CLASS 2

QUIZ 1

a) $f_X(x,y) = 2 \quad 0 \le x \le y \le 1$

~~b)~~ $f(x,y) = 2 . \quad \boxed{0 \le x \le 1 \quad 0 \le y \le 1}$

c) $f(x,y) = xy . \quad$ " "

d) $f(x,y) = 4xy . \quad$ " "

e) $f(x,y) = x + y \quad$ " "

$$\iint f(x,y)\,dx\,dy = 1$$

b) $\int_0^1 \int_0^1 2\,dx\,dy + 0 = \int_0^1 [2x]_0^1\,dy$

$= \int_0^1 2\,dy$

$= 2$ $\square$

~~c~~ $\int_0^1 \int_0^1 xy \, dx \, dy = \int_0^1 y \left[\int_0^1 x \, dx \right] dy$

$\int_0^1 x \, dx = \left[\frac{1}{2} x^2 \right]_0^1$

$= \frac{1}{2} \cdot 1^2 - \frac{1}{2} \cdot 0^2$

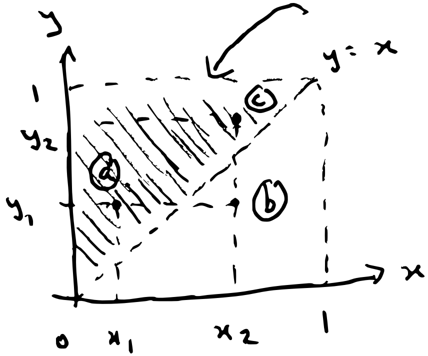
$= \frac{1}{2}.$

$= \int_0^1 y \cdot \frac{1}{2} \, dy$

$= \frac{1}{4}$

d .

a ~~b~~ ~~c~~ d e

a) 

$\underbrace{x \le y}$

$\underbrace{f(x,y) > 0} \quad = \quad \operatorname{supp} f$

sps. $f(x,y) = f_X(x)\, f_Y(y)$ for some f_X, f_Y

(a): $f(a) > 0 \Rightarrow f_X(x_1)\, f_Y(y_1) > 0 \Rightarrow f_X(x_1) > 0$ and $\underbrace{f_Y(y_1) > 0}$

(c): $f(c) > 0 \Rightarrow f_X(x_2)\, f_Y(y_2) > 0 \Rightarrow \underbrace{f_X(x_2) > 0}$ and $f_Y(y_2) > 0$

$\therefore \quad f(b) = f_X(x_2)\, f_Y(y_1) > 0 \quad \times$

$\square$

d) $f_X(x) = 2x$

$f_Y(y) = 2y$

e) $f(x)\,f(y) \boxed{= \cancel{?}^{\,x} + y}$

$x = 0: \quad \underbrace{f(0)}_{c_1}\,f(y) = 0 + y \quad \Rightarrow \quad f(y) = c_1 \cdot y$

$y = 0: \quad f(x) = c_2 \cdot x$

$f(x)\,f(y) = \boxed{c_1 c_2 x y}$

QUIZ 8

$$f(x,y,z) = (x+y)\,e^{-z} \qquad \text{for} \quad 0 \le x \le 1, \quad 0 \le y \le 1, \quad z > 0$$

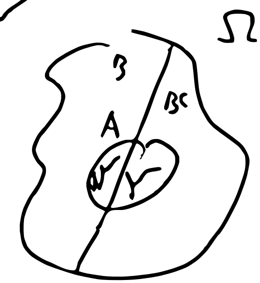
i) $f_Z(z)$ ii) $f_{X,Y}(x,y)$ iii) $f_X(x)$

$$P(A) = P(A \cap B) + P(A \cap B^c)$$

$$P(X=x) = \sum_{y} P(X=x \cap Y=y)$$

$$= \int f(x,y)\,dy$$

"Integrate out" y



$$\text{i)}\ f(z) = \int_0^1\int_0^1 (x+y)\,e^{-z}\,dx\,dy \qquad 2$$

$$= e^{-z}\int_0^1\int_0^1 (x+y)\,dx\,dy$$

$$= e^{-z}\left[\int_0^1 \left[\tfrac{1}{2}x^2\right]_0^1 + \left[xy\right]_0^1\ dy\right]$$

$$= e^{-z}\left\{\int_0^1 \tfrac{1}{2} + y\ dy\right]$$

$$= e^{-z}\left\{\left[\tfrac{1}{2}y\right]_0^1 + \left[\tfrac{1}{2}y^2\right]_0^1\right]$$

