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Response trajectories reveal the temporal dynamics of fraction representations☆

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ABSTRACT

Previous studies on mental arithmetic with fractions have been equivocal with respect to the nature of mental representations that are formed with fractions. It is not clear from present evidence whether fractions form perceptual primitives independent from components or whether component magnitudes must be processed in addition to the holistic magnitude. In the present study, we attempt to resolve this issue by using computer mouse-tracking. We analyzed the dynamics of participants' hand movements as they compared presented fractions to 1/2. We found that before settling to the correct answer, hand trajectories showed competitive influences of component magnitude and overall fraction magnitude, but the influence of components happened much earlier. These data support the idea that in fraction comparison, component magnitudes and holistic magnitude are processed together in a continuous, competitive manner.

1. Introduction

When skilled adults think about fractions, what do their representations look like? For instance, suppose that on the first day of a university course, the professor said that a class project would count for 3/7 of the final grade. A student then might be interested to know whether that fraction (3/7) was more or less than one half of his/her grade. The focus of this paper concerns how skilled adults make this decision.

The mathematics education literature is replete with examples of how multiple representations of fraction can be deliberately formed depending on context (Lamon, 2012). However, we are more concerned with the automatic, unconscious mental representations that are formed when comparing fractions. Previous studies along this line have inferred different types of numerical representations from the patterns of performance that have been found in numerical comparison tasks. For example, one of these predictable patterns is the numerical distance effect (Moyer & Landauer, 1967), where participants tend to respond faster in a comparison task to numbers that are farther apart (e.g., 2 vs. 8) compared to numbers that are closer together (e.g., 2 vs. 3). This is often taken as evidence for a mental number line that houses a fuzzy analog code for number magnitudes (Dehaene, Bossini, & Giraux, 1993); in this model, close number pairs suffer from representational overlap that delays the comparison decision process, a problem which pairs that are farther apart suffer from to a lesser extent.

Even though such distance effects are robust with respect to whole number comparisons (e.g., Dehaene, Dupoux, & Mehler, 1990), comparison tasks with fractions have not typically yielded such clean interpretations. Indeed, many recent studies have yielded equivocal results with respect to this issue. In one of the first studies to examine mental representations of fractions, Bonato, Fabbri, Umilta, and Zorzi (2007) had participants compare unit fractions (e.g., 1/3 versus 1/5) and found no numerical distance effect on fraction magnitudes. Rather, they found a distance effect on the components, leading them to conclude that adults do not represent fractions holistically as a single number, but instead form component-based representations.

Kallai and Tzelgov (2009) reached similar conclusions, albeit with a different method. Using the size-congruity effect (Henik & Tzelgov, 1982) as a marker of automatic processing, Kallai and Tzelgov (2009) found that physical comparison of unit fractions yielded a size congruity effect on the denominators, but not on the fraction magnitudes. That is, the numerical values of the denominators were automatically activated in long-term memory, but not the fraction magnitudes themselves. Further, Kallai and Tzelgov (2009) found no size-congruity effect when comparing non-unit fractions, but they did find both size-congruity and distance effects when comparing fractions to natural numbers. This led them to conclude that adults form representations of fractions based on components instead of holistic magnitudes, although there may be an automatic representation of a generalized fraction as a quantity less than one.

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Meert, Grégoire, and Noël (2009, 2010) extended these results by having adults compare fraction pairs that exhibited more variation than the unit-fraction stimuli used by Bonato et al. (2007). In their first study, Meert et al. (2009) used same numerator pairs (e.g., 2/9 vs. 2/3) and same denominator pairs (e.g., 2/7 vs. 5/7) to prime a natural number comparison. They found a distance effect on the numerators in same denominator pairs, suggesting a component-based representation on these types of fractions. However, contrary to Bonato et al. (2007), Meert et al. (2009) found a distance effect on fraction magnitudes in same numerator pairs, indicating that adults were accessing a representation of magnitude. Furthermore, the presence of a negative priming effect on natural number comparisons when presented after same numerator fraction pairs suggested that the denominators were compared automatically, after which the component-based representation had to be inhibited to facilitate selection of the correct fraction. Meert et al. (2010) extended this work to include fractions with no common components (e.g., 5/7 vs. 3/8) and found that performance was again explained by distance between fraction magnitudes, indicating holistic magnitude representations, although congruity effects on components led them to conclude that component magnitudes were also processed. The results of these studies together indicate that adults may form a hybrid representation of fractions that depends on task demands.

Several other recent studies have obtained mixed evidence for the nature of mental representations in fraction comparison tasks. Schneider and Siegler (2010) showed that adults form holistic magnitude representations across a wide variety of stimuli and populations. Several other behavioral studies have presented similar evidence for magnitude representations (DeWolf & Vosniadou, 2014; Gabriel, Szücs, & Content, 2013; Ganor-Stern, 2012; Zhang et al., 2011). Two recent imaging studies have supported this view: Ischebeck, Schocke, and Delazer (2009) and Jacob and Nieder (2009) found that fraction magnitude modulated activity within the intraparietal sulcus, which they interpreted as evidence for holistic magnitude representations of fractions.

