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(* Legendre Polynomials Pn(x) and Legendre Functions of the Second Kind Qn(x) *)
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In[1]:=
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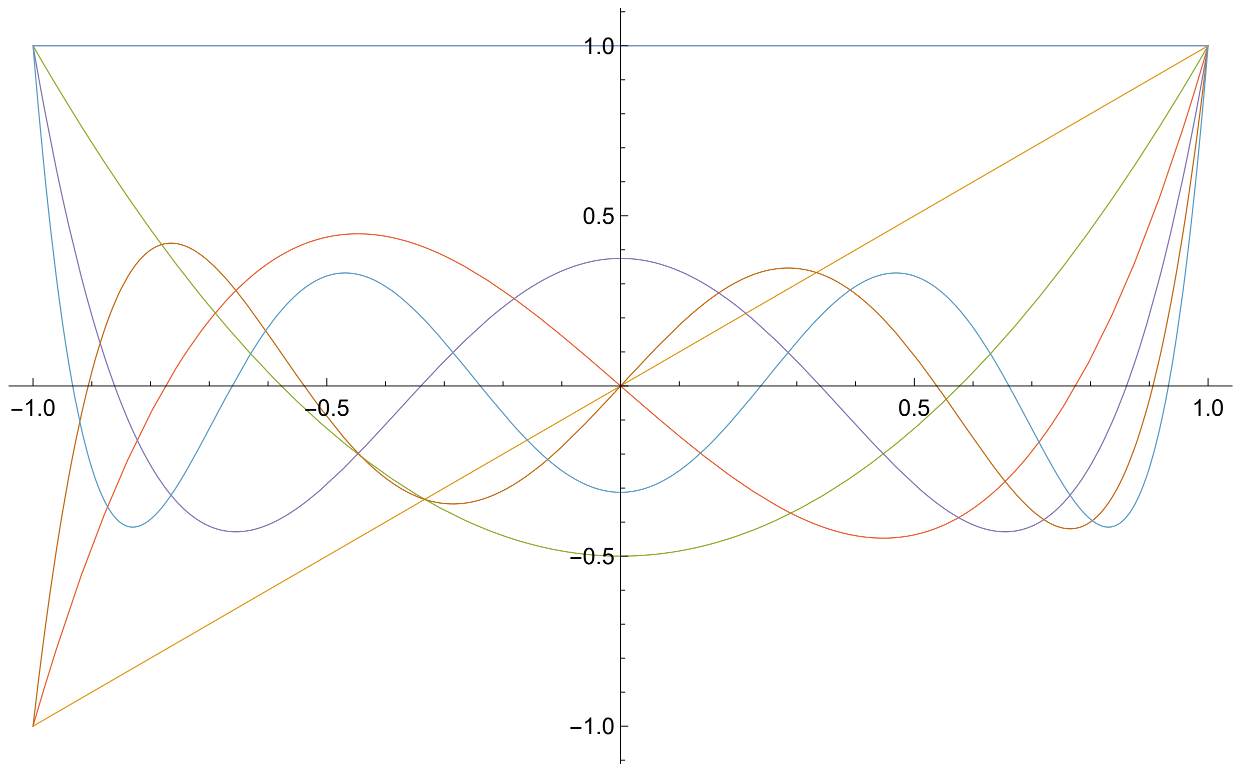
Table[LegendreP[n, x], {n, 0, 6}] // MatrixForm
Plot[%, {x, -1, 1}, PlotStyle → Thickness[0.001], TicksStyle → 12]
Table[LegendreQ[n, x], {n, 0, 6}] // FullSimplify // MatrixForm
Plot[%, {x, -1, 1}, PlotStyle → Thickness[0.001], TicksStyle → 12]
```

Out[1]//MatrixForm=

$$\begin{pmatrix} 1 \\ x \\ \frac{1}{2}\left(-1+3x^2\right) \\ \frac{1}{2}\left(-3x+5x^3\right) \\ \frac{1}{8}\left(3-30x^2+35x^4\right) \\ \frac{1}{8}\left(15x-70x^3+63x^5\right) \\ \frac{1}{16}\left(-5+105x^2-315x^4+231x^6\right) \end{pmatrix}$$

