Lecture 8 – Linear regression models

Example: Six people are given a math pretest (x) and then a one-month instructional intervention. Their posttest scores (Y) are recorded below.

Person	X = pretest	Y = posttest
A	1	3
B	2	5
C	7	8
D	6	10
E	4	4
F	4	6

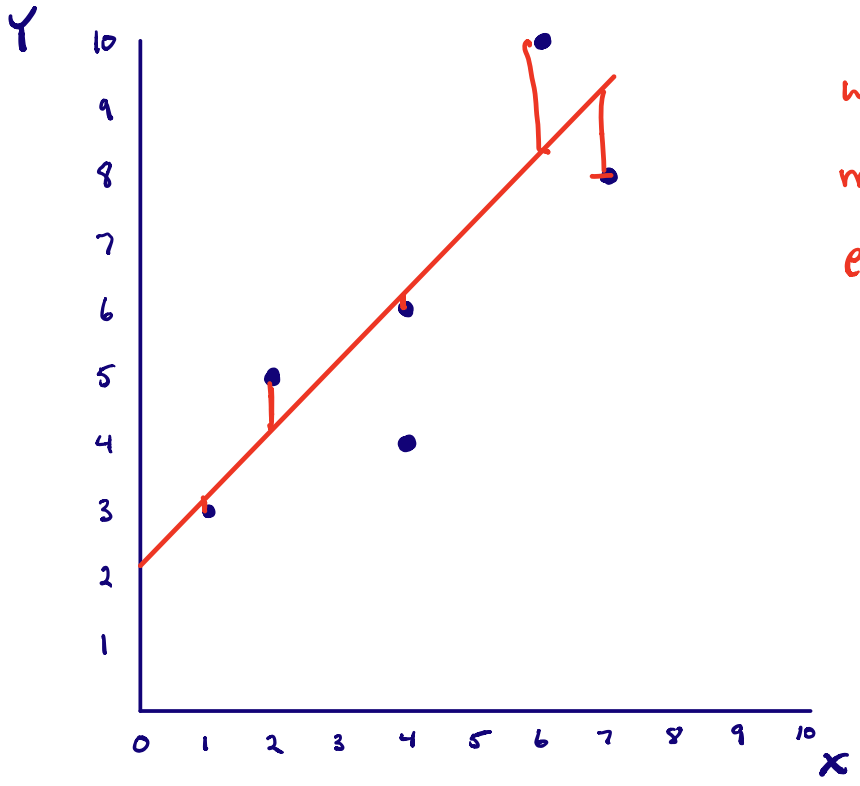
Some questions:

(1) How can I predict posttest scores directly from pretest scores?

↳ the "regression equation"

(2) Does this regression equation represent a real relationship between X and Y?

Linear regression: the "line of best fit"



we want the line that minimizes this "residual error" across all points.

Method: Least squares regression

* Equation of line: $Y = a + bX$

  ↳ a = intercept

  b = slope

* Fit:

$$b = \frac{SP}{SS_x}$$

$$a = \overline{Y} - b\overline{X}$$

Lets work out the regression equation:

X	Y	$X-\bar{X}$	$Y-\bar{Y}$	$(X-\bar{X})^2$	$(Y-\bar{Y})^2$	$(X-\bar{X})(Y-\bar{Y})$
1	3	-3	-3	9	9	9
2	5	-2	-1	4	1	2
7	8	3	2	9	4	6
6	10	2	4	4	16	8
4	4	0	-2	0	4	0
4	6	0	0	0	0	0
$\bar{X}=4$	$\bar{Y}=6$			$SS_x = 26$	$SS_y = 34$	$SP = 25$

$$\text{Slope} = b = \frac{SP}{SS_x} = \frac{25}{26} = 0.962$$

$$\text{Intercept} = a = \bar{Y} - b\bar{X}$$

$$= 6 - 0.962(4)$$

$$= 2.152$$

So, the regression equation is

$$Y = 2.152 + 0.962X$$

Example: what is the predicted posttest score for a pretest score of 5?

