

Take a long and short regression:

OVB notes

$$y_i = \alpha^L + \beta^L T_i + \gamma X_i + \varepsilon_i \quad (\text{long})$$

$$y_i = \alpha^S + \beta^S T_i + u_i \quad (\text{short})$$

$$X_i = \alpha^A + \delta T_i + v_i \quad (\text{auxiliary})$$

The OVB formula is a connection between the coefficients:

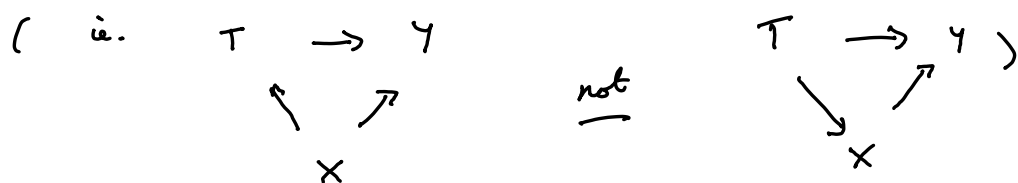
$$\beta^S = \beta^L + \gamma \delta$$

The use of it: • when we don't have data on X_i , but would ideally control for it (i.e., it's a confounder), we can reason that if (for example) we think $\gamma > 0$ and $\delta > 0$ then $\beta^L < \beta^S$

- To argue the signs of γ and δ we need to argue the correlation is positive (between X_i and y_i , fixing T_i , for γ ; and between T_i and X_i for δ)
- But we only use the OVB formula is

this way if we care about β^L instead of β^S .

This happens when X_i is a confounder - and to argue X_i is a confounder we need a causal story



remember: correlation between X and T cannot distinguish these!

why does the OVB formula hold?

The formal proof is in the lecture slides. The intuition I presented in class was that the OVB formula breaks the way that T_i correlates with Y_i in the short regression into two parts:

- A direct effect, holding X_i constant (β^L)
- An indirect effect "via" the way it correlates with X_i ($\gamma\delta$)

NOTE: NOTHING CAUSAL;
ALL CORRELATION

The reason I talked about the chain rule is because this has the same intuition (breaking into a direct and indirect part). note that you cannot derive the OVB formula from the chain rule: they're just analogous to each other.

The analogy is that:

$$\beta^S = \frac{dY_i}{dT_i} \rightarrow \text{The total way in which different } T_i \text{ is associated with different } Y_i$$

$$\beta^L = \frac{\partial Y_i}{\partial T_i} \rightarrow \text{The partial effect: keeping } X_i \text{ constant}$$

$$\gamma = \frac{\partial Y_i}{\partial X_i} \rightarrow \text{same as above.}$$

$$\delta = \frac{\partial X_i}{\partial T_i} \rightarrow \text{same as above}$$

Then just like the chain rule:

$$\underbrace{\frac{dY_i}{dT_i}}_{\text{total}} = \underbrace{\frac{\partial Y_i}{\partial T_i}}_{\text{direct}} + \underbrace{\frac{\partial Y_i}{\partial X_i} \cdot \frac{\partial X_i}{\partial T_i}}_{\text{indirect}}$$

$$\Leftrightarrow \beta^S = \beta^L + \gamma \delta$$