Lecture 9 – Mediation analysis

Example: do prerequisite courses impact subsequent course grades?

Consider a regression model where technical writing grade (X) predicts research methods grade (Y).

c = slope of regression line

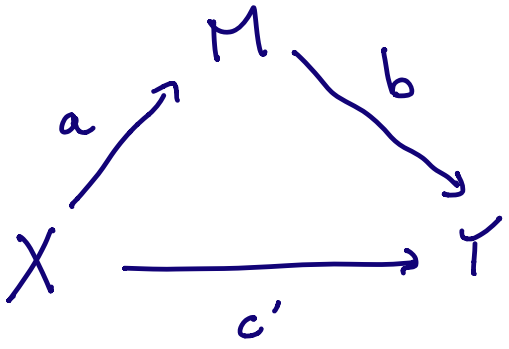
= "total effect" of X on Y

From JASP:

* $c = 0.402$
* writing grade is significant predictor of research grade

So prerequisite matters ... right?

Consider an alternative model, where the effect of writing grade is mediated (or partially explained) by GPA:



Notation: c' = "direct effect" (after controlling for M)

ab = "indirect effect" (through M)

Fundamental idea: mediation occurs when the direct effect (after controlling for mediator) is less than the total effect c.

So, in mediation analysis, we have two goals:

(1) estimate the direct effect c' and the indirect effect ab

(2) test whether mediation occurs (i.e., $c' < c$).

Goal 1 - computing direct and indirect effects.

To do this, we need the fundamental equation of mediation analysis:

total effect = direct effect + indirect effect

or

$$c = c' + ab$$

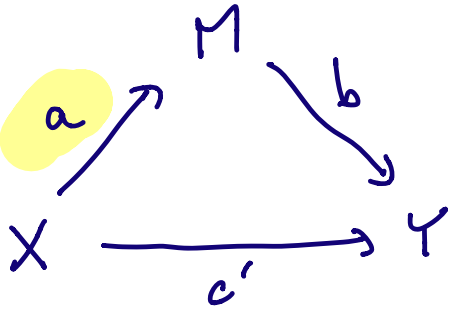
Step 1: compute total effect c

↳ linear regression: $X \longrightarrow Y$

c = slope / effect of X on Y

= 0.402

Step 2: compute path a



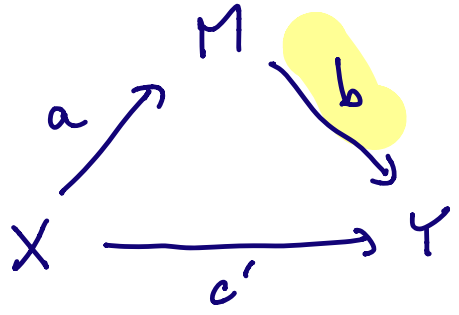
* linear regression: $X \longrightarrow M$

a = slope / effect of X on M

= 0.244

(Note: SE_a = 0.027 ... we'll need this later)

Step 3: compute path b



* linear regression: $X + M \rightarrow Y$

b = slope / effect of M on Y

$= 1.300$

$(SE_b = 0.144)$

Step 4: compute direct effect c'

Use fundamental eqn. of mediation analysis:

$$c = c' + ab$$

$$\hookrightarrow \quad c' = c - ab$$

$$= 0.402 - (0.244)(1.300)$$

$$= 0.0848$$

Notice that the direct effect c' is much less than the total effect c. This tells us that GPA mediates the relationship between writing grade and research grade!

Goal 2 - Is the mediation significant?

To test whether $c' < c$, we can equivalently test whether $ab > 0$. This is called the Sobel (1982) test.

H_0: $ab = 0$ (i.e., no mediation)

H_1: $ab > 0$ (i.e., mediation occurs).

To test, we compute $z = \dfrac{ab}{SE_{ab}}$

where $SE_{ab} = \sqrt{a^2 SE_b^2 + b^2 SE_a^2}$.

When n is large, these z-scores are normally distributed.

Example:

$$SE_{ab} = \sqrt{a^2 SE_b^2 + b^2 SE_a^2}$$

$$= \sqrt{0.244^2 \cdot 0.144^2 + 1.30^2 \cdot 0.027^2}$$

$$= 0.0497$$

So

$$z = \frac{ab}{SE_{ab}} = \frac{(0.244)(1.300)}{0.0497}$$

$$= 6.38$$

From normal distribution calculator, we can see $p < 0.001$.

So, GPA significantly mediates the relationship between writing grades and research grades.

Note: David Kenny has a wonderful website about mediation analysis, including more resources:

http://davidakenny.net/cm/mediate.htm