Lecture 6 - Analysis of Covariance

Consider the following quiz scores Y for students in a statistics class, randomly assigned to one of two types of instruction:

Group 1 — Traditional lecture: Y_1	Group 2 — Small-group tutorials: Y_2
5	5
6	7
7	8
7	9
8	9
9	10
$\bar{Y}_1 = 7$	$\bar{Y}_2 = 8$

Are these different?

No!

t-test: $t(10) = 1.074$

ANOVA: $F(1,10) = 1.154$

$p = 0.308$

$BF_{01} = 1.50$

So instruction type doesn't matter?

Consider also that we recorded pre-instruction aptitude scores (X) for each participant:

Group 1 Traditional lecture		Group 2 Small-group tutorials	
Y_1	X_1	Y_2	X_2
5	5	5	2
6	4	7	4
7	5	8	2
7	6	9	4
8	7	9	5
9	9	10	7
$\bar{Y}_1 = 7$	$\bar{X}_1 = 6$	$\bar{Y}_2 = 8$	$\bar{X}_2 = 4$

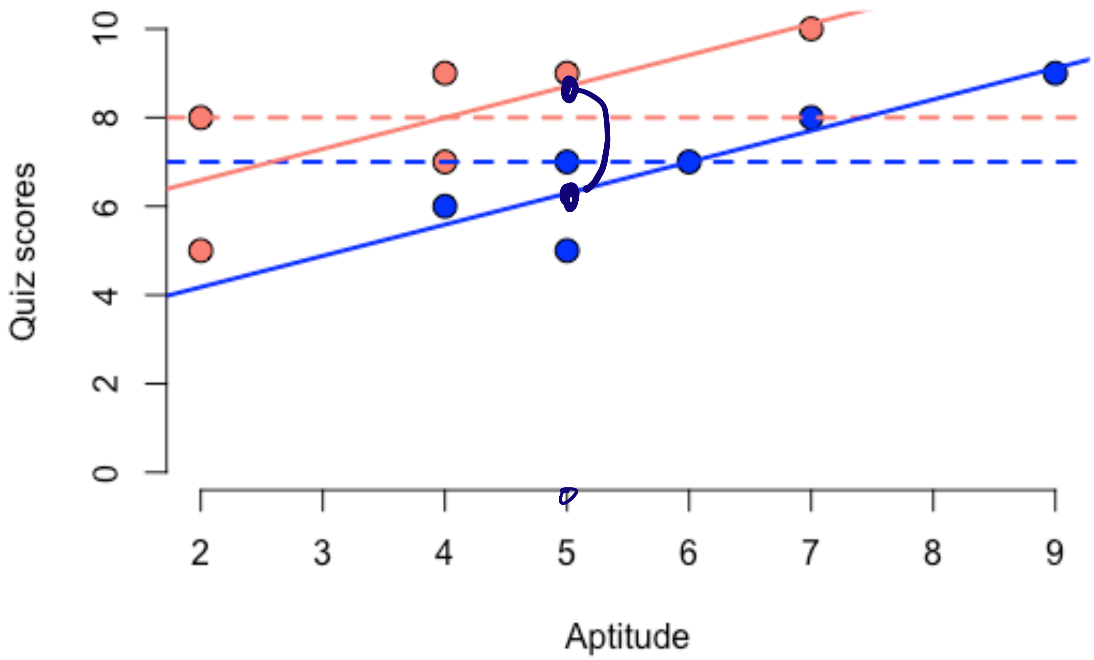
Clearly, the groups are not equivalent in terms of aptitude.

What if we could also account for these aptitude scores in our analysis?

That is, how would the effect of instruction type change if we could make both groups have equivalent aptitude? (i.e., "control" for aptitude)

Here's how do it!

* fit parallel regression lines to each group.
* use the slopes to calculate adjusted means.



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● – small groups

The effect after "controlling" for aptitude is the vertical distance between the regression lines.

Note that it is larger than the original effect.

Lets calculate the adjusted group means

Calculating adjusted group means:

$$\boxed{\bar{Y}_j' = \bar{Y}_j - \beta(\bar{X}_j - \bar{X})}$$

So for our data:

Group 1 (trad lecture): $\bar{Y}_1' = \bar{Y}_1 - \beta(\bar{X}_1 - \bar{X})$

$$= 7 - 0.706(6-5)$$

$$= 6.29$$

Group 2 (small groups): $\bar{Y}_2' = \bar{Y}_2 - \beta(\bar{X}_2 - \bar{X})$

$$= 8 - 0.706(4-5)$$

$$= 8.70$$

Note: 1. the adjusted means are farther apart

2. the adjustment literally removes the effect of aptitude

3. this procedure is called "analysis of covariance"

Doing ANCOVA in JASP

1. perform ANOVA on Y as function of grouping variable.

2. perform linear regression on Y with X and group variable as covariates

   * this gives you the slope (ie, covariate effect) needed to calculate adjusted means

   * use "unstandardized" coefficient for X

3. perform ANCOVA on Y as function of group with covariate X.

Note – we'll do Bayesian ANCOVA in the next lecture video!

How to write results:

"We performed a one-way ANOVA on final quiz scores as a function of instruction type.

Mean quiz scores were not significantly different between traditional lecture ($\bar{Y} = 7$) and small group tutorial instruction ($\bar{Y} = 8$); $F(1,10) = 1.15$, $p = 0.308$.

However, when we accounted for pre-instruction aptitude scores as a predictor in our model, the difference between adjusted mean quiz scores increased ($\bar{Y}' = 6.29$ for traditional lecture compared to $\bar{Y}' = 8.70$ for small groups).

This difference in covariate-adjusted group means was statistically significant, as confirmed by an ANCOVA including aptitude as a covariate, $F(1,9) = 12.81$, $p = 0.006$."