Reversing the manual digit bias in two-digit number comparison

Thomas J. Faulkenberry
Tarleton State University

Alexander Cruise
Ariel University

Samuel Shaki
Ariel University

Author Note

Please address correspondence to Thomas J. Faulkenberry, Department of Psychological Sciences, Tarleton State University, Box T-0820, Stephenville, TX 76402. Email: faulkenberry@tarleton.edu. All raw data and analysis code are available at http://github.com/tomfaulkenberry

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Abstract

Though recent work in numerical cognition has supported the embodiment of number representations (e.g., a mental number line), little is known about the embodiment of multi-digit number representations. Along this line, Bloechle, Huber, and Moeller (2015) found that pointing positions in two-digit number comparison were biased leftward toward the decade digit. Moreover, this bias was reduced in unit-decade incompatible pairs. In the present study, we used computer mouse tracking to investigate response dynamics as participants completed two-digit number comparisons by clicking "Smaller" or "Larger" depending on how presented numbers compared to 55. Similar to Bloechle et al. (2015), we found that trajectories exhibited a leftward bias that was reduced for unit-decade incompatible comparisons. However, when positions of response labels were reversed (Larger-Smaller), the biases reversed. That is, we found a rightward bias for compatible pairs that was reduced for incompatible pairs. This result calls into question a purely embodied representation of place value structure and instead supports a competition model of two-digit number representation.

Keywords: Two-digit comparison, place value, response competition, computer mouse tracking

Reversing the manual digit bias in two-digit number comparison

Making mental comparisons of two-digit numbers can be a daily task for many people. For example, when driving an automobile, one must hold a fixed standard in memory (i.e., remembering that the speed limit is 65 miles per hour) while simultaneously perceiving a symbolic representation of the current speed from the speedometer and quickly deciding whether that speed is above or below the speed limit. Though this task is fairly automatic for practiced drivers, the cognitive mechanisms underlying such two-digit comparison are not well-understood. In the present paper, we examine the dynamics of how people compare two-digit numbers to a fixed standard.

One of the earliest attempts to understand the processes involved in such a comparison was Hinrichs, Yurko, and Hu (1981), who asked participants to compare a two-digit number to a fixed standard (e.g., 55). They found that reaction times decreased as numerical distance from the standard increased. Going back to Moyer and Landauer (1967), this numerical distance effect has been considered a signature of an analog representation of number. Put another way, this distance effect implies that "the internal representation of number may be an 'analog' of some physical property and that the number comparison process is similar to an internal psychophysical comparison process" (Hinrichs et al., 1981, p. 890). In addition, Hinrichs et al. (1981) found that the unit digit (e.g., the "8" in the number 28) continued to affect processing times, even though paying attention to only the decade digit (e.g., the "2" in 28) is sufficient to make the comparison decision. This inability of participants to ignore the unit digit led Hinrichs et al. (1981) to conclude that two-digit numbers are processed as holistic, integrated quantities.

In a similar experiment, Dehaene, Dupoux, and Mehler (1990) also found that reaction times decreased as numerical distance from the standard increased. Further, Dehaene et al. (1990) found that presenting the unit digit before the decade digit did not increase or decrease the influence of the unit digit on reaction times. Dehaene et al. (1990) concluded that symbolic two-digit numbers must first be encoded into an internal, analog magnitude code on a mental number line (e.g., Restle, 1970; Moyer & Landauer, 1967). In other words, early evidence pointed to the claim that skilled adults collapse the base-10 structure of a two-digit number into a single, holistic magnitude code.

However, Nuerk, Weger, and Willmes (2001) challenged this claim on the basis of the unit-decade compatibility effect. Such an effect occurs when it takes longer to choose the larger of the pair 36_52 than to choose the larger of 52_68. One possible reason for this is because in the pair 36_52, separate comparisons of the decades ($3 < 7$) and the units ($6 > 2$) lead to opposite decisions (thus 36_52 is called a unit-decade incompatible pair). On the other hand, for the pair 52_68, separate comparisons lead to the same decision ($5 < 6$ and $2 < 8$), so 52_68 is called a unit-decade compatible pair. As the overall distance between numbers in each pair is 16, the presence of such a compatibility effect presents problems for a purely holistic account, which would predict no difference in reaction times between the two pairs. In addition to this compatibility effect, several researchers (e.g., Nuerk et al., 2001, 2004; Wood, Nuerk, Freitas, Freitas, & Willmes, 2006) found that overall distance predicted reaction times, leading Nuerk and Willmes (2005) to propose a hybrid model, where two-digit numbers activate both holistic and decomposed representations in parallel.

In addition to the holistic and hybrid models of two-digit number representation, there is a third class of models which posits that two-digit numbers are decomposed so that the magnitude of each digit is represented separately, with no integrated holistic representation (e.g., McCloskey, 1992; Verguts & De Moor, 2005). Verguts and De Moor (2005) found evidence for such a decomposed model by manipulating overall distance (small versus large) and decade distance (same decade, different decade). They found that for same decade pairs, there was a numerical distance effect; that is, large distance pairs took less time to compare than small distance pairs. However, no such distance effect emerged for different decade pairs, presumably due to the fact that the larger of a different decade pair can be selected based on a comparison of the decade digit alone. Verguts and

De Moor (2005) interpreted their results in terms of a model of single-digit number comparison (Verguts, Fias, & Stevens, 2005), arguing that decomposed representations allow recycling of efficient single-digit number processing to solve the inherently harder problem of two-digit number comparison.

Whereas the holistic versus decomposed debate has dominated the literature on two-digit number representation, other recent work has situated two-digit number comparison within an embodied cognition framework (Wilson, 2002; Barsalou, 2008). For example, Bloechle et al. (2015) found evidence for a spatial-numerical association between two-digit numbers and horizontal space, where the decade digit is associated with leftward space and the unit digit is associated with rightward space. In their study, Bloechle et al. (2015) had participants point to the larger of a pair of two-digit numbers on a touch screen. They found that the landing position of the finger on the target number was significantly left of the midline between the decade and unit digit. Further, this "decade bias" was reduced on unit-decade incompatible trials, presumably because on such trials the interfering magnitude information from the unit digit resulted in increased unit processing. Bloechle et al. (2015) interpreted this response pattern as reflective of a long-term memory association of decades with leftward space and units with rightward space, possibly due to the past experience of reading and writing multi-digit Arabic numerals.

