Lecture 5 - Estimating reliability coefficients

Recall: reliability =

(1) proportion of variance in observed scores that is due to variability among true scores.

$$\rho_{XX'} = \frac{\sigma_T^2}{\sigma_X^2}$$

(2) squared correlation between observed and true scores

$$\rho_{XX'} = \rho_{XT}^2$$

(3) what is left over after removing error variability

$$\rho_{XX'} = 1 - \frac{\sigma_E^2}{\sigma_X^2}$$

(4) lack of correlation between observed and error scores

$$\rho_{XX'} = 1 - \rho_{XE}^2$$

But these are all theoretical - how do I compute reliability from observed measurements?

Today, we'll discuss how to use parallel forms to compute reliability coefficients.

1. split into two parallel halves and compute correlation (Spearman-Brown formula)

2. if halves are not parallel, we can estimate reliability coeff. with Cronbach's alpha (α)

3. split into multiple subtests (generalized Spearman-Brown and Cronbach α)

Method 1 - split our test X into two "parallel" halves Y, Y'

↳ parallel tests have same true scores and observed variance

So $X = Y + Y'$

Then $\sigma_X^2 = \sigma_Y^2 + \sigma_{Y'}^2 + 2\sigma_{YY'} = \sigma_Y^2 + \sigma_{Y'}^2 + 2\rho_{YY'}\sigma_Y\sigma_{Y'}$

Parallel $\longrightarrow$ $\sigma_{Y'} = \sigma_Y$, so

$$\sigma_X^2 = \sigma_Y^2 + \sigma_Y^2 + 2\rho_{YY'}\sigma_Y\sigma_Y$$

$$= 2\sigma_Y^2 + 2\rho_{YY'}\sigma_Y^2$$

$$= 2\sigma_Y^2(1 + \rho_{YY'})$$

Now consider the true score variance for X, which we denote T_x

Then
$$\sigma^2_{T_x} = \sigma^2_{T_Y} + \sigma^2_{T_{Y'}} + 2\sigma_{T_Y T_{Y'}}$$
$$= \sigma^2_{T_Y} + \sigma^2_{T_{Y'}} + 2\rho_{T_Y T_{Y'}}\sigma_{T_Y}\sigma_{T_{Y'}}$$

parallel $\longrightarrow$ $\sigma_{T_{Y'}} = \sigma_{T_Y}$

and $\rho_{T_Y T_{Y'}} = 1$ (because true scores for each subject must be equal on both parallel forms)

Thus
$$\sigma^2_{T_x} = \sigma^2_{T_Y} + \sigma^2_{T_{Y'}} + 2\rho_{T_Y T_{Y'}}\sigma_{T_Y}\sigma_{T_{Y'}}$$
$$= \sigma^2_{T_Y} + \sigma^2_{T_Y} + 2(1)\sigma_{T_Y}\sigma_{T_Y}$$
$$= \sigma^2_{T_Y} + \sigma^2_{T_Y} + 2\sigma^2_{T_Y}$$
$$= 4\sigma^2_{T_Y}$$

So reliability $= \dfrac{\sigma^2_{T_X}}{\sigma^2_X} = \dfrac{4\sigma^2_{T_Y}}{2\sigma^2_Y(1+\rho_{YY'})}$

$$= 2 \cdot \frac{\sigma^2_{T_Y}}{\sigma^2_Y} \cdot \frac{1}{1+\rho_{YY'}}$$

Spearman - Brown formula

$$\boxed{= \frac{2\rho_{YY'}}{1+\rho_{YY'}}}$$

This formula allows us to compute reliability by splitting a test into two parallel halves and computing the correlation between them!

<u>Example:</u> suppose we observe a correlation between two parallel halves equal to 0.90. What is the reliability of your test?

Use S-B: $\rho_{XX'} = \dfrac{2\rho_{YY'}}{1+\rho_{YY'}} = \dfrac{2(0.90)}{1+0.90} = \dfrac{1.8}{1.9} = 0.95$

OK, great! But – Spearman-Brown only works if the two halves are <u>parallel</u>.

If not, we can use the alpha (α) coefficient to get a lower bound for reliability coefficient (due to Cronbach, 1950):

$$\boxed{\begin{aligned} \rho_{XX'} \geq \alpha &= \frac{2 \times \text{covariance between } Y, Y'}{\text{total test variance}} \\ &= \frac{2\left[\sigma_X^2 - (\sigma_Y^2 + \sigma_{Y'}^2)\right]}{\sigma_X^2} \end{aligned}}$$

<u>Example:</u> Suppose you split a test into two halves. The variance of the first half is 7, and the variance of the second half is 5, and the total test variance is 17.9.

Then $\rho_{XX'} \geq \alpha = \dfrac{2\left[17.9 - (7 + 5)\right]}{17.9} = 0.66$

The form of Cronbach's α can be extended to allow computation with item variances. Suppose X is composed of N items Y_i.

$$\rho_{xx'} \geq \alpha = \left(\frac{N}{N-1}\right)\left[\frac{\sigma_X^2 - \left(\sigma_{Y_1}^2 + \sigma_{Y_2}^2 + \cdots + \sigma_{Y_N}^2\right)}{\sigma_X^2}\right]$$

Example:

Subject	Q1	Q2	Q3	X
1	5	5	7	17
2	3	2	1	6
3	1	3	7	11
4	8	7	9	24
5	7	5	3	15
σ_i^2	8.2	3.8	10.8	45.3

Then

$$\rho_{xx'} \geq \alpha = \left(\frac{3}{3-1}\right)\left[\frac{45.3 - (8.2 + 3.8 + 10.8)}{45.3}\right]$$

$$= \left(\frac{3}{2}\right)\left(\frac{22.5}{45.3}\right) = 0.745$$

Final thoughts - what happens to reliability if we make the test longer or shorter?

Recall: Spearman - Brown for 2 parallel halves:

$$\rho_{xx'} = \frac{2\rho_{YY'}}{1 + \rho_{YY'}}$$

It turns out that Spearman-Brown generalizes to N parallel forms

$$\boxed{\rho_{xx'} = \frac{N\rho_{YY'}}{1 + (N-1)\rho_{YY'}}}$$

Example: suppose you have a five-item test with an estimated reliability of 0.6. What happens to the reliability if you make the test 3 times longer?

Generalized S-B: $\rho_{xx'} = \frac{3(0.6)}{1 + (3-1)(0.6)} = \frac{1.8}{2.2} = 0.82.$