

EC2020 Session 9 class 1

2) Y_t is % change in GDP between years $t-1$ and t

$Y_{1980}, Y_{1991} \dots Y_{2020}$

~~$Y_t = \alpha + \beta_1 O_t + \beta_2 O_{t-1} + \epsilon_t$~~

$O_t = \max\{0, \% \text{ change in oil prices}\}$

we have $t = 1, \dots, 184$

\rightarrow ~~$Y_t = \alpha + \beta_1 O_t + \beta_2 O_{t-1} + \epsilon_t$~~

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a, b) ~~$t = 5$~~
 $t = 6$

$O_t = 25$
 $O_t = 0$

- if $O_5 \uparrow$ by 1,
 $Y_5 \uparrow$ by β_1
- if $O_5 \uparrow$ by 1,
 $Y_6 \uparrow$ by β_2

c) give 95% CI for the effect after 1 period

25 pt

$$Y_t = \alpha + \beta_1 O_t + \beta_2 \underline{O_{t-1}} + \varepsilon_t$$

$t = 1, \dots, 184 \rightarrow$ we have Y_1, \dots, Y_{184}
and O_1, \dots, O_{184}

At time $t=1$: $\underline{Y_1} = \alpha + \beta_1 \underline{O_1} + \beta_2 \underline{O_0} + \varepsilon_1$

at time $t=2$: $\underline{Y_2} = \alpha + \beta_1 \underline{O_2} + \beta_2 \underline{O_1} + \varepsilon_2$ ✓

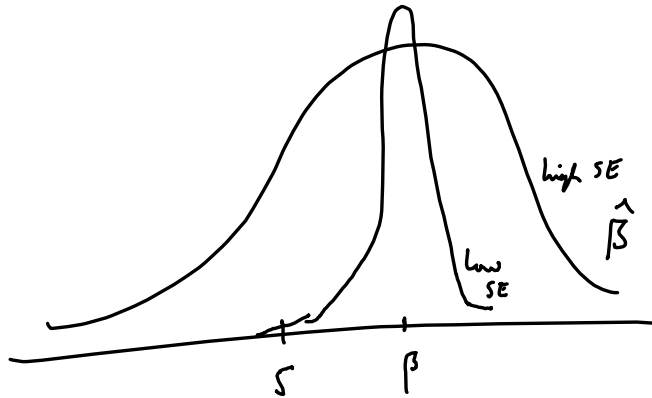
I can run OLS using Y_2, \dots, Y_{184}

~~178~~ is ~~21846-8~~

184

176

d)



$$T = \frac{\hat{\beta} - \beta_0}{\text{se}(\hat{\beta})}$$

3a) $y_t = \beta_1 y_{t-1} + \varepsilon_t$ with $|\beta_1| < 1$

find the OLS estimator for β_1

$$\hat{\beta}_1 = \underset{\beta}{\text{argmin}} \sum (y_t - \beta y_{t-1})^2$$

↙
 $y_t = \beta x_t + \varepsilon_t$

$$\begin{aligned} \hat{\beta} &= \underset{\beta}{\text{argmin}} \sum \hat{\varepsilon}_t^2 \\ &= \underset{\beta}{\text{argmin}} \sum (y_t - \beta x_t)^2 \end{aligned}$$

$$\hat{\beta}_1 = \underset{\beta}{\operatorname{argmin}} \sum (y_t - \beta y_{t-1})^2$$

$$\text{FOC} \quad -2 \sum_t (y_t - \hat{\beta}_1 y_{t-1}) y_{t-1} = 0$$

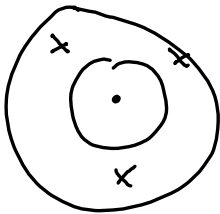
$$\Rightarrow \sum y_t y_{t-1} - \sum \hat{\beta}_1 y_{t-1}^2 = 0$$

and so

$$\boxed{\hat{\beta}_1 = \frac{\frac{1}{n} \sum y_t y_{t-1}}{\frac{1}{n} \sum y_{t-1}^2}}$$

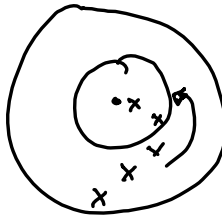
↑

b)



UNBIASED

$$\mathbb{E}(\hat{\beta}) = \beta$$



CONSISTENT

$$\hat{\beta} \xrightarrow{p} \beta \Leftrightarrow \lim_{p \rightarrow \infty} \hat{\beta} = \beta$$

$$y_t = \beta x_t + \varepsilon_t$$

$$\underline{x_t = y_{t-1}}$$

OLS unbiased: $E(\varepsilon_t | X_t) = 0$

$$\uparrow$$

$$E(\varepsilon_t | x_1, x_2, \dots, x_T) = 0 \quad \swarrow \text{[MLR 4]} \quad \underline{\text{"strict exogeneity"}}$$

\Downarrow LIE

OLS ~~consistent~~ consistent: $E(\varepsilon_t x_t) = 0 \Leftrightarrow \underline{\text{cov}(x_t, \varepsilon_t) = 0} \quad \square$

$$E(\varepsilon_t) = 0 \quad E(\varepsilon_t | x_1, x_2, \dots, x_T)$$

y_t IR at time t

$$=$$

$$E(\varepsilon_t | y_2, \dots, y_{T-1}) = 0$$

\Downarrow

$$\underline{E(\varepsilon_t | y_t)} = 0$$

$$y_t = \beta IR_t + \varepsilon_t$$