To our knowledge, the study of Bloechle et al. (2015) is the only such study to find this decade bias in manual pointing positions. However, we are aware of one other study that investigated the dynamics of manual responses in two-digit number comparison. Dotan and Dehaene (2013) tracked participants' finger trajectories as they pointed to the approximate location of a single two-digit number on a number line. Dotan and Dehaene (2013) concluded that when performing the number-to-position task, adults form a fast representation of the unit digit magnitude, followed by both holistic and decomposed representations of the two-digit number. Their results indicate that both holistic and decomposed representations play a role in two-digit number representation (supporting the hybrid model, e.g., Nuerk et al., 2001). Further, their data indicate that unit processing may override decade processing (at least through the first 550 ms of the response execution).

Given the inability of previous studies to discriminate between the holistic, decomposed, and hybrid models, the studies of Bloechle et al. (2015) and Dotan and Dehaene (2013) are an important next step in the research program, particularly because they explore the dynamics of the hand movements that people make when making decisions about two-digit numbers. Tracking these hand movements is becoming an increasingly popular method for testing various models in numerical cognition (Fischer & Hartmann, 2014; Faulkenberry & Rey, 2014). Because these hand movements reflect the projection of internal cognitive processes onto an observable behavioral output (Spivey, Grosjean, & Knoblich, 2005; Spivey, 2007), such hand tracking provides a window into the internal processes that evolve during a numerical decision. Several recent studies have exploited this technique in various forms, such as directly tracking hand/finger movements (Song & Nakayama, 2008; Santens, Goossens, & Verguts, 2011) and tracking the movement of a computer mouse (Marghetis, Núñez, & Bergen, 2014; Faulkenberry, 2014; Ganor-Stern & Goldman, 2014; Faulkenberry, Montgomery, & Tennes, 2015; Haslbeck, Wood, & Witte, 2015; Faulkenberry, Cruise, Lavro, & Shaki, 2016; Faulkenberry, 2016).

In the present study, we asked participants to use a computer mouse to choose whether two-digit numbers were smaller or larger than 55. To measure the dynamics of participants' choices, we recorded the streaming positions of the mouse cursor on the computer screen. Particularly we looked for dynamic signatures of holistic and decomposed processing. For holistic processing, such signatures have already been found in multiple studies (Song & Nakayama, 2008; Santens et al., 2011; Faulkenberry, 2016), at least for single digit numbers. In one of the first hand tracking studies with numbers, Song and Nakayama (2008) had participants choose whether a single digit number was less than or greater than 5. They found that trajectories became more curved toward the center of the screen as the target numbers became closer in magnitude to the comparison standard (e.g., trajectories for the target 4 were more curved than trajectories for the target 1). Song and Nakayama (2008) argued that the increase in curvature obtained in such a number comparison task reflects a direct mapping from an internal "mental number line" to external space. Their explanation was that with numbers that are close to the comparison standard, the hand is drawn toward the external projection of the stimulus number, which would necessarily be represented closer to the middle of the mental number line. This would result in the appearance of more curvature in the response trajectories. Numbers that are far from the standard would be represented closer to the ends of the mental number line, and hence, their external projections would draw the hand more directly toward the response option (resulting in less curvature).

Santens et al. (2011) argued instead that the increase in trajectory curvature that one sees with close number pairs is due to increased competition between parallel and partially active response options. They also pointed out that the two views (competition versus direct mapping) would yield divergent predictions for trajectories in the incongruent response mapping (LARGER - SMALLER). The competition view, they argued, would predict that trajectory curvature would decrease as a function of numerical distance, since larger distance pairs would have less representational overlap, and hence less response competition, leading to less curvature toward the incorrect response option. The direct mapping view, on the other hand, would predict that trajectory curvature would increase as a function of numerical distance, because with an incongruent response mapping, the external projection of a number that is far from the comparison standard would be at the opposite end of where the response is to be made. For example, 89 would be projected to the far right of physical space, but due to the incongruent response mapping, the decision LARGER would be made on the left side of the screen. Hence, the initial hand movement would be toward the right, but the hand would have to move all the way to the left of the screen to make the response. This overall movement would have a large amount of curvature compared to a number whose external projection was closer to the middle. In their paper, Santens et al. (2011) tested between these alternatives and found that in the incongruent response mapping condition, curvatures decreased, supporting the competition view (see also Faulkenberry, 2016).

For decomposed signatures, predictions are less clear. As already mentioned, we are aware of only two other studies that have investigated manual responses in two-digit number comparison, and only one of these (Dotan & Dehaene, 2013) tracked the continuous dynamics of the manual response. Both Bloechle et al. (2015) and Dotan and Dehaene (2013) have found a response bias reflecting increased activation of a particular digit. Also, Bloechle et al. (2015) found that on unit-decade incompatible trials, the bias toward the decade digit was reduced, implying a greater activation of the unit digit. As such, we predict to obtain a similar result with mouse trajectories. However, it is not clear at present what the source of this bias may be. Though Bloechle et al. (2015) explained that such bias may be the result of long term memory associations between digits and space (e.g., a direct mapping account), such bias may also be obtained as a result of response competition (Santens et al., 2011; Faulkenberry, 2016).

To test between these two possibilities, we had participants perform comparison judgments with the labels SMALLER - LARGER displayed (a spatially congruent response mapping) as well as LARGER - SMALLER (a spatially incongruent response mapping). If the manual digit bias observed in Bloechle et al. (2015) and Dotan and Dehaene (2013) is due to an embodied representation of place value, then trajectories should exhibit less leftward bias on unit-decade incompatible trials; critically, this reduction of bias will not depend on the type of response mapping used. On the other hand, if the manual digit bias is due to response competition (Santens et al., 2011; Faulkenberry, 2016), then the bias patterns (e.g., reduction of leftward bias on unit-decade incompatible trials) should reverse when using the spatially incongruent response mapping (LARGER - SMALLER). Such reversal would indicate that the unit digit bias observed in Bloechle et al. (2015) and

Dotan and Dehaene (2013) may be due to task demands rather than an inherently embodied representation of place value.

Method

Participants

Thirty-seven undergraduate students (36 female, mean age = 24.1 years, age range 18 to 60) participated in this experiment in exchange for partial course credit in their psychology courses. Four participants reported being left hand-dominant, but all reported that they used their right hand for the computer mouse. The experiment was reviewed and approved by the institutional review board at Tarleton State University.

