Workshop: Bayesian Statistics with JASP

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Plan for today:

1. what is Bayesian inference?
2. Using JASP with examples:
   - *t*-tests
   - correlation
   - estimating Bayes factors from summary statistics

All materials can be found at:

`http://github.com/tomfaulkenberry/bayesAngeloState`

The problem of inference

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(think sampling distributions)

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- bases decision criterion on controlling long-run error rates (i.e., α)

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- "posterior belief in model $\mathcal{M}$"
- notation: $p(\mathcal{M} \mid \text{data})$
- no accept/reject decision

Bayes' Rule

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Natural action in science is to *compare* two models $\mathcal{M}_1$ and $\mathcal{M}_2$.

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Bayes factor

The predictive updating factor

$$B_{12} = \frac{p(\text{data} \mid \mathcal{M}_1)}{p(\text{data} \mid \mathcal{M}_2)}$$

tells us how much better $\mathcal{M}_1$ predicts our observed data than $\mathcal{M}_2$.

This ratio is called the **Bayes factor**

Bayes factors

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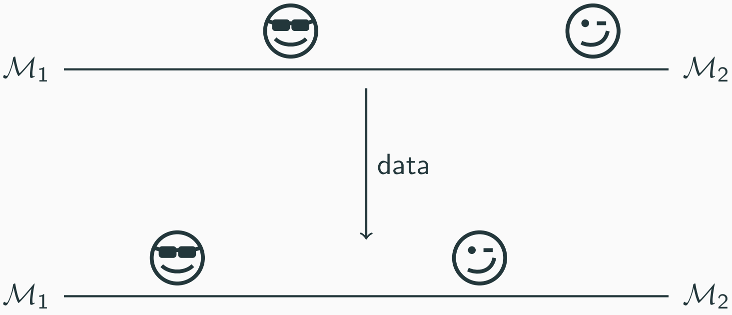
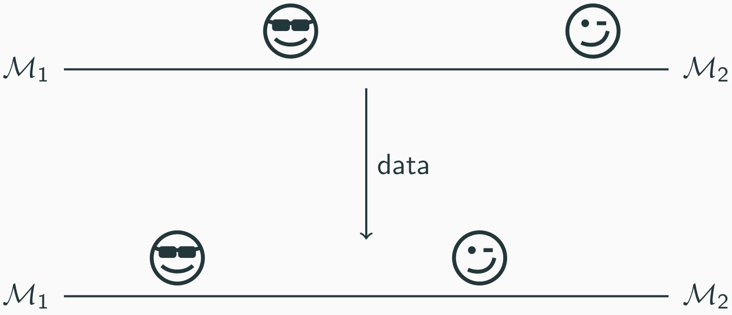


Bayes factors



Although 😎 and 😉 have different prior beliefs, they both shift their belief an equal amount toward $\mathcal{M}_1$.

Interpreting Bayes factors

Example 1: suppose $B_{12} = 10$.

Interpretation: the observed data are 10 times more likely under $\mathcal{M}_1$ than $\mathcal{M}_2$.

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Example 2: suppose $B_{12} = \frac{1}{10}$. Then $B_{21} = 10$.

Interpretation: the observed data are 10 times more likely under $\mathcal{M}_2$ than $\mathcal{M}_1$.

Example 3: suppose $B_{12} = 1$.

Interpretation: the observed data are equally likely under $\mathcal{M}_1$ and $\mathcal{M}_2$.

Bayes factors

Jeffreys (1961) proposed the following thresholds for evidence:

Bayes factor	Evidence
1-3	anecdotal
3-10	moderate
10-30	strong
30-100	very strong
> 100	extreme

Models $\leftrightarrow$ hypotheses

Full Bayesian inference requires specification of generative models for data. This is often difficult.

Also, we are typically trained to evaluate hypotheses about effects.

To reconcile the two, several teams (e.g., Rouder, Morey, Wagenmakers, et al.) have developed *default* Bayesian hypothesis tests. The key idea is that we define models on effect size.

