EC2020 Elements of Econometrics — Session 1

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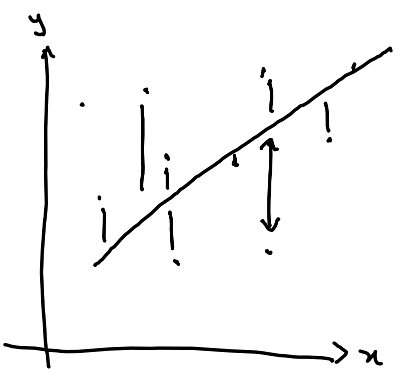
Wooldridge

x independent y dependent $x_1 \dots x_n$

→ regressor $\frac{1}{n}\sum x_i$

explanatory variable

covariate



$$y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$$

$$\hat{\beta}_1 = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sum (x_i - \bar{x})^2}$$

Q1

$price_i$

bdr_i

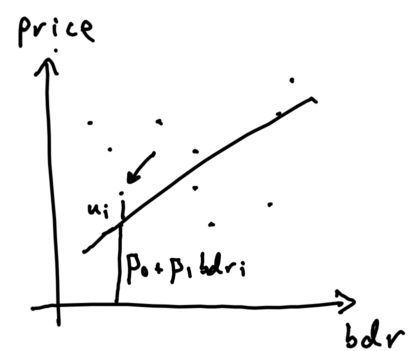
$$price_i = \beta_0 + \beta_1 \, bdr_i + u_i$$

$\hat{\beta}_1$ to be unbiased $\hat{\beta}_1$ estimates β_1

Unbiasedness: $\mathbb{E}\,\hat{\beta}_1 = \beta_1$

Efficiency: $Var(\hat{\beta}_1)$ "low"

(1) $\hat{\beta}_1$ is unbiased if $\boxed{bdr_i,\ u_i}$ uncorrelated

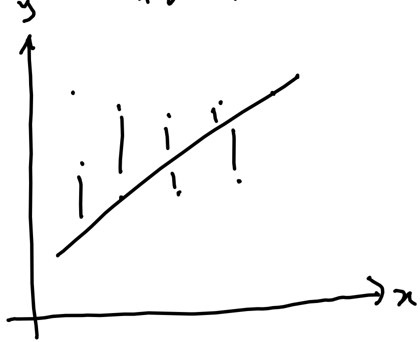


$bdr \to price$

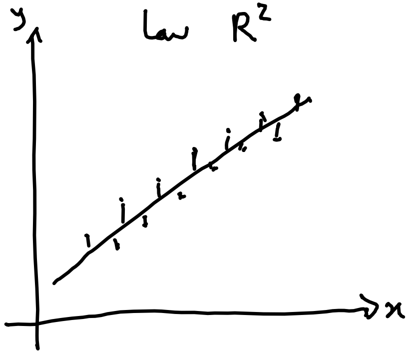
u_i

R^2

High R^2



Low R^2



$$price_i = \underbrace{\hat{\beta}_0 + \hat{\beta}_1 bdr_i} + \underbrace{\hat{u}_i}$$

$$R^2 = \frac{SSE}{SST} = 1 - \frac{\sum \hat{u}_i^2}{\sum (y_i - \bar{y})^2}$$

$$R^2 = 0.17$$

$$price_i = \overbrace{\beta_0 + \beta_1 sqft_i} + \overbrace{u_i}$$

$$\underline{R^2 = 0.99}$$

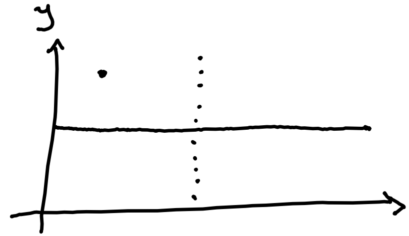
<u>$sqft_i$</u> price

<u>colour front door</u>

Q4

$$y_i = b_0 + u_i$$

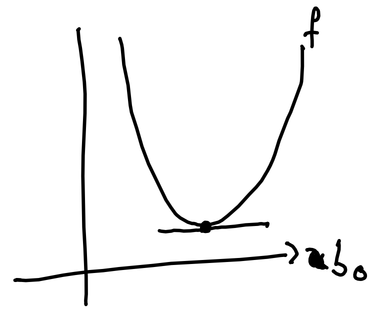
$$\boxed{\min_{b_0} \sum_{i=1}^{n} (y_i - b_0)^2} = f(b_0)$$