However, other studies indicate that this magnitude representation may be accessed only under special conditions. For example, Sprute and Temple (2011) concluded that magnitude representations are formed when fraction pairs are such that component comparison is discouraged. Similarly, Obersteiner, Van Dooren, Van Hoof, and Verschaffel (2013) found that even expert mathematicians gain access to fraction magnitude only when it is absolutely necessary (e.g., when there are no common components). Using trial-by-trial strategy reports, Faulkenberry and Pierce (2011) found that the type of representation formed depended on the strategy used. Particularly, if participants used a holistic comparison strategy, numerical distance between fraction magnitudes was the best predictor of performance. However, when component-based strategies were used (e.g., the cross-multiplication strategy, where cross products are computed and then compared), the numerical distance effect disappeared, but a problem-size effect on the cross-products emerged, giving further evidence for a component-based representation. These studies resonate with the notion of a hybrid representation that is neither purely component-based nor purely holistic (Meert et al., 2009, 2010) and that is likely dependent upon strategy and/or task requirements (Ganor-Stern, 2012; Ganor-Stern, Karasik-Rivkin, & Tzelgov, 2010; Huber, Moeller, & Nuerk, 2014; Zhang, Fang, Gabriel, & Szücs, 2014).

In light of these equivocal findings, an open question remains. Do adults mentally represent fractions in terms of holistic processing, where the primitive object of representation is the fraction's magnitude, or do adults rely on componential processing, where fractions are represented by first attending to component magnitudes and then estimating their ratio. More specifically, we ask the following: in the 1–2 s that it takes an adult to compare a presented fraction to a fixed standard (e.g., 1/2), how does the mental representation that is formed in response to the task change over time?

With the existing evidence, it becomes difficult to make solid predictions. However, it may be possible to make predictions by bridging these seemingly disparate findings by appealing to dual-process theories of cognition (e.g., Evans & Stanovich, 2013). Dual-process theories have been developed to explain performance in reasoning tasks that results from intuitive biases that influence processing before a slower, more analytic system takes over. Such dual-process theories have been recently applied to mathematical reasoning processes (DeWolf & Vosniadou, 2014; Gillard, Dooren, Schaeken, & Verschaffel, 2009; Obersteiner et al., 2013), and generally predict that there is an initial, automatic representation of number that is later overtaken by a second representation that is influenced by intentionality, resources, task demands, etc. Based on previous studies on automaticity of fraction representations (e.g., Kallai & Tzelgov, 2009, 2012), we predict that the initial, automatic representation formed is directly tied to the components of the fraction, whereas the later, more intentional representation uses magnitude information.

We directly test this hypothesis in the present study. Critically, we used a hand-tracking paradigm (Fischer & Hartmann, 2014; Freeman & Ambady, 2010; Spivey, Grosjean, & Knoblich, 2005) to gain insight into the online formation of fraction representations. Variants of this paradigm have been successfully employed to answer questions regarding the mental representations of whole numbers (Faulkenberry, 2014; Santens, Goossens, & Verguts, 2011; Song & Nakayama, 2008). This should be a useful approach to the present problem, as previous studies lead us to predict that reaction time alone may not be sensitive enough to test selective influence of component size and numerical distance. In other words, we will likely see reaction time effects when manipulating component size (Bonato et al., 2007) as well as distance from the comparison fraction (Schneider & Siegler, 2010). Thus, it will be useful to perform a more fine-grained analysis of the *timecourse* of the decision as opposed to relying solely on the *duration* of the decision. If it is the case that both component size and distance show significant effects on performance, the timecourse analysis will allow us to determine *when* the influences of these factors occur.

In the present study, we asked participants to quickly decide whether a presented fraction was smaller or larger than 1/2. During the task, we collected the streaming (x, y) coordinates of a computer mouse pointer as the participants clicked on the correct response. By separately manipulating component size and distance from 1/2, we tested the selective influence of both factors on the trajectories of participants' hands as they made their decisions, providing a valuable window into the online formation of their mental representations (Freeman, Dale, & Farmer, 2011). Specifically, if participants represent fractions in a componential manner as well as gaining access to fraction magnitude (i.e., the dual-processing framework), then both manipulations should result in deflections of the average hand trajectories. That is, when component size is inconsistent with magnitude (e.g., a large fraction with small components, such as 3/4) or when the to-be-compared fraction is closer to 1/2, we should see a deflection of the hand trajectory toward the incorrect answer before moving to the correct answer. However, the deflection due to component size should occur *earlier* in the timecourse of the decision compared to the deflection that stems from manipulating the distance from 1/2. If, on the other hand, participants' immediate representations are based solely on magnitude without the need for processing component magnitude (i.e., a direct access model of fraction representation), then the earlier influence should come from distance from 1/2 with no influence of component size.