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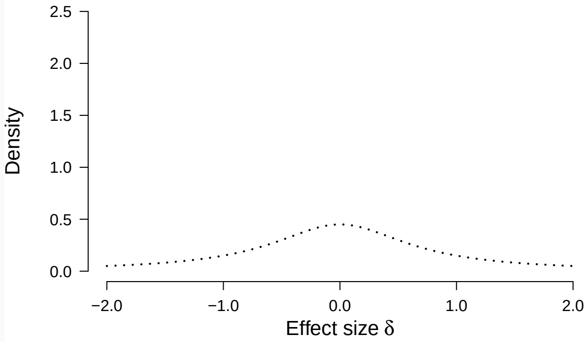
Models $\leftrightarrow$ hypotheses

Specifying models on effect size

- let $\delta = \frac{\mu}{\sigma}$ (think Cohen's d, but at the population level)
- define competing models on δ:
    - $\mathcal{H}_0 : \delta = 0$ (the effect size is 0)
    - $\mathcal{H}_1 : \delta \neq 0$ (the effect size is not 0)
- use Bayes' rule to compute

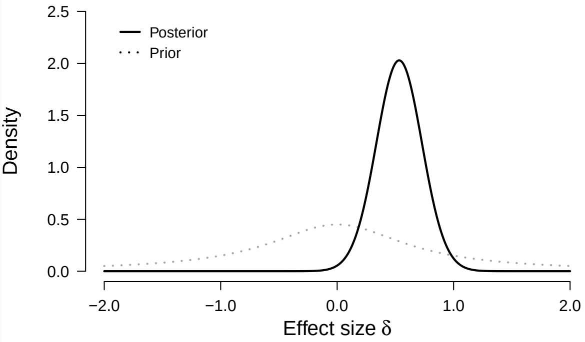
$$p(\mathcal{H}_1 \mid \text{data}) = p(\mathcal{H}_1) \times \frac{p(\text{data} \mid \mathcal{H}_1)}{p(\text{data})}$$

Generic default Bayesian test



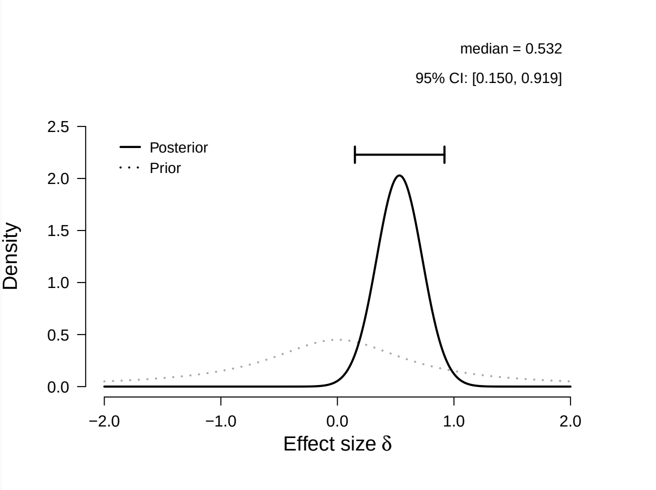
Start with prior belief about expected effect sizes δ.

Generic default Bayesian test



Observing data updates our prior to a posterior.

Generic default Bayesian test



We can extract posterior estimates of δ

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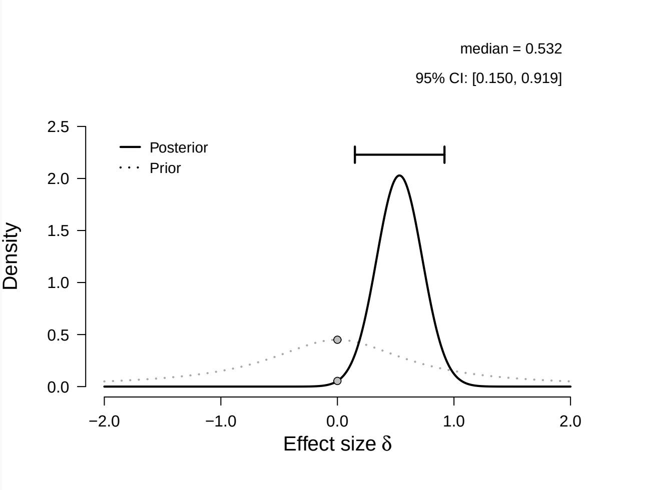

The Bayes factor is the ratio of the densities of $\delta = 0$ in the posterior and prior.