Apparatus

The experiment was implemented using the MouseTracker software package (Freeman & Ambady, 2010). The experimental trials were presented on a 20 inch iMac desktop computer with a screen resolution of 1,280 x 1,024 pixels. We ran the MouseTracker program on the iMac using a virtual Windows XP environment via Parallels. Following the recommendations of Fischer and Hartmann (2014), we disabled the "dynamic acceleration" option and lowered the speed of the mouse movements on the screen to the second-lowest possible speed in the mouse settings dialog. This is done to prevent quick and erratic mouse movements, resulting in a smooth and more reliable record of participants' hand movements. The resulting displacement ratio of the mouse to screen movement was 1 cm to 100 pixels.

Stimuli and procedure

Our stimulus set consisted of 64 two-digit numbers adapted from Reynvoet, Notebaert, and Van den Bussche (2011). We asked participants to perform a two-digit number comparison using 55 as the comparison standard. We excluded all pure decade numbers (e.g., 60) and tie numbers (i.e., numbers where unit and decade digits were equal, such as 44). We manipulated size (smaller than 55, larger than 55), holistic distance (close to 55, far from 55), and unit-decade compatibility (compatible, incompatible). See Table 1 for the complete list of stimuli.

Participants were told that on every trial a number would appear in the center of the screen, and they would be asked to choose, as quickly as possible, whether the number was less than 55 or greater than 55. Each trial started with a blank screen presented for 1000 ms, followed by a screen that displayed the response labels SMALLER and LARGER at the top left and right of the screen, respectively. Each response label was presented in Arial font with point size 24.

The order of the response labels was switched once midway through the experiment; half of the participants started with the SMALLER-LARGER ordering, while the other half began with the LARGER-SMALLER ordering. After 1000 ms, a START button appeared. Once the START button was pressed, one of the stimulus numerals appeared in the center of the screen, presented in Arial font with point size 48. Participants then clicked on the correct of these two options; while doing so, the software recorded the streaming (x, y) coordinates of the computer mouse approximately 70 times per second.

We manipulated the spatial congruity of the response labels SMALLER and LARGER: in the congruent condition, SMALLER appeared in the upper left corner and LARGER appeared in the upper right corner. In the incongruent condition, these labels were reversed. In half of the trials, the correct answer was on the left side, whereas on the other half of the trials, the correct answer was on the right side.

For incorrect responses, the program displayed an “X” for 1000 ms. To ensure that trajectories reflected online processing, participants were encouraged to begin their movements as early as possible and were warned if initiated movement later than 400 ms following stimulus presentation. This instruction is customarily included in mousetracking studies so that trajectories reflect the dynamics of a decision process rather than simply reflecting the kinematics of a response choice after the choice has already been made (Freeman & Ambady, 2009; Spivey et al., 2005).

For each of the two counterbalanced spatial congruity conditions, participants completed two blocks of 64 trials (2 repetitions of each stimulus number, randomly presented), with a short break in between each block. In all, each participant completed 256 experimental trials in a single 30 minute session.

Results

General performance

Participants completed a total of 9,472 experimental trials. Of these, we removed 40 trials that contained an error response (0.42%). Over the remaining correct trials, participants exhibited a mean response time of 1198 ms (SD=356 ms). Each manual response consisted of an initiation time (time to initiate mouse movement after pressing the start button; M=118 ms, SD=93 ms) and movement time (time span from movement initiation to response; M=1080 ms, SD=362 ms). We removed an additional 168 trials for which response time exceeded 3 standard deviations from the mean response time, as well as any trials for which response time was less than 200 ms (5 trials). All subsequent analyses were performed on the remaining 9,259 trials.

Holistic processing signatures

Time analyses. Mouse movement times (MT) were submitted to a 2 (Distance: close, far) x 2 (Response side: left, right) x 2 (Response mapping: congruent, incongruent) repeated measures analysis of variance. There was a significant main effect of distance, $F(1, 36) = 123.1$, $p < 0.001$, $\eta_p^2 = 0.77$. As can be seen in Figure 1, movement times were longer for two-digit numbers that were close to 55 (M=1085 ms) compared to trials with numbers far from 55 (M=1024 ms). No other terms in the ANOVA model were statistically significant (all F-values less than 0.89).

Similarly, initiation times were submitted to a 2 (Distance: close, far) x 2 (Response side: left, right) x 2 (Response mapping: congruent, incongruent) repeated measures analysis of variance. There was a small, but statistically significant main effect of response mapping, $F(1, 36) = 4.80$, $p = 0.04$, $\eta_p^2 = 0.12$. Trials with a congruent response mapping (SMALLER - LARGER) took slightly longer to initiate mouse movement (M=120 ms) compared to trials with an incongruent response mapping (LARGER - SMALLER; M=116 ms). No other terms in the ANOVA model were statistically significant (all F-values less than 0.89).

Trajectory analyses. Mouse movement trajectories were measured by recording the streaming x, y - coordinates of the computer mouse during each trial. In order to compare trajectory characteristics across trials of differing response times, all raw mouse trajectories were pre-processed in MouseTracker (Freeman & Ambady, 2010) so that all trajectories were rescaled onto a standard coordinate space of $[-1, 1] \times [0, 1.5]$ and normalized via linear interpolation to consist of exactly 101 timesteps.

In Figure 2, we present average mouse trajectories as a function of distance (close, far), response mapping (congruent, incongruent), and decision (larger than 55, smaller than 55). As can be seen in the figure, trajectories are more curved toward the incorrect response alternative for numbers that are close to the comparison standard of 55, compared to trajectories for numbers that are far from 55. This increase in trajectory curvature is in line with patterns reported in several previous studies (Song & Nakayama, 2008; Santens et al., 2011).

As mentioned earlier, there are two possible explanations for this increased trajectory curvature, a direct mapping view (Song & Nakayama, 2008) and a competition view (Santens et al., 2011; Faulkenberry, 2016). Recall that the discriminating signature comes from the pattern of response curvatures with distance in the incongruent response mapping; increasing curvatures with distance support the direct mapping view, whereas decreasing curvatures with distance support the competition view. To test this, we computed area under the curve (AUC) as an index of curvature for each trajectory. As with the time measures above, AUC values were submitted to a 2 (Distance: close, far) x 2 (Response side: left, right) x 2 (Response mapping: congruent, incongruent) repeated measures analysis of variance. As can be seen in Figure 3, there was a significant main effect of distance, $F(1, 36) = 81.4$, $p < 0.001$, $\eta_p^2 = 0.69$. AUC values for numbers that were close to the comparison standard were larger ($M = 0.68$) than for numbers which were far from the comparison standard (M=0.45). No other terms in the ANOVA model were statistically significant (all F-values less than 3.8). Critically, the lack of an interaction between distance and response mapping ($F(1, 36) = 0.99$, $p = 0.33$) implies that the pattern of decreasing curvatures as a function of distance holds when restricted to incongruent trials. This result supports the competition model over the direct mapping model. Whereas the distance effect reflects a basic signature of holistic representation (at least at the input level), the hand movements reflect a competition between competing response nodes instead of a direct mapping from an internal mental number line to external space.

