Bayesian Statistics – Lecture 4

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Goals for today

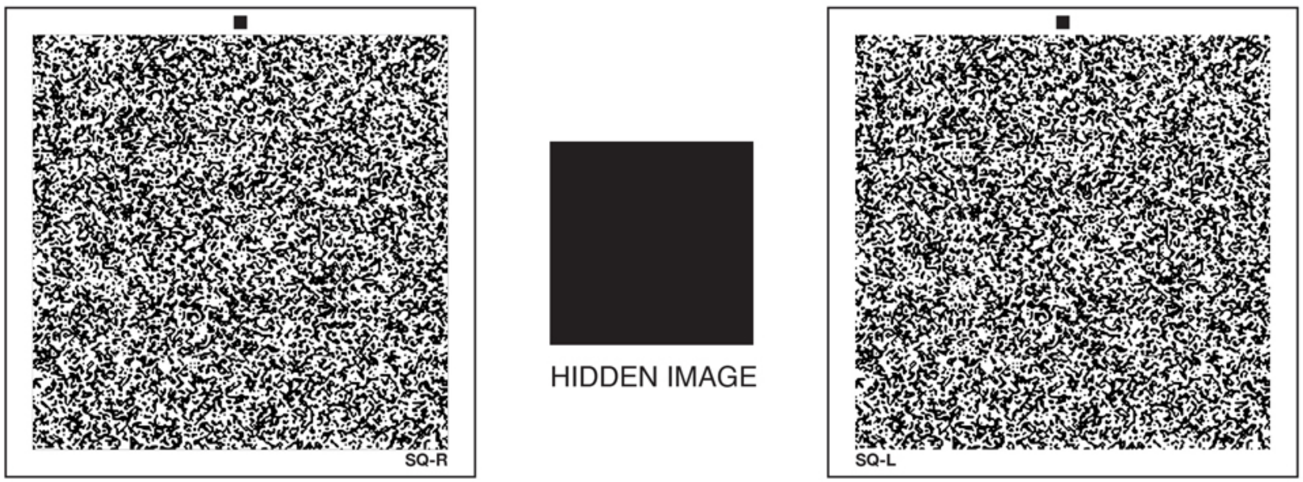
1. Introduce the Bayesian t-test

2. Explore some different priors

3. Work some examples in JASP

A new example

A random-dot stereogram is a pair of images of random dots that, when viewed with the eyes focused on a point behind the images, produces a sensation of depth (stereopsis) with the object appearing to be *floating* in front of the display level



A new example

One classic example comes from Frisby and Clatworthy (1975), who presented random dot stereograms to 78 participants. Each participant was asked to report how long it took them to perceive ("fuse") the object (in this case, a spiral staircase). 35 of the participants were given extra visual information about the target image – the remaining were given no extra information.

Question – did the extra visual information affect fusion times?

Review of the t-test

Consider two independent samples $x_1, \ldots, _{N\,1}$ and $y_1, \ldots, y_{N2}$, assumed to be sampled from *normal* distributions with means μ_1 and μ_2, respectively.

The t-test is used to test whether the population means differ

Models to be compared:

- $\mathcal{H}_0 : \mu_1 = \mu_2$
- $\mathcal{H}_1 : \mu_1 \neq \mu_2$

Classical t-test in JASP

Let's perform a classical t-test in JASP

Bayesian t-test

The Bayesian version of the t-test reparameterizes the model in terms of *effect size*: $\delta = \frac{\mu_1 - \mu_2}{\sigma}$.

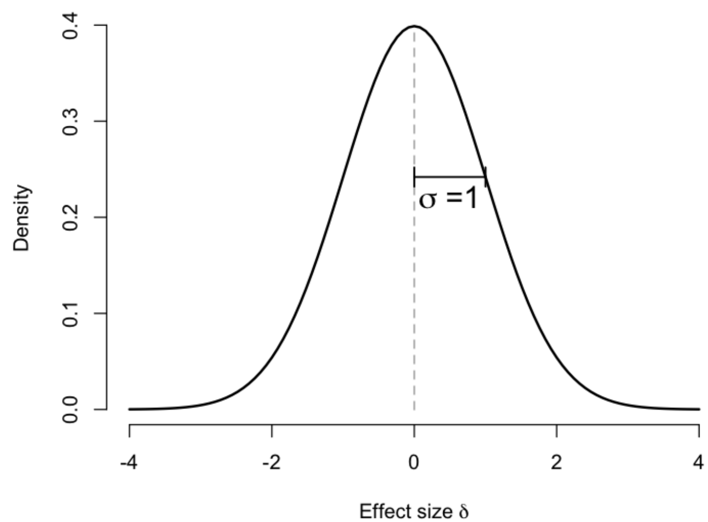
- $\mathcal{H}_0 : \delta = 0$
- $\mathcal{H}_1 : \delta \neq 0$

Thus, we must specify a prior distribution for δ under $\mathcal{H}_1$.

