EC2020 Session 4

2) C - consumption

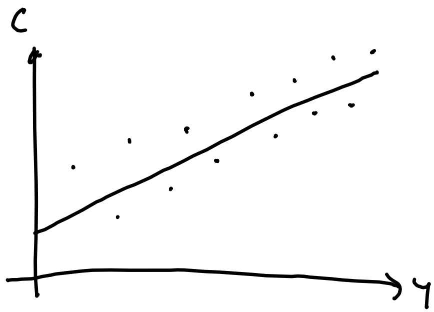
Y - income

$$\boxed{C_i = \beta_0 + \beta_1 Y_i + u_i}$$

Y_i not random

$\mathbb{E} u_i = 0$

functional form of a regression

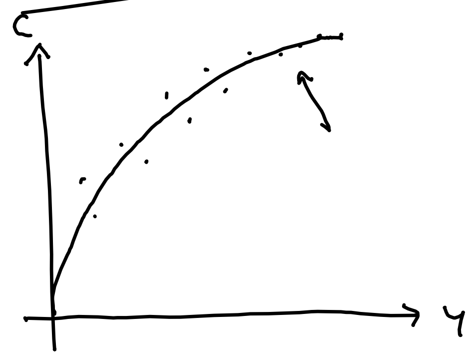


MPC is constant

$\frac{dC}{dY}$

Instead:

$$C_i = \beta_0 + \beta_1 Y_i + \beta_2 Y_i^2 + u_i$$



$$\begin{array}{l} \qquad\quad [14] \quad [15] \quad\ \ [16] \\ a) \quad C_i = \underline{\beta_0} + \underline{\beta_1} Y_i + \underline{\beta_2} Y_i^2 + u_i \end{array}$$

data: £ (C, Y)

new data: £1000 $(\tilde{C}, \tilde{Y})$

old: (C, Y)

new: $(\tilde{C}, \tilde{Y})$

Workflow of Econometrics
- Assumptions
- Interpretation
- Estimation

$$\begin{bmatrix} C = 5000 & , \tilde{C} = 5 \\ Y = 8000 & , \tilde{Y} = 8 \end{bmatrix} \leftarrow$$

$$\tilde{C} = \frac{C}{1000} \qquad \tilde{Y} = \frac{Y}{1000}$$

$$1000\,\tilde{C}_i = \beta_0 + \beta_1 (1000)\tilde{Y}_i + \beta_2 (1000)^2 \tilde{Y}_i^2 + u_i$$

$$\tilde{C}_i = \frac{\beta_0}{1000} + \beta_1 \tilde{Y}_i + \beta_2 (1000)\tilde{Y}_i^2 + u_i$$

$$\left[\frac{14}{1000}\right] \qquad [15] \qquad [16\,000]$$

Short

b) $C_i = \beta_0^s + \beta_1^s Y_i + e_i$

Long

$C_i = \beta_0 + \beta_1 Y_i + \beta_2 Y_i^2 + u_i$

OVB formula

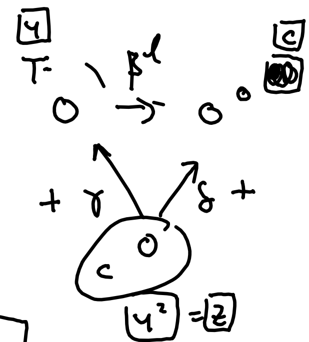
Treatment → T : Health insurance

O : How healthy

C : family income

$T \xrightarrow{\beta^s} O$

$O_i = \alpha + \beta^s T_i + \varepsilon_i$



$O_i = \alpha^l + \beta^l T_i + \delta C_i + u_i$

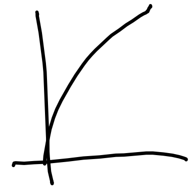
OVB: $\boxed{\beta^s = \beta^l + \underbrace{\gamma\delta}_{+}}$

and so: $\beta^s > \beta^l$

$y_i^2 = z_i$

Short: $c_i = \beta_0^s + \beta_1^s y_i + e_i$

Long: $c_i = \beta_0^l + \beta_1^l y_i + \beta_2^l y_i^2 + u_i$



~~steps~~

step 1: write out estimator

step 2: plug in the truth (long reg.)

