Introduction

Digital Signal Processing

August 26, 2025

What is Digital Signal Processing?

What is Digital Signal Processing?

What is a signal?

What a Signal *Can* Be

A signal can be...

- a function that carries **information**
- a quantity that **varies** over time and/or space
- a collection of measurements from a **sensor**
- a collection of **synthesized** values

Examples:

- audio
- images
- video
- radar
- sonar
- lidar
- voltage
- radio waves
- gamma / x-rays

Mathematical Definition of a Signal

Mathematically, a signal is a *function*, $x(a)$, from an input set A, to an output set B:

$$x : A \to B$$

The input set, A, is typically time, space, or both time and space

The output set, B, is typically integers ($\mathbb{Z}$), real numbers ($\mathbb{R}$), or complex numbers ($\mathbb{C}$)

What is Digital Signal Processing?

What is digital?

Digital = Discrete

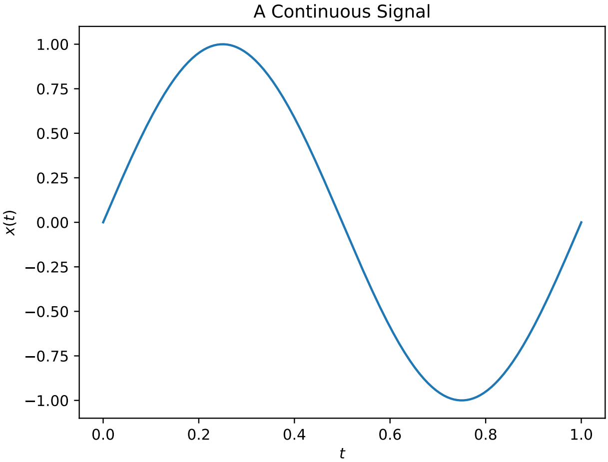
A **digital signal** is a function

$$x : A \to B,$$

where both the input set, A, and output set, B, are discrete.

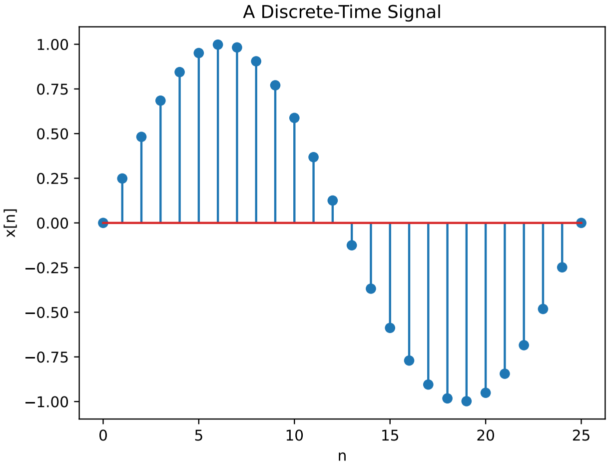
Notation:

$$x[n], \quad n \in \mathbb{Z}$$



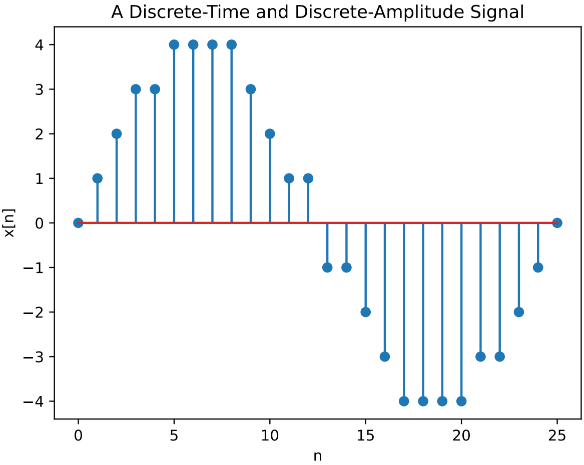
$$x : \mathbb{R} \to \mathbb{R}$$

$$x(t) = \sin(2\pi t)$$



$$x : \mathbb{Z} \to \mathbb{R}$$

$$x[n] = \sin\left(2\pi \frac{n}{25}\right)$$



$$x : \mathbb{Z} \to \mathbb{Z}$$

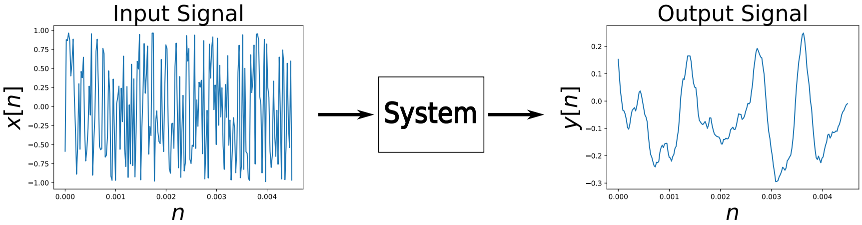
$$x[n] = \text{round}\left(4 \sin\left(2\pi \frac{n}{25}\right)\right)$$

What is Digital Signal Processing?