Decomposed processing signatures

Time analyses. Mouse movement times (MT) were submitted to a 2 (Unit-decade compatibility: compatible, incompatible) x 2 (Response side: left, right) x 2 (Response mapping: congruent, incongruent) repeated measures analysis of variance. There was a significant main effect of unit-decade compatibility, $F(1, 36) = 17.9$, $p < 0.001$, $\eta_p^2 = 0.33$. As shown in Figure 4, movement times for incompatible trials were faster (M=1044 ms) than compatible trials (M=1065 ms). There was a significant three-way interaction between compatibility, response mapping, and response side, $F(1, 36) = 24.3$, $p < 0.001$, $\eta_p^2 = 0.40$. Unit-decade compatibility interacted with response mapping for both leftward and rightward trajectories. However, the nature of this interaction depended on response side (see Figure 4): for rightward trajectories, both response mappings resulted in a unit-decade compatibility effect (albeit in opposite directions). On the other hand, for leftward trajectories, only the incongruent response mapping resulted in a unit-decade compatibility effect.

We also submitted initiation times to a 2 (Distance: close, far) x 2 (Response side: left, right) x 2 (Response mapping: congruent, incongruent) repeated measures analysis of variance. No terms in the ANOVA model were statistically significant (all F-values less than 1.0).

Trajectory analyses. In Figure 5, we present average mouse trajectories as a function of unit-decade compatibility (compatible, incompatible), response mapping (congruent, incongruent), and response side (left, right). To index trajectory curvatures, we submitted AUC values to a 2 (Unit-decade compatibility: compatible, incompatible) x 2 (Response side: left, right) x 2 (Response mapping: congruent, incongruent) repeated measures analysis of variance. While the effect of compatibility approached significance ($p = 0.06$), no main effects or two-way interactions in the ANOVA model were significant (all F-ratios less than 3.7). However, as we can see in Figure 6, there was a significant three-way interaction between compatibility, response mapping, and response side, $F(1, 36) = 52.7$, $p < 0.001$, $\eta_p^2 = 0.59$. Inspection of Figures 5 and 6 shows that in the congruent response mapping (SMALLER - LARGER), trajectories exhibited reduced leftward bias on incompatible trials. This is indexed by a smaller AUC value for incompatible trials in rightward trajectories (compatible, M=0.65; incompatible, M=0.49) and a larger AUC value for incompatible trials in leftward trajectories (compatible, M=0.50; incompatible, M=0.53). Such a result reflects increased activation of the unit digit for incompatible trials throughout the response process, which is consistent with the findings of both Bloechle et al. (2015) and Dotan and Dehaene (2013).

One explanation for this unit bias on incompatible trials may be that the unit digit is the one whose decomposed comparison is opposite of the correct decision. Such inconsistency may result in increased attention being dedicated to this digit (Moeller, Fischer, Nuerk, & Willmes, 2009). As Bloechle et al. (2015) suggest, this increased attention may lead to a direct interaction between basic representation of place value and motor processing (e.g., an embodied, direct mapping view), leading to the hand being drawn rightward (toward the spatial position of the unit digit) on these trials. On the other hand, the bias may be due to increased competition between response nodes (Santens et al., 2011; Faulkenberry, 2016). If the unit bias we observed on incompatible trials is the result of an embodied representation of place value, then we should see the same rightward bias on incompatible trials even with an incongruent response mapping (LARGER - SMALLER). On the other hand, if the unit bias is the result of competition, then the unit bias should be reversed with an incongruent response mapping. That is, trajectories should be drawn leftward instead of rightward. As can be seen in Figure 6, this is exactly what we found. Rightward trajectories exhibited a larger AUC value for incompatible trials than for compatible trials. Also, leftward trajectories exhibited a smaller AUC value for incompatible trials than for compatible trials. Both of these patterns are evidence for a leftward shift in trajectories on incompatible trials. Such a shift cannot be explained by the embodied perspective of Bloechle et al. (2015), and rather reflects an interaction between motor processing and place value that is tied to specific task demands.

Discussion

The purpose of the present study was to examine the dynamics of adults' two-digit number representations. Using computer mousetracking, we tracked and explored the characteristics of participants' hand movements as they chose whether presented two-digit numbers were either larger or smaller than 55. Our participants showed evidence of both holistic and decomposed representations. First, we saw that trajectories became more curved toward the incorrect response as targets became numerically closer to the comparison standard of 55. This dynamic numerical distance effect extends the findings of several recent experiments with single digit number comparison (Song & Nakayama, 2008; Santens et al., 2011; Faulkenberry, 2016) and implies that participants are forming holistic representations when making magnitude decisions about two-digit numbers. Further, we found evidence of decomposed processing by showing that trajectories for unit-decade incompatible exhibited a directional bias that was modulated by response mapping. In all, these results support a hybrid model of two-digit number processing (e.g., Nuerk & Willmes, 2005).

In addition to providing evidence supporting the hybrid model of Nuerk and Willmes (2005), we were able to go further and test between competing explanations of some recent results with response dynamics in two-digit number comparison. With regard to holistic magnitude processing, some researchers have claimed that response dynamics in a comparison task reflect a direct mapping between response effectors (e.g., hands or fingers) and a mental number line (Song & Nakayama, 2008). However, using a line of reasoning similar to Santens et al. (2011) and Faulkenberry (2016), we showed that response curvatures increased with decreasing comparison distance regardless of the response mapping used. Such a result is not predicted by the direct mapping model and instead supports a competition model of number processing (e.g., Verguts et al., 2005). Though their model is not specifically designed for two-digit number tasks, our results indicate that the model of Verguts et al. (2005) is at least conceptually plausible to explain magnitude processing in the two-digit number case.

