EC221 class #3

- $y = X_1\beta_1 + X_2\beta_2 + \varepsilon$
  $= X\beta + \varepsilon$

$$\underline{\begin{bmatrix} X_1' & X_2' \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}} = \underline{X_1' X_1 + X_2' X_2}$$

OVB formula

$y = X_1\beta_1 + u \rightarrow$ as if this was true

$$\hat{\beta}_1^s = (X_1'X_1)^{-1} X_1' y \quad \leftarrow \text{write down } \hat{\beta}^s$$

$$= (X_1'X_1)^{-1} X_1' (X_1\beta_1 + X_2\beta_2 + \varepsilon) \quad \leftarrow \text{plug in truth}$$

$$= \beta_1 + (X_1'X_1)^{-1} X_1' X_2 \beta_2 + (X_1'X_1)^{-1} X_1' \varepsilon$$

$$\boxed{\mathbb{E}\, \varepsilon | X = 0}$$

$$\mathbb{E}\, \hat{\beta}_1^s = \beta_1 + (X_1'X_1)^{-1} X_1' X_2 \beta_2 + 0$$

$$\mathbb{E}\hat{\beta}_1^s = \beta_1 + \underbrace{(X_1'X_1)^{-1}X_1'X_2\bar{\beta}_2}_{\text{OVB}}$$

$$X_2 = \gamma X_1 + u \qquad \hat{\gamma} = (X_1'X_1)^{-1}X_1'X_2 \qquad \square$$

Q1)

$$y_i = \beta_1 \text{summer}_i + \beta_2 \text{autumn}_i + \beta_3 \text{winter}_i + \beta_4 \text{spring}_i + \varepsilon_i$$

$$\mathbb{E}(y_i \mid \text{summer}_i) = \beta_1 \cdot 1 + \beta_2 \cdot 0 + \beta_3 \cdot 0 + \beta_4 \cdot 0$$
$$= \beta_1$$

$$y_i = \alpha + \beta_1 \text{summer}_i + \beta_2 \text{autumn}_i + \beta_3 \text{spring}_i + u_i$$

$$\mathbb{E}(y_i \mid \text{winter}_i) = \alpha \qquad \mathbb{E}(y_i \mid \text{summer}_i) = \alpha + \beta_1 \cdot 1 + \cancel{\beta_2 \cdot 0}$$

$$\beta_1 = \mathbb{E}(y_i \mid \text{summer}_i) - \mathbb{E}(y_i \mid \text{winter}) \qquad \square$$

- $y_i = \beta_1 \text{ summer}_i + \underbrace{\beta_2 \text{ autumn}_i + \beta_3 \text{ winter}_i + \beta_4 \text{ spring}_i}_{\delta} + \varepsilon_i$

- $\underline{y_i = \gamma \text{ summer}_i + u_i}$

$$\hat{\gamma} = \frac{\sum y_i \text{ summer}_i}{\sum \text{summer}_i^2}$$

RTP:

$$\mathbb{E}\hat{\gamma} = \beta_1 + (X_1'X_1)^{-1} \overbrace{\underline{X_1'X_2}}^{\text{orthogonal}} \delta$$

$$X_2 = \begin{bmatrix} \text{autumn}_i \\ \text{winter}_i \\ \text{spring}_i \end{bmatrix} \qquad X_1 = [\text{summer}_i]$$

$$\text{summer}_i = m_1 \text{ autumn}_i + m_2 \text{ winter}_i + m_3 \text{ spring}_i + v_i$$

$$X_1'X_2 = \begin{bmatrix} \text{autumn}_i \times \text{summer}_i \\ \text{winter}_i \times \text{summer}_i \\ \text{spring}_i \times \text{summer}_i \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

2a

$$y = X_1 \beta_1 + X_2 \beta_2 + \varepsilon \qquad (\hat\beta_1, \hat\beta_2)$$

$$= X\beta + \varepsilon \qquad X = [X_1 \ X_2] \quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \end{bmatrix}$$

1 $y = X\beta + \varepsilon$

$$\hat\beta = \arg\min_\beta \varepsilon'\varepsilon \quad \cancel{= (y - X\beta)'(y - X\beta)}$$

$$= (y - X_1\beta_1 - X_2\beta_2)'(y - X_1\beta_1 - X_2\beta_2)$$

$$(X'X)\hat\beta = X'y$$

$$\begin{pmatrix} X_1'X_1 & X_1'X_2 \\ X_2'X_1 & X_2'X_2 \end{pmatrix} \begin{pmatrix} \hat\beta_1 \\ \hat\beta_2 \end{pmatrix} = \begin{pmatrix} X_1'y \\ X_2'y \end{pmatrix}$$

FWL proof

(2) $y = X_1\tilde{\beta}_1 + X_2\beta_2 + \varepsilon$

$$\underbrace{M_2 y}_{\text{"}y\text{"}} = \underbrace{M_2 X_1}_{\text{"}X\text{"}}\beta_1 + \cancel{M_2 X_2 \beta_2} + M_2\varepsilon$$

"$\hat{\beta} = (X'X)^{-1}X'y$"

"X" $= M_2 X_1$

"y" $= M_2 y$

$$\hat{\beta}_1 = \left((M_2X_1)'(M_2X_1)\right)^{-1}(M_2X_1)'M_2 y$$

$$= \left(\underline{X_1'M_2'M_2X_1}\right)^{-1}\left(X_1'M_2'M_2 y\right)$$

$$= \left(\underline{X_1'M_2X_1}\right)^{-1}\left(X_1'M_2 y\right) \quad \square$$

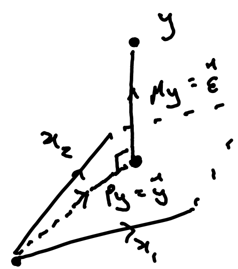
$(AB)' = B'A'$

- M_2 sym.
- M_2 idem.

$$M_2'M_2 = M_2M_2 = M_2$$

$\uparrow$ sym. $\qquad \uparrow$ idem.

$$\underline{\text{rank}(X) = \text{rank}(X'X)}$$



$Py + My = y$

$y = X\beta + \varepsilon$

$$y = \underbrace{X\hat{\beta}}_{Py = \hat{y}} + \underbrace{\hat{\varepsilon}}_{My}$$