Financial Engineering and Risk Management Futures

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Problems with forward contracts

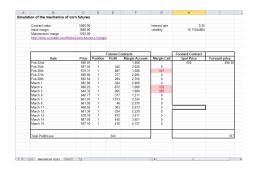
- Not organized through an exchange.
- Consequently, no price transparency!
- Double-coincidence-of-wants: need someone to take the opposite side!
- Default risk of the counterparty.

Futures contract

- Solves the problem of a multitude of prices for the same maturity by marking-to-market
 - disbursing profits/losses at the end of each day
- Now contracts can be organized through an exchange.
- Can be written on any underlying security with a settlement price
 - Commodities
 - Broad based indicies, e.g. S & P 500, Russel 2000, etc.
 - Volatility of the market, e.g. VIX futures
- $\bullet \ \, \text{http://www.cmegroup.com/market-data/delayed-quotes/commodities.html} \\$

Mechanics of a futures contract

- Individuals open a margin account with a broker
- ullet Enter into N futures contracts with price F_0
- ullet Deposit initial margin into the account pprox 5-10% of contract value
- All profit/loss settled using margin account
- Margin call if balance is low



Pros/cons of futures

Pros:

- High leverage: high profit
- Very liquid
- Can be written on a wide variety of underlying assets

Cons:

- High leverage: high risk
- Futures prices are approximately linear function of the underlying only linear payoffs can be hedged
- May not be flexible enough; back to Forwards!

Pricing futures

- Need martingale pricing formalism
- Deterministic interest rates: forward price = futures price
- ullet At maturity futures price $F_T=$ price of underlying S_T

Hedging using Futures: Long hedge

Today is Sept. 1st. A baker needs 500,000 bushels of wheat on December 1st. So, the baker faces the risk of an uncertain price on Dec. 1st.

Hedging strategy: buy $100~\mathrm{futures}$ contracts maturing on Dec. 1st – each for $5000~\mathrm{bushels}$

Cash flow on Dec. 1st

- Futures position at maturity: $F_T F_0 = S_T F_0$
- ullet Buy in the spot market: S_T
- Effective cash flow: $S_T F_0 S_T = -F_0$

Price fixed at $F_0!$

Did this cost anything? Cash flows associated with margin calls.

Perfect hedges are not always possible

Why?

- The date T may not be a futures expiration date.
- ullet P_T may not correspond to an integer number of futures contracts
- A futures contract on the underlying may not be available
- The futures contract might not be liquid
- ullet The payoff P_T may be nonlinear in the underlying

$\mathsf{Basis} = \mathsf{Spot} \; \mathsf{price} \; \mathsf{of} \; \mathsf{underlying} \; \mathsf{-} \; \mathsf{futures} \; \mathsf{price}$

- Perfect hedge: basis = 0 at time T
- Basis risk: basis \neq 0 at time T
- Basis risk arises because the futures contract is on a related but different asset, or expires at a different time.

Hedging problem with basis risk

Today is Sept. 1st. A taco company needs 500,000 bushels of kidney beans on December 1st. So, the taco company faces the risk of an uncertain price.

Problem: No kidney bean futures available. Basis risk inevitable.

Hedge: Go long \boldsymbol{y} soybean futures each for 5000 bushels of soybeans

Cash Flow in 90 days

- Futures position at maturity: $(F_T F_0)y$
- ullet Buy kidney beans in the spot market: P_T
- Effective cash flow: $C_T = y(F_T F_0) + P_T$

 $P_T \neq yF_T$ for any y: Perfect hedge impossible!

Minimum variance hedging

Variance of the cash flow

$$\mathbf{var}(C_T) = \mathbf{var}(P_T) + \mathbf{var}(y(F_T - F_0))$$

$$+ 2\mathbf{cov}(y(F_T - F_0), P_T)$$

$$= \mathbf{var}(P_T) + y^2\mathbf{var}(F_T) + 2y\mathbf{cov}(F_T, P_T)$$

Set the derivative with respect to y to zero:

$$\frac{d\mathbf{var}(C_T(y))}{dy} = 2y\mathbf{var}(F_T) + 2\mathbf{cov}(F_T, P_T) = 0$$

Optimal number of Futures contracts:

$$y^* = -\frac{\mathbf{cov}(F_T, P_T)}{\mathbf{var}(F_T)}$$