With regard to decomposed processing, we found that participants did exhibit certain response signatures that were indicative of their attending to the decades and units of two-digit numbers separately. Though many studies have previously shown this to be the case, the present study is one of a very small number of studies (Dotan & Dehaene, 2013; Bloechle et al., 2015) to examine the response dynamics of such decomposed processing. Both Dotan and Dehaene (2013) and Bloechle et al. (2015) showed that the unit digit has a significant influence on manual responses. Specifically, Bloechle et al. (2015) found an overall leftward bias in manual pointing positions that was reduced for unit-decade incompatible numbers, where separate comparison of decades and units leads to opposite decisions. Bloechle et al. (2015) hypothesized that this was due to an embodied representation of two-digit number, where long term memory traces have decades linked spatially with leftward space and units with rightward space. We demonstrated a version of this bias in our mouse trajectories, with trajectories for unit decade incompatible targets tracing rightward of trajectories for compatible targets. Critically, though, we were able to reverse this bias when the response mapping was reversed from LEFT=SMALLER to LEFT=LARGER. That is, trajectories for unit decade incompatible targets traced to the left of the trajectories for compatible targets. Such behavior cannot be explained by a purely embodied representation of two-digit numbers (Bloechle et al., 2015), and instead supports a model where decision dynamics stem from competition among parallel and partially active responses (Freeman, Dale, & Farmer, 2011). Of course, the present work is not sufficient to completely specify the architecture of such a model, so this will likely be an exciting future avenue for research.

While the present study provides an important step toward the description of response dynamics in two-digit number comparison, some inherent limitations persist. First, though we were able to test between competing hypotheses for the response signatures that were indicative of each of two main types of number representation (holistic and decomposed), the design of the study did not allow us to explicitly measure the relative contribution of each type of processing. Indeed, it is currently unclear to what extent the formation of two-digit number representations are driven by holistic and decomposed processing. Certainly, the fact that we saw robust signatures of both holistic and decomposed processing likely rules out both a purely holistic model (e.g., Hinrichs et al., 1981; Dehaene et al., 1990) and a strictly decomposed model (e.g., Verguts & De Moor, 2005). As such, the support that the current work lends to a hybrid model (Nuerk & Willmes, 2005) is purely by elimination, so future work should set out to explicitly test this model in light the predictions it might make about response dynamics.

More broadly, the present work adds to the growing body of literature in numerical cognition that shows support for competitive processing in number tasks ranging among single digit number comparison (Faulkenberry, 2016; Santens et al., 2011), numerical parity (Faulkenberry, 2014), fraction comparison (Faulkenberry et al., 2015), comparison of number pairs (Ganor-Stern & Goldman, 2014), and physical size comparison (Faulkenberry et al., 2016). This work provides empirical data in support of recent computational models of number representation (Verguts et al., 2005; Gevers, Verguts, Reynvoet, Caessens, & Fias, 2006) and showing that a purely embodied perspective is likely insufficient to account for the observed associations between symbolic number and space (Santens & Gevers, 2008; Gevers et al., 2010). Futher, the present study adds to a growing body of work on the dynamics of cognitive processing, representing diverse topics such as stereotype formation (Freeman & Ambady, 2009), voice processing (Sulpizio et al., 2015), language comprehension (Spivey et al., 2005; Incera & McLellan, 2015), memory (Abney, McBride, Conte, & Vinson, 2014; Papesh & Goldinger, 2012), and face processing (Freeman & Ambady, 2011; Hehman, Carpinella, Johnson, Leitner, & Freeman, 2014).

In summary, the present study shows that in two-digit number comparison, adults show evidence of both holistic and decomposed processing. Though this is not the first time such response dynamics have been observed, our study did provide the first evidence that these response patterns are not due to a direct mapping between an embodied number representation and external space, but are rather likely the result of task specific demands that promote competition between parallel and partially active response options.

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Table 1

Two-digit stimuli used in the comparison task

	Holistic distance	
	Close	Far
Unit-decade compatible targets		
Less than 55	31 32 34 35 41 42 43 45	12 13 14 15 21 23 24 25
Greater than 55	56 57 58 59 65 67 68 69	75 76 78 79 85 86 87 89
Unit-decade incompatible targets		
Less than 55	36 37 38 39 46 47 48 49	16 17 18 19 26 27 28 29
Greater than 55	61 62 63 64 71 72 73 74	81 82 83 84 91 92 93 94