What is processing?

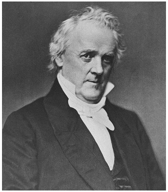
What is Digital Signal Processing?

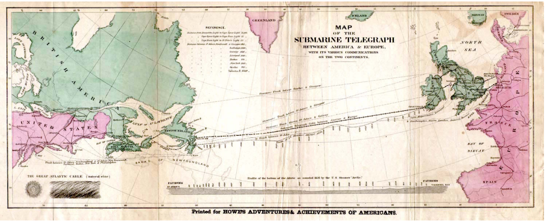
A **system** is any process that takes a signal as input and returns a signal as output.



Historical Example: Communications

First Transatlantic Telegraph, 1858





U.S. President Buchanan

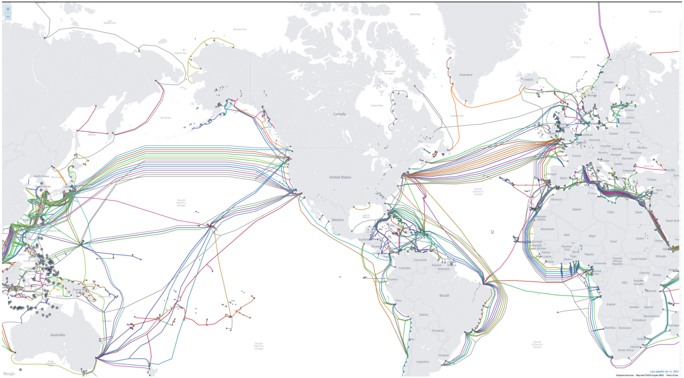
“an additional link between the nations whose friendship is founded on their common interest and reciprocal esteem”

U.K. Queen Victoria

This message took 17 hours and 40 minutes to transmit!

How Does Information (Internet) Travel Across Continents Today?

Still using underwater cables!



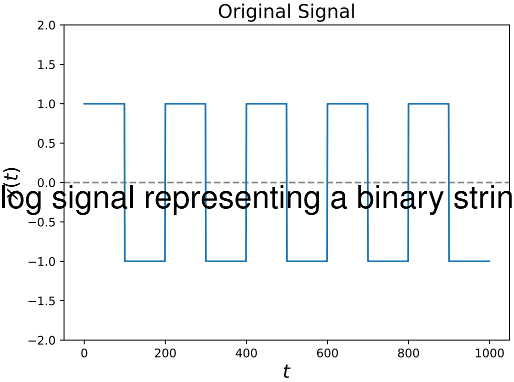
https://www.submarinecablemap.com/

History of Transatlantic Speeds

Cable	Year	Type	Signaling	Capacity
	1858	Coax	telegraph	a few words per hour
	1866	Coax	telegraph	6-8 words per minute
	1928	Coax	telegraph	2500 characters/min.
TAT-1	1956	Coax	telephone	36 voice channels
TAT-3	1963	Coax	telephone	138 voice channels
TAT-5	1970	Coax	telephone	845 voice channels
TAT-6	1976	Coax	telephone	4000 voice channels
TAT-8	1988	Fiber	data	280 Mbit/s (40K v.c.)
TAT-14	2000	Fiber	data	640 Gbit/s (9,7M v.c.)
GTT	2015	Fiber	data	53 Tbit/s
Google	2021	Fiber	data	250 Tbit/s

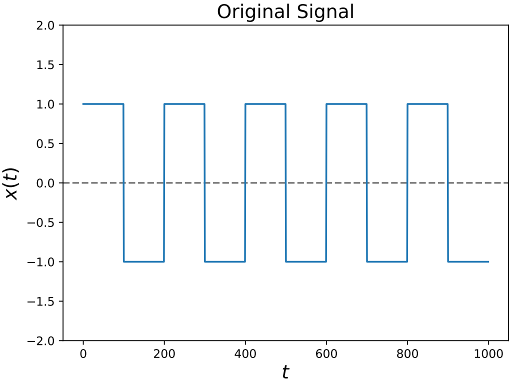
From https://atlantic-cable.com/Cables/speed.htm

