EC2020 Elements of Econometrics

tomglinnon.github.io/EC2020

t.m.glinnon@lse.ac.uk

$$\boxed{y = \beta_0 + \beta_1 x + u \qquad \mathbb{E}u = c}$$

$$y = (\beta_0 + c) + \beta_1 x + \underbrace{u - c}_{v} \qquad \mathbb{E}v = 0$$

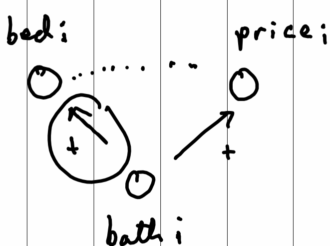
$$y = \beta_2 + \beta_1 x + v \quad \text{where} \quad \mathbb{E}v = 0$$

bathrooms

Q1: $$\boxed{price_i = \beta_0 + \beta_1 \underbrace{bdr_i}_{x_i} + \underbrace{u_i}_{u_i}}$$

$$\mathbb{E}\, price_i = \beta_0 + \beta_1 \mathbb{E}\, bed_i + \dots$$

$$\beta_1 = \frac{\partial \mathbb{E}\, price_i}{\partial \mathbb{E}\, bed_i}$$



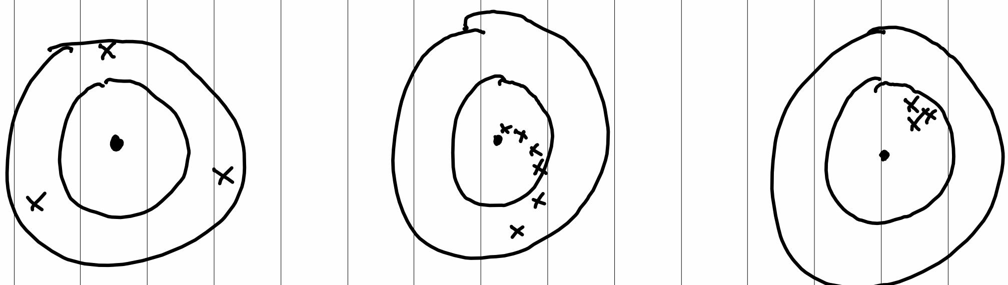
$$\text{price}_i = \beta_0 + \beta_1 \, bdr_i + u_i$$

$$R^2 = 0.17$$

$$R^2 = \frac{SSE}{SST} = \frac{\sum \hat{u}_i^2}{\sum (y_i - \bar{y})^2}$$

$\hat{\beta}_1$ unbiased : $E(\hat{\beta}_1) = \beta_1$

$$\hat{u}_i = p_i - \hat{\beta}_0 - \hat{\beta}_1 b_i$$



unbiased   consistency   efficiency

Q 7 a) $\underset{b_0}{\arg\min} \sum_{i=1}^{n} (y_i - b_0)^2$

$\underline{y_i = b_0 + \varepsilon_i}$

~~RTP:~~ $\cancel{\hat{b}_0 = \bar{y}}$

$$\frac{\partial}{\partial b_0} : \sum_{i=1}^{n} -2(y_i - \hat{b}_0) = 0$$

$$\sum_{i=1}^{n} (y_i - \hat{b}_0) = 0$$

$$\sum_{i=1}^{n} y_i - n\hat{b}_0 = 0$$

$$\hat{b}_0 = \frac{1}{n}\sum_{i=1}^{n} y_i = \bar{y} \qquad \square$$

$$\frac{d}{dx}(a+b) = \frac{da}{dx} + \frac{db}{dx}$$

b) $\hat{u}_i = y_i - \hat{\beta}_0 \qquad \sum \hat{u}_i = 0$

$$\sum \hat{u}_i = \sum (y_i - \hat{\beta}_0) = \sum y_i - n\hat{\beta}_0$$

$$= \sum y_i - \cancel{n} \cdot \frac{1}{\cancel{n}} \cdot \sum y_i = 0 \qquad \square$$

Q5a) $\boxed{y_i = \beta_1 x_i + u_i \qquad \text{find } \hat{\beta}_1}$

$$\hat{\beta}_1 = \underset{b_1}{\text{argmin}} \sum (y_i - b_1 x_i)^2$$

$$\frac{\partial}{\partial b_1}: \quad \cancel{-2} \sum (y_i - \hat{b}_1 x_i) x_i = 0 \qquad \text{(FOC)}$$

$$0 = \sum (x_i y_i - \hat{b}_1 x_i^2)$$

$$= \sum x_i y_i - \hat{b}_1 \sum x_i^2 \quad \Rightarrow \quad \boxed{\hat{b}_1 = \frac{\sum x_i y_i}{\sum x_i^2} \quad \square}$$

b) $$Var(\hat{\beta}_1) = Var\left(\frac{\sum x_i y_i}{\sum x_i^2}\right) = Var\left(\frac{\sum x_i(\beta_1 x_i + u_i)}{\sum x_i^2}\right)$$

$Var(ax) = a^2 Var(x)$

$a = \frac{1}{\sum x_i^2}$

$$= Var\left(\frac{\cancel{\beta_1 \sum x_i^2} + \sum x_i u_i}{\sum x_i^2}\right)$$

$$= Var\left(\cancel{\beta_1} \frac{\sum x_i u_i}{\sum x_i^2}\right) = \frac{1}{(\sum x_i^2)^2} Var(\sum x_i u_i)$$

$$\mathrm{Var}(\hat{\beta}_1) = \left(\frac{1}{\sum x_i^2}\right)^2 \mathrm{Var}\left(\sum x_i u_i\right)$$

$$= \left(\frac{1}{\sum x_i^2}\right)^2 \sum \mathrm{Var}(x_i u_i)$$

$$= \left(\frac{1}{\sum x_i^2}\right)^2 \sum x_i^2 \, \mathrm{Var}(u_i)$$

$$= \sigma^2 \left(\frac{1}{\sum x_i^2}\right)^2 \sum x_i^2 \cancel{\sigma^2} = \frac{\sigma^2}{\sum x_i^2} \qquad \square$$