

PSYC 2317 - Lecture 10 - Bayesian Hypothesis Testing.

Suppose we are interested in assessing effectiveness of some mathematics instruction program.

After implementing the program, we measure mathematical ability using the Scale for Advancing Mathematical Ability (SAMA), a national assessment with a known mean score of 50.

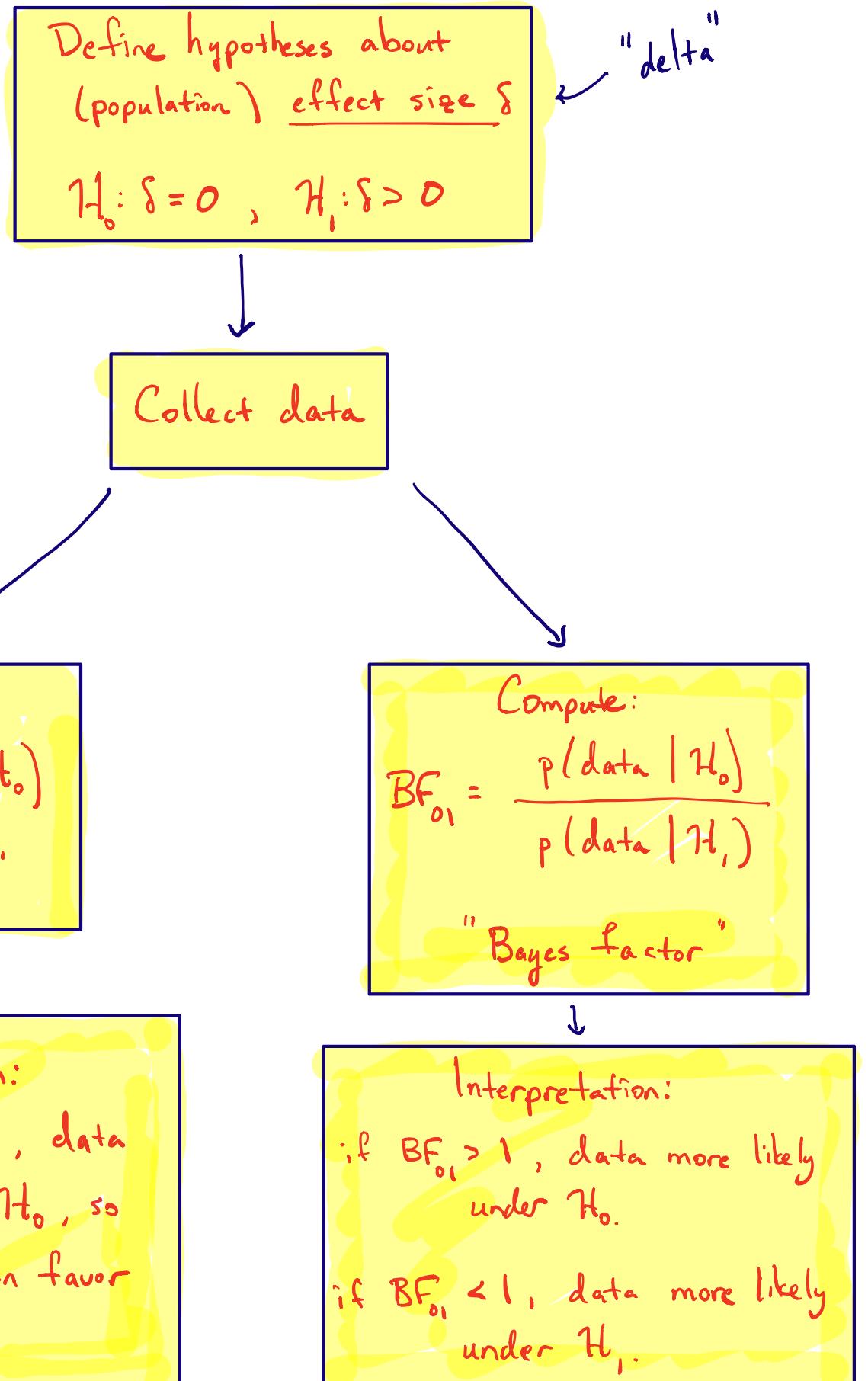
Our sample (who had the training) had a mean of 54.4 and a standard deviation of 10.

Did the training work?

Basic statistical framework:

- (1) define two competing models ("hypotheses") that could possibly generate our observed data.
 - one where the training worked (H_1)
 - one where it didn't (H_0)
- (2) assess the fit of these models against the observed data.