Analog vs. Digital Transmission



$x(t)$ is an analog signal representing a binary string

Analog vs. Digital Transmission



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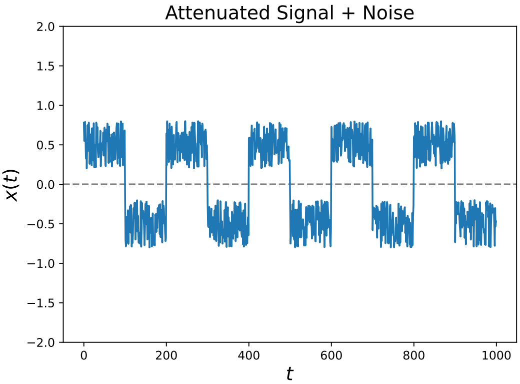
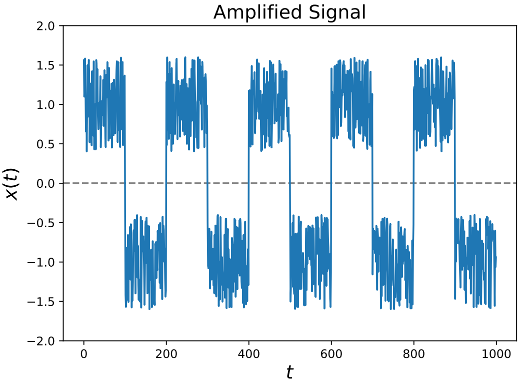
Analog vs. Digital Transmission



$x(t)$ becomes attenuated and noisy:

$$x_c(t) = \alpha x(t) + \epsilon(t), \qquad \alpha < 1, \epsilon(t) = \text{noise}$$

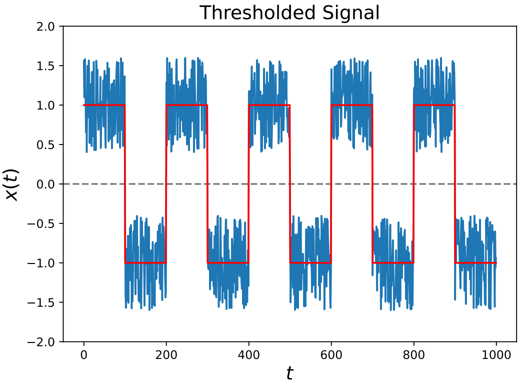
Analog vs. Digital Transmission



Amplifying $x_c(t)$ recovers amplitude, but magnifies noise:

$$x_a(t) = \frac{1}{\alpha}x_c(t) = \frac{1}{\alpha}(\alpha x(t) + \epsilon(t)) = x(t) + \frac{1}{\alpha}\epsilon(t)$$

Analog vs. Digital Transmission



Recover "perfect" $x(t)$ by thresholding:

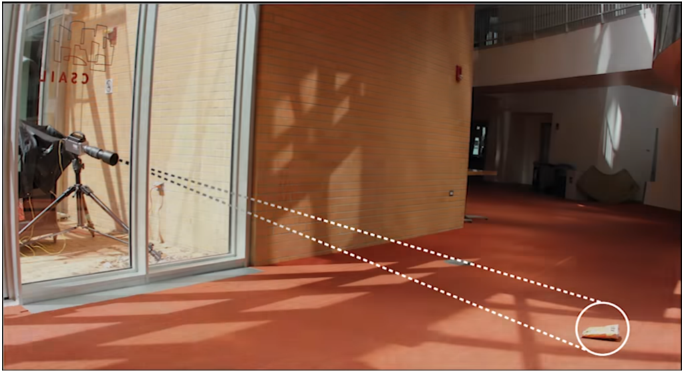
$$\tilde{x}(t) = \begin{cases} -1 & \text{if } x_a(t) < 0 \\ +1 & \text{if } x_a(t) \geq 0 \end{cases}$$

Digital Data Compression

High-speed data transfer is also made possible by compression

Data Type	Compression Ratio
Image	10:1 (JPEG)
Audio	12:1 to 25:1 (MPEG)
Video	50:1 to 2000:1 (H.264)

Spying With a Bag of Chips?



https://www.youtube.com/watch?v=FKXOucXB4a8

Textbooks on DSP

(These are not required for this course!)

- Oppenheim and Schafer, *Discrete-Time Signal Processing*
  Classic reference, mathematically demanding, not free
- Downey, *Think DSP*
  Python, less mathematics, free to download
- Prandoni and Vetterli, *Signal Processing for Communication*
  Linear algebra perspective, free to download