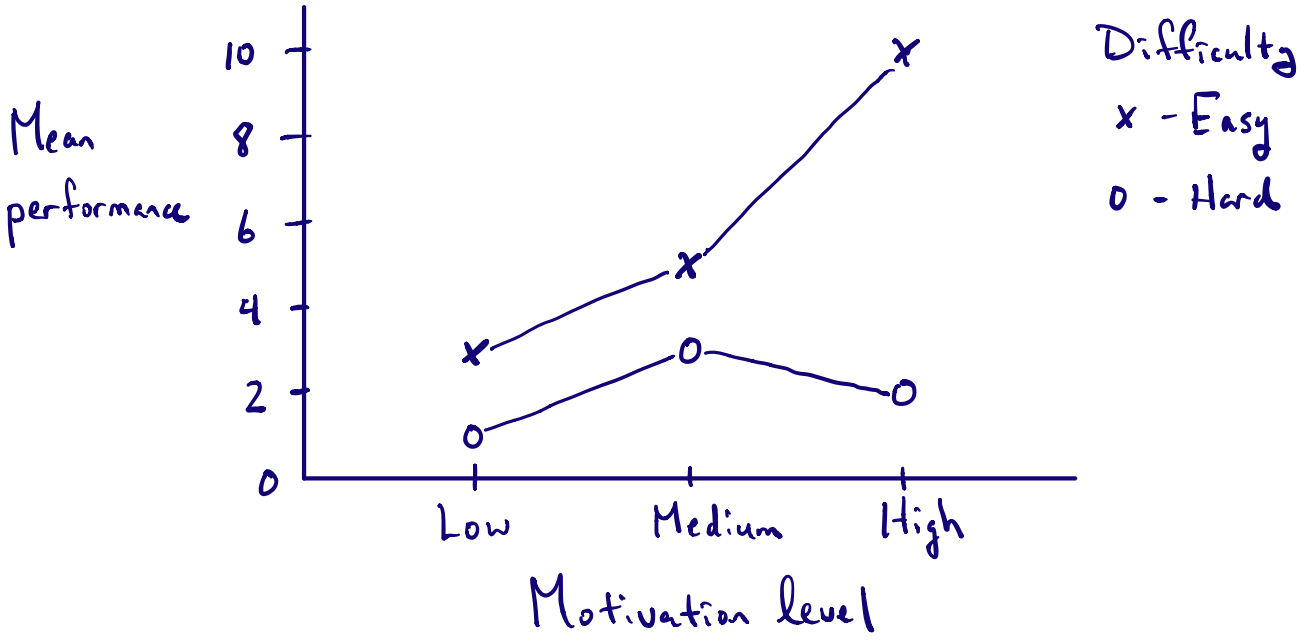
Lecture 5 - Factorial Designs

Consider a study that measures performance as a function of motivation level (low, medium, high) and task difficulty (easy, hard).

Motivation level	Difficulty: Easy	Difficulty: Hard
low	3, 1, 1, 6, 4	0, 2, 0, 0, 3
medium	1, 4, 8, 6, 6	2, 7, 2, 2, 2
high	10, 10, 14, 7, 9	1, 1, 1, 6, 1



So what is the story?

- does motivation level matter?
- does difficulty matter?
- anything else?

Factorial designs give us a structured way to answer these questions.

* "main effects"
* "interactions"

To see what these are, consider the following table of means:

Motivation level	Difficulty: Easy	Difficulty: Hard	M
Low	3	1	2
Medium	5	3	4
High	10	2	6
M	6	2	4

Main effect of Motivation level

↳ are there diffs in these marginal means?

Main effect of difficulty

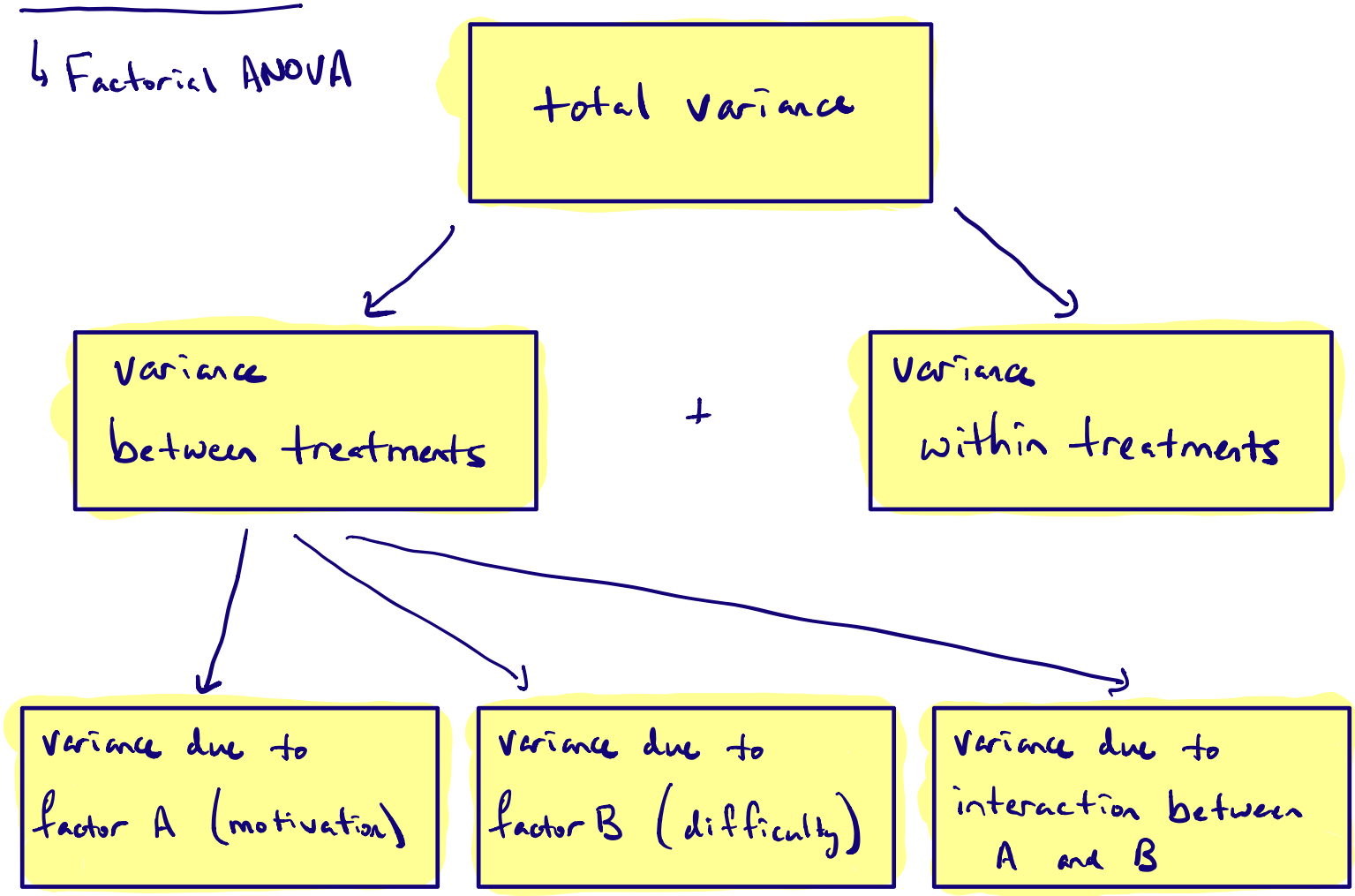
↳ are there diffs in these marginal means?

