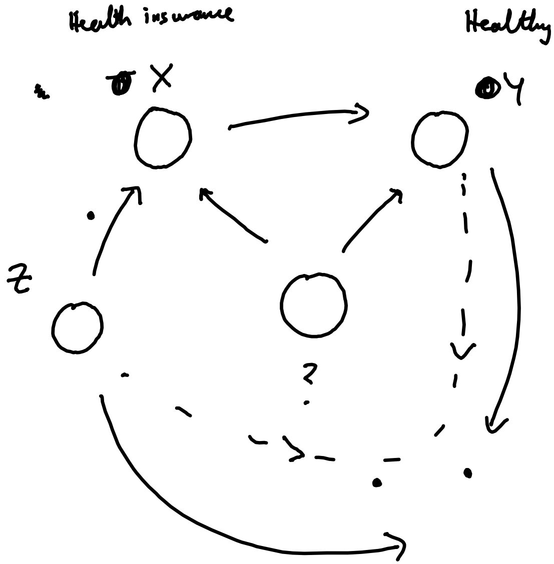
EC2020 - Session 6



OLS vs. IV

conditions for IV

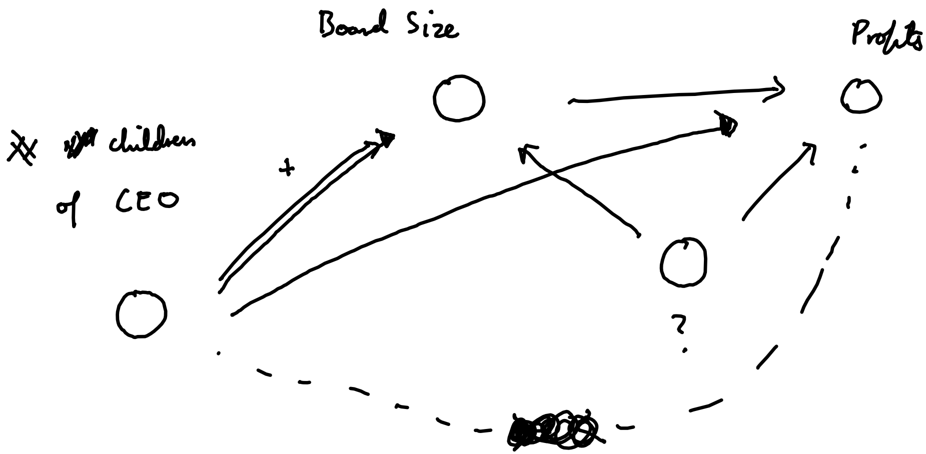
$y_i = x_i\beta + \varepsilon_i$

$$\begin{cases} \operatorname{cov}(x_i, z_i) \neq 0 & \text{(1)} \\ \text{and} \quad \operatorname{cov}(z_i, \varepsilon_i) = 0 & \text{(2)} \end{cases}$$

(1) FIRST STAGE

(2) EXCLUSION RESTRICTION

"AS GOOD AS RANDOMLY ASSIGNED"



$$y_i = \hat{\beta}_0 + \hat{\beta}_1 x_i + \hat{\varepsilon}_i$$

<u>First stage</u>: ✓ — $cor(x_i, z_i) \neq 0$ — $\widehat{cor}(x_i, z_i)$ — Testable

<u>Exclusion</u>: ✓

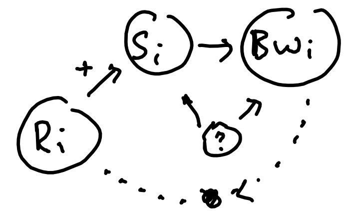
<u>AGARA</u>: ✓ — $cor(z_i, \varepsilon_i) = 0$ — $\widehat{cor}(z_i, \varepsilon_i) = 0$ — Untestable

Q3:

Birth weight $\quad\quad$ smoked.

$$BW_i = \beta_0 + \beta_1 S_i + u_i$$

$$R_i = \begin{cases} 1 & \text{got leaflet} \\ 0 & \text{didn't} \end{cases}$$



$\alpha_1 = 5$

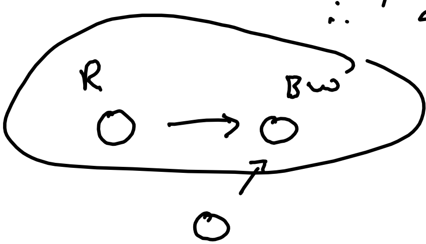
$$BW_i = \alpha_0 + \alpha_1 R_i + \varepsilon_i$$

How to interpret coefficients

$E(BW_i)$ for those who got $= \alpha_0 + \alpha_1 \cdot 1$

$E(BW_i)$ for those who didn't $= \alpha_0 + \cancel{\alpha_1 \cdot 0}$

$\therefore\ \alpha_1 = \underbrace{E \text{ for those who did}}_{\alpha_0 + \alpha_1} - \underbrace{E \text{ for those who didn't}}_{\alpha_0}$



$$y_i = \beta_0 + \beta_1 x_i + u_i \qquad cor(x_i, u_i) \neq 0$$

Relevance / FS:

$$cor(x_i, z_i) \neq 0$$

Exogeneity:

$$cor(z_i, u_i) = 0$$

$$cor(y_i, z_i) = cor(\beta_0 + \beta_1 x_i + u_i, z_i)$$

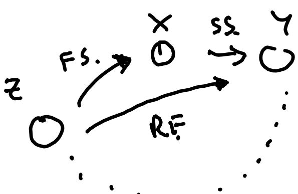
$$= \cancel{cor(\beta_0, z_i)} + cor(\beta_1 x_i, z_i) + \cancel{cor(u_i, z_i)}$$

$$= \beta_1 \, cor(x_i, z_i)$$

$$\boxed{\hat{\beta}_1 = \frac{\widehat{cor}(y_i, z_i)}{\widehat{cor}(x_i, z_i)}} \qquad \beta_1 = \frac{cor(y_i, z_i)}{cor(x_i, z_i)}$$

$$y_i = \beta_1 x_i + u_i$$

$$\hat{\beta}_1 = \frac{\widehat{cov}(x_i, y_i)}{\widehat{Var}(x_i)}$$



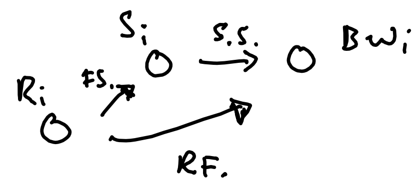
$$\boxed{\hat{\beta}_1 = \frac{\widehat{cov}(y_i, z_i) / \widehat{Var}(z_i)}{\widehat{cov}(x_i, z_i) / \widehat{Var}(z_i)}}$$

$$y_i = \alpha_0 + \alpha_1 z_i + v_i$$

$$x_i = \gamma_0 + \gamma_1 z_i + \varepsilon_i$$

Bristol: $Bw_i,\ R_i,\ \cancel{S_i}$ $\quad$ } $\underline{RF}$ ✓ $\quad$ FS ✗

Bath: $\cancel{Bw_i},\ R_i,\ S_i$ $\quad$ } RF ✗ $\quad$ $\underline{FS}$ ✓



$$\hat{\beta}_{IV} = \frac{\hat{RF}}{\hat{FS}}$$