Financial Engineering and Risk Management

Options

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Options

Definition. A European Call Option gives the buyer the right but not the obligation to purchase 1 unit of the underlying at specified price K (strike price) at a specified time T (expiration).

Definition. An American Call Option gives the buyer the right but not the obligation to purchase 1 unit of the underlying at specified price K (strike price) at any time until a specified time T (expiration).

Definition. A European Put Option gives the buyer the right but not the obligation to sell 1 unit of the underlying at specified price K (strike price) at a speficied time T (expiration).

Definition. An American Put Option gives the buyer the right but not the obligation to sell 1 unit of the underlying at specified price K (strike price) at any time until a specified time T (expiration).

Payoff and intrinsic value of a call option

Payoff of a European call option at expiration T

- price $S_T < K$: do not exercise the option, payoff $= 0$
- price $S_T > K$: exercise option and sell in the spot market, payoff $= S_T - K$

Payoff $= \max\{S_T - K, 0\}$ ... nonlinear in the price S_T

Intrinsic value of a call option at time $t \leq T = \max\{S_t - K, 0\}$

- In the money: $S_t > K$
- At the money: $S_t = K$
- Out of the money: $S_t < K$

Everything works the other way for put options.

Payoff and intrinsic value of a put option

Payoff of a European put option at expiration T

- price $S_T < K$: exercise the option and see in spot market, payoff $= K - S_T$
- price $S_T > K$: do not exercise option, payoff $= 0$

Payoff $= \max\{K - S_T, 0\}$ ... nonlinear in the price S_T

Intrinsic value of a put option at time $t \leq T = \max\{K - S_t, 0\}$

- In the money: $S_t < K$
- At the money: $S_t = K$
- Out of the money: $S_t > K$

Pricing options

Nonlinear payoff ... cannot price without a model for the underlying

Prices of options

- European put/call with strike K and expiration T: $p_E(t; K, T)$, $c_E(t; K, T)$
- American put/call with strike K and expiration T: $p_A(t; K, T)$, $c_A(t; K, T)$

European put-call parity at time t for non-dividend paying stock:

$$p_E(t; K, T) + S_t = c_E(t; K, T) + Kd(t, T)$$

Trading strategy

- At time t buy European call with strike K and expiration T
- At time t sell European put with strike K and expiration T
- At time t (short) sell 1 unit of underlying and buy at time T
- Lend $K \cdot d(t, T)$ dollars up to time T

No-arbitrage argument

- Cash flow at time T: $\max\{S_T - K, 0\} - \max\{K - S_T, 0\} - S_T + K = 0$
- Cash flow at time t: $-c_E(t; K, T) + p_E(t; K, T) + S_t - Kd(t, T) = 0$

Bounds on prices of European options

Price of American option $\geq$ Price of European option

- $c_A(t; K, T) \geq c_E(t; K, T)$, and $p_A(t; K, T) \geq p_E(t; K, T)$

Lower bound on European options as a function of stock price S_t

- $c_E(t; K, T) = \max\{S_t + p_E(t; K, T) - Kd(t, T), 0\} \geq \max\{S_t - Kd(t, T), 0\}$
- $p_E(t; K, T) = \max\{Kd(t, T) + c_E(t; K, T) - S_t, 0\} \geq \max\{Kd(t, T) - S_t, 0\}$

Upper bound on European options as a function of stock price S_t

- $\max\{S_T - K, 0\} \leq S_T$ implies $c_E(t; K, T) \leq S_t$
- $\max\{K - S_T, 0\} \leq K$ implies $p_E(t; K, T) \leq Kd(t, T)$

Effect of dividends $p_E(t; K, T) + S_t - D = c_E(t; K, T) + Kd(t, T)$

- $D =$ present value of all dividends until maturity

Bounds on prices of American options

Price of American call as function of stock price S_t:

- $c_A(t, K, T) \geq c_E(t; K, T) \geq \max\left\{S_t - Kd(t, T), 0\right\} > \max\left\{S_t - K, 0\right\}$

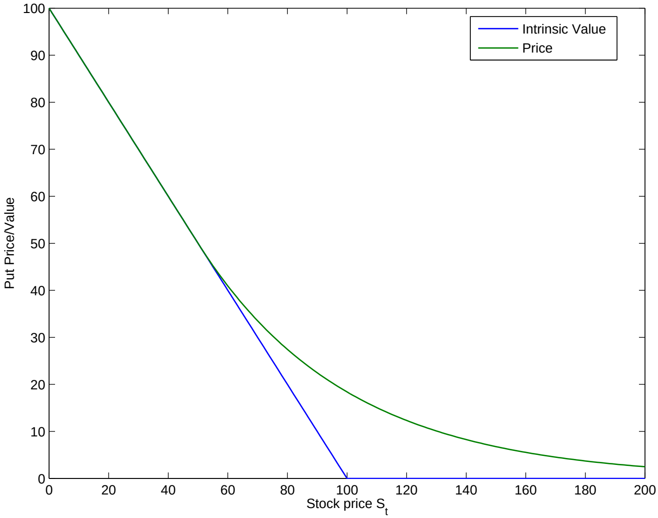
Thus, the price of an American call is always strictly greater than the exercise value of the call option.

Never optimal to exercise an American call on a non-dividend paying stock early!
$c_A(t; K, T) = c_E(t, K, T)$

Price of American put as function of stock price S_t:

- Bound $p_A(t, K, T) \geq p_E(t; K, T) \geq \max\left\{Kd(t, T) - S_t, 0\right\}$
- But the exercise value of a American put option is $\max\left\{K - S_t, 0\right\}$

Bounds do not tell us much!


Intrinsic Value
Price
Put Price/Value
Stock price S_t
0
10
20
30
40
50
60
70
80
90
100
0
20
40
60
80
100
120
140
160
180
200