Substitute $X = 5$ into regression equation:

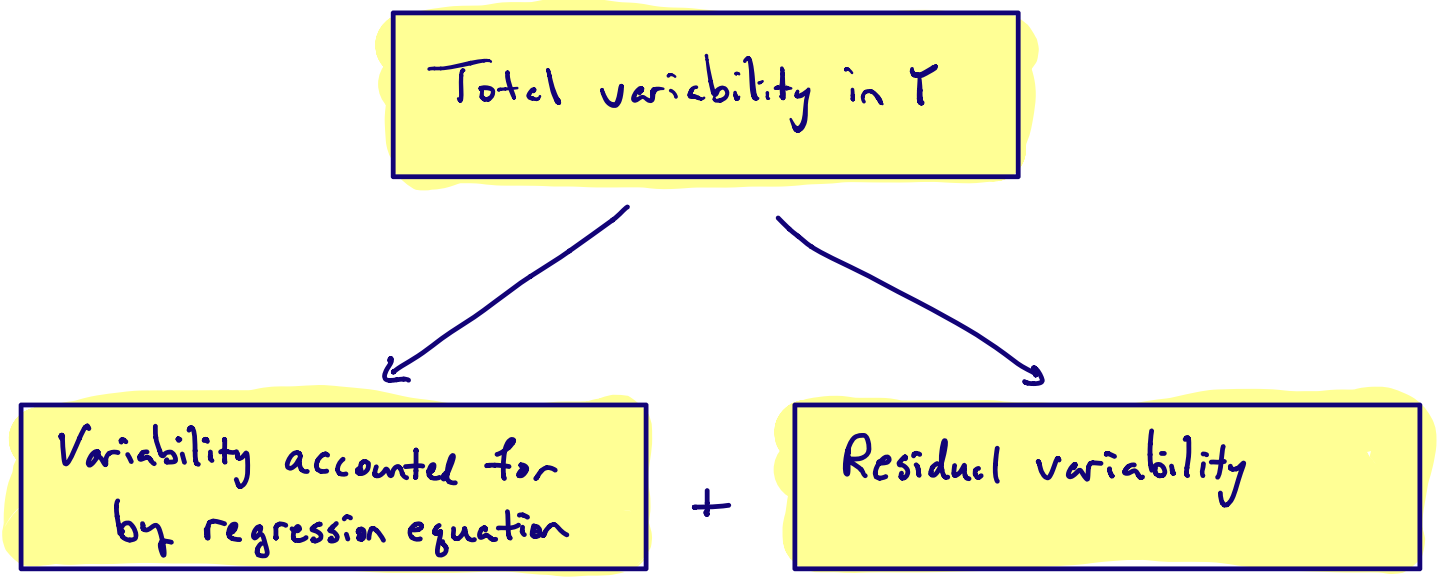
$$Y = 2.152 + 0.962(5)$$

$$= 6.962$$

Does this regression equation represent a real relationship?

↳ "Is pretest a significant predictor of posttest?"

↳ use ANOVA



How to compute:

* Find correlation coefficient: $r = \frac{SP}{\sqrt{SS_x SS_y}}$

* $SS_{regression} = r^2 \cdot SS_y$

* $SS_{residual} = SS_y - SS_{regression}$

Back to our example:

* $r = \frac{SP}{\sqrt{SS_x SS_y}} = \frac{25}{\sqrt{26 \cdot 34}} = 0.841$

* $SS_{regression} = r^2 \cdot SS_y = (0.841)^2 (34)$

$= 24.04$

ANOVA table

Source	SS	df	MS	F
regression	24.04	1	24.04	9.65
residual	9.96	4	2.49	
total	34	5		

$F(1, 4) = 9.65, \quad p = 0.036$

"Pretest is a significant predictor of posttest."