EC2020 Session 7

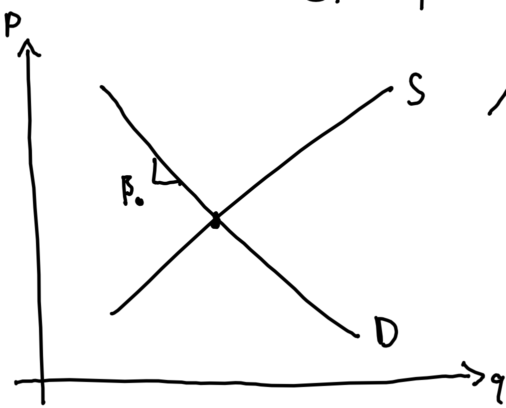
tomglinnan.github.io/EC2020

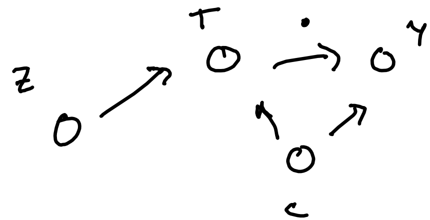
simultaneous eq^{n}s.

(p_i, q_i)

$$p_i = \alpha + \beta q_i + \varepsilon_i$$

$$\hat{\beta} \simeq 0$$





$$y_i = \alpha + \beta x_i + u_i$$

$$\mathrm{cor}(x_i, u_i) \neq 0$$

z_{1i},

measurement error

a) $y_i = \beta x_i^* + u_i$

$x_i = x_i^* + e_i$

y_i = health

x_i^* = income

x_i = self-reported income

we see (x_i, y_i) not (x_i^*, y_i)

CEV assumptions

- $cor(x_i^*, e_i) = 0$
- $cor(e_i, u_i) = 0$

b) Ideally, we would do OLS of y_i on x_i^* because of G-M assumptions

As we can't, we try OLS of y_i on x_i

$\hat{\beta} \not\xrightarrow{p} \beta$

$$\begin{cases} y_i = \beta x_i^* + u_i \\ x_i = x_i^* + e_i \end{cases} \Rightarrow x_i^* = x_i - e_i$$

$$y_i = \beta x_i \underbrace{- \beta e_i + u_i}_{v_i} = \beta x_i + v_i$$

$$\hat{\beta} = \frac{\sum x_i y_i}{\sum x_i^2} = \frac{\sum x_i (\beta x_i + v_i)}{\sum x_i^2}$$

$$= \frac{\beta \sum \cancel{\beta} x_i^2 + \sum x_i v_i}{\sum x_i^2}$$

$$= \beta + \frac{\frac{1}{n} \sum x_i v_i}{\frac{1}{n} \sum x_i^2}$$

$$\hat{\beta} \xrightarrow{p} \beta + \frac{\mathbb{E}\, x_i v_i}{\mathbb{E}\, x_i^2}$$

$\hat{\beta} \xrightarrow{p} \beta$

$$\boxed{y_i = \beta x_i + v_i}$$

$$\frac{1}{n} \sum f(x_i) \xrightarrow{p} \mathbb{E} f(x_i)$$

$$\frac{1}{n} \sum x_i v_i \xrightarrow{p} \mathbb{E}\, x_i v_i$$

$$\frac{1}{n} \sum x_i^2 \xrightarrow{p} \mathbb{E}\, x_i^2$$

$$\hat{\beta} \xrightarrow{p} \beta + \frac{\mathbb{E}x_i v_i \ ①}{\mathbb{E}x_i^2 \ ②}$$

$$\begin{cases} y_i = \beta x_i^* + u_i \\ x_i = x_i^* + e_i \end{cases}$$

$\mathbb{E}x_i^* u_i = 0$

$v_i = u_i - \beta e_i$

CEV: $\operatorname{cov}(e_i, x_i^*) = 0 = \mathbb{E}e_i x_i^*$

$\operatorname{cov}(e_i, u_i) = 0 = \mathbb{E}e_i u_i$

$$\operatorname{cov}(A,B) = \mathbb{E}(AB) - \cancel{(\mathbb{E}A)(\mathbb{E}B)}$$

$$② : \mathbb{E}(x_i^2) = \mathbb{E}\left[(x_i^* + e_i)^2\right] = \mathbb{E}\left[(x_i^*)^2 + 2x_i^* e_i + e_i^2\right]$$