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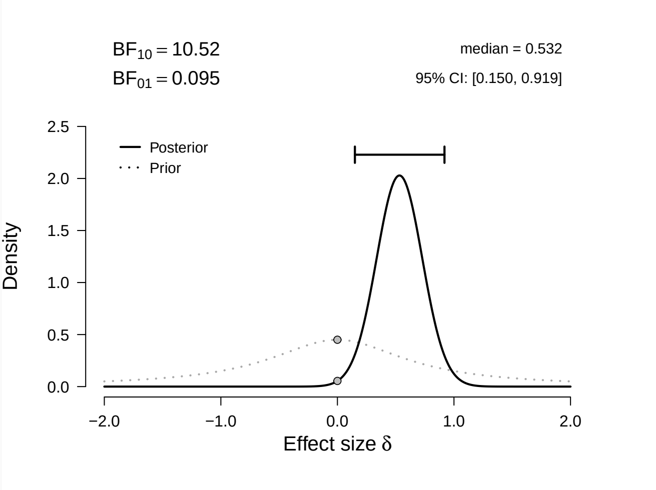

Observing data reduced our belief that $\delta = 0$ by a factor of 10.52

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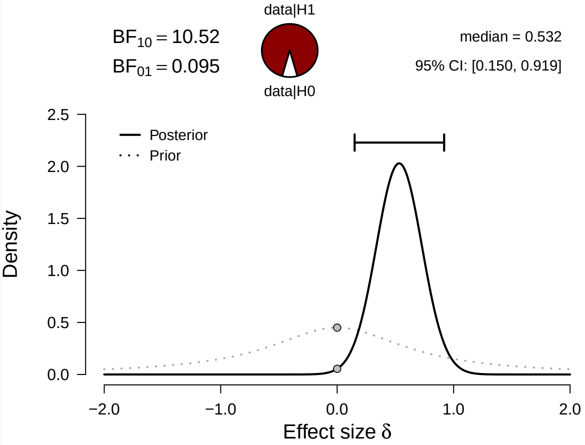

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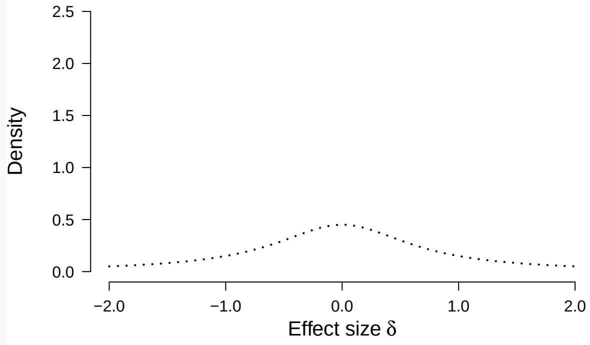

What happens if the null is supported instead?

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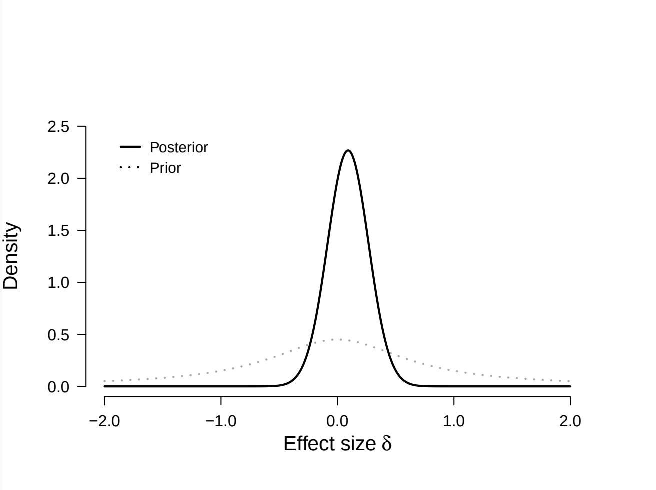

