Bayesian Statistics – Lecture 3

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Goals for today

1. Introduce the Bayesian correlation test
2. Discuss the stretched Beta prior
3. Work some examples in JASP
4. Discuss *sensitivity analyses*

Recall our example from Lecture 1

The NEO Personality Inventory-Revised (NEO PI-R) is a 240-item questionnaire that measures the "Big 5" dimensions of personality: Agreeableness (A), Conscientiousness (C), Neuroticism (N), Extraversion (E), and Openness to Experience (O).

Dolan et al. (2009) administered a Dutch version of the NEO PI-R to 500 first-year psychology students at the University of Amsterdam.

Two examples

Today, we'll perform a Bayesian correlation test in JASP to explore the associations between:

- Neuroticism & Agreeableness
- Extraversion & Conscientiousness

Review: Bayes' theorem

In a (Bayesian) correlation test, we're interested in the population correlation ρ. Bayes' theorem says:

$$\underbrace{\pi(\rho \mid \text{data})}_{\substack{\text{Posterior}\\ \text{distribution}\\ \text{for } \rho}} = \underbrace{\pi(\rho)}_{\substack{\text{Prior}\\ \text{distribution}\\ \text{for } \rho}} \times \underbrace{\frac{p(\text{data} \mid \rho)}{p(\text{data})}}_{\substack{\text{predictive updating}\\ \text{factor}}}$$

Informally – to get the **posterior** distribution for ρ:

1. start with a **prior** distribution for ρ

2. change/update the prior distribution at each value of ρ by an **updating factor** that depends on the fit between ρ and your observed data

Model comparison via the Bayes factor

In a Bayesian model comparison, we're interested in the relative predictive performance of two models $\mathcal{H}_0$ and $\mathcal{H}_1$. This is given by the *Bayes factor*:

$$\mathsf{BF}_{10} = \frac{p(\mathsf{data} \mid \mathcal{H}_1)}{p(\mathsf{data} \mid \mathcal{H}_0)}$$

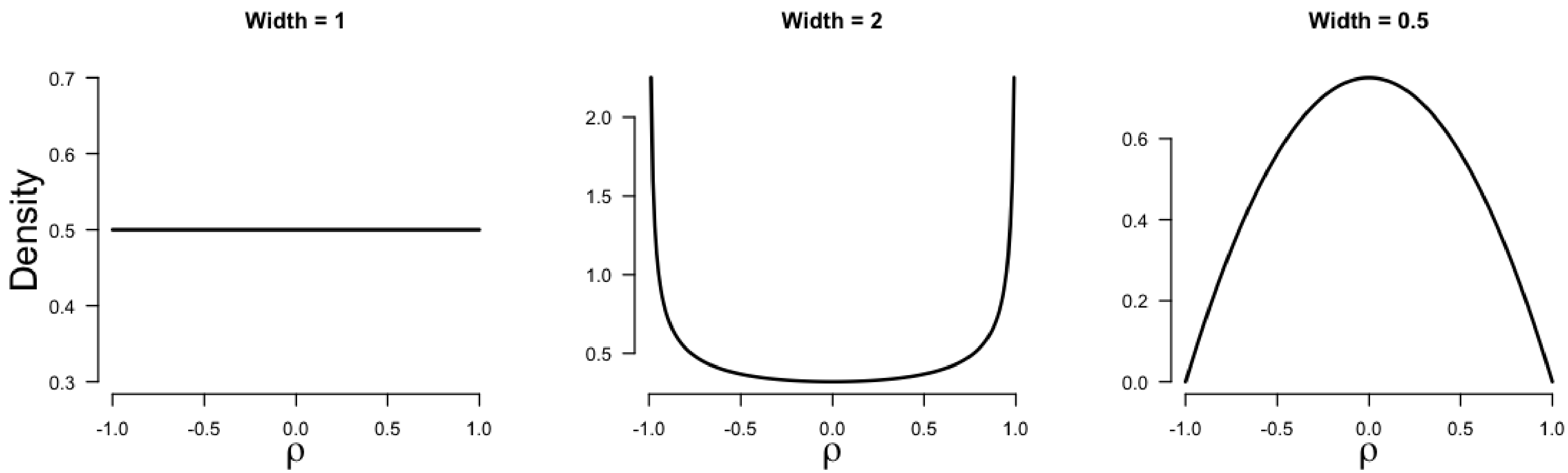
- if $\mathsf{BF}_{10} > 1$, then the data is better predicted by $\mathcal{H}_1$
- if $\mathsf{BF}_{10} < 1$, then the data is better predicted by $\mathcal{H}_0$
- BF_{10} is a measure of the *evidence* for the winning model

Choosing a prior

Any Bayesian computation requires us to *first* define a prior distribution for the parameter of interest:

- for last week's binomial probability θ, we needed a distribution that ranged from 0 to 1
  - we used a *Beta* prior
- for this week's correlation ρ, we need a prior that ranges between -1 and 1
  - the Beta prior doesn't work for this, as it is only defined between 0 and 1
  - instead, we can use a *stretched Beta prior* (Ly, Marsman, & Wagenmakers, 2018)

Stretched Beta Prior



Stretched Beta Prior

The stretched Beta prior is defined on ρ between -1 and 1, and has one shape parameter κ, sometimes called "width"

- κ must be between 0 and 2
- $\kappa = 1$ is equivalent to a uniform distribution
- $\kappa < 1$ places more prior mass in the middle $(\rho = 0)$
- $\kappa > 1$ places more prior mass at the ends $(\rho = \pm 1)$

Examples in JASP

Let's perform a Bayesian correlation test in JASP to explore the associations between:

- Neuroticism & Agreeableness
- Extraversion & Conscientiousness

Aside - Jeffreys' (1961) Evidence Categories

Bayes factor	Evidence category
> 100	Extreme evidence for $\mathcal{H}_1$
30 - 100	Very strong evidence for $\mathcal{H}_1$
10 - 30	Strong evidence for $\mathcal{H}_1$
3 - 10	Moderate evidence for $\mathcal{H}_1$
1 - 3	Anecdotal evidence for $\mathcal{H}_1$
1	No evidence
1/3 - 1	Anecdotal evidence for $\mathcal{H}_0$
1/10 - 1/3	Moderate evidence for $\mathcal{H}_0$
1/30 - 1/10	Strong evidence for $\mathcal{H}_0$
1/100 - 1/30	Very strong evidence for $\mathcal{H}_0$
$< 1/100$	Extreme evidence for $\mathcal{H}_0$

Summary

1. We introduced the Bayesian correlation test
2. We discussed the stretched Beta prior
3. We worked some examples in JASP and discussed Jeffreys' (1961) evidence categories
4. Because Bayes factors depend on the choice of prior, we discussed the need for *sensitivity analyses*