield duration vs curve duration

My summary:

☐ Macaulay duration, modified duration, approximate modified duration, approximate Macaulay duration, effective duration, money duration, price value of a basis point, duration gap

My summary:

☐ Duration as the measure of interest rate risk

My summary:

☐ Characteristics of Macaulay and modified durations

My summary:

☐ Effective convexity, approximate convexity, money convexity

My summary:

☐ Duration and convexity as measures of interest rate risk

My summary:

☐ Portfolio duration

My summary:

Reviewing formulas:

$$\mathbf{MacD} = \left(\frac{C}{1+r} + \frac{2 \times C}{(1+r)^2} + \cdots + \frac{(n-1) \times C}{(1+r)^{n-1}} + \frac{n \times C}{(1+r)^n} + \frac{n \times FV}{(1+r)^n}\right) \times \frac{1}{P}$$

$$\mathbf{MacD} = 1 \times \frac{\frac{C}{1+r}}{P} + 2 \times \frac{\frac{C}{(1+r)^2}}{P} + \cdots + (n-1) \times \frac{\frac{C}{(1+r)^{n-1}}}{P} + n \times \frac{\frac{C+FV}{(1+r)^n}}{P}$$

Write down the formula:

$$\mathbf{MacD} = \frac{1+r}{r} - \frac{1+r+n \times (c-r)}{c \times [(1+r)^n - 1] + r} - \frac{t}{T}$$

Write down the formula:

$$\mathbf{ModD} = \frac{MacD}{1+r}$$

Write down the formula:

$$\textbf{Price change } (\%) = -\mathbf{ModD} \times (\textbf{yield change})$$

Write down the formula:

$$\mathbf{duration\ gap = MacD - investment\ horizon}$$

Write down the formula:

$$\mathbf{approximate\ modified\ duration = \frac{P_{-} - P_{+}}{2 \times P \times \Delta r}}$$

Write down the formula:

$$\mathbf{MacD = ModD \times (1 + r)}$$

Write down the formula:

$$\mathbf{effective\ duration = \frac{P_{-} - P_{+}}{2 \times P \times \Delta r_{B}}}$$

Write down the formula:

$$\textbf{price value of a basis point} = \frac{\mathbf{P_- - P_+}}{\mathbf{2}}$$

Write down the formula:

$$\textbf{approximate convexity} = \frac{\mathbf{P_- + P_+ - 2 \times P}}{\mathbf{P \times (\Delta r)^2}}$$

Write down the formula:

$$\textbf{effective convexity} = \frac{\mathbf{P_- + P_+ - 2 \times P}}{\mathbf{P \times (\Delta r_B)^2}}$$

Write down the formula:

$$\textbf{Price change (\%)} = -\textbf{ModD} \times (\textbf{yield change}) + \mathbf{0.5} \times \textbf{Convexity} \times (\textbf{yield change})^{\mathbf{2}}$$

Write down the formula:

Keeping myself accountable:

TABLE 1 | STUDY

When you sit down to study, you may want to **try the Pomodoro Technique** to handle your study sessions: study for 25 minutes, then take a 5-minute break. Repeat this 25+5 study-break sequence all throughout your daily study session.

✔ Tick off as you proceed.

POMODORO TIMETABLE: study-break sequences (25′ + 5′)													
date		date		date		date		date		date		date	
25′		25′		25′		25′		25′		25′		25′	
5′		5′		5′		5′		5′		5′		5′	
25′		25′		25′		25′		25′		25′		25′	
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5′		5′		5′		5′		5′		5′		5′	

TABLE 2 | REVIEW

Never ever neglect revision! Though it's not the most popular thing among CFA candidates, regular revision is what makes the difference. If you want to pass your exam, **schedule & do your review sessions.**

REVIEW TIMETABLE: When did I review this Reading?													
date		date		date		date		date		date		date	
date		date		date		date		date		date		date	