2. Experiment

2.1. Participants

Twenty-six undergraduate students (mean age = 23.1 years, 14 female) participated in exchange for partial course credit. All participants reported being right-hand dominant. The study was approved by a local ethics committee, and each participant provided written informed consent prior to taking part in the study.

2.2. Apparatus

All stimuli were presented using a MacBook Pro 15 inch laptop computer connected to a Dell 17-inch external display with a resolution of 1024×768. A Dell optical mouse was used as the primary response device. To record mouse trajectories during responses, we used the MouseTracker software package (Freeman & Ambady, 2010), freely available as a download from http://psych.nyu.edu/freemanlab/mousetracker/. We ran the program on the MacBook Pro using a virtual Windows XP environment via Parallels. Following the recommendations of Fischer and Hartmann (2014), we disabled the "dynamic acceleration" option and lowered the speed of the mouse movements on the screen to the second-lowest possible speed in the mouse settings dialog. This is done to prevent ballistic mouse movements and get a more reliable measure of participants' hand movements. The resulting displacement ratio of the mouse to screen movement was 1 cm to 100 pixels.

2.3. Stimuli and design

We constructed 8 fractions with single-digit components (see Table 1). The specific choice of fractions was obtained by crossing the factors of component-size (small: less than 5; large: greater than 5) and fraction magnitude (small: less than 1/2; large: greater than 1/2). Note that with single-digit components, it is impossible to construct fractions smaller than 1/2 with both components larger than 5. In this condition, only the denominators are greater than 5.

This choice of stimuli allowed us to symmetrically manipulate the consistency of component size and magnitude. That is, for large fractions, components could either be small (2/3, 3/4) or large (6/8, 6/9). Similarly for small fractions, components could either be small (1/3, 1/4) or large (2/8, 3/9). In addition, we were able to manipulate the distance between the stimulus fraction and the comparison fraction 1/2. This distance was either small (1/3, 3/9, 2/3, 6/9, with each fraction being a distance of $1/6 = 0.17$ from the comparison) or large (1/4, 2/8, 3/4, 6/8, with each fraction being a distance of $1/4 = 0.25$ away from the comparison).

2.4. Procedure

Participants were seated approximately 70 cm from the computer screen and held the mouse in their right hand. They were told that for each trial, they would be asked to quickly and accurately choose whether the presented fraction was greater or smaller than the target fraction 1/2. Each trial began with a blank screen presented for 1000 ms, followed by a screen that displayed the response labels SMALLER and LARGER at the top left and right of the screen, respectively. The order of these labels was switched once midway through the experiment; half of the participants started with the SMALLER–LARGER ordering, while the other half began with the LARGER–SMALLER ordering. After 1000 ms, a START button appeared. When participants clicked the START button, one of the 8 stimulus fractions randomly appeared in the center of the screen. Participants were then required to quickly click on the response label appropriately designating whether the presented fraction was larger or smaller than 1/2. During these responses, we recorded the streaming (x, y)-coordinates of the participants' computer mouse movements (with a sampling rate of approximately 70 Hz).

In order to ensure that mouse trajectories reflected online processing, we instructed participants to begin moving their computer mouse as quickly as possible. In the event that the mouse initiation time exceeded 250 ms, a message appeared on the screen after the participant's response, instructing them to start moving earlier on future trials, even if they were not completely sure of their response. In total, each participant completed 240 trials (120 in each response label ordering).

Table 1
Fraction stimuli used in the experiment.

	Component size	
Magnitude	Small	Large
Small	$\frac{1}{3}, \frac{1}{4}$	$\frac{2}{8}, \frac{3}{9}$[a]
Large	$\frac{2}{3}, \frac{3}{4}$	$\frac{6}{8}, \frac{6}{9}$

[a] With single-digit components, it is impossible to construct a fraction that has a large numerator (greater then 5) while still having a magnitude larger than 1/2. In this case, only denominators are greater than 5.

3. Results

Participants completed a total of 6240 fraction comparison trials. Across these trials, there were 250 errors, resulting in an error rate of 4.0%. These trials were removed from further analysis. In addition, we performed an outlier screening whereby we removed 35 additional trials for which reaction times exceeded 3 standard deviations from the overall mean reaction time of 1248 ms. This resulted in retaining 5955 trials for all subsequent analyses (95.4% of the original data). All statistical analyses were performed using the R statistical package (R Development Core Team, 2011). All figures shown were created in R using the ggplot2 package (Wickham, 2009).