$\cancel{y_i = \beta_0 + \beta_1 x_i + u_i}$

~~(price_i, ...)~~

$$\boxed{\min_{x} f(x)}$$



- $\frac{d}{dx}\left(f(x) + g(x)\right) = \frac{df}{dx} + \frac{dg}{dx} \rightarrow$

$$\frac{d}{dx}\left(\sum f_i(x)\right) = \sum \frac{d}{dx}\left(f_i(x)\right)$$

$$\frac{d}{db_0}\left(\sum (y_i - b_0)^2\right) = \sum \frac{d}{db_0}(y_i - b_0)^2 = \sum_{i=1}^{n} -2(y_i - b_0)$$

$$\sum_{i=1}^{n} -2(y_i - \hat{b}_0) = 0$$

$$\sum_{i=1}^{n} (y_i - \hat{b}_0) = 0 \quad \rightarrow \quad \cancel{\sum y_i - \sum \hat{b}_0 = 0}$$

$$\sum_{i=1}^{n} y_i - n\hat{b}_0 = 0$$

$$\rightarrow \boxed{\hat{b}_0 = \frac{1}{n}\sum_{i=1}^{n} y_i} \quad \square$$

4b

$$y_i = b_0 + u_i \qquad \text{Errors: } u_i$$

$$y_i = \hat{b}_0 + \hat{u}_i \qquad \text{Residuals: } \hat{u}_i, \bar{u}_i$$

RTP: $\boxed{\sum \hat{u}_i = 0}$

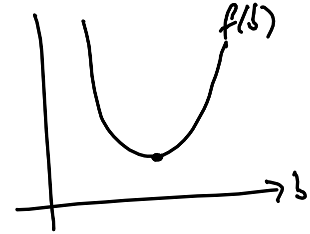
$$\sum \hat{u}_i = \sum (y_i - \boxed{\hat{b}_0})$$

$$= \sum y_i - \sum \hat{b}_0 = n\hat{b}_0 - \sum \hat{b}_0 = n\hat{b}_0 - n\hat{b}_0 = 0 \quad \square$$

Q5 a

$$y_i = \beta_1 x_i + u_i$$

$$y_i = \hat{\beta}_1 x_i + \hat{u}_i$$



$$\sum \hat{u}_i^2 = \sum (y_i - \hat{\beta}_1 x_i)^2$$

$$\min_b \sum (y_i - b x_i)^2$$

$$\sum_i \frac{d}{db} (y_i - b x_i)^2$$

$$= \sum_i 2 (y_i - b x_i)(-x_i) = -2 \sum_i x_i (y_i - b x_i)$$

$$\sum_i x_i (y_i - x_i \hat{b}) = 0$$

$$\boxed{\sum a x_i = a \sum x_i}$$

$$\sum_i (x_i y_i - x_i^2 \hat{b}) = 0$$

$$\sum_i x_i y_i = \sum_i (x_i^2 \hat{b}) = \hat{b} \sum_i x_i^2$$

$$\boxed{\hat{b} = \frac{\sum x_i y_i}{\sum x_i^2}}$$

$$y_i = b x_i + u_i$$

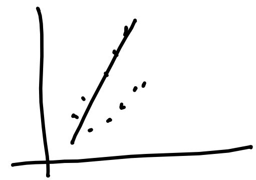
$$\simeq \boxed{\frac{\mathrm{cov}(x_i, y_i)}{\mathrm{Var}(x_i)}} \quad \mapsto \quad y_i = a + b x_i + u_i$$

$$\boxed{\sum x_i^2 \neq 0}$$

when is $\sum x_i^2 = 0$

$\Leftrightarrow$

all x_i are 0



SLR3

5b

$$\boxed{\hat{\beta} = \frac{\sum x_i y_i}{\sum x_i^2}}$$

$$\boxed{y_i = b x_i + u_i}$$

SLR

$$Var(\hat{\beta}) = Var\left(\frac{\sum x_i y_i}{\sum x_i^2}\right)$$

$$= Var\left(\frac{\sum x_i (b x_i + u_i)}{\sum x_i^2}\right)$$

$$= Var\left(b \frac{\cancel{\sum x_i^2}}{\cancel{\sum x_i^2}} + \frac{\sum x_i u_i}{\sum x_i^2}\right)$$

$Var(a + X) = Var(X)$ $\rightarrow$

$$= Var\left(\frac{\sum x_i u_i}{\sum x_i^2}\right)$$

$Var(aX) = a^2 Var(X)$

$+ x_i$ fixed $\rightarrow$

$$= \left(\frac{1}{\sum x_i^2}\right)^2 Var\left(\sum x_i u_i\right)$$

u_i independent

$Var(\sum x_i) = \sum Var(x_i)$ $\rightarrow$

$$= \left(\frac{1}{\sum x_i^2}\right)^2 \sum x_i^2 \, Var(u_i)$$

$$Var(\hat{\beta}) = \left(\frac{1}{\sum x_i^2}\right)^2 \sum x_i^2 \, Var(u_i)$$

$$= \left(\frac{1}{\sum x_i^2}\right)^2 \sum x_i^2 \sigma^2 = \frac{\sigma^2}{\sum x_i^2} \quad \square$$

SLRS

HOMOSCEDASTICITY