Welcome to:

EC400 Probability and Statistics

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OH: Just email me

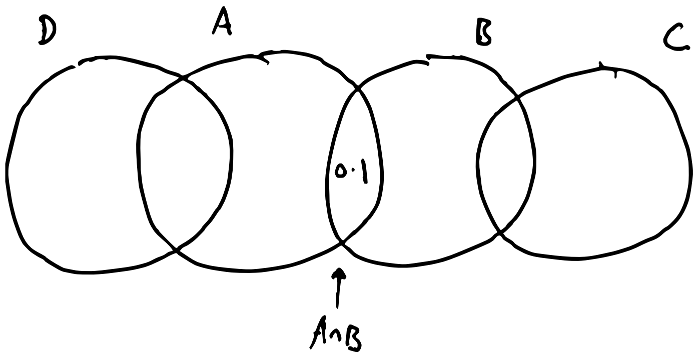
After classes, I'll upload these notes to:

tomglinnan.github.io/EC400

QUIZ Q2

4 events A, B, C, D. $P(A) = P(B) = P(C) = P(D) = 0.4$

$P(C \cap D) = P(C \cap A) = P(D \cap B) = 0$ $P(A \cap B) = 0.1$



a) A and D are independent

Is it: $A \cap D = \emptyset$? $\longrightarrow$ A and D are mutually exclusive

Independence: $P(A,B) = P(A)P(B)$

$\Longleftrightarrow$

$\underbrace{P(B|A) = P(B)}$ ← Independence.

Bayes Rule

$P(B|A) = \dfrac{P(A|B)P(B)}{P(A)}$

B: Hate statistics

A: EC400

Ω: All students at LSE

Independence: $P(\underbrace{\text{~~Take~~ Hate statistics} \mid \text{Take EC400}}_{\text{I take all EC400 students. How many hate statistics}}) = P(\underbrace{\text{Hate statistics}}_{\text{All LSE students.}})$

" "

Mutually exclusive: $P(A \cap B) = 0$

$P(\text{Hate statistics and taking EC400}) = 0$

ME: $P(A \cap B) = 0$

Independent: $P(A \cap B) = P(A) \cdot P(B)$

$P(A)P(B) = 0$

$\Leftrightarrow$ either $P(A) = 0$

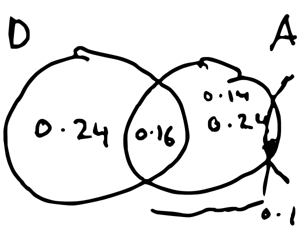
or $P(B) = 0$

Independence

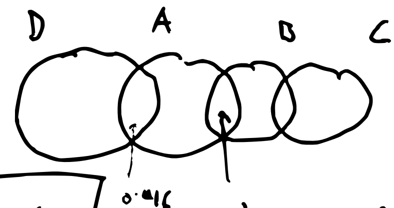
$P(A|B) = P(A)$

$\uparrow$ if $P(B) > 0$

a) A, D are independent:



$\boxed{P(A \cup D) = 0.64}$



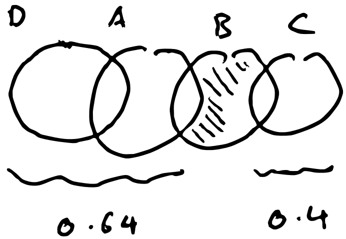
$0.1 = P(A \cap B)$

Then $P(A \cap D) = P(A)P(D) = 0.4^2 = 0.16$

$P(A \cup D) = 0.64$

$P(C \cap A) = P(C \cap D) = 0$

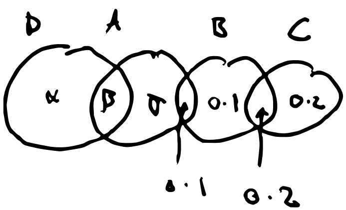
$P(C) = 0.4$



$\therefore \quad P(D \cup A \cup C) \geqslant 0.64 + 0.4$

> 1

b)



