Welcome to:

EC400 Probability and Statistics

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Dates: 10^{th} - 16^{th} sept

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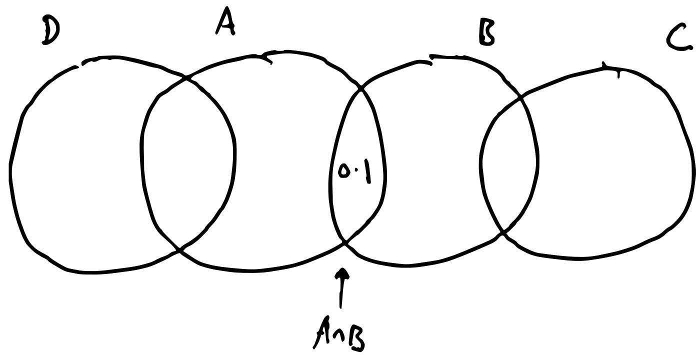
After classes, I'll upload these notes to:

tomglinnan.github.io/EC400

QUIZ Q2

4 events A, B, C, D. $P(A) = P(B) = P(C) = P(D) = 0.4$

$P(C \cap D) = P(C \cap A) = P(D \cap B) = 0$ $P(A \cap B) = 0.1$



a) A and D are independent

Is it: $A \cap D = 0$? $\longrightarrow$ A and D are mutually exclusive

Independence: $P(A, B) = P(A)P(B)$

$\Longleftrightarrow$

$P(B|A) = P(B)$ Independence.

Bayes Rule

$$P(B|A) = \frac{P(A|B)P(B)}{P(A)}$$

B: Hate statistics

A: EC400

Ω: All students at LSE

Independence: P(Hate statistics | Take EC400) = P(Hate statistics)

I take all EC400 students. How many hate statistics

All LSE students. " "

Mutually exclusive: $P(A \cap B) = 0$

P(Hate statistics and taking EC400) = 0

ME: $P(A \cap B) = 0$

Independent: $P(A \cap B) = P(A) \cdot P(B)$

$P(A)P(B) = 0$

$\Leftrightarrow$ either $P(A) = 0$

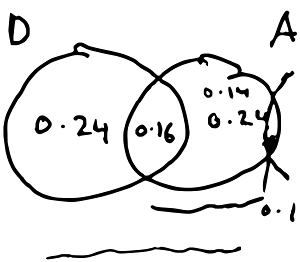
or $P(B) = 0$

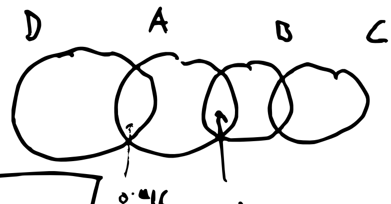
Independence

$P(A|B) = P(A)$

$\uparrow$ if $P(B) > 0$

a) A, D are independent:





$= P(A \cap B)$

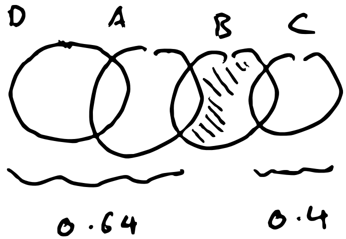
$\boxed{P(A \cup D) = 0.64}$

Then $P(A \cap D) = P(A)P(D) = 0.4^2 = 0.16$

$P(A \cup D) = 0.64$

$P(C \cap A) = P(C \cap D) = 0$

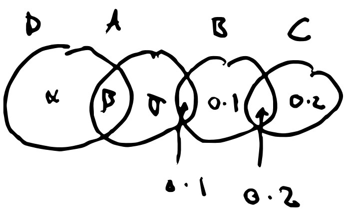
$P(C) = 0.4$



$\therefore \quad P(D \cup A \cup C) \geq 0.64 + 0.4$

> 1

b)