$$= e^{-z}\left\{\tfrac{1}{2} + \tfrac{1}{2}\right\} = e^{-z}$$

ii) $f(x,y) = \int_0^\infty (x+y)e^{-z}\,dz$

$$= (x+y)\int_0^\infty e^{-z}\,dz$$

$$= (x+y)\left[-e^{-z}\right]_0^\infty$$

$$= (x+y)\left[0 - (-e^{-0})\right]$$

$$= (x+y)$$

iii) $f(x) = \int_0^\infty \int_0^1 (x+y)e^{-z}\,dy\,dz = \int_0^\infty \int_0^1 xe^{-z} + \boxed{y}e^{-z}\,dy\,dz$

$$= \int_0^\infty xe^{-z} + \left[\int_0^1 y\,dy\right]e^{-z}\,dz$$

$$= \int_0^\infty xe^{-z} + \tfrac{1}{2}e^{-z}\,dz = \left[-xe^{-z} - \tfrac{1}{2}e^{-z}\right]_0^\infty$$

$$= 0 + x + \tfrac{1}{2}(0+1) = x + \tfrac{1}{2} \qquad \square$$

$$\int_0^\infty \int_0^1 \underbrace{(x+y)\,e^{-z}} \, dy \, dz$$

$$= \int_0^\infty \int_0^1 x e^{-z} + y e^{-z} \, dy \, dz$$

$$= \int_0^\infty \left[x e^{-z} y \right]_0^1 + \left[\tfrac{1}{2} y^2 e^{-z} \right]_0^1 dz$$

QUIZ 14 (/4)

- B: do something n times, thing "success or failure" if $P(\text{succes}) = p$ and iid. Then # successes $\sim B(n, p)$

iid = Independent + Identically distributed.

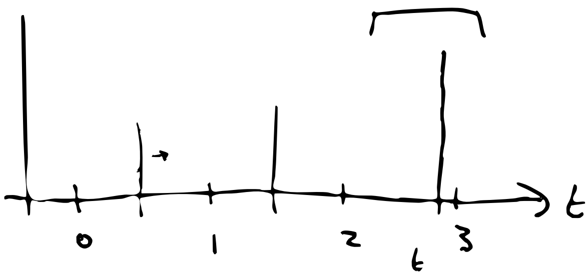
$$p(X = x) = \binom{n}{x} p^x (1-p)^{n-x}$$

Score 3 goals from 5

$$P(X = 3) = \underbrace{\binom{5}{3}} \underbrace{p^3}_{\text{score } 3} \underbrace{(1-p)^2}_{\text{miss } 2}$$

Binomial : n S or F $\iff$ n minutes; 1 per minute

Poisson : n minutes; ~~any~~ on the continuum $[0, n]$



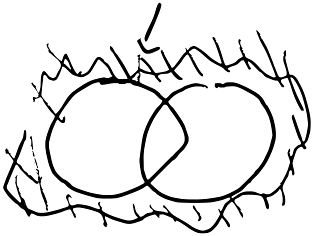
$$y_i = x_i'\beta + \varepsilon_i$$

QUIZ 15

$A \cup B$ $\qquad\qquad\qquad\qquad$ $A^c \cap B^c$

$$P(\text{call} \geq 1 \text{ of them}) = 1 - P(\text{neither call})$$

$$= 1 - P(X_m = 0) \cdot P(X_F = 0).$$



$(A \cup B)^c = A^c \cap B^c$

$$P(X_m = 0) = \frac{e^{-2/7} \cdot \cancel{\left(\frac{2}{7}\right)^0}}{\cancel{0!}} \qquad \boxed{\lambda_m = \frac{2}{7}}$$

$$= e^{-2/7}$$

$$P(X_F = 0) = e^{-1/7}$$

$$P(\geq 1 \text{ calls}) = 1 - e^{-2/7} \cdot e^{-1/7} = 1 - e^{-3/7} \qquad \square$$

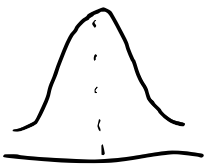
~~Binomial~~

Geometric: $\left\{ \begin{array}{l} \text{2 states (success, failure)} \\ P(\text{success}) = p \quad \text{iid} \end{array} \right.$ → same as Binomial

B : ~~#~~ successes.

G: How long until 1st success.