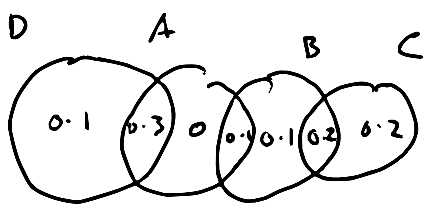
Possible that $P(B \cap C) = 0.2$

Assigned 0.6 so far

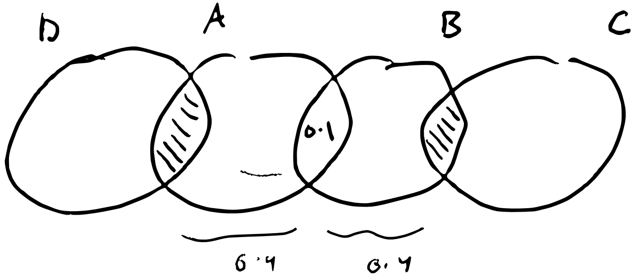
- $\beta + \gamma + 0.1 = 0.4$
- $\alpha + \beta = 0.4$

$$\left.\begin{array}{l}\beta+\gamma+0.1=0.4\\ \alpha+\beta=0.4\end{array}\right\}\Rightarrow\left\{\begin{array}{l}\beta+\gamma=0.3\\ \alpha+\beta=0.4\end{array}\right.$$

$$\left\{\begin{array}{l}\beta=0.3\\ \gamma=0\\ \alpha=0.1\end{array}\right.$$



c)



$P((B\cap C)\cup(A\cap D)) = 0.45$

$P(C\cap D) = 0$

$P(C\cup D) = P(C) + P(D) - P(\cancel{C\cap D})$

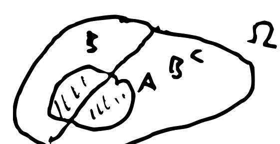
i) $P(C\cup D) = 0.8 \Rightarrow P((C\cup D)^c) = 0.2$

ii) $P((B\cap C)\cup(A\cap D)) = 0.45$

$\Rightarrow P((B\cup A)\cap(C\cup D)) = 0.45$

iii) $P(A\cup B) = 0.7$

$$\boxed{P(A) = P(A\cap B) + P(A\cap B^c)}$$



iv) $P(A\cup B) = P((A\cup B)\cap(C\cup D))$

$+ P((A\cup B)\cap(C\cup D)^c)$

$$0.7 = P(A \cup B) = \underbrace{P((A \cup B) \cap (C \cup D))}_{0.45} + \underbrace{P((A \cup B) \cap (C \cup D)^c)}_{0.25}$$

we know that $\underbrace{P((C \cup D)^c)}_{0.2} \geq \underbrace{P((A \cap B) \cap (C \cup D)^c)}_{0.25}$ $\square$

pro →

QUIZ Q4

If "$P(A|B) < P(A)$".

- $P(A) < P(B)$
- $P(\emptyset) < P(A)$
- ~~$P(A)$~~ $\geq$ $P(A \cap B)$ ✗
- $P(A|B) < P(A \cap B)$
- $P(B|A) < P(B)$.

1 person takes EC400

Hate stats

A : Hate stats

B : Take EC400

Ω : All LSE students.

: "$P(\text{Hate stats} \mid \text{Take EC400}) < P(\text{Hate stats})$"

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

$$E(y|x) \sim x'\beta$$

$$\cancel{P(A \cap B) = P(A|B)\,P(B) = P(B|A)\,P(A)}$$

Suppose: $\underbrace{P(A|B)} < P(A)$

$$\frac{P(B|A)\,P(\cancel{A})}{P(B)} < P(\cancel{A}) \implies P(B|A) < P(B) \qquad \square$$

