Interaction Effects

The main idea: we might be interested if correlations (or even causations, if we have experiments!) might be different for different groups of people.

For example, in class we looked at a dataset which tried to answer if having a more attractive class teacher means you like the class more.

↳ We might want to see if, for example, this effect is larger for women than men. Are they judged more on their looks in this way?

Idea 1: Run the regression

$$\text{Course evaluation}_i = \alpha + \text{Beauty}_i \beta + \varepsilon_i$$

on men and women separately, then compare $\hat{\alpha}$ and $\hat{\beta}$ in each case.

This is valid for this simple case. However, when we do more complicated things, we want more flexibility than assuming men's and women's data are completely separate (which we're implicitly doing here). Ideally, we want to do one regression on all the data

↳ This provides a basis for more complicated modelling of these 'hetrogenous' effects, when idea 1 is not valid.

↳ "different for different people"

Idea 2: create a dummy variable for 'female'

$$\text{female}_i = \begin{cases} 0 & \text{if not female} \\ 1 & \text{if female} \end{cases}$$

Then try:

$$\underset{(CE_i)}{\text{course eval}_i} = \alpha + \beta\,\text{female}_i + \gamma\,\text{beauty}_i + \delta\,\text{beauty}_i \times \text{female}_i + \varepsilon_i$$

why? look at conditional expectations:

$$\begin{aligned} \mathbb{E}(CE_i \mid \text{female}_i = 0) &= \alpha + \beta \cdot 0 + \gamma \cdot \mathbb{E}\,\text{beauty}_i + 0 + 0 \\ &= \alpha + \gamma \cdot \mathbb{E}(\text{beauty}_i \mid \text{female}_i = 0) \end{aligned}$$

$$\mathbb{E}(CE_i \mid female_i = 1) = (\alpha + \beta) + (\gamma + \delta)\,\mathbb{E}(beauty_i \mid female_i = 1)$$

So:

- α = intercept if not female
- $\alpha + \beta$ = intercept if female
- $\gamma = \dfrac{\partial \mathbb{E}(CE_i \mid fem. = 0)}{\partial \mathbb{E}(beauty_i \mid fem. = 0)}$
- $\gamma + \delta = \dfrac{\partial \mathbb{E}(CE_i \mid fem. = 1)}{\partial \mathbb{E}(beauty_i \mid fem. = 1)}$

So β and δ are like the 'bonus' effects (remember: in the correlation sense. no causality here!) of being female.

↳ women being more judged on attractiveness (if the regression had a causal interpretation)

is $\delta > 0$

↳ courses taught by women are liked more on average is $\beta > 0$

A question: why not $CE_i = \alpha + \beta\, female_i + \varepsilon_i$?

Then $\mathbb{E}(CE_i \mid fem. = 0) = \alpha$

$\mathbb{E}(CE_i \mid fem. = 1) = \alpha + \beta$

↳ only a model of intercepts!

why not $CE_i = \alpha + \beta\, fem_i + \gamma\, beauty_i + \varepsilon_i$?

Then $\mathbb{E}(CE_i \mid fem = 0) = (\alpha + \beta) + \gamma\, \mathbb{E}(beauty_i \mid fem = 0)$

$\mathbb{E}(CE_i \mid fem = 1) = \alpha + \gamma\, \mathbb{E}(beauty_i \mid fem = 0)$

$$\therefore \quad \frac{\partial \mathbb{E}(CE_i \mid fem. = 0)}{\partial \mathbb{E}(beauty_i \mid fem = 0)} = \gamma = \frac{\partial \mathbb{E}(CE_i \mid fem. = 1)}{\partial \mathbb{E}(beauty_i \mid fem = 1)}$$

↳ You're forcing these to be the same!

↳ saturating the model by adding $female_i \times beauty_i$ means you're not forcing this!

different kind of 'saturating' to the kind talked about for multicollinearity

In the sense of: you're estimating the model presuming they're the same. If not: bad assumption $\therefore$ estimator has bad properties why not:

$$CE_i = \alpha + \beta\, beauty_i + \gamma\,(beauty_i \times female_i) + \varepsilon_i$$

$$\mathbb{E}(CE_i \mid fem_i = 0) = \alpha + \beta\, \mathbb{E}(beauty_i \mid fem_i = 0)$$

$$\mathbb{E}(CE_i \mid fem_i = 1) = \alpha + (\beta + \gamma)\, \mathbb{E}(beauty_i \mid fem_i = 1)$$

↳ forcing the intercept to be the same!

Finally note that for $fem_i = 0$, $\left\{ \begin{array}{l} \text{intercept} = \alpha \\ \text{slope} = \gamma \end{array} \right.$

So why not run

$$CE_i = \alpha + \gamma\, beauty_i + \varepsilon_i \quad ?$$

Here: $\mathbb{E}(CE_i) = \alpha + \gamma\, \mathbb{E}(beauty_i)$

So $$\gamma = \frac{\partial\, \mathbb{E}(CE_i)}{\partial\, \mathbb{E}(beauty_i)}$$

In other words, not the same γ as in the saturated regression. This is OVB! Because this is a 'short' regression.