3.1. Reaction time analysis

First, we looked for behavioral signatures of component-based representations of fractions. To this end, we investigated the effects of component size on fraction performance. Mean reaction time and movement initiation time were separately analyzed via a 2 (component size: small versus large) × 2 (fraction magnitude: small versus large) repeated measures analysis of variance (see Table 2). There was a significant main effect of component size on RT, $F(1, 25) = 9.51$, $p = 0.004$, $\eta_p^2 = 0.28$; participants completed trials with small components ($M = 1189$ ms) more quickly than trials with large components ($M = 1257$ ms). The main effect of fraction magnitude on RT was not significant, $F(1, 25) = 0.045$. Critically, there was a significant interaction between component size and fraction magnitude on RT, $F(1, 25) = 97.97$, $p < 0.001$, $\eta_p^2 = 0.80$. This reflected a congruency effect between component size and fraction magnitude; participants were faster to respond to trials in which component size and fraction magnitude were congruent (e.g., small components and small magnitude, such as 1/3) compared to trials in which component size and fraction magnitude were incongruent (e.g., small components with large magnitude, such as 3/4. There were no significant effects on movement initiation time (all F-ratios less than 0.85).

Table 2
RT and initiation time by component size and distance from 1/2.

	Component size[a]		Distance from 1/2[b]	
	RT	Init	RT	Init
Large fractions				
Small	1257(243)	75(21)	1237(247)	77(25)
Large	1191(258)	76(25)	1211(252)	73(21)
Small fractions				
Small	1121(215)	76(24)	1233(236)	75(27)
Large	1324(303)	75(25)	1205(258)	75(22)

Note: RTs are measured in ms. Standard deviations are presented in parentheses.
[a] Component size: small = less than 5, large = greater than 5.
[b] Distance: small = actual distance equal to 1/6, large = actual distance equal to 1/4.

Next, we looked for signatures of magnitude representations of fractions. To this end, we investigated the effects of numerical distance on fraction performance. Mean RT and movement initiation time were separately analyzed via a 2 (distance from 1/2: small versus large) × 2 (fraction magnitude: small versus large) repeated measures analysis of variance (also see Table 2). There was a significant main effect of distance on RT, $F(1, 25) = 5.40$, $p = 0.028$, $\eta_p^2 = 0.18$. Participants were faster to respond when fractions were farther from 1/2 ($M = 1208$ ms) compared to when fractions were closer to 1/2 ($M = 1235$ ms). This replicates the predicted numerical distance effect (Moyer & Landauer, 1967), which is a common marker of magnitude representations. No other effects on RT were significant (all F-ratios less than 0.4). Similar to above, there were no significant effects on movement initiation time (all F-ratios less than 1.4).

Together, these results suggest that both component/magnitude consistency and distance from 1/2 have an overall effect on the mental representations of fractions that are formed in a fraction comparison task, echoing most recent studies in fraction cognition (Faulkenberry & Pierce, 2011; Meert et al., 2009, 2010; Schneider & Siegler, 2010). As such, RT alone is likely insufficient to provide clear evidence regarding the relative importance of components and magnitude on the formation of fraction representations. To overcome this limitation, we also analyzed hand trajectories.

3.2. Analysis of hand trajectories

To prepare the raw mouse trajectory data for analysis, we performed an initial default preprocessing with the MouseTracker software package (Freeman & Ambady, 2010). All mouse trajectories were rescaled into a standard coordinate space (x-coordinate range: -1 to 1; y-coordinate range: 0 to 1.5). In addition, to remove the influence of varying response times, all raw trajectories were normalized (via linear interpolation) to consist of 101 time steps. This step was critical in order to allow us to average across trials with differing time durations. For each trial we measured the degree to which the incorrect response alternative influenced participants' decisions by computing the maximum deviation (MD), the largest perpendicular deviation between the actual trajectory and the ideal response trajectory, represented by a straight line from the trajectory's starting point and the correct response. We also measured area under the curve (AUC), the geometric area between the actual trajectory and the ideal response trajectory. These measures typically yield the same overall results, but AUC provides a better measure of attraction toward the incorrect response alternative, whereas MD provides a better measure of maximum attraction (Freeman & Ambady, 2010).

For ease of visualization and interpretation of these hand trajectories, all trajectories involving responses to fractions larger than 1/2 were remapped to the right side of the display, and those involving fractions smaller than 1/2 were remapped to the left side of the display.