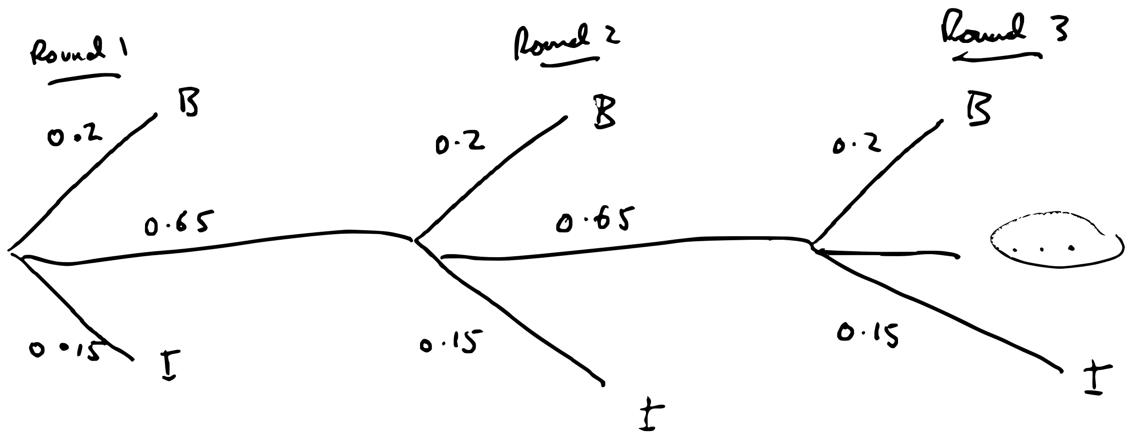
Bayes:

$$P(B|A) = \underbrace{\frac{P(A|B)}{P(A)}}_{>1 \text{ if}} \cdot P(B)$$

posterior

prior

QUIZ 6



$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

$P(\text{B scores}) = 0.8$

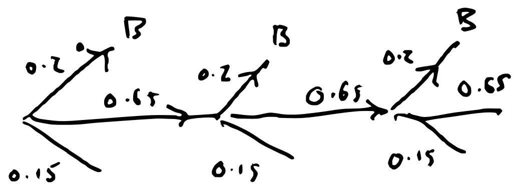
$P(\text{I scores}) = 0.75$

$P(\text{B score and I don't}) = P(\text{B score}) \times P(\text{I don't})$

$= 0.8 \times 0.25$

$= 0.2$

$P(\text{I score; B don't}) = 0.75 \times 0.2 = 0.15$



FOR INDEPENDENT & M.E. EVENS

"A and B" $\quad P(A \cap B) = P(A)P(B)$

"A or B" $\quad P(A \cup B) = P(A) + P(B)$

So loosely

AND $=$ $\times$

OR $=$ $+$

$P(\text{Brazil win})$ ~~+ ... 0.2~~

$= \underbrace{0.2}_{P(\text{win in round 1})} + \underbrace{(0.65 \times 0.2)}_{P(\text{win in round 2})} + \underbrace{(0.65^2 \times 0.2)}_{P(\text{win in round 3})}$

$+ \underbrace{(0.65^3 \times 0.2)}_{P(\text{win in round 4})} \ \ldots$

"need" $|r| < 1$

Geometric series:

$a = 0.2$
$r = 0.65$

$P(\text{Brazil win}) = 0.2 + (0.2 \times 0.65) + (0.2 \times 0.65^2) \ \ldots$

$a + ar + ar^2 + ar^3 \ \ldots$

$$P(\text{Brazil win}) = \sum_{n=0}^{\infty} a r^n \quad \text{where } a = 0.2,\ r = 0.65$$

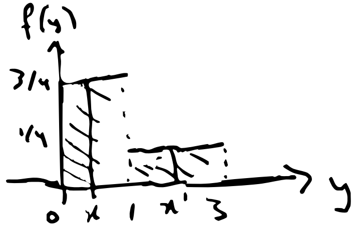
$$= \frac{a}{1-r} = \frac{0.2}{1-0.65} = \frac{4}{7} \quad \square$$

WHY CARE ABOUT PROBABILITY?

- $\left[\ \underline{y_i = x_i'\beta + \varepsilon_i} \quad \mathbb{E}\, x_i \varepsilon_i = 0\ \right]$ → RESTRICTION ON PDF OF (x_i, y_i)
- So is econ. model:

  eg $GDP_i = f(\text{Inf}, \text{Unemp} \ldots)$ → random f.
- Aim: To test theories by looking at reduced <u>form</u> (fewer assumptions)
- Point of formalism: prove when estimators works well or badly

1i) $f(y) = \begin{cases} 3/4 & 0 \le y \le 1 \\ 1/4 & 1 \le y \le 3 \\ 0 & \text{otw.} \end{cases}$



$F(y) = \begin{cases} 0 & y < 0 \\ \frac{3}{4} \cdot y & 0 \le y \le 1 \\ \frac{1}{2} + \frac{1}{4} y & 1 \le y \le 3 \\ 1 & y > 3 \end{cases}$

