Bayesian Statistics – Lecture 5

Thomas J. Faulkenberry, Ph.D.

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Goals for today

1. Introduce the Bayesian analysis of variance

2. Talk about *Bayesian model averaging* and *inclusion Bayes factors*

3. Work an example in JASP

Example – the classic "tooth growth" dataset

THE GROWTH OF THE ODONTOBLASTS OF THE INCISOR TOOTH AS A CRITERION OF THE VITAMIN C INTAKE OF THE GUINEA PIG [1]

E. W. CRAMPTON

Department of Nutrition, Macdonald College, McGill University, P.O., Prov. Quebec, Canada

FIVE FIGURES

(Received for publication November 22, 1946)

After describing briefly 2 physical methods, 21 chemical methods, and 1 biochemical method, Rosenberg ('45) stated "Although the chemical, and to a small extent also, physical methods are replacing more and more the biological determinations of vitamin C, the biological tests maintain their place as the ultimate and most correct method of determining vitamin C." The problem of the assay of this vitamin was of particular concern to the Canadian Government during the war years because of the difficulty of providing natural sources of vitamin C to the armed forces for a considerable portion of the year. Inasmuch as different chemical procedures frequently gave different results as to the potency of a food in which the armed forces were interested, this laboratory was requested in 1942 to undertake the establishment of a vitamin C bioassay which might be used as a check against chemical procedures.

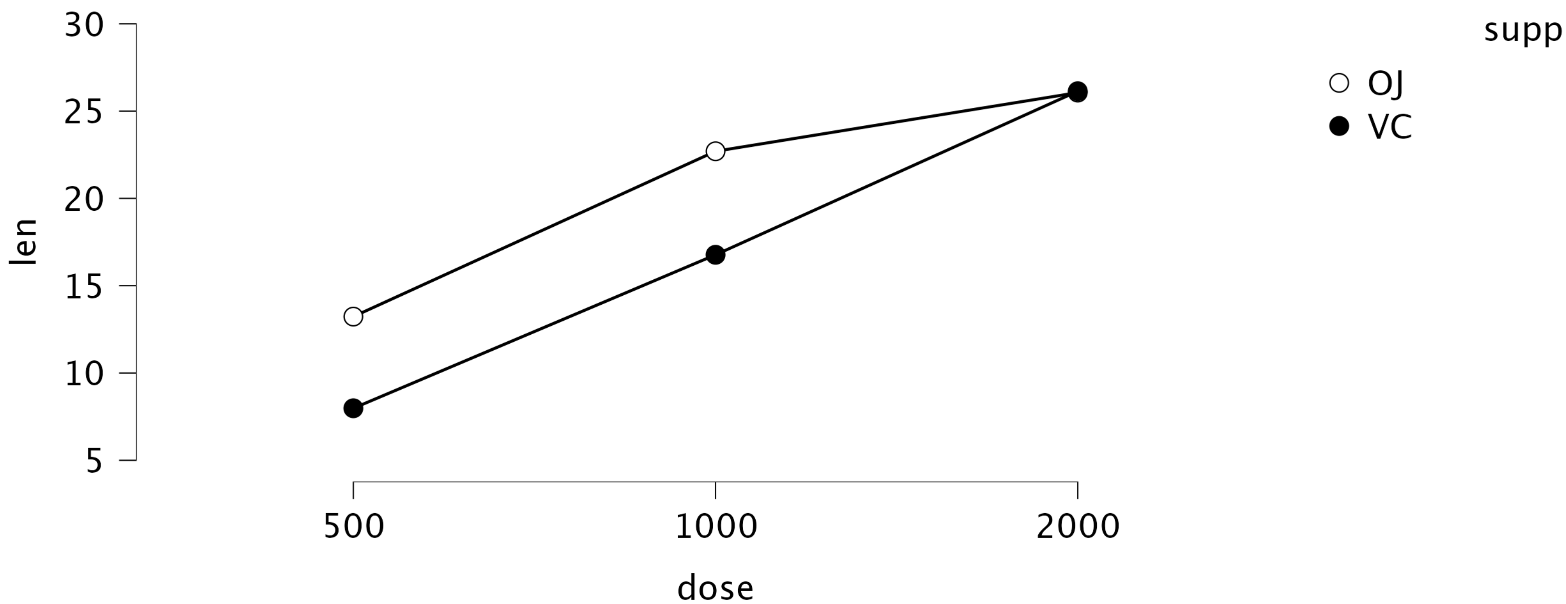
Example – the classic "tooth growth" dataset

Crampton (1947) measured tooth growth (in mm) of $N = 60$ guinea pigs who were each given a dose of vitamin C

Manipulations:

- *supplement*: orange juice (OJ) or ascorbic acid (VC)
- *dose*: 0.5, 1, or 2 mg per day

Example – the classic "tooth growth" dataset



Classical ANOVA in JASP

Let's perform a classical ANOVA in JASP

Bayesian ANOVA

The Bayesian ANOVA compares the fits of *five* models:

- Null model
- Main effect 1 model
- Main effect 2 model
- Additive model (Main effects 1 + 2)
- Interactive model (Main effects 1+2 + interaction)

Bayesian ANOVA

The Bayesian ANOVA compares the fits of *five* models:

- $\mathcal{M}_0$: len ~ 1
- $\mathcal{M}_1$: len $\sim$ supp
- $\mathcal{M}_2$: len $\sim$ dose
- $\mathcal{M}_3$: len $\sim$ supp $+$ dose
- $\mathcal{M}_4$: len $\sim$ supp $+$ dose $+$ supp $\cdot$ dose

Bayesian ANOVA in JASP

Let's walk through the procedure in JASP and talk about the output

Bayesian ANOVA in JASP

Model Comparison

Models	P(M)	P(M|data)	BF_M	BF_{10}	error %
supp + dose + supp ∗ dose	0.200	0.723	10.459	1.000	
supp + dose	0.200	0.272	1.494	0.376	2.467
dose	0.200	0.005	0.019	0.006	1.881
supp	0.200	1.125×10^{-15}	4.501×10^{-15}	1.556×10^{-15}	1.881
Null model	0.200	9.387×10^{-16}	3.755×10^{-15}	1.298×10^{-15}	1.881

Bayesian ANOVA in JASP

Prior model probability

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Bayesian ANOVA in JASP

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Posterior model probability

Bayesian ANOVA in JASP

Prior model probability

Change from prior to posterior model odds

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Bayesian ANOVA in JASP

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Posterior model probability

Bayes factor against best fitting model

Bayesian ANOVA in JASP

Prior model probability

Change from prior to posterior model odds

Numeric error

Model Comparison

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Posterior model probability

Bayes factor against best fitting model

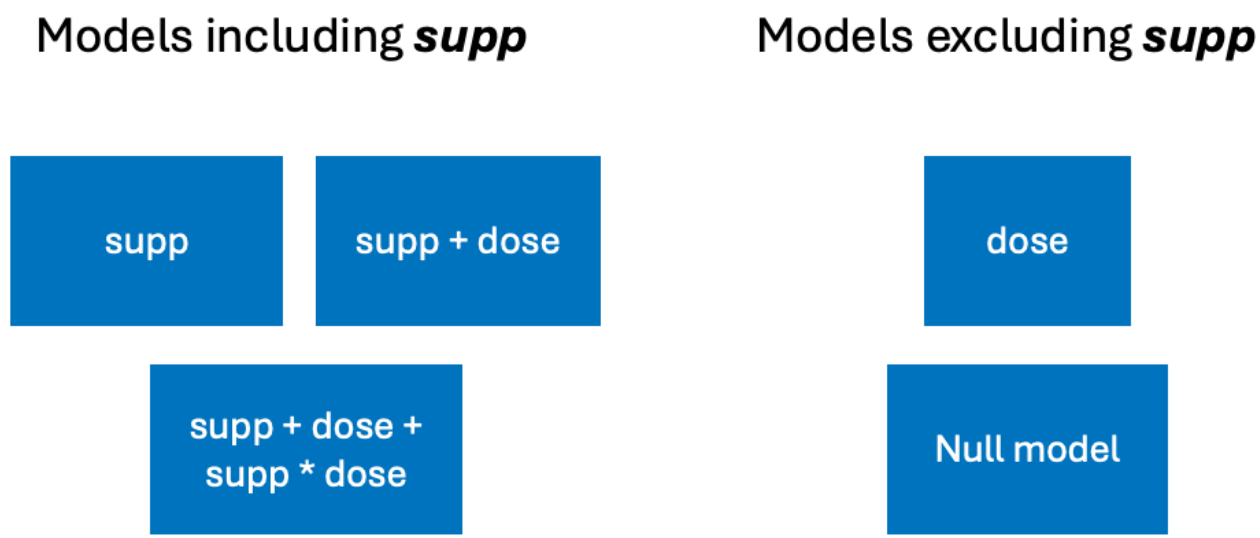
A big difference from classical ANOVA

In classical ANOVA, we are accustomed to reporting "significance" of *main effects* and *interactions*.

Is there a Bayesian version of this?

Bayesian model averaging

As it happens, yes. Since each "effect" (i.e., one of the possible terms in the model) appears in *multiple* models, we can measure the relative evidence of **including** the effect versus **not including** the effect



JASP "effects table"

Analysis of Effects - len

Effects	P(incl)	P(excl)	P(incl|data)	P(excl|data)	BF_{incl}
supp	0.600	0.400	0.995	0.005	141.833
dose	0.600	0.400	1.000	3.553×10^{-15}	$1.876 \times 10^{+14}$
supp $*$ dose	0.200	0.800	0.723	0.277	10.459

JASP "effects table"

Prior probability of including effect

Analysis of Effects – len

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JASP "effects table"

Prior probability of including effect

Posterior probability of including effect

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JASP "effects table"

Prior probability of including effect

Posterior probability of including effect

Bayes factor for including the effect

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Reporting a Bayesian ANOVA

- describe the models that are defined by your experimental design
  - with N manipulations, there are $2^N + 1$ models

Reporting a Bayesian ANOVA

- report the best fitting model from the Model Comparison table
  - minimally, report the posterior probability of the model.
  - even better, report the "model Bayes factor" - i.e., the factor by which prior odds are updated to posterior odds after observing data

Reporting a Bayesian ANOVA

- report inclusion Bayes factors for each main effect and interaction
  - "the data were XXX times more likely under models including the main effect of YYY than under models excluding the main effect of YYY".
  - note – this same template can be used to report evidence *against* including an effect