Prove OVB formula:

$$\boxed{\hat{\beta}_1^s = \frac{\sum (y_i - \bar{y})(c_i - \bar{c})}{\sum (y_i - \bar{y})^2} = \frac{\hat{cov}(y_i, c_i)}{\hat{var}(y_i)}}$$

$$= \frac{\sum (y_i - \bar{y})(\cancel{\beta_0} + \beta_1^l y_i + \beta_2^l y_i^2 + u_i - \cancel{\beta_0} - \beta_1^l \bar{y} - \beta_2^l \bar{y}^2 - \bar{u})}{\sum (y_i - \bar{y})^2}$$

$$= \beta_1^l \frac{\sum (y_i - \bar{y})^2 \cancel{(y_i - \bar{y})}}{\sum (y_i - \bar{y})^2} + \beta_2^l \frac{\sum (y_i - \bar{y})(z_i - \bar{z})}{\sum (y_i - \bar{y})^2} + \frac{\sum (y_i - \bar{y})(u_i - \bar{u})}{\sum (y_i - \bar{y})^2}$$

$$\boxed{\beta^s = \beta^l + \gamma\delta}$$

$\beta_1^s \quad \beta_1^l \quad \beta_2^l$

coefficient of y_i^2 on y_i

$$= \beta_1^l + \beta_2^l \frac{\hat{cov}(y_i, y_i^2)}{\hat{var}(y_i)} + \frac{\sum (y_i - \bar{y})(u_i - \bar{u})}{\sum (y_i - \bar{y})^2}$$

$$\hat{\beta}_1^s = \beta_1^l + \beta_2^l \hat{\gamma} + \frac{\sum (y_i - \bar{y})(u_i - \bar{u})}{\sum (y_i - \bar{y})^2}$$

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$$\mathbb{E}\,\hat{\beta}_1^s = \beta_1^l + \mathbb{E}\left[\beta_2^l \hat{\gamma}\right] + \mathbb{E}\left[\frac{\sum (y_i - \bar{y})(u_i - \bar{u})}{\sum (y_i - \bar{y})^2}\right]$$

① • u is independent of $Y_1 \dots Y_n$ $\longrightarrow$ $\mathbb{E}\left[u_i \mid Y_1 \dots Y_n\right] \underset{①}{=} \mathbb{E}(u_i) \underset{②}{=} 0$

② • $\mathbb{E}(u) = 0$

LAW OF ITERATED EXPECTATIONS

$$\mathbb{E}\left[\frac{\sum (y_i - \bar{y})(u_i - \bar{u})}{\sum (y_i - \bar{y})^2}\right] \overset{\text{LIE}}{=} \mathbb{E}\left[\mathbb{E}\left[\frac{\sum (y_i - \bar{y})(u_i - \bar{u})}{\sum (y_i - \bar{y})^2} \,\middle|\, Y\right]\right]$$

$$= \mathbb{E}\left[\frac{\sum (y_i - \bar{y})(\cancel{\mathbb{E}} u_i - \cancel{\mathbb{E}} \bar{u})}{\sum (y_i - \bar{y})^2}\right] = 0$$

$$\mathbb{E}\,\hat{\beta}_1^s = \beta_1^l + \overbrace{\mathbb{E}\left[\underbrace{\beta_2^l}_{-}\, \underbrace{\hat{\gamma}}_{+}\right]}^{\text{negative}}$$

3a) • N° affairs A

• Rates Marriage (1–5) RM

• Religion (1–5) R

• N° years married Y

$\boxed{C}$

$$A_i = \beta_0 + \beta_1 RM_i + \beta_2 R_i + \beta_3 Y_i + u_i$$

$$\frac{\hat{\beta}_1 = -0.678}{se(\hat{\beta}_1) = 0.161}$$

Interpret:

• what does the variable mean

• statistical significance

$H_0: \beta_1 = 0$

$H_1: \beta_1 \neq 0$

$T \approx 6$

• Think of confounders

economic significance