Evolutionary Stable Strategies - Simple Example

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1 Introduction

This project was inspired by the Chapter *Agression: Stability and the Selfish Machine* in [1]. The concept of an Evolutionary Stable Strategy (ESS) was first introduced by Maynard Smith in [2]. It is defined as a strategy which, if adopted by the majority of a population, cannot be beaten by an alternative, intially rare, strategy. In other words no deviant individual can outsmart the system. The best strategy for an individual depends on the strategy of the majority of the population. To better understand it, we will consider a model of hawks and doves, described by Maynard Smith.

2 Doves versus Hawks

2.1 The Model

In a hypothetical population each individual adopts one of the two fighting strategies:

1. a *dove* strategy,

2. or a *hawk* strategy.

The life of each individual consists of three phases:

1. the duel phase,

2. the reproduction phase,

3. the extinction phase.

2.1.1 The Duel Phase

This phase consists of the duels between the pairs of individuals. Each individual participates in exactly one duel with a randomly chosen opponent. The results of the duels depend on the fighting strategy adopted by the participants in the following way:

- When a hawk fights a hawk, they do not stop until one of them is seriously injured. The injured hawk looses 100 HP ($p_{defeat} = -100$) while the one which won gets 50 HP ($p_{win} = 50$).

- When a hawk fights a dove, the dove runs away and therefore does not loose any points. The hawk is awarded $p_{win} = 50$ HP for winning.

	after a fight with a	
	dove	hawk
dove's HP	90 or 140	100
hawk's HP	150	0 or 150

Table 1: Possible HPs of representatives of each strategy after the duels.

- When a dove fights a dove, they do not attack each other but try to intimidate their opponent until one of them flies away. They both loose 10 HP for wasting time ($p_{waste} = -10$). The one which stayed is awarded $p_{win} = 50$ HP for winning.

Let each individual start with $p_{start} = 100$ HP.[1] The possible HPs of representatives of each strategy after the duels are presented in the table 1.

2.1.2 The Reproduction and Extinction Phases

After the duels, every individual passes to the reproduction phase. The number of its offspring is directly propotional to its HPs. Therefore the HPs of an individual are directly convertible into the survival of its genes.

Having reproduced, the first generation dies out. Now the second generation goes through the same phases. The process is repeated for a given number of generations (n_{epochs}).

2.2 Stable Ratio

Let us find an average HP of an individual in a population consisting of d doves and h hawks. When a dove fights a dove, in 50% of cases it ends up having 90 HP and in the remaining 50% - 140 HP. Therefore an average HP of a dove which fought a dove is 115 HP. The dove which fought a hawk always ends up having 100 HP. Therefore an average result for a dove in a population of d doves and h hawks is:

$$\overline{p_d} = \frac{115d + 100h}{d + h} \tag{1}$$

The average HP of a hawk can be calculated similarly and is given by the following formula:

$$\overline{p_h} = \frac{150d + 75h}{d + h} \tag{2}$$

Let us assume that the propotiality coefficient α between the number of offpring and individual's HP is the same for all members of the population and independent of their fighting strategy. Therefore if a population consisted of d doves and h hawks in the next generation it will - on average - consist of $d' = \alpha \cdot \overline{p_d} \cdot d$ doves and $h' = \alpha \cdot \overline{p_h} \cdot h$ hawks. The new ratio of doves to hawks will be as follows:

$$\frac{d'}{h'} = \frac{\overline{p_d} \cdot d}{\overline{p_h} \cdot h}. \tag{3}$$

[1] This will guarantee that after the duels the HP of each individual will be positive, therefore it will be possible to define the number of its offstring as propotional to the individual's HP.

From that formula we can see that if

$$\overline{p_d} = \overline{p_h}, \tag{4}$$

the ratio will remain unchanged. Using the formulas 2 and 3 we obtain the stable ratio of doves to hawks:

$$\frac{d}{h} = \frac{5}{7}. \tag{5}$$

2.3 Simulation results

Fig. 1 shows the evolution of the population, initially consisting of 10 doves and 90 hawks. In the beginning an average dove is more succesful than an average hawk. In a population dominated by hawks a dove has an advantage of barely ever loosing points as hawks are its main opponents. A hawk on the other hand is likely to loose points in a fight with another hawk (it has a 50% chance of loosing and the penalty for it is twice as large as the reward for winning). Hence the number of doves increases at the cost of hawks. When the number of doves is sufficiently large, the hawks quite frequently have a chance of an easy win in a fight with a dove. The numbers of both species stabilize. One can notice some oscillation in the proportion of doves to hawks, but in general the ratio remains stable around the predicted value of $\frac{5}{7}$.

In the Fig. 2 the evolution of ratios of doves to hawks is presented for various initial compositions of the population. It is clearly seen that the final proportion does not depend on the initial conditions and is consistent with the prediction.