To analyze the influence of components and magnitude on the dynamic formation of fraction representations, we computed mean hand trajectories for responses to fractions as a function of component size (small versus large) and distance from 1/2 (small versus large). As can be seen in Fig. 1, panels (a) and (c), trajectories for trials in which there is a mismatch between component size and fraction magnitude exhibit a great deal of continuous attraction toward the incorrect alternative. This congruency effect was statistically significant; when submitting MD and AUC values separately to a 2 (component size: small versus large) × 2 (fraction magnitude: small versus large) repeated measures analysis of variance, there was a significant interaction between component size and fraction magnitude on MD values, $F(1, 25) = 103.1$, $p < 0.001$, $\eta_p^2 = 0.80$, as well as a significant interaction between component size and fraction magnitude on AUC values, $F(1, 25) = 74.03$, $p < 0.001$, $\eta_p^2 = 0.75$. Such a results indicate that components exert much influence on participants' dynamic decision processes as they judge whether fractions are larger than 1/2.[1]

In addition, we measured the degree of dynamic attraction toward the incorrect alternative throughout the decision process by computing the differences between x-coordinates over each of the 101 timesteps. We found that these differences were statistically significant between the 32nd and the 80th timestep for large fractions, and between the 28th and 92nd timestep for small fractions. Both results indicate early and sustained influence of component size on fraction representations.

In order to evaluate our claim that component-based representations are formed even earlier than magnitude representations, we then performed a similar trajectory analysis when manipulating distance from 1/2. As can be seen in Fig. 1, panels (b) and (d), trajectories for fractions that are closer to 1/2 exhibit a small degree of dynamic attraction toward the incorrect alternative. This "dynamic numerical distance effect" is statistically significant; when submitting MD and AUC values separately to a 2 (distance from 1/2: small versus large) × 2 (fraction magnitude (small versus large) repeated measures analysis of variance, there was a significant main effect of distance on MD, $F(1, 25) = 27.83$, $p < 0.001$, $\eta_p^2 = 0.53$, as well as AUC, $F(1, 25) = 19.31$, $p < 0.001$, $\eta_p^2 = 0.44$.

Similar to above, we measured the degree of dynamic attraction toward the incorrect alternative throughout the decision process by again comparing x-coordinates over each of the 101 timesteps. Critically, we found that the differences in x-coordinates were statistically significant only between the 47th and the 77th timesteps for large fractions, and between the 43rd and the 74th timesteps for small fractions. This indicates that while there was significant influence of distance on fraction representations (providing evidence for a magnitude-based representation), the influence happened later in the decision process compared to that of component-size (which showed influence as early as the 28th timestep).

Our critical claim is that even though RT data shows influences of both component size and numerical distance (providing evidence that representations are both component- and magnitude-driven), our hand-trajectory data shows that components seem to be driving the initial representations and magnitude is driving the later representation. Though we have already shown that the trajectories "split" earlier when manipulating components than when manipulating distances, this picture comes from averaging across all trials. A more principled approach to this issue would need to consider the participant as the appropriate unit of analysis; if each participant exhibits trajectory deviations for component size earlier than for distance, then we could capture this by submitting the deviation *times* to a simple hypothesis test (c.f., Freeman, Ma, Han, & Ambady, 2013).

To do this, we recorded for each participant the time at which their average hand trajectories began to show influence of (1) components and (2) distance. We did this by analyzing the first significant trajectory deviation timestep (i.e., the first time the x-coordinates of competing trajectories differed significantly via an independent-samples t-test) via a 2 (manipulation type: component-size versus distance) × 2 (fraction magnitude: small versus large) repeated measures analysis of variance. Critically, there was a significant main effect of manipulation type, $F(1, 25) = 35.7$, $p < 0.001$, $\eta_p^2 = 0.59$. Participants began showing the

[1] With our limited set of stimuli being repeated many times throughout the experiment, it is possible that participants might have developed a memory for the answers and that later trials may not reflect numerical processing. To test this, we split the data into *early* versus *late* trials (where trials 1–120 were classified as early and trials 121–240 were classified as late). We then compared the MD values for consistent trials (i.e., small fraction with small components, large fraction with large components) and inconsistent trials (i.e., small fraction with large components, large fraction with small components). On EARLY trials, inconsistent trials (mean MD = 0.68) showed a larger deviation than consistent trials (mean MD = 0.50). Using Cohen's d as a measure of this effect, we then constructed a 95% confidence interval on this effect size to be (0.32, 0.53). Doing the same analysis on LATE trials, we got a 95% confidence interval on effect size to be (0.31, 0.52). In other words, the effect of component/magnitude consistency on hand trajectories is no different between early and late trials.