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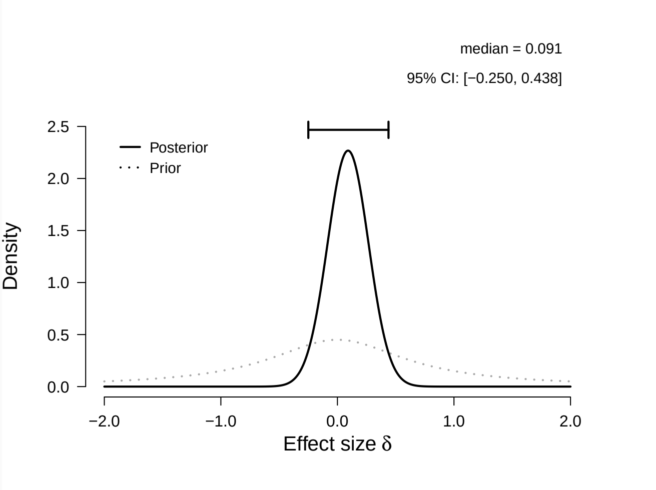

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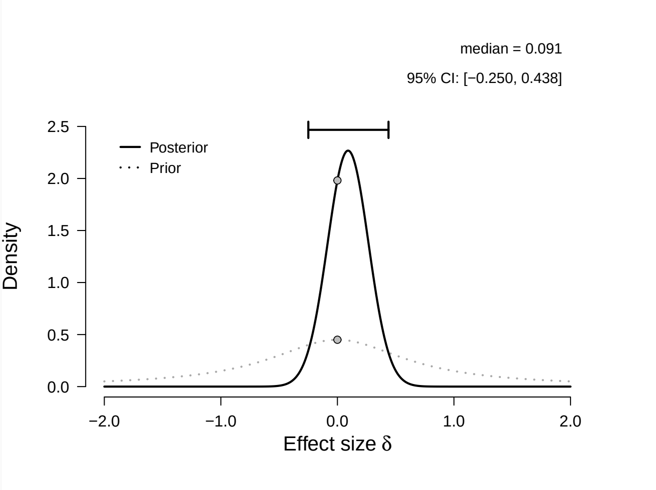

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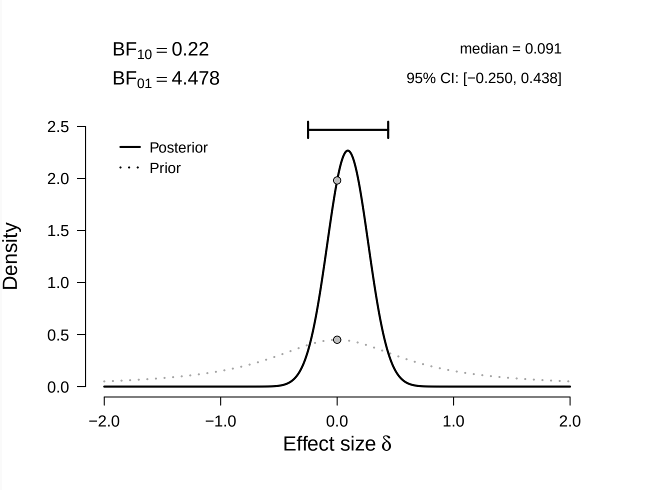

Observing data increased our belief that $\delta = 0$ by a factor of 4.478

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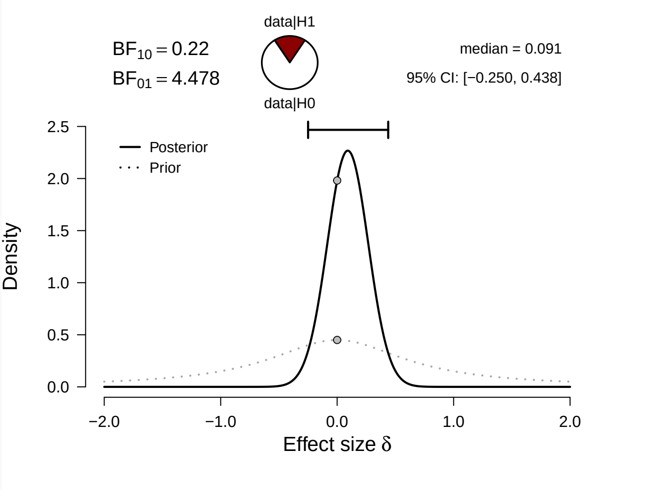

Questions?

Now let's work some examples together.

All datasets can be downloaded at
http://github.com/tomfaulkenberry/bayesAngeloState

http://jasp-stats.org

Thank you!

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