Bayesian Statistics

1. Consider the binomial experiment from the lecture ($N = 10$ trials, $x = 8$ successes).
   (a) In the JASP "Learn Stats" module, build the three models listed below in the "Hypothesis" section:
      - $\mathcal{H}_1 : \theta \sim \text{Beta}(2, 2)$
      - $\mathcal{H}_2 : \theta \sim \text{Beta}(2, 4)$
      - $\mathcal{H}_3 : \theta \sim \text{Beta}(4, 2)$

   (b) Plot each of the resulting posterior distributions in one figure. An easy way to do this is to go to "Prior and Posterior", choose "Posterior Distribution + Joint + Overlying".
   (c) How are the posteriors similar? How are they different?
   (d) Now add a null (spike) model to your list: $\mathcal{H}_0 : \theta = 0.5$. Use the prior predictive distributions to calculate the following Bayes factors for models 1, 2, and 3 against this null model.
   (e) Are the Bayes factors similar, or are there differences among them? (*Hint: the point of this exercise is to demonstrate that inference about the posterior distribution doesn't depend too much on the prior used, but Bayes factors do*).

2. In an experiment on ESP (extrasensory perception), participants are required to predict the color (red or black) of a card drawn randomly from a complete deck. Suppose that a set of participants correctly identifies the color on 38 out of 75 trials. Consider a hypothesis test on two models for these data: $\mathcal{H}_0 : \theta = 0.5$ versus $\mathcal{H}_1 : \theta \sim \text{Beta}(1, 1)$.
   (a) Visually estimate $\pi(\theta = 0.5)$ and $\pi(\theta = 0.5 \mid \text{data})$ from the "Prior and posterior distribution" plot in the JASP Learn Bayes module. Calculate the (estimated) updating factor.
   (b) Calculate the predictive performance of each model against the observed data (that is, $p(\text{data} \mid \mathcal{H}_0)$ and $p(\text{data} \mid \mathcal{H}_1)$). Which model predicted the observed data ($x = 38$ successes) best? How much better was it compared to the other model?
   (c) What do you notice about the answers you obtained in parts (a) and (b)? (*Hint: if you notice that they match, good job...you just demonstrated something called the Savage-Dickey density ratio!*)