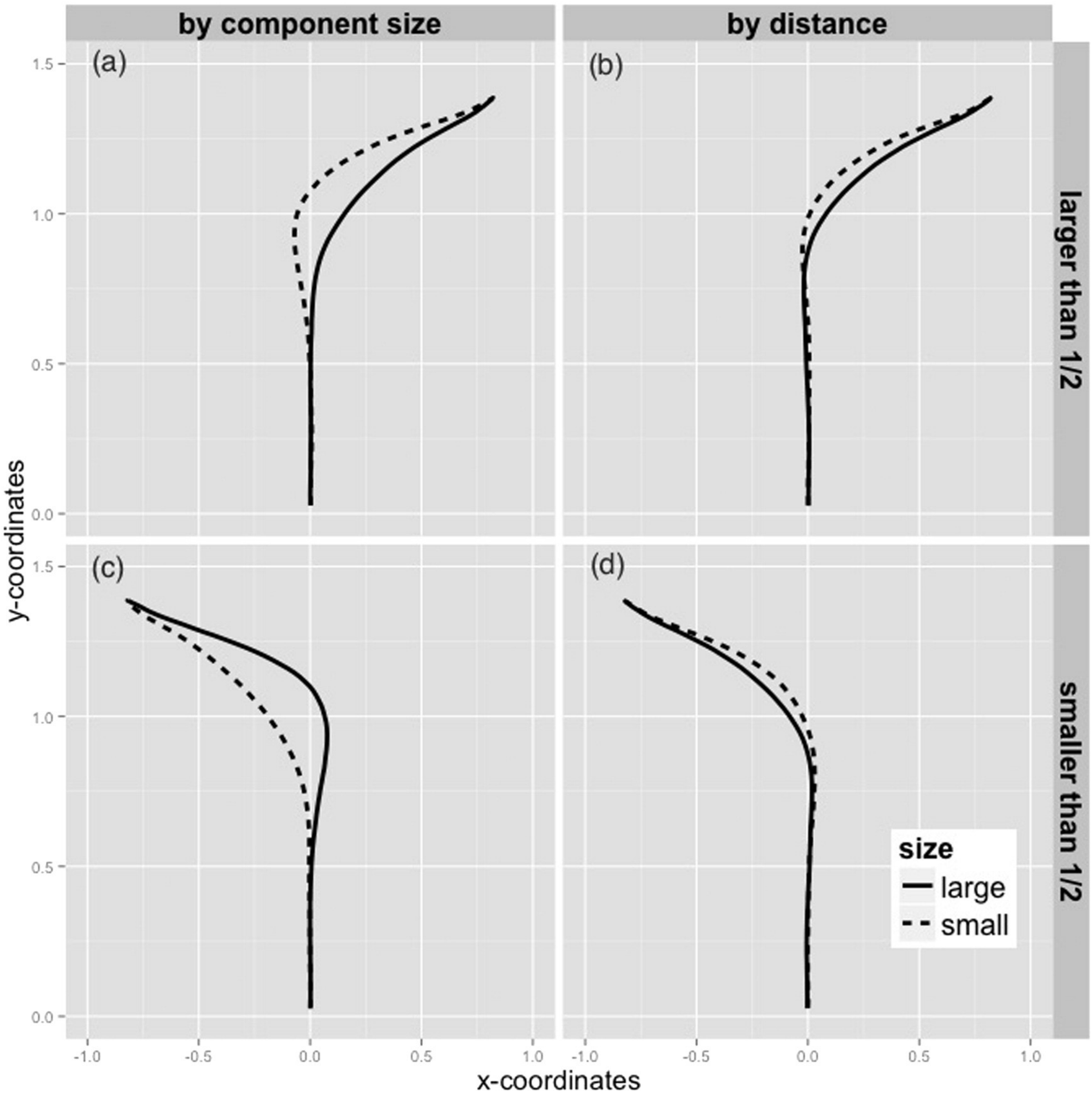

Fig. 1. Mean hand trajectories as a function of component size (panels (a) and (c)) and distance from 1/2 (panels (b) and (d)) show continuous competition from alternative response options. Critically, the trajectory deviations happen earlier for components than for distance, indicating earlier competitive influence of components.

influence of components an average of 18.5 steps earlier than the onset of influence from distance. This supports the intuition gleaned from looking at the average hand trajectories collapsed over all participants, and thus provides solid evidence in support of our hypothesis that components are driving the initial representations formed in a fraction comparison task, whereas magnitude begins to show influence later during the task.

3.3. Distribution of response trajectories

To cement our claims that participants' initial representations are tied to continuous competition from the automatic activation of fraction components, we need to consider the possibility that the wide trajectory deviations seen in Fig. 1 stem from averaging across two quite disparate categories of response trajectories: one response category in which participants headed straight for the correct answer (with no influence of the other response alternative) and the other category in which participants headed straight for the incorrect alternative and then sharply corrected their response midflight. This kind of discrete switching behavior, when averaged across many hundreds of trials, could produce average trajectories much like those in Fig. 1(a) and (c) (see Freeman & Dale, 2012 for a further discussion of this issue). We want to rule this out, however, since such behavior would indicate that the differences in trajectories that we've witnessed stem only from stochastic task switching and not necessarily from the influence of initial representations of fractions. Our claim is the latter.