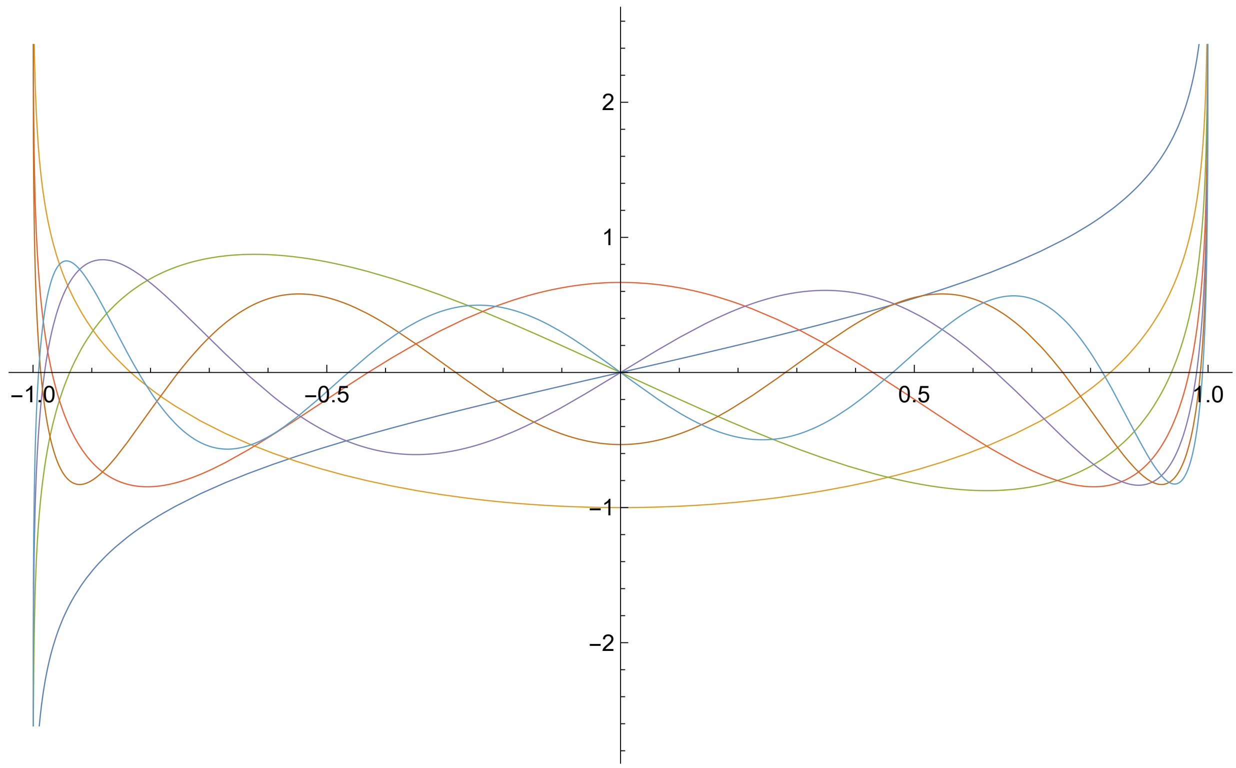
Out[2]=



Out[3]//MatrixForm=

$$\begin{pmatrix} \text{ArcTanh}[x] \\ -1+x\,\text{ArcTanh}[x] \\ \frac{1}{4}\left(-6x+\left(-2+6x^2\right)\text{ArcTanh}[x]\right) \\ \frac{1}{6}\left(4-15x^2+3x\left(-3+5x^2\right)\text{ArcTanh}[x]\right) \\ \frac{1}{24}\left(55x-105x^3+3\left(3-30x^2+35x^4\right)\text{ArcTanh}[x]\right) \\ -\frac{8}{15}-\frac{7}{8}x^2\left(-7+9x^2\right)+\frac{1}{8}x\left(15-70x^2+63x^4\right)\text{ArcTanh}[x] \\ \frac{1}{80}\left(-7x\left(33-170x^2+165x^4\right)+5\left(-5+21x^2\left(5-15x^2+11x^4\right)\right)\text{ArcTanh}[x]\right) \end{pmatrix}$$

Out[4]=



In[5]:= $\text{ArcTanh}[x] - \frac{1}{2}\,\text{Log}\left[\frac{1+x}{1-x}\right]$ // PowerExpand // FullSimplify

Out[5]= 0