

Lecture 7 - Bayesian Analysis of Covariance

Recall: the goal of ANCOVA is to test for group differences while simultaneously controlling for the influence of some covariate.

Steps: (1) do the ANOVA (JASP ANOVA module)

(2) calculate adjusted means by finding slope (effect) of covariate

(JASP Linear Regression module)

(3) do the ANCOVA (JASP ANCOVA module)

Three different procedures!

Bayesian ANCOVA does it all at once

(and even more!)

Example: does synchronous attendance matter in hybrid courses?

- see my JASP blog post
- for 33 students in my Fall 2020 statistics course I recorded:
 - * final course grade (max 100)
 - * mode of attendance (0 = asynchronous,
1 = synchronous)
 - * average standardized viewing time for recorded lectures (in minutes - max 75)

Date available at <https://osf.io/yf2sb>

Let's perform a Bayesian ANCOVA:

- * Dependent variable = "grade"
- * Fixed factors = "sync"
- * Covariates = "avgView"

Output from JASP:

- Model comparison: just like in Unit 5, this table gives prior / posterior model probabilities, sorted from best fit to worst fit.

Model Comparison

Models	P(M)	P(M data)	BF _M	BF ₁₀	error %
avgView	0.250	0.663	5.890	1.000	
sync + avgView	0.250	0.307	1.331	0.464	1.055
sync	0.250	0.020	0.063	0.031	0.004
Null model	0.250	0.010	0.030	0.015	0.004

Interpretation: the model containing only average viewing time is the most probable after observing data

Note:

(1) all models are set to be equally likely, a priori.

(2) BF_M: change in model odds after observing data.

Ex: for "avgView" model

$$\text{Prior odds} = \frac{0.25}{0.25 + 0.25 + 0.25} = 0.333$$

$$\text{Posterior odds} = \frac{0.663}{0.307 + 0.020 + 0.010} = 1.967$$

$$\text{so } \text{BF}_M = \frac{1.967}{0.333} = 5.89$$

(3) BF_{10} = relative predictive adequacy against best model

* including "sync" in the model gives $BF_{10} = 0.464$

→ the data are only 0.464 times as likely
if we include the effect of attendance mode.

OR: the data are $1/0.464 = 2.16$ times more
likely if we exclude the effect of attendance
mode.

2. **Inclusion Bayes factors:** just like in Unit 5, we can
do Bayesian model averaging and compute Bayes factors
for each "effect".

Analysis of Effects - grade

Effects	P(incl)	P(excl)	P(incl data)	P(excl data)	BF_{incl}
sync	0.500	0.500	0.328	0.672	0.487
avgView	0.500	0.500	0.970	0.030	32.106

3. Estimates: whereas the first two tables perform model comparison, this table gives us (model averaged) estimates of the effects

Model Averaged Posterior Summary

Variable	Level	Mean	SD	95% Credible Interval	
				Lower	Upper
Intercept		78.209	3.068	72.158	84.143
sync	0	-1.255	3.483	-8.915	5.364
	1	1.255	3.483	-5.447	8.832
avgView		0.393	0.146	0.100	0.680

Consider the estimate for avgView:

* mean = 0.393 points per additional minute.

* can interpret this as: "each additional minute of recorded lecture viewing time increased the predicted grade by 0.393 points,
 95% credible interval = (0.100, 0.680)"

Putting it all together:

We performed a Bayesian analysis of covariance on final course grades, including attendance mode (synchronous, asynchronous) as a fixed factor and average viewing time of recorded lectures as a covariate.

The Bayesian ANCOVA works by comparing four models with varying predictors of final course grade: (1) a null model; (2) a model containing only attendance mode as a predictor; (3) a model containing only viewing time as a predictor; and (4) a model containing both attendance mode and viewing time as predictors.

Only models 3 and 4 had their model odds increased after observing data ($BF_H = 5.890$ and $BF_H = 1.331$, respectively). Of these, model 3 was most probable, $P(M | \text{data}) = 0.663$, and the observed data was 2.16 times more likely under Model 3 than Model 4.

To account for model uncertainty, we performed Bayesian model averaging to test the "effects" of both predictors.

The data were 32.106 times more likely under models containing viewing time as a predictor, but only 0.487 times as likely when including attendance mode.

Thus, we believe that only average viewing time impacts course grade (mean effect = 0.393 points per additional minute, 95% credible interval = $[0.100, 0.680]$), and critically, attendance mode has no effect.