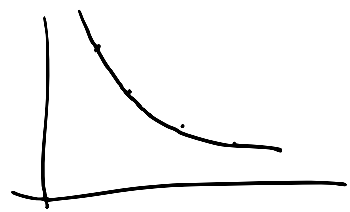
B:



np

$$p(X=x) = \binom{n}{x} p^x (1-p)^{n-x}$$

G:



$p = \frac{1}{2}$

$\binom{5}{3}$ ways to score 3 penalties

$\binom{5}{3} = 10$

$\binom{5}{0} = 1$

$\binom{5}{5} = 1$

$\binom{5}{1} = 5$

Poisson + Exp.

Poisson: N° of successes

Exponential: How long until 1st success.

P iid

$\Longleftrightarrow$

constant "rate" of

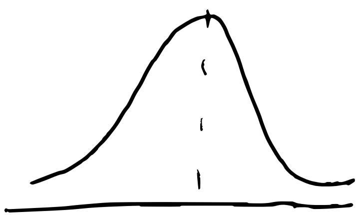
success.

Normal:

$X_i \overset{iid}{\sim}$

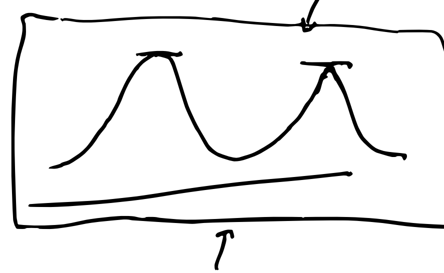
$$\sum_{i=1}^{n} X_i \simeq N\left(\mu, \frac{\sigma^2}{n}\right) \quad \text{as} \quad n \to \infty$$

CLT.



Same med. mode and mean

BIMODAL



$\hat{\beta}$ $\beta = 0$

$\boxed{\text{BOOTSTRAP HYPOTHESIS TEST}}$

QUIZ 9

(4?)

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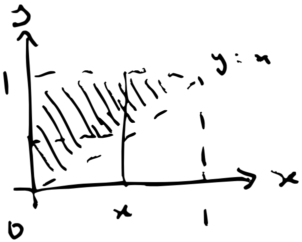
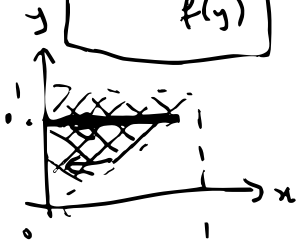


$$f(x,y) = \begin{cases} 2 & y > x \\ 0 & \text{otw} \end{cases}$$

$\mathrm{Var}(Y \mid X = x)$?

$f(y \mid X = x)$

MARGINAL $f(y)$



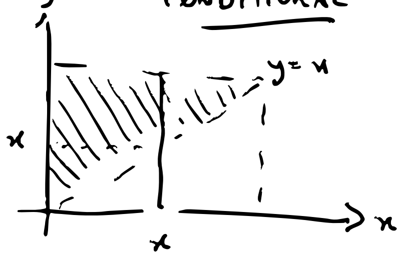
$$f(y) = \int f(x,y)\,dx$$

PDF

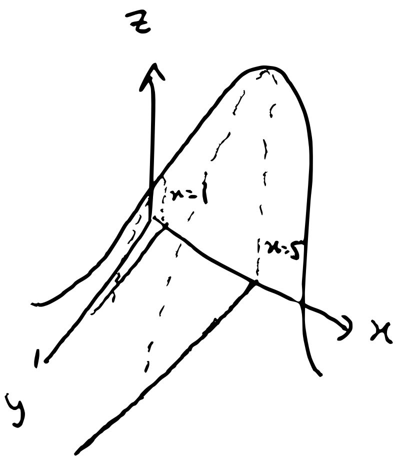
$$f(y \mid x) = \frac{f(x,y)}{f(x)}$$

MARGINAL

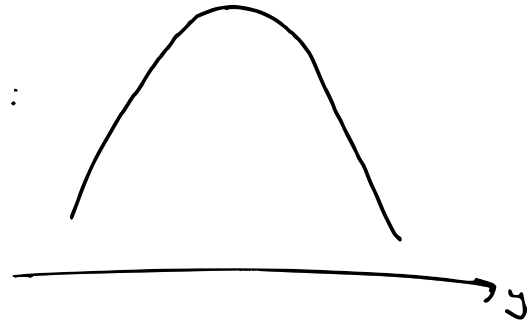
CONDITIONAL



$$f(y \mid X = x) = \begin{cases} 0 & y < x \\ k & y \geq x \end{cases}$$



$\Rightarrow f(y|X=5)$:



