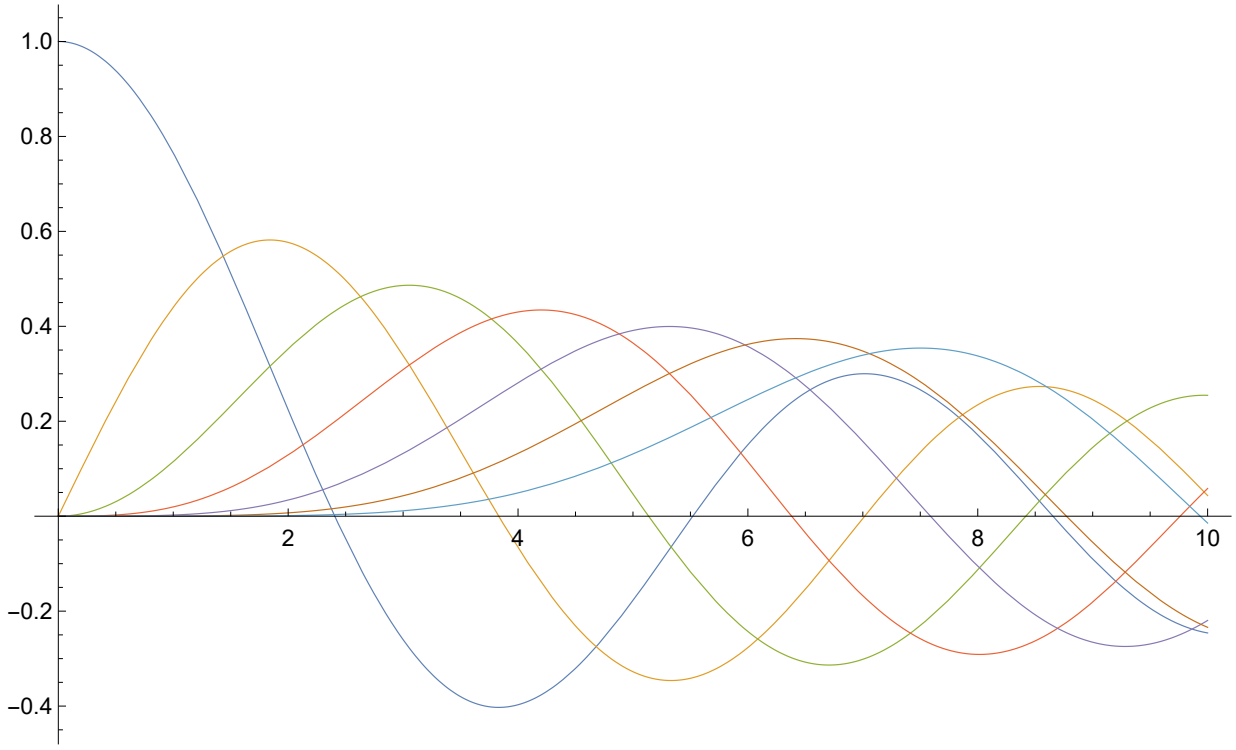
```
(* Bessel Functions (of the First Kind) Jn(x) and Bessel Functions of the Second Kind Yn(x) *)
```

In[1]:=
```
Table[BesselJ[n, x], {n, 0, 6}] // MatrixForm
Plot[%, {x, 0, 10}, PlotStyle → Thickness[0.001], TicksStyle → 12]
Table[BesselY[n, x], {n, 0, 6}] // FullSimplify // MatrixForm
Plot[%, {x, 0, 10}, PlotStyle → Thickness[0.001], TicksStyle → 12]
```

Out[1]//MatrixForm=

$$\begin{pmatrix} \text{BesselJ}[0, x] \\ \text{BesselJ}[1, x] \\ \text{BesselJ}[2, x] \\ \text{BesselJ}[3, x] \\ \text{BesselJ}[4, x] \\ \text{BesselJ}[5, x] \\ \text{BesselJ}[6, x] \end{pmatrix}$$

