

# Financial Engineering and Risk Management

## Swaps

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# Swaps

**Definition.** Swaps are contracts that transform one kind of cash flow into another.

**Example.**

- Plain vanilla swap: fixed interest rate vs floating interest rates
- Commodity swaps: exchange floating price for a fixed price. e.g. gold swaps, oil swaps.
- Currency swaps

Why swaps?

- Change the nature of cash flows
- Leverage strengths in different markets

# Example of leveraging strengths

Two companies

Company	Fixed	Floating
A	4.0%	LIBOR + 0.3%
B	5.2%	LIBOR + 1.0%

Company A is “better” in both but relatively weaker in the floating rate market

Company A

- Borrows in fixed market at 4.0%
- Swap with B: pays LIBOR and receives 3.95%

Company B

- B borrow in the floating market at LIBOR + 1.0%
- Swap with A: pays 3.95% and receives LIBOR

Effective rates:

- A:  $-4\% + 3.95\% - \text{LIBOR} = -(\text{LIBOR} + 0.05\%)$
- B:  $-\text{LIBOR} - 1\% + \text{LIBOR} - 3.95\% = -4.95\%$

Both gain!

# Role of financial intermediaries

Same two companies

Company	Fixed	Floating
A	4.0%	LIBOR + 0.3%
B	5.2%	LIBOR + 1.0%

Financial intermediary that constructs the swap.

Company A

- Borrows in fixed market at 4.0%
- Swap with **Intermediary**: pays LIBOR and receives 3.93%

Company B

- B borrow in the floating market at LIBOR + 1.0%
- Swap with **Intermediary**: pays 3.97% and receives LIBOR

Financial intermediary makes 0.04% or 4 basis points. Why?

- Compensation for taking on counterparty risk and providing a service

# Pricing interest rate swaps

$r_t$  = floating (unknown) interest rate at time  $t$

Cash flows at time  $t = 1, \dots, T$

- Company  $A$  (long): receives  $Nr_{t-1}$  and pays  $NX$
- Company  $B$  (short): receives  $NX$  and pays  $Nr_{t-1}$

Value of swap to company  $A$

- $N(r_0, \dots, r_{T-1})$  = Cash flow of floating rate bond - Face value. Therefore, value of swap to company  $A$

$$V_A = N(1 - d(0, T)) - NX \sum_{t=1}^T d(0, t)$$

- Set  $X$  so that  $V_A = 0$ , i.e.

$$X = \frac{1 - d(0, T)}{\sum_{t=1}^T d(0, t)}$$