$\uparrow$

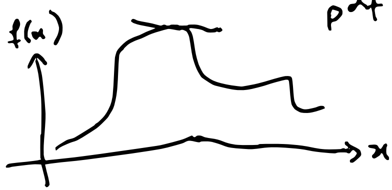
$P(Y \le y)$

PLOT FUNCTIONS

$\int_0^x f(y)\,dy = \int_0^x 3/4\,dy = \frac{3}{4} x$

$\frac{3}{4} + \int_1^{x'} \frac{1}{4}\,dy = \frac{3}{4} + \frac{1}{4} x' - \frac{1}{4}$

$= \frac{1}{2} + \frac{1}{4} x'$

3) 

pdf

MLE

a) $f(x) = \left(\frac{1}{2}\right)^x$ for $x = 1, 2, 3 \ldots$ 0 elsewhere.

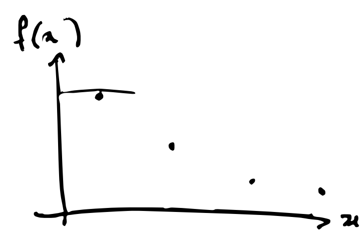
$\uparrow$ DISCRETE

$P(X = x)$

$P(X=1) = \frac{1}{2}$

$P(X=2) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$P(X=3) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$



consider n. $P(X = n+1) = \left(\frac{1}{2}\right)^{n+1} = \left(\frac{1}{2}\right)^n \cdot \frac{1}{2} < \left(\frac{1}{2}\right)^n = P(X=n)$

b) $f(x) = \frac{1}{2}x^2 e^{-x}$ $\quad x > 0$ $\quad$ 0 elsewhere.

$\longrightarrow \max_x \left(\underbrace{\tfrac{1}{2}x^2}_{u}\right)\left(\underbrace{e^{-x}}_{v}\right)$ $\quad$ Product rule: $(uv)' = uv' + u'v$

FOC: $\frac{\partial}{\partial x} : \frac{1}{2}x^2(-e^{-x}) + xe^{-x}$

$$= \cancel{\tfrac{1}{2}e^{-x}}(2x - x^2) = 0$$

$$2x - x^2 = 0$$

$$x(2-x) = 0 \Rightarrow x = 0$$

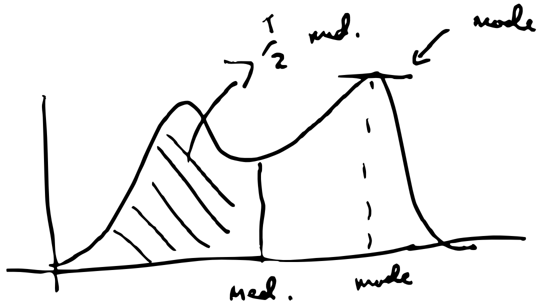
or $\boxed{x = 2.}$

SOC: $\frac{\partial^2}{\partial x^2} : x(-e^{-x}) + \frac{1}{2}x^2e^{-x} + e^{-x} - xe^{-x}$

$$= e^{-x}\left[-x + \tfrac{1}{2}x^2 + 1 - x\right]$$

$$= \tfrac{1}{2}e^{-x}\left[x^2 - 4x + 2\right]$$

4a)



$$f(x) = \underbrace{\frac{4!}{x!\,(4-x)!}}_{\binom{4}{x} \quad {}^4C_x} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{4-x} \qquad x = 0, 1, \ldots, 4 \qquad 0 \text{ elsewhere.}$$

$$f(0) = \binom{4}{0} \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{4-0} = \left(\frac{3}{4}\right)^4 \approx \frac{81}{256} < \frac{1}{2}$$

$$f(1) = \binom{4}{1} \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^3 = \frac{108}{256} \qquad f(0) + f(1) = \frac{81}{256} + \frac{108}{256} = \frac{191}{256} > \frac{1}{2}$$

$$P(X \leq 0) = f(0) \qquad P(X \leq 1) = f(0) + f(1)$$

$$P(X \le 0) \le \frac{1}{2} \le P(X \le 1)$$

med = 1

$$P(X \le 0) = P(X < 1) = \frac{81}{256} < \frac{1}{2}$$

$$P(X \le 1) = \frac{191}{256} > \frac{1}{2}$$

$\square$