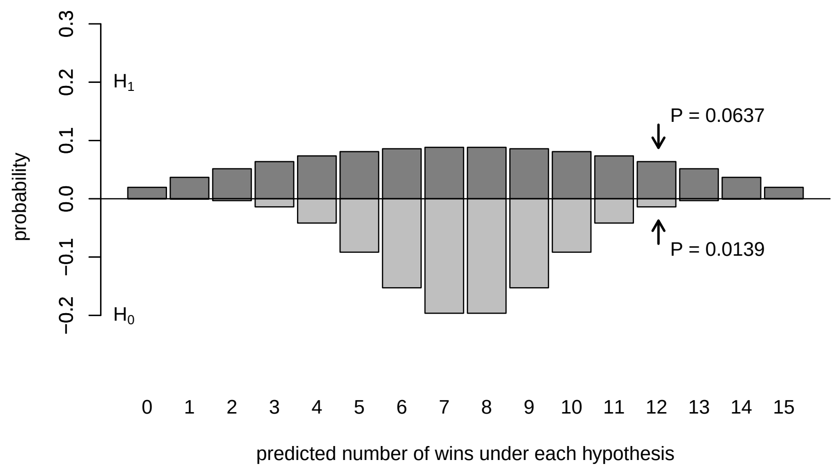
PSYC 4390/5090: Bayesian Statistics

Tarleton State University | Homework 1

1. This exercise will get you to think about the relationship between BF_{10} and BF_{01}.
   (a) Suppose that a paper reports $\text{BF}_{10} = 0.04$. Which model ($\mathcal{H}_0$ or $\mathcal{H}_1$) is supported in this case?
   (b) Compute BF_{01} (Hint: think about the definition of Bayes factor as a *ratio*).
   (c) What is the mathematical relationship between BF_{10} and BF_{01}?

2. Consider a binomial model (with parameter w) for an experiment with 15 trials. The plot below shows the predictive distributions (e.g., Etz, Haaf, Rouder, & Vandekerckhove, 2018) for two models: $\mathcal{H}_0 : w = 0.5$ (bottom plot) versus $\mathcal{H}_1 : w \neq 0.5$ (top plot). The prior for w under $\mathcal{H}_1$ is the "smooth peaked prior" I described in the lecture (note: if you're curious, it is a beta(2,2) function).



   (a) The values displayed are the likelihoods of observing $x = 12$ successes under each model. Which model better supports this data? Use these likelihoods to compute the Bayes factor for the winning model.
   (b) What data would have to be observed to support $\mathcal{H}_0$ over $\mathcal{H}_1$? Of these potential data, which would give the largest support for $\mathcal{H}_0$?

3. Suppose we have two competing models $\mathcal{H}_0$ and $\mathcal{H}_1$ for some observed data. Suppose the Bayes factor BF_{10} is found to be 10.72.
   (a) Which model is supported: $\mathcal{H}_0$ or $\mathcal{H}_1$? Explain what the Bayes factor means in this context.
   (b) Suppose you believe that $\mathcal{H}_0$ and $\mathcal{H}_1$ are equally likely, *a priori*. That is, $P(\mathcal{H}_0) = 0.50$ and $P(\mathcal{H}_1) = 0.50$. What is the posterior probability for the winning model?
   (c) Suppose instead that you believe that $\mathcal{H}_1$ is three times as likely, *a priori*, as $\mathcal{H}_0$. What is the posterior probability for the winning model in this case?