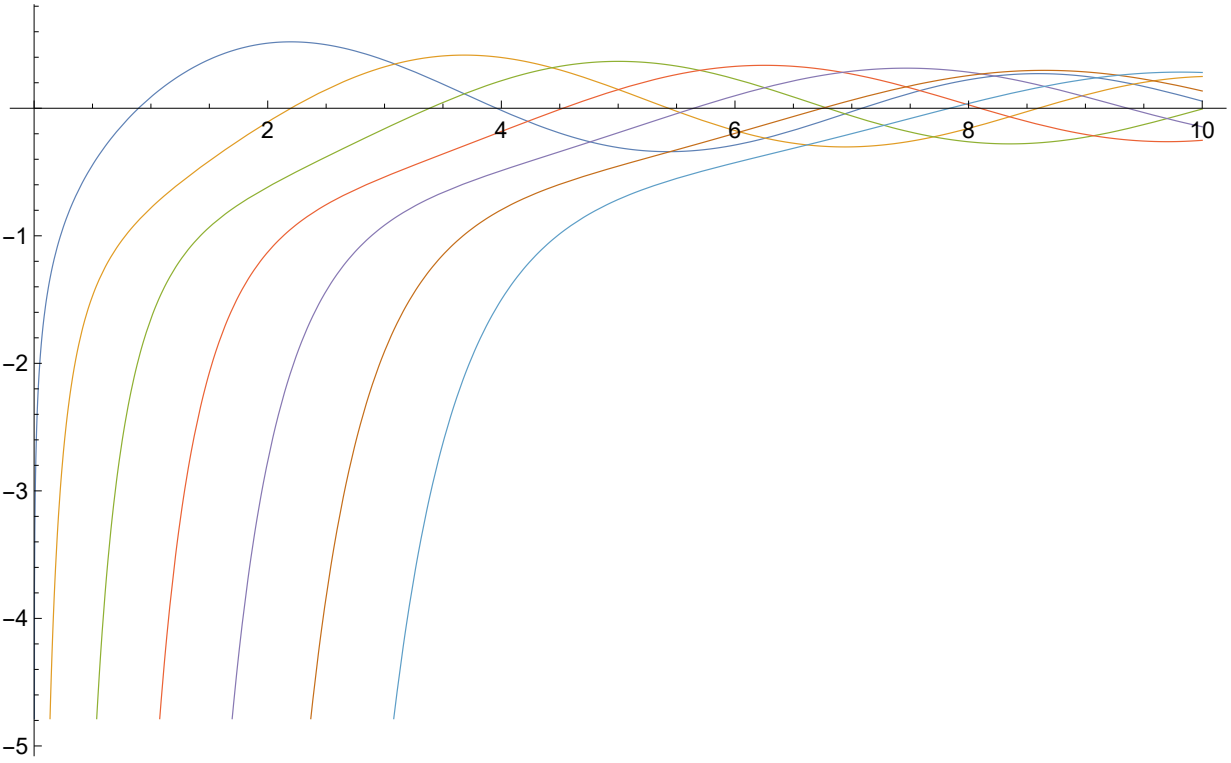
Out[2]=



Out[3]//MatrixForm=

$$\begin{pmatrix} \text{BesselY}[0, x] \\ \text{BesselY}[1, x] \\ \text{BesselY}[2, x] \\ \text{BesselY}[3, x] \\ \text{BesselY}[4, x] \\ \text{BesselY}[5, x] \\ \text{BesselY}[6, x] \end{pmatrix}$$

Out[4]=




```
(* Half Integer Bessel Functions are Elementary functions *)
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In[5]:=
```
BesselJ[1/2, x] // Simplify
BesselJ[3/2, x] // Simplify
BesselY[1/2, x] // Simplify
BesselY[3/2, x] // Simplify
```

Out[5]=
$$\frac{\sqrt{\frac{2}{\pi}}\ \text{Sin}[x]}{\sqrt{x}}$$

Out[6]=
$$\frac{\sqrt{\frac{2}{\pi}}\ \left(-x\,\text{Cos}[x]+\text{Sin}[x]\right)}{x^{3/2}}$$

Out[7]=
$$-\frac{\sqrt{\frac{2}{\pi}}\ \text{Cos}[x]}{\sqrt{x}}$$

Out[8]=
$$-\frac{\sqrt{\frac{2}{\pi}}\ \left(\text{Cos}[x]+x\,\text{Sin}[x]\right)}{x^{3/2}}$$