References

[1] R.Dawkins *The Selfish Gene*, Oxford University Press (1989)

[2] J. Maynard Smith, *Game Theory and the Evolution of Behaviour*, Proceedings of the Royal Society B, Volume 205, issue 1161 (1979) (DOI: 10.1098/rspb.1979.0080)

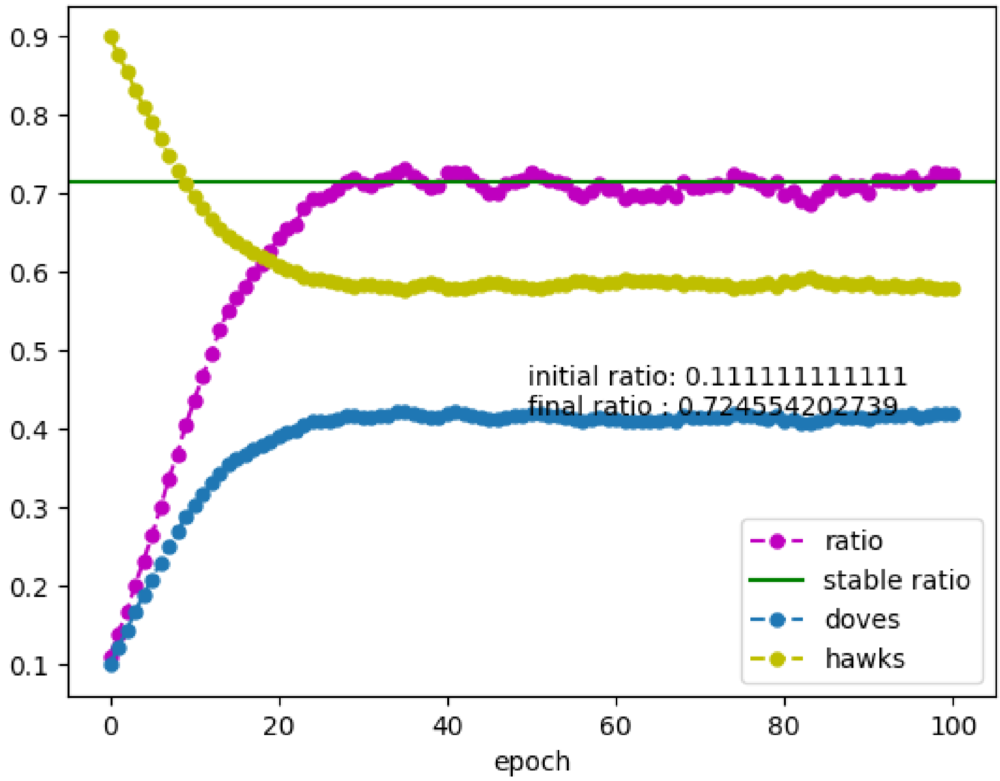

Figure 1: Evolution of proportion of doves and hawks in the population and the ratio of doves to hawks. Initially there were 10 doves and 90 hawks. Green line represents the expected stable ratio of $\frac{5}{7}$.

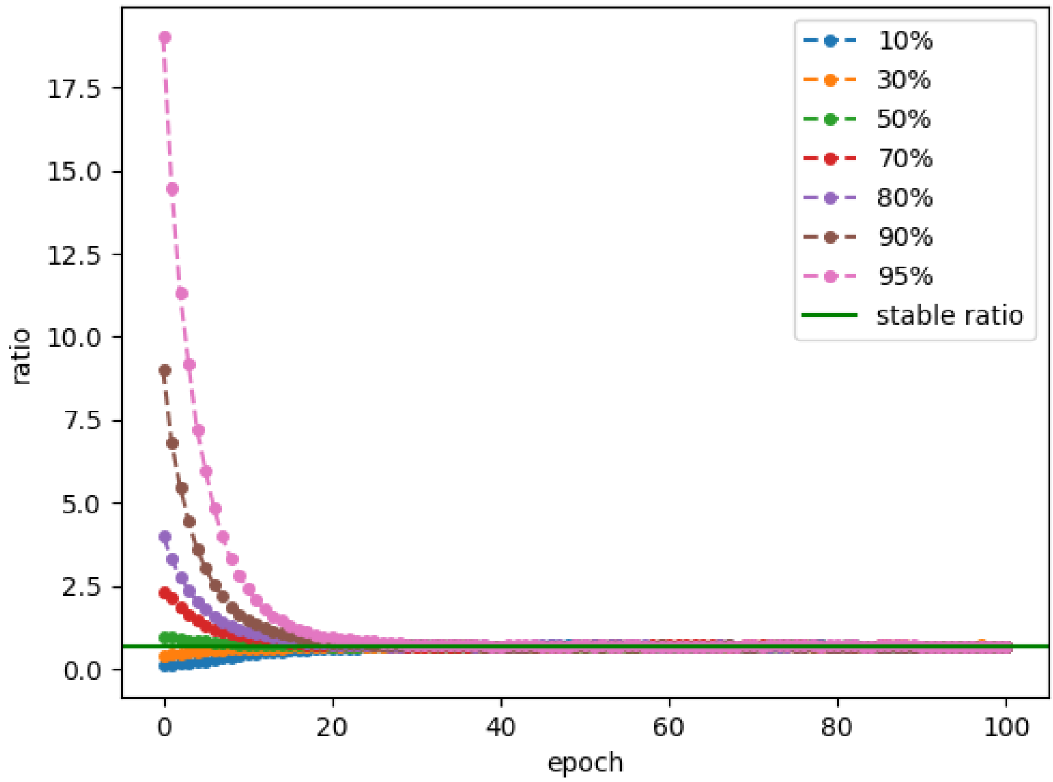

Figure 2: Evolution of ratios for different initial proportions (percentage of doves with respect to whole population). Green line represents the stable ratio of $\frac{5}{7}$.