In order to rule out this task-switching possibility, we converted the maximum deviation value (MD) from each trial to a z-score (separately by participant) and inspected this z-distribution of MD values (see Fig. 2). If participants are task switching on trials, then there should be two peaks in the z-distribution of MD values: one peak corresponding to direct trajectories (small MD values), and another peak corresponding to those trials in which the midflight correction occurred (large MD values). Visual inspection of Fig. 2 indicates that this is not the case. To confirm, we computed the bimodality coefficient (SAS Institute Inc., 2012) for this z-distribution. The bimodality coefficient (BC) is computed as:

$$BC = \frac{s^2+1}{k+\frac{3(n-1)^2}{(n-2)(n-3)}}$$

where s represents skewness, k represents kurtosis, and n is the size of the distribution. The BC for the z-distribution of MD values was

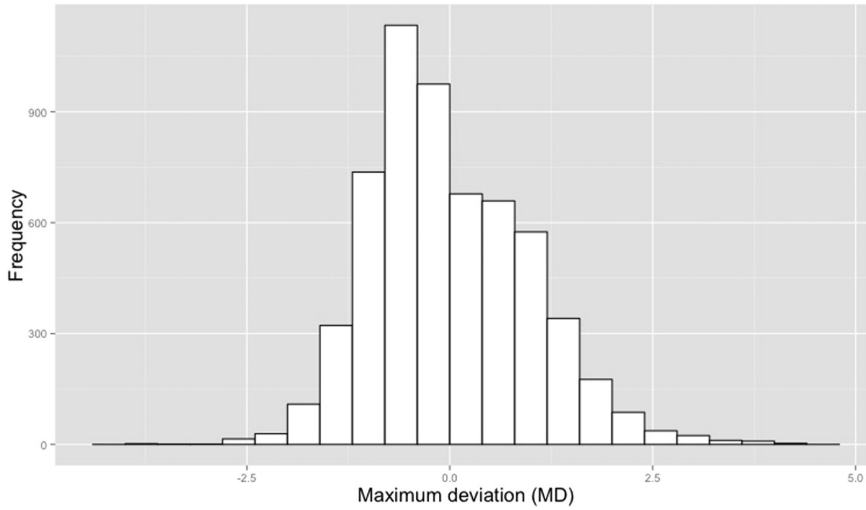

Fig. 2. The z-distribution of MD values is unimodal, indicating parallel activations of response alternatives during a trial.

computed to be 0.20 (skewness = 0.52 kurtosis = 3.42), which was well within the $BC < 0.555$ bimodality-free region (SAS Institute Inc., 2012). In addition, we computed Hartigan's dip statistic D (Hartigan & Hartigan, 1985). The advantage of this statistic is that it is inferential; if $p < 0.05$, the distribution is considered to be multimodal (Freeman & Dale, 2012). Using the R package diptest (Maechler, 2013), we computed $D = 0.0025$, $p > 0.99$, confirming that the distribution is indeed not bimodal. Taken together, these results speak against the hypothesis that participants did not engaged in sharp midflight corrections of their trajectories. Rather, it seems that during a trial, both response alternatives were partially activated until a clear winner emerged and the correct answer was selected (Faulkenberry, 2014; Santens et al., 2011).

4. Discussion

In the present study we asked participants to quickly judge whether presented fractions were greater or less than 1/2. By manipulating both component/magnitude consistency and distance from 1/2 and recording hand trajectories via a hand-tracking paradigm, we were able to test the influence of both components and numerical distance on fraction representations. As predicted, participants' initial representations were directly tied to the components of the fraction, which supports the conclusions of several recent studies on fractions (e.g., Bonato et al., 2007; Kallai & Tzelgov, 2009).

The evidence for this claim comes from the consistent effect that component size exerted on participants' hand trajectories in the fraction comparison task: when component size was inconsistent with the overall magnitude of the fraction (e.g., large components, but small overall magnitude), participants hands tended to drift away toward the incorrect answer before eventually settling in picking the correct one. As a concrete example, the fractions 2/3 and 6/9 are both equivalent in terms of numerical magnitude, but the pattern of hand trajectories differed between them. Participants showed a more direct path toward clicking LARGER for 6/9, but a wider, deflected path when clicking LARGER for 2/3. This supports the notion of an automatic representation of components that has been previously found by Kallai and Tzelgov (2009) and is consistent with a dual-process model of numerical representation (Obersteiner et al., 2013). Such dual-process theories hypothesize two parallel cognitive systems; one that is responsible for fast processing based on elementary numerical intuitions, and a second, slower system that is based on more methodical, analytic processing that is influenced by intentionality, resources, task demands, etc. In our study, the initial, automatic representation of components may reflect this fast, intuition-based processing.