How to proceed?



p-values versus Bayes Factors

$$\text{p-value} = p(\text{data} \mid H_0)$$

1) only considers fit of H_0 as a potential model for data

2) ignores fit of H_1 ,

Thus, "support" for H_1 is only indirect

$$\text{Bayes factor} = \frac{p(\text{data} \mid H_0)}{p(\text{data} \mid H_1)}$$

1) considers relative adequacy of both models as predictors of data.

2) can directly index support for either H_0 or H_1 .

Ex: $\frac{BF}{H_0} = 9 \rightarrow$ "The observed data are 9 times more likely under H_1 than H_0 ".

$BF_{01} = 9 \rightarrow$ "The observed data are 9 times more likely under H_0 than H_1 ".

Interpreting Bayes factors:

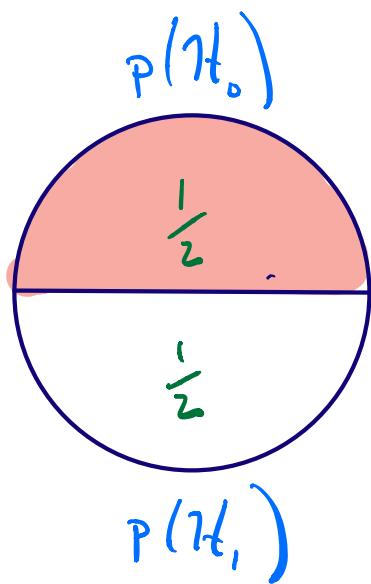
(1) relative predictive adequacy of models

* $BF_{01} = 9 \rightarrow$ "The observed data are 9 times more likely under H_0 than H_1 ".

(2) updating factor

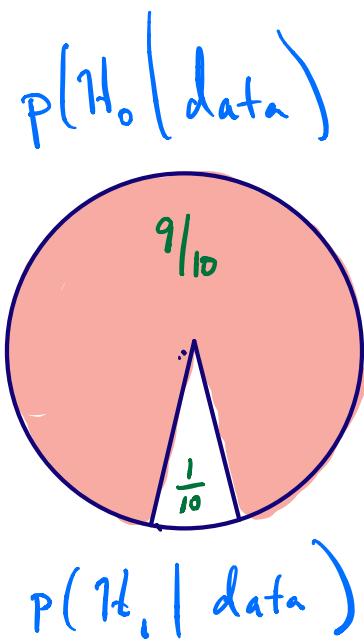
* $BF_{01} = 9 \rightarrow$ "After observing data, my prior odds for H_0 over H_1 have been increased by a factor of 9"

Example:



observe
data

$\xrightarrow{} \quad BF_{01} = 9$



Prior odds = 1:1

Posterior odds = 9:1

Let's continue our working example. Recall that we tested $N = 65$ participants and observed a sample mean of 54.4 with $sD = 10$.

Step 1 - convert our observed data to test statistic

$$t = \frac{\bar{x} - \mu}{\hat{s}/\sqrt{N}} = \frac{54.4 - 50}{10/\sqrt{65}} = \frac{4.4}{1.24} = 3.55$$

Step 2 - convert t-score to Bayes factor

- * <https://tomfaulkenberry.shinyapps.io/bayesFactorCalc>
- * or, use "Summary Stats Module" in JASP.

Elements to report:

- define H_0, H_1

"Under the null hypothesis we expect an effect size of 0. Thus, we define $H_0: \delta = 0$. Under the alternative, we expect a positive effect.

That is, $H_1: \delta > 0$ "

- report and interpret Bayes factor

"We found a Bayes factor of $BF_{10} = 69.4$, which means that the observed data are approximately 70 times more likely under H_1 than H_0 ."

- calculate and report posterior model probability for preferred model.

- "Assuming prior odds of 1-1 for H_1 and H_0 , our observed data updated these odds to 69.4 -to- 1 in favor of H_1 . This is equivalent to a posterior model probability of $p(H_1 | \text{data}) = 0.986$ "

What about "statistical significance"?

- we usually don't use this phrase in Bayesian context.
- instead, we speak of model evidence from data

Jeffreys (1961) :

BF	Evidence*
1 - 3	anecdotal
3 - 10	moderate
10 - 30	strong
30 - 100	very strong
> 100	extreme

* these are
only guidelines!

"In summary, these data provide very strong evidence in favor of $\eta_{t,j}$; that is, the training program had a positive effect on maths performance."