Some priors for effect size δ

Option 1: the *unit information prior* (Kass & Raftery, 1995) assumes δ is distributed as a normal distribution with mean 0 and standard deviation 1

$\delta \sim \text{Normal}(0, 1)$



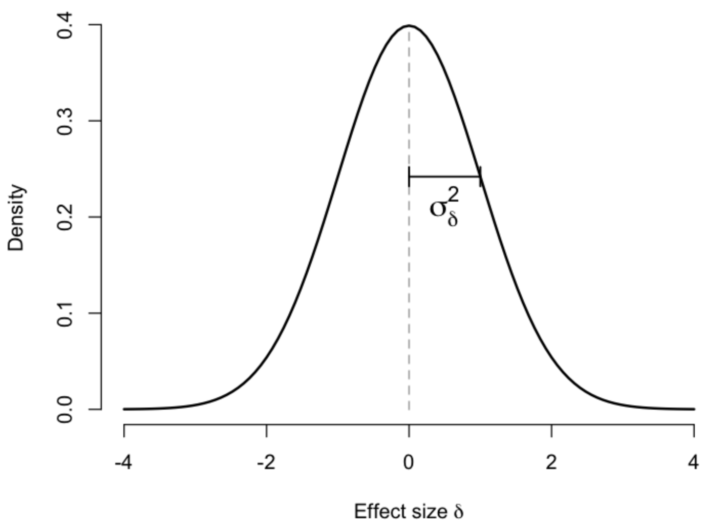
Some priors for effect size δ

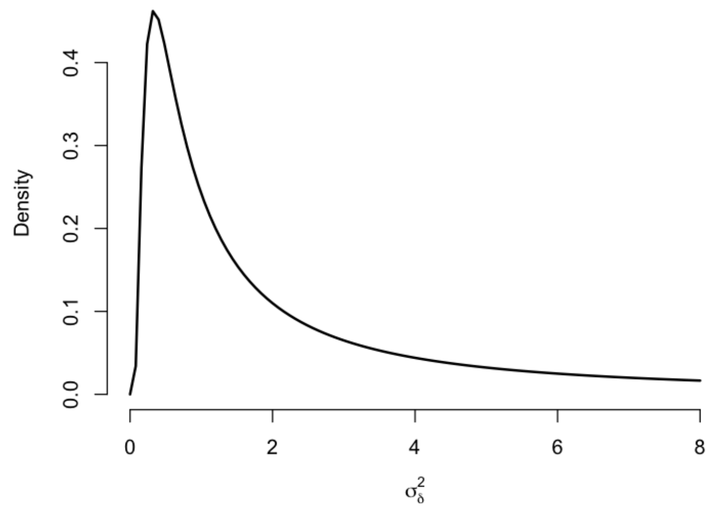
Option 2: Even less information would be provided if σ was not specified. Zellner & Siow (1980) proposed a hierarchical model:

- $\delta \sim$ Normal$(0, \sigma_\delta^2)$
- $\sigma_\delta^2 \sim$ inverse $\chi^2(1)$

Some priors for effect size δ

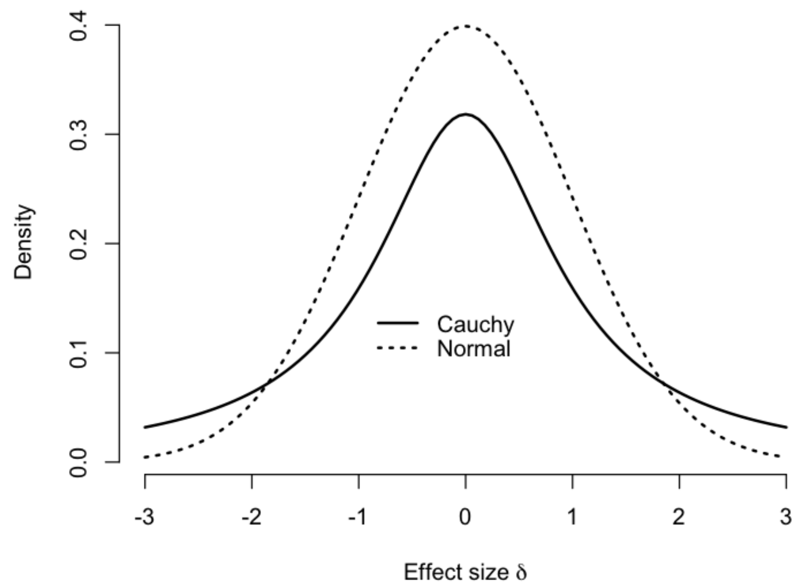
Option 2: Zellner & Siow (1980) hierarchical prior:





Some priors for effect size δ

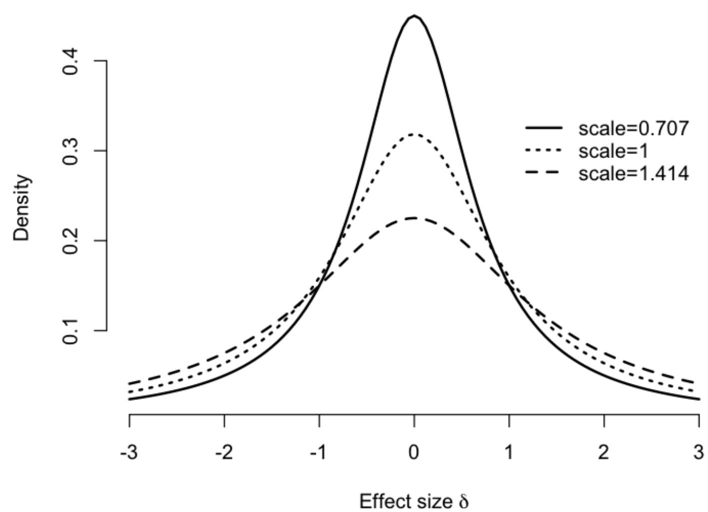
Option 3: Liang et al. (2008) noted that the Zellner & Siow (1980) prior is equivalent to a *Cauchy* distribution



Some priors for effect size δ

JASP uses the (scaled) Cauchy distribution as the default prior for δ:

- $\delta \sim \text{Cauchy}(r)$



Bayesian t-test in JASP

Now let's perform a Bayesian t-test in JASP:

- prior and posterior plot
- sensitivity analysis