$$= \mathbb{E}(x_i^*)^2 + \mathbb{E}e_i^2$$

$$\operatorname{Var}(e_i) = \mathbb{E}e_i^2 \cancel{- (\mathbb{E}e_i)^2}$$

$$= \mathbb{E}(x_i^*)^2 + \operatorname{Var}(e_i) > 0$$

$$① : \mathbb{E}(x_i v_i) = \mathbb{E}\left[(x_i^* + e_i)(u_i - \beta e_i)\right] = \cancel{\mathbb{E}x_i^* u_i} + \cancel{\mathbb{E}e_i u_i}$$

$$- \beta \cancel{\mathbb{E}x_i^* e_i} - \beta \mathbb{E}e_i^2 = -\beta \operatorname{Var}(e_i)$$

$$\hat{\beta} \xrightarrow{p} \beta + \frac{-\beta \, Var(e_i)}{\mathbb{E}(x_i^*)^2 + Var(e_i)}$$

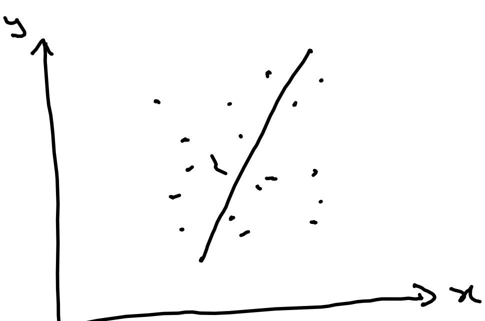
$$= \beta \left[1 - \frac{Var(e_i)}{\mathbb{E}(x_i^*)^2 + Var(e_i)} \right]$$

$A = Var(e_i)$

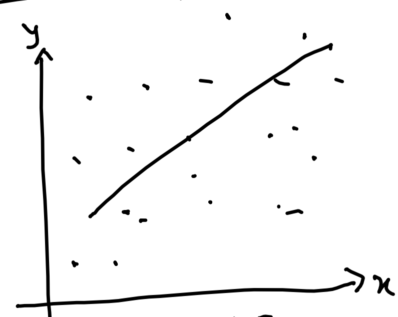
$B = \mathbb{E}(x_i^*)^2$

$$1 - \frac{A}{A+B} = \frac{A+B-A}{A+B} = \frac{B}{A+B} \in [0,1]$$

$\hat{\beta} \xrightarrow{p} \lambda \beta$ where $\lambda \in [0,1]$ Attenuation Bias



no ME



some ME

2) $\underline{\text{support}_i} = \alpha_1 + \alpha_2 \underline{\text{visits}_i} + \alpha_3 \text{inc}_i + \alpha_4 \text{remor1}_i + \alpha_5 \text{dist}_i + \varepsilon_i^1$

$\underline{\text{visits}_i} = \beta_1 + \beta_2 \text{support}_i + \beta_3 \text{remor2}_i + \beta_4 \text{dist}_i + \varepsilon_i^2$

$\boxed{\text{STRUCTURAL EQNS}}$

show $\underline{\text{that}}$ support_i is $\underline{\text{endogenous}}$ in the second eq^n.

$$\text{support}_i = \alpha_1 + \alpha_2 \left(\beta_1 + \beta_2 \text{support}_i + \beta_3 \text{remor2}_i + \beta_4 \text{dist}_i + \varepsilon_i^2\right) + \alpha_3 \text{inc}_i + \alpha_4 \text{remor1}_i + \alpha_5 \text{dist}_i + \varepsilon_i^1$$

$$(1 - \alpha_2 \beta_2)\, \text{support}_i = \underline{\alpha_2 \beta_1} + \alpha_2 \beta_3 \text{remor2}_i + (\alpha_2 \beta_4 + \alpha_5)\, \text{dist}_i + \alpha_3 \text{inc}_i + \alpha_4 \text{remor1}_i + \underbrace{\frac{\alpha_2 \varepsilon_i^2 + \varepsilon_i^1}{1 - \alpha_2\beta_2}}_{v_i}$$

$\alpha_2 \beta_2 \neq 1$

$\pi_1 := \dfrac{\alpha_2 \beta_2}{1 - \alpha_2 \beta_2}$

$$\boxed{\text{Support}_i = \pi_1 + \pi_2 \text{remor2}_i + \pi_3 \text{dist}_i + \pi_4 \text{inc}_i + \pi_5 \text{remor1}_i + v_i}$$

$\boxed{\text{REDUCED FORM}}$

$$\operatorname{cov}(\text{support}_i, \varepsilon_i^2) = \operatorname{cov}(v_i, \varepsilon_i^2)$$

$$= \operatorname{cov}(\alpha_2 \varepsilon_i^2 + \varepsilon_i^1, \varepsilon_i^2)$$

$$= \alpha_2 \operatorname{Var}(\varepsilon_i^2) \neq 0 \qquad \square$$