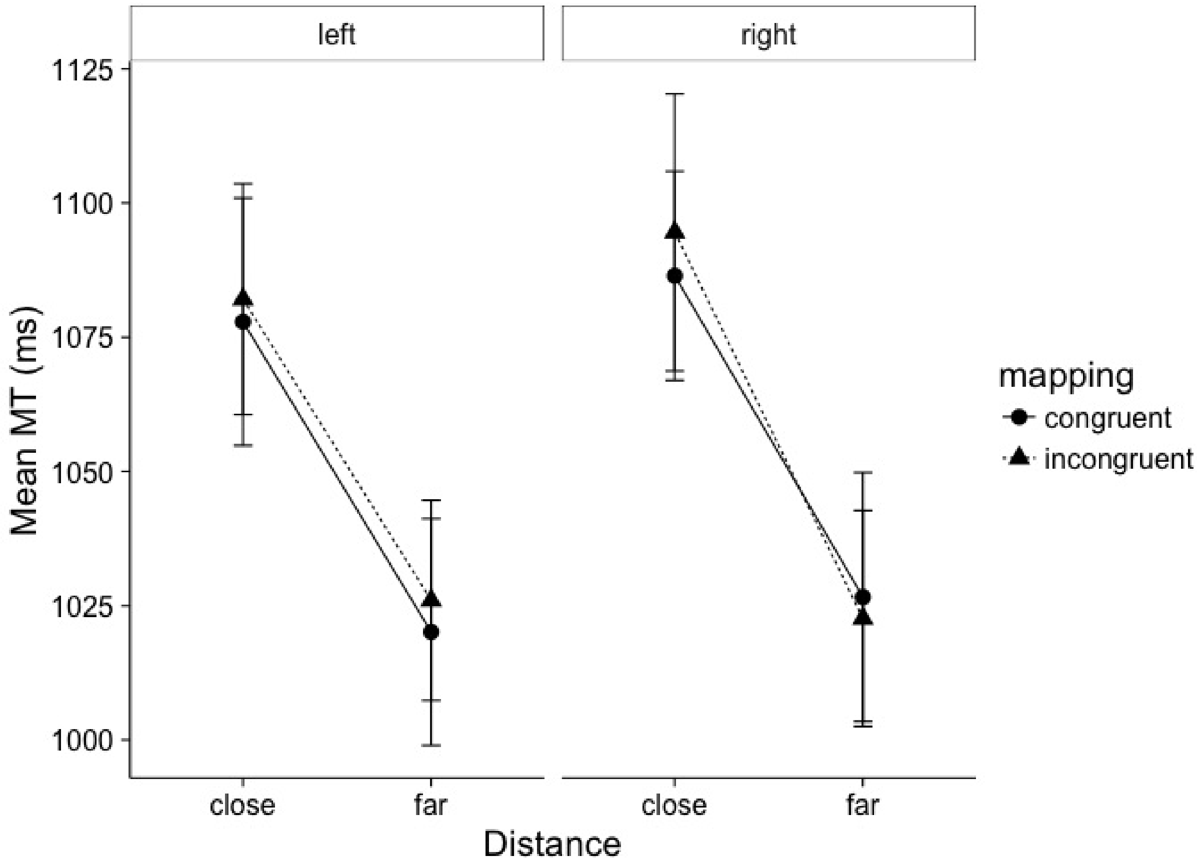

Figure 1. Mean movement times as a function of distance (close, far), response side (left, right), and response mapping (congruent, incongruent). Error bars represent within-subject 95% confidence intervals as recommended by Morey (2008).

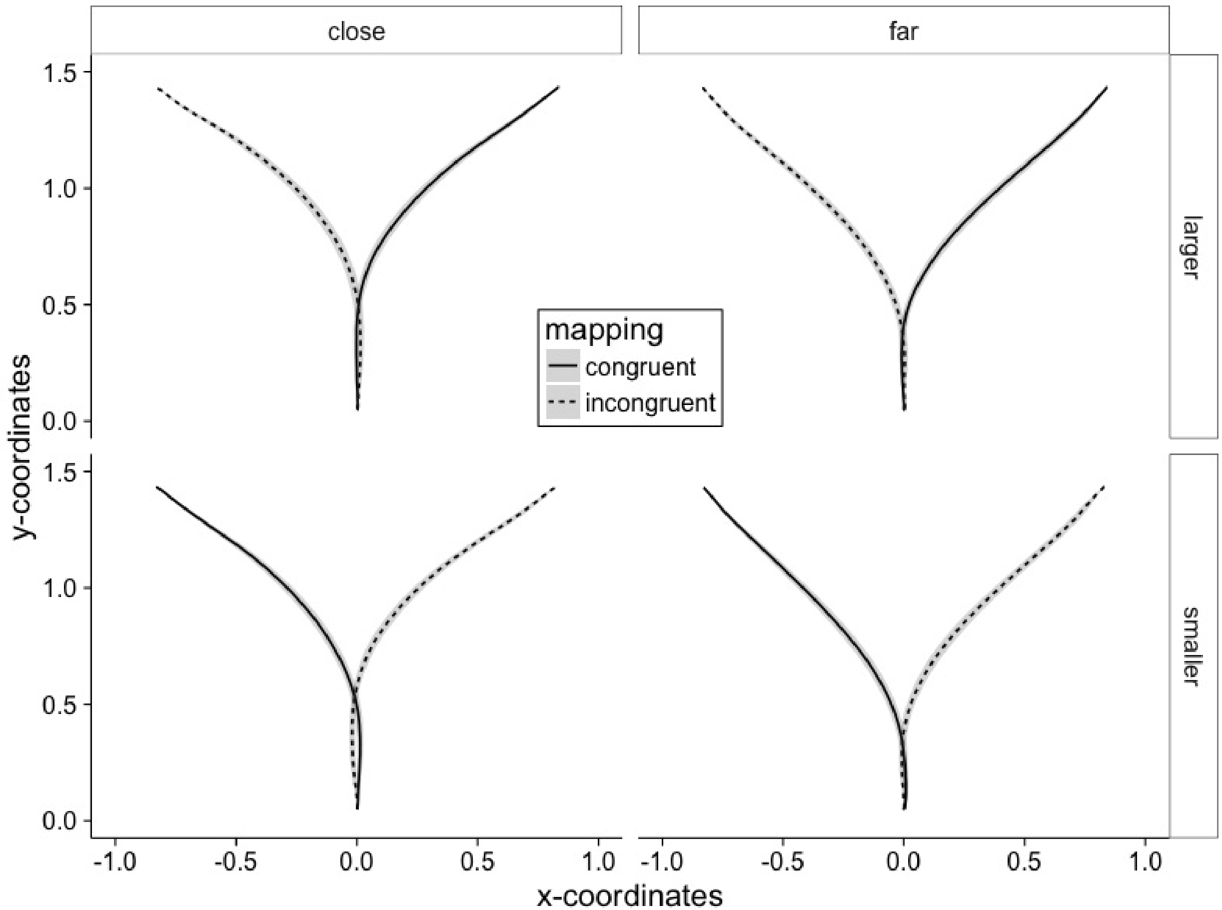

Figure 2. Average computer mouse trajectories as a function of distance (close, far), response mapping (congruent, incongruent), and decision (larger than 55, smaller than 55). Shading represents one standard error, computed from the mean x-coordinates of trajectories over the sample of 37 participants.

