Q1

1. $m(a+bX) = \frac{1}{N}\sum_{i=1}^{N} a+bx_i = \frac{1}{N}\sum_{i=1}^{N} a + \frac{1}{N}\sum_{i=1}^{N} bx_i$

$$\frac{Na}{N} + b\left(\frac{1}{N}\sum_{i=1}^{N} x_i\right)$$

$$= a + bM(X)$$

2. $cov(X, a+bY) = \frac{1}{N}\sum_{i=1}^{N}(x_i - M(x))(a+by_i - m(a+bY))$

$$a+by_i - (a+bM(Y))$$

$$\frac{1}{N}\sum_{i=1}^{N}(x_i - M(x))(b(y_i - M(Y)))$$

$$b\boxed{\frac{1}{N}\sum_{i=1}^{N}(x_i - M(x))(y_i - M(y))}$$

X

Y

$$b\,cov(X, Y)$$

3. $cov(a+bX, a+bX) = \frac{1}{N}\sum_{i=1}^{N}(a+bx_i - m(a+bX))(a+bX_i - m(a+bx))$

$$(b(x_i - M(x))(b(x_i - M(x))$$

$$\frac{1}{N}\sum_{i=1}^{N} b^2(x_i - M(x))^2$$

$$b^2\frac{1}{N}\sum_{i=1}^{N}(x_i - M(x))^2 \rightarrow = s^2$$

$$b^2 cov(X, X)$$

4. Non-decreasing transformations preserve order, so the transformation of the median is equal to the median of the transformation. This same logic can be applied to any quantile as each data point is transformed it will remain in the same quantile because the relative position of each point is preserved. However, for statistics that measure distance between points is not preserved, therefore IQR and range do not apply.

5. No, means rely on the actual numerical values of the data, therefore transforming the data first then taking the mean is not equal to the tranformation of the original mean as some data points may have a greater change in value as a result of the transformation, leading to said differences.