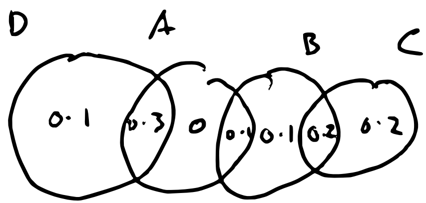
Possible that $P(B \cap C) = 0.2$

Assigned 0.6 so far

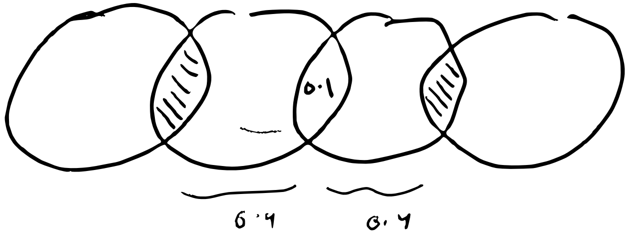
- $\beta + \gamma + 0.1 = \cancel{0.4}$
- $\alpha + \beta = 0.4$

$$\left.\begin{aligned} \beta+\gamma+0.1 &= 0.4 \\ \alpha+\beta &= 0.4 \end{aligned}\right\} \Rightarrow \begin{cases} \beta+\gamma = 0.3 \\ \alpha+\beta = 0.4 \end{cases}$$

$$\begin{cases} \beta = 0.3 \\ \gamma = 0 \\ \alpha = 0.1 \end{cases}$$



c)



$P((B\cap C)\cup(A\cap D)) = 0.45$

$P(C\cap D) = 0$ $\quad$ $P(C\cup D) = P(C) + P(D) - \cancel{P(C\cap D)}$

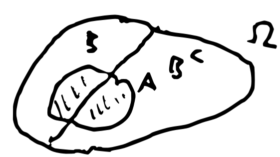
i) $P(C\cup D) = 0.8 \Rightarrow P((C\cup D)^c) = 0.2$

ii) $P((B\cap C)\cup(A\cap D)) = 0.45$

$\Rightarrow P((B\cup A)\cap(C\cup D)) = 0.45$

iii) $P(A\cup B) = 0.7$

$\boxed{P(A) = P(A\cap B) + P(A\cap B^c)}$



iv) $P(A\cup B) = P((A\cup B)\cap(C\cup D))$

$+ P((A\cup B)\cap(C\cup D)^c)$

$$0.7 = P(A\cup B) = \underbrace{P((A\cup B)\cap(C\cup D))}_{0.45} + \underbrace{P((A\cup B)\cap(C\cup D)^c)}_{0.25}$$

we know that $\underbrace{P((C\cup D)^c)}_{0.2} \geq \underbrace{P((A\cap B)\cap(C\cup D)^c)}_{0.25}$

$\square$

pto →

Quiz Q4

If "$P(A|B) < P(A)$".

- $P(A) < P(B)$
- $P(B) < P(A)$
- $\cancel{P(A) \geq P(A \cap B)}$ ✗
- $P(A|B) < P(A \cap B)$
- $P(B|A) < P(B)$.

1 person takes EC400

Hate stats

A : Hate stats

B : Take EC400

Ω : All LSE students.

: "P(Hate stats | Take EC400) < P(Hate stats)"