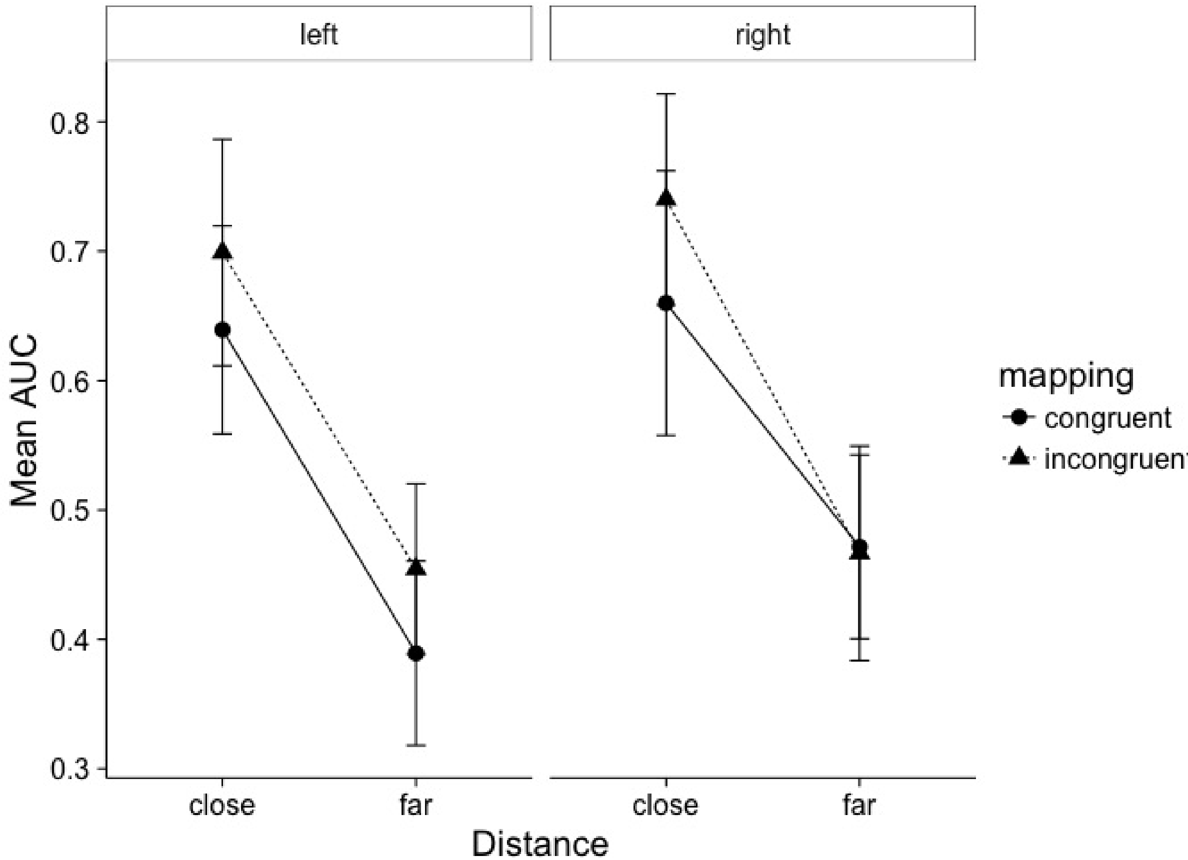

Figure 3. Mean AUC values as a function of distance (close, far), response side (left, right), and response mapping (congruent, incongruent). Error bars represent within-subject 95% confidence intervals as recommended by Morey (2008).

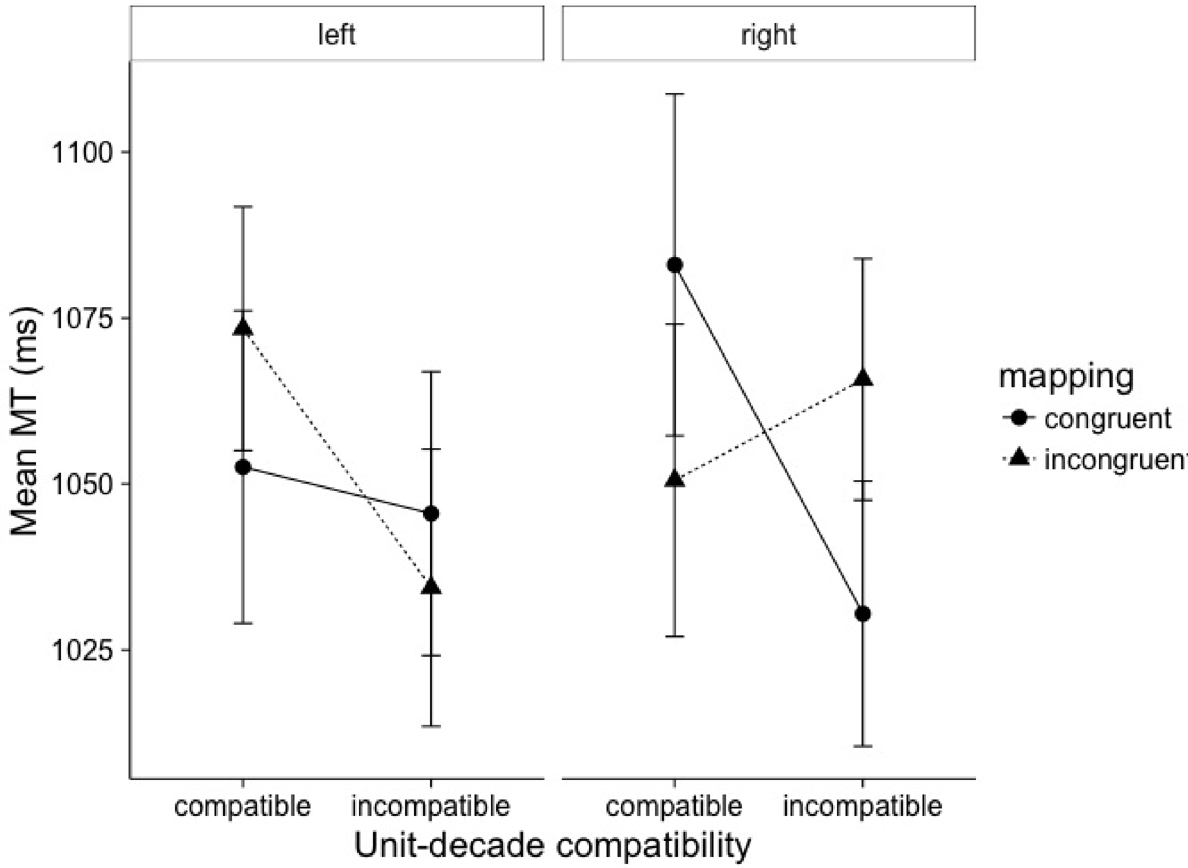

Figure 4. Mean movement times as a function of unit-decade compatibility (compatible, incompatible), response side (left, right), and response mapping (congruent, incongruent). Error bars represent within-subject 95% confidence intervals as recommended by Morey (2008).