Interaction

↳ does the pattern in one variable depend on specific level of the other?

How to test?

↳ Factorial ANOVA



Computations similar to other ANOVAs — but tedious.

↳ we'll use JASP instead.

↳ data file:

* spreadsheet saved as CSV
* one column for dependent variable
* one column for each independent variable
* one row per observation

Interpreting JASP output

ANOVA - performance

Cases	Sum of Squares	df	Mean Square	F	p
difficulty	120.000	1	120.000	24.000	< .001
motivation	80.000	2	40.000	8.000	0.002
difficulty * motivation	60.000	2	30.000	6.000	0.008
Residuals	120.000	24	5.000		

Note. Type III Sum of Squares

Main effect of difficulty:

↳ $F(1, 24) = 24.00$, $p < 0.001$

↳ "significant" - i.e, difficulty affects performance

Main effect of motivation level:

↳ $F(2, 24) = 8.00$, $p = 0.002$

↳ "significant" - i.e, motivation level affects performance

Interaction:

↳ $F(2, 24) = 6.00$, $p = 0.008$

↳ "significant" - i.e, effect of motivation depends on difficulty.

Bayesian ANOVA

* JASP builds 5 models (all equally likely, a priori) then computes their **posterior probabilities** $p(M \mid data)$

Model Comparison

Models	P(M)	P(M|data)	BF_M	BF_{10}	error %
difficulty + motivation + difficulty * motivation	0.200	0.817	17.870	1.000	
difficulty + motivation	0.200	0.154	0.726	0.188	2.009
difficulty	0.200	0.026	0.108	0.032	1.838
motivation	0.200	0.002	0.008	0.002	1.838
Null model	0.200	0.001	0.004	0.001	1.838

How do we compute Bayes factors for main effects / interaction?

Answer: "**inclusion Bayes factors**"

↳ see Faulkenberry, Ly, & Wagenmakers (2020) - Journal of Numerical Cognition for details.

Def: an **inclusion Bayes factor** for an effect is the factor by which prior odds for including the effect in a model are updated after observing data.

To compute:

1. compute prior odds for including the effect
2. compute posterior odds for including the effect
3. divide posterior odds by prior odds

Lets consider the main effect of difficulty:

Model Comparison

Models	P(M)	P(M|data)
difficulty + motivation + difficulty ✻ motivation	0.200	0.817
difficulty + motivation	0.200	0.154
difficulty	0.200	0.026
motivation	0.200	0.002
Null model	0.200	0.001

1. Prior odds for including difficulty: $= \frac{P(\text{including difficulty})}{P(\text{not including difficulty})}$

$$= \frac{0.2 + 0.2 + 0.2}{0.2 + 0.2}$$

$$= \frac{0.6}{0.4}$$

$$= 1.50$$

2. Posterior odds for including difficulty $= \frac{0.817 + 0.154 + 0.026}{0.002 + 0.001}$

$$= \frac{0.997}{0.003}$$

$$= 332.33$$

3. Inclusion Bayes factor = posterior odds / prior odds

$$= 332.33 \,/\, 1.50 = 221.55$$

Interpretation: "the observed data are 221.55 more likely under a model which contains a main effect of task difficulty."

Note: JASP will produce these inclusion Bayes factors for you!

↳ just check "Effects" under "Tables".

Analysis of Effects – performance

Effects	P(incl)	P(excl)	P(incl|data)	P(excl|data)	BF_{incl}
difficulty	0.600	0.400	0.997	0.003	225.765
motivation	0.600	0.400	0.973	0.027	23.717
difficulty ✻ motivation	0.200	0.800	0.817	0.183	17.870

Summary:

for factorial ANOVA, we test the following:

* main effects (at least two)
* interaction(s) (at least one)

Frequentist test — use p-value

Bayesian test — use inclusion Bayes factor