$$P(B|A) = \frac{P(A|B)\,P(B)}{P(A)}$$

$$E(y|x) \sim x'\beta$$

$$\cancel{P(A \cap B) = P(A|B)\,P(B) = P(B|A)\,P(A)}$$

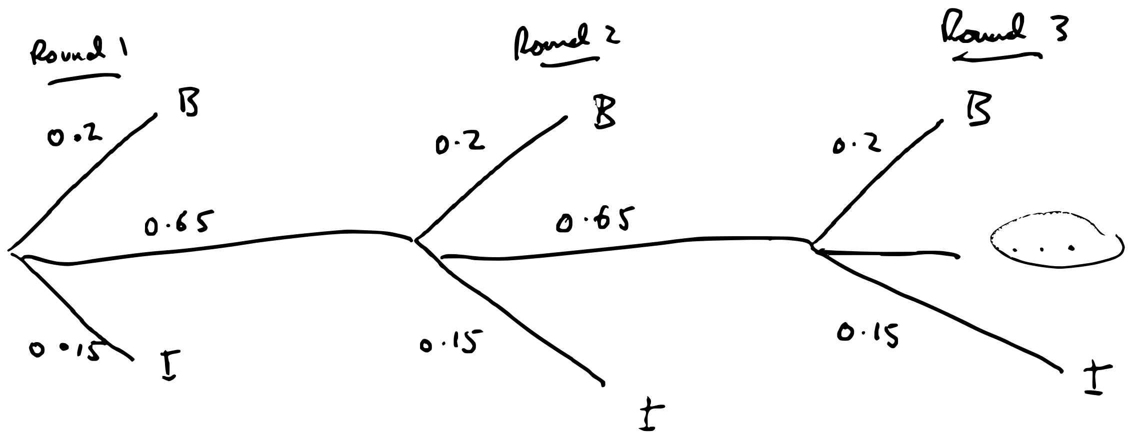
Suppose: $P(A|B) < P(A)$

$$\frac{P(B|A)\,P(\cancel{A})}{P(B)} < P(\cancel{A}) \Rightarrow P(B|A) < P(B) \quad \square$$

Bayes:

$$\underset{\text{posterior}}{P(B|A)} = \underbrace{\frac{P(A|B)}{P(A)}}_{>1 \text{ if}} \cdot \underset{\text{prior}}{P(B)}$$

QUIZ 6



$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

$P(B \text{ scores}) = 0.8$

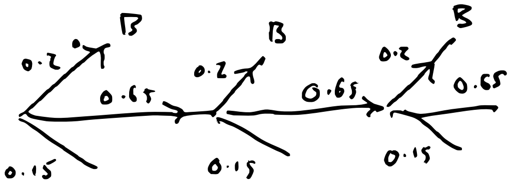
$P(I \text{ scores}) = 0.75$

$P(B \text{ score and } I \text{ don't}) = P(B \text{ score}) \times P(I \text{ don't})$

$= 0.8 \times 0.25$

$= 0.2$

$P(I \text{ score}, B \text{ don't}) = 0.75 \times 0.2 = 0.15$



P(Brazil win)

$= \underbrace{0.2}_{P(\text{win in round } 1)} + \underbrace{(0.65 \times 0.2)}_{P(\text{win in round } 2)} + \underbrace{(0.65^2 \times 0.2)}_{P(\text{win in round } 3)}$

$+ \underbrace{(0.65^3 \times 0.2)}_{P(\text{win in round } 4)} \ldots$

FOR INDEPENDENT & M.E. EVENS

"A and B" $\quad P(A \cap B) = P(A)P(B)$

"A or B" $\quad P(A \cup B) = P(A) + P(B)$

So loosely

AND = $\times$

OR = $+$

"need" $|r| < 1$

Geometric series:

$a = 0.2$

$r = 0.65$

$P(\text{Brazil win}) = 0.2 + (0.2 \times 0.65) + (0.2 \times 0.65^2) \ldots$

$a + ar + ar^2 + ar^3 \ldots$

$$P(\text{Brazil win}) = \sum_{n=0}^{\infty} a r^n \qquad \text{where} \quad a = 0.2, \ r = 0.65$$

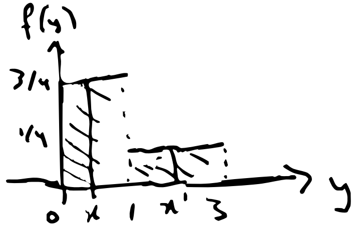
$$= \frac{a}{1-r} = \frac{0.2}{1-0.65} = \frac{4}{7} \qquad \square$$

WHY CARE ABOUT PROBABILITY?

- $\left.\underline{y_i = x_i'\beta + \varepsilon_i} \quad \mathbb{E}\, x_i \varepsilon_i = 0 \right]$ → RESTRICTION ON PDF OF (x_i, y_i)
- so is econ. model:

  eg $GDP_i = f(\text{Inf}, \text{Unemp} \ldots)$ → random f.
- Aim: To test theories by looking at reduced <u>form</u> (fewer assumptions)
- Point of formalism: prove when estimators works well or badly

1i) $f(y) = \begin{cases} 3/4 & 0 \le y \le 1 \\ 1/4 & 1 \le y \le 3 \\ 0 & \text{otw.} \end{cases}$



$F(y) = \begin{cases} 0 & y < 0 \\ \frac{3}{4} \cdot y & 0 \le y \le 1 \\ \frac{1}{2} + \frac{1}{4} y & 1 \le y \le 3 \\ 1 & y > 3 \end{cases}$