```
(* Asymptotic Behaviour of Bessel Functions *)
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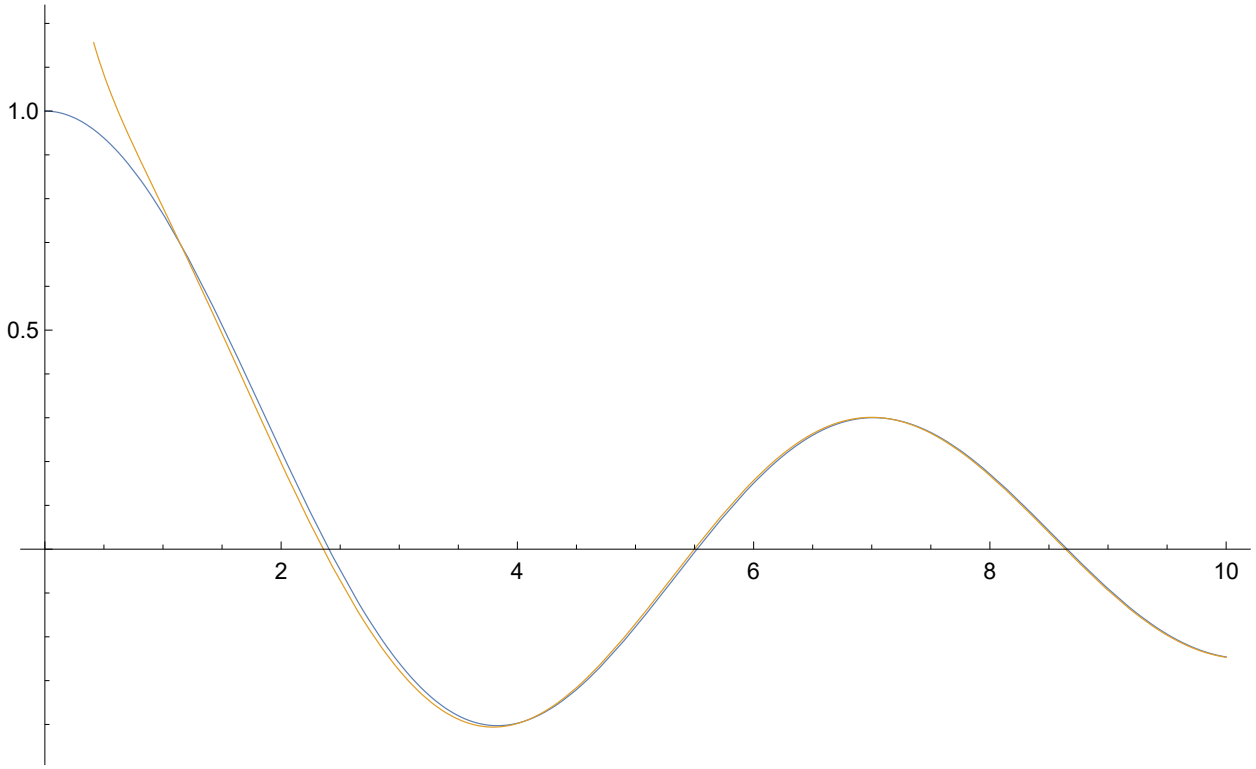
In[9]:=
```
Plot[{BesselJ[0, x], Sqrt[2/(π x)] Sin[x + π/4]},
 {x, 0, 10}, PlotStyle → Thickness[0.001], TicksStyle → 12]
```

Out[9]=

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(* Series Expansion of Bessel Functions *)
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In[10]:=
```
Series[BesselJ[0, x], {x, 0, 10}]
Series[BesselJ[1, x], {x, 0, 10}]
```

$$\sum_{n=0}^{\infty} \frac{(-x^2)^n}{4^n\,(n!)^2}$$

$$\frac{1}{2}\sum_{n=0}^{\infty} \frac{(-1)^n\,x^{2n+1}}{4^n\,n!\,(n+1)!}$$

Out[10]= $1 - \frac{x^2}{4} + \frac{x^4}{64} - \frac{x^6}{2304} + \frac{x^8}{147\,456} - \frac{x^{10}}{14\,745\,600} + O[x]^{11}$

Out[11]= $\frac{x}{2} - \frac{x^3}{16} + \frac{x^5}{384} - \frac{x^7}{18\,432} + \frac{x^9}{1\,474\,560} + O[x]^{11}$

Out[12]= BesselJ[0, x]

Out[13]= BesselJ[1, x]