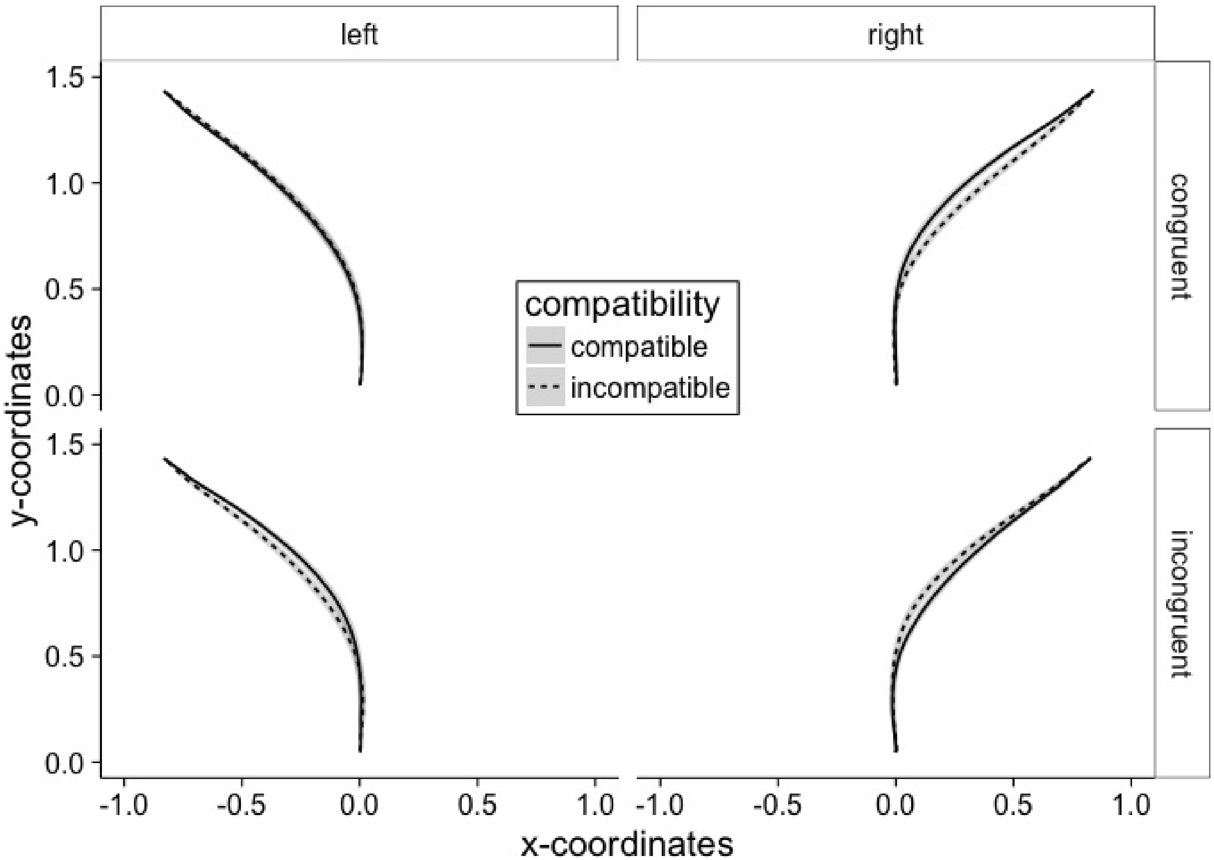

Figure 5. Average computer mouse trajectories as a function of unit-decade compatibility (compatible, incompatible), response mapping (congruent, incongruent), and response side (left, right). Shading represents one standard error, computed from the mean x-coordinates of trajectories over the sample of 37 participants.

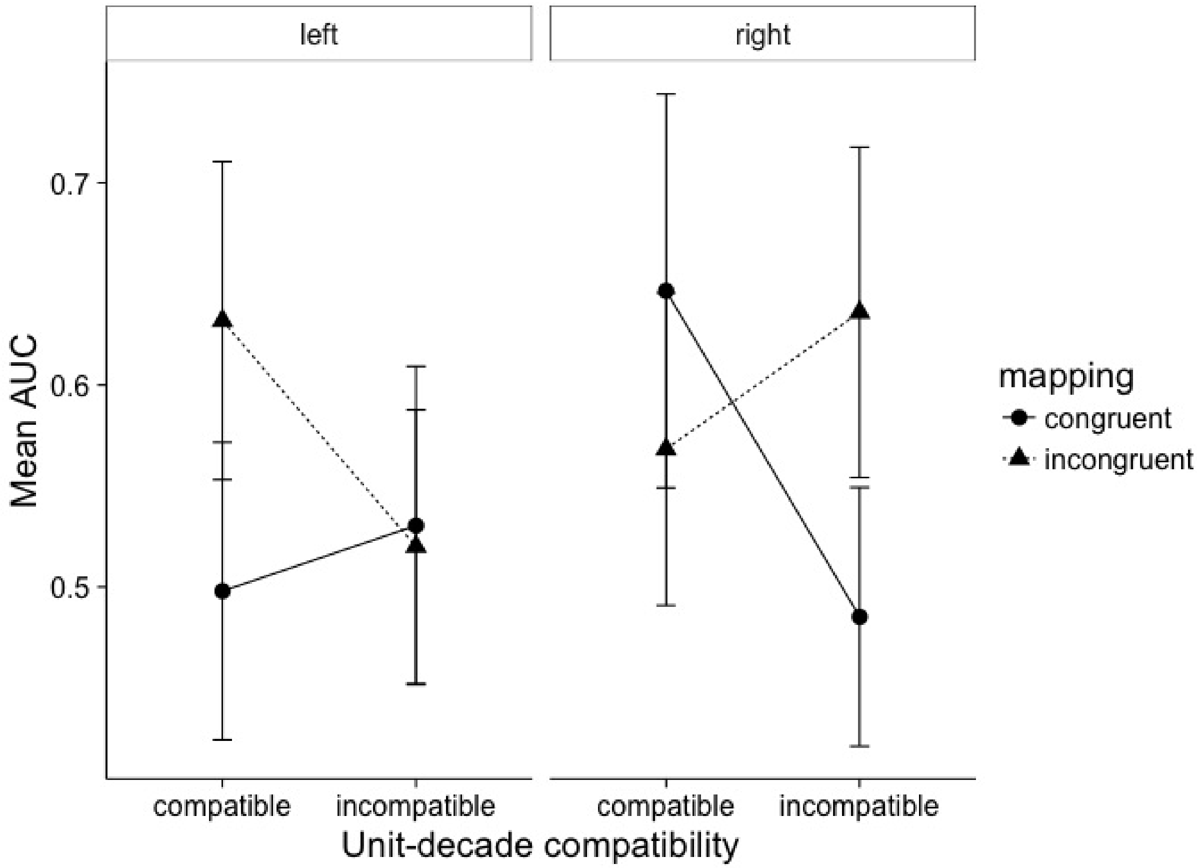

Figure 6. Mean AUC values as a function of unit-decade compatibility (compatible, incompatible), response side (left, right), and response mapping (congruent, incongruent). Error bars represent within-subject 95% confidence intervals as recommended by Morey (2008).