$f(y|X=1)$:

$\mathrm{Var}(y|X=x)$

9): $f(x,y) = \begin{cases} 2 & 0 \le x < y \le 1 \\ 0 & \text{otw} \end{cases}$

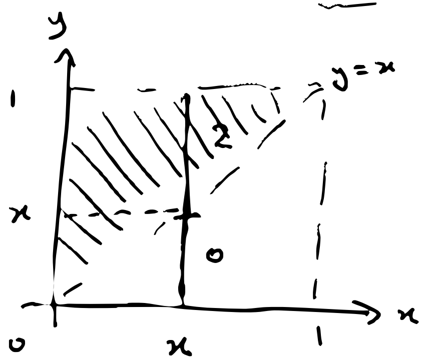
CONDITIONAL

$$f(y|x) = \frac{f(x,y)}{f(x)}$$

← MARGINAL OF ~~X~~ X

Bayes' Rule

$$f(x) = \int f(x,y)\,dy$$



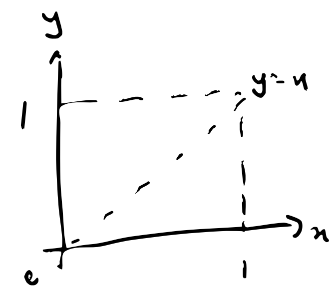
$$f(x) = \int_0^x 0\,dy + \int_x^1 2\,dy$$

$$= 2(1-x)$$

$$f_{x}(y|x) = \frac{f(x,y)}{f(x)} = \begin{cases} \frac{1}{1-x} & x<y \\ 0 & x \geq y \end{cases}$$

$$f(x,y) = \begin{cases} 2 & x<y \\ 0 & \text{otw} \end{cases}$$

$\cancel{V(y|X=x)}$



$$\mathbb{E}(y|X=x) = \int y\, f(y|X=x)\, dy$$

$$= \int_x^1 y \cdot \frac{1}{1-x}\, dy$$

$$= \frac{1}{1-x}\int_x^1 y\, dy = \frac{1}{1-x}\left[\frac{1}{2}y^2\right]_x^1 = \frac{\frac{1}{2}(1-x^2)}{1-x}$$

$$\mathbb{E}(Y \mid X=x) = \frac{\frac{1}{2}(1-x^2)}{1-x} = \frac{\frac{1}{2}(\cancel{1-x})(1+x)}{\cancel{1-x}} = \underbrace{\tfrac{1}{2}(1+x)}$$

$$\mathbb{E}(Y^2 \mid X=x) = \int_x^1 y^2 f(y \mid X=x)\,dy$$

$$= \frac{1}{1-x} \int_x^1 y^2\,dy$$

$$= \frac{1}{1-x}\left[\tfrac{1}{3}y^3\right]_x^1 = \ldots \quad \underbrace{\frac{1-x^3}{3(1-x)}}$$

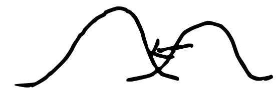
$$\mathrm{Var}(Y \mid X=x) = \mathbb{E}(Y^2 \mid X) - \mathbb{E}(Y \mid Y)^2$$

$$\frac{(1-x)^2}{12}$$

11) $X \sim N[12.5, 0.2^2] \leftarrow N(\mu, \sigma^2)$

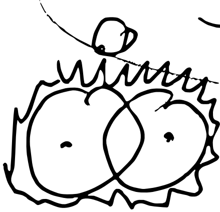
$X - 12.5 \sim N[0, 0.2^2]$

Doesn't fit if $X < 12$ or $X > 13$



$$P(\underbrace{X<12 \text{ or } X>13}_{A \cup B}) = 1 - P(12 < X < 13)$$

$$\boxed{\frac{X-12.5}{0.2}} \sim N[0,1]$$



A: $X < 12$

B: $X > 13$

$(A \cup B)^c = A^c \cap B^c$

$P(A \cup B) = 1 - P[A^c \cap B^c]$

$$P(12 < X < 13)$$

$$= P(-0.5 < X - 12.5 < 0.5)$$

$$= P\left(-\frac{0.5}{0.2} < \underbrace{\frac{X-12.5}{0.2}}_{Z} < \frac{0.5}{0.2}\right)$$

$$= P(Z < 5/2) - P\left(Z < -\frac{5}{2}\right)$$

$Var(aX)$

$= a^2 Var(X)$

$a = \frac{1}{\sigma}$

where

$Z \sim N[0,1]$