We hypothesized that the slower, analytic processing would be in the form of a magnitude representation, as previously demonstrated in several studies (DeWolf & Vosniadou, 2014; Gabriel et al., 2013; Ganor-Stern, 2012; Schneider & Siegler, 2010; Zhang et al., 2011). Indeed, the current data supports this; we found that fractions farther from 1/2 took less time to respond to than did fractions that were close to 1/2. This is a classic marker of magnitude-based representations (Moyer & Landauer, 1967). Crucial to the present study, however, is that by analyzing the continuous, competitive dynamics that resulted in this numerical distance effect, we were able to show that the effects of distance did not occur until much later in the response timecourse than did the effects of component size. In other words, when forming fraction representations, adults seem to process the components first and possibly compute their ratio rather than gaining direct access to fraction magnitude without first processing components.

To explain our conclusion better, consider the following argument. If one hypothesized a mechanism by which our participants could immediately retrieve a holistic representation of fraction from long-term memory without having to first process the component magnitudes, then the magnitudes of the individual components would be irrelevant to the decision, or at the very least, the components would only serve as perceptual stimuli to trigger a retrieval of holistic magnitude from long-term memory. Along this line, one might further argue that a fraction displayed in lowest terms, such as 3/4, would be more familiar than the same fraction expressed as 6/8. As such, this familiarity should lead to a processing advantage whereby 3/4 would be judged as greater than 1/2 more quickly than 6/8. We found exactly the opposite; 6/8 was processed more quickly than 3/4. Rather than familiarity, it is the consistency between holistic magnitude and component magnitude that provides a processing advantage. That is, when both component magnitude and holistic magnitude are small (or large), the decision is made more quickly than when component magnitude and holistic magnitude are inconsistent. Thus, it must be the case that the magnitudes of the fraction components are being processed.

Implicit in our interpretation is the notion that the curved hand trajectories result from a dynamic competition of partially active representations (Freeman et al., 2011; Spivey et al., 2005). It is entirely possible that such curved trajectories can result from two distinct response patterns, whereby on some trials participants exhibit straight-line paths toward the correct answer, and on other trials, participants sharply correct an initially incorrect response path midflight (Freeman & Dale, 2012). Such response patterns would necessarily result in a bimodal distribution of trajectories, and we demonstrated that this does not seem to be the case. This is in line with the architecture of the computational model of Gevers, Verguts, Reynvoet, Caessens, and

Fias (2006), in which whole number decisions result from dynamic competition between partially active response nodes. We note that there is debate in the present literature about whether some trajectory signatures indeed provide evidence of competition among response options, especially in the case where trajectories do not widely deviate from the vertical midline, as in Fig. 1, panels (b) and (d) (Fischer & Hartmann, 2014; but see Faulkenberry & Rey, 2014). In any case, such a conceptualization of numerical processing may provide a rich avenue of future research in numerical cognition, particularly in terms of processes that have traditionally relied on reaction time measures alone (Faulkenberry, 2014; Santens et al., 2011; Song & Nakayama, 2008).

The present research may provide a bridge between some seemingly contradictory findings in recent research on fraction representations. Whereas we have solid evidence that componential representations are formed, we also have evidence that magnitude representations are formed, albeit later. The requirement of such a representation is not always obvious. For instance, in the present task there is no reason, a priori, for someone to gain direct access to fraction magnitude. The goal in this task was to decide whether a presented fraction is less than or greater than 1/2. With such a task, it is possible that participants could use a rough estimation strategy on components alone. Indeed, all that is required is for the participant to quickly ascertain whether the numerator is less than or greater than half the denominator. For example, one could hypothetically judge 2/9 to be less than half simply by noting that 2 is quite a bit less than 9. Such a strategy could easily be taught to a child without symbolic knowledge of fractions. When participants use component-based strategies for calculation, such component representations can displace the holistic magnitude representation (Jacob, Vallentin, & Nieder, 2012), which has been shown to be the case when participants use purely component-based strategies such as cross-multiplication (Faulkenberry & Pierce, 2011).

However, the present data indicates that magnitude does indeed play a part in our mental processing of fractions, particularly for skilled adults. It just comes into play at a later time, only after adults have first attended to the magnitude of the components. This has important ramifications for teaching: since magnitude is a critical part of successful adult representations of fractions, it is important that children gain a knowledge of fractions not only from a symbolic, component-driven view, but also their underlying numerical values (Gabriel, Coché, Szücs, Carette, & Rey, 2012; Siegler, Fazio, Bailey, & Zhou, 2013).

In sum, through the use of computer mouse-tracking to study the dynamics of numerical processes, we found that when adults are asked to perform a simple magnitude comparison task with fractions, components magnitudes seem to be processed. However, later representational refinements seem to tap into numerical magnitude, even though the representation of such is not explicitly required for the task. Such a conclusion about the timecourse of a numerical representation highlights a distinct advantage of using computer mouse-tracking to study numerical processing. Future studies along this line should continue to manipulate task requirements (e.g., Huber et al., 2014) to determine when and how representations of holistic magnitude and components contribute to fraction representations.

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