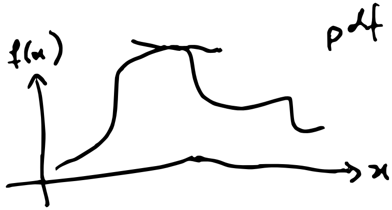
$P(Y \le y)$

PLOT FUNCTIONS

$$\int_0^x f(y)\,dy = \int_0^x 3/4\,dy = \frac{3}{4} x$$

$$\frac{3}{4} + \int_1^{x'} \frac{1}{4}\,dy = \frac{3}{4} + \frac{1}{4} x' - \frac{1}{4}$$

$$= \frac{1}{2} + \frac{1}{4} x'$$

3) 

MLE

a) $f(x) = \left(\frac{1}{2}\right)^x$ for $\underbrace{x = 1, 2, 3 \ldots}$ 0 elsewhere.

$\uparrow$ DISCRETE

$P(X = x)$

$P(X=1) = \frac{1}{2}$

$P(X=2) = \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

$P(X=3) = \left(\frac{1}{2}\right)^3 = \frac{1}{8}$

consider n. $P(X = n+1) = \left(\frac{1}{2}\right)^{n+1} = \left(\frac{1}{2}\right)^n \cdot \frac{1}{2} < \left(\frac{1}{2}\right)^n = P(X=n)$

b) $f(x) = \frac{1}{2}x^2 e^{-x}$ $\quad x > 0 \quad$ 0 elsewhere.

$\longrightarrow \max_{x} \left(\underset{u}{\frac{1}{2}x^2}\right)\left(\underset{v}{e^{-x}}\right)$ $\qquad$ Product rule: $(uv)' = uv' + u'v$

FOC: $\frac{\partial}{\partial x}$: $\frac{1}{2}x^2(-e^{-x}) + xe^{-x}$

$$= \frac{1}{2}\cancel{e^{-x}}(2x - x^2) = 0$$

$$2x - x^2 = 0$$

$$x(2-x) = 0 \quad \Rightarrow \quad x = 0$$

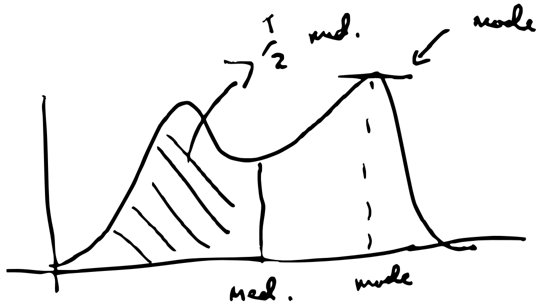
$$\boxed{\text{or} \quad x = 2.}$$

SOC: $\frac{\partial^2}{\partial x^2}$: $x(-e^{-x}) + \frac{1}{2}x^2 e^{-x} + e^{-x} - xe^{-x}$

$$= e^{-x}\left[-x + \frac{1}{2}x^2 + 1 - x\right]$$

$$= \frac{1}{2}e^{-x}\left[x^2 - 4x + 2\right] =$$

4 a)



$$f(x) = \underbrace{\frac{4!}{x!\,(4-x)!}}_{\binom{4}{x} \quad {}^4C_x} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{4-x} \qquad x = 0, 1, \ldots, 4 \qquad 0 \text{ inclusive.}$$

$$f(0) = \binom{4}{0} \left(\frac{1}{4}\right)^0 \left(\frac{3}{4}\right)^{4-0} = \left(\frac{3}{4}\right)^4 \approx \frac{81}{256} < \frac{1}{2}$$

$$f(1) = \binom{4}{1} \left(\frac{1}{4}\right)^1 \left(\frac{3}{4}\right)^3 = \frac{108}{256} \qquad f(0) + f(1) = \frac{81}{256} + \frac{108}{256} = \frac{191}{256} > \frac{1}{2}$$

$$P(X \le 0) = f(0) \qquad P(X \le 1) = f(0) + f(1)$$

$$P(X \le 0) \le \frac{1}{2} \le P(X \le 1)$$

$\underline{\text{med} = 1}$

$$P(X \le 0) = P(X < 1) = \frac{81}{256} < \frac{1}{2}$$

$$P(X \le 1) = \frac{191}{256} > \frac{1}{2}$$

$\square$