$\{X_i\}$

H_0: Innocent

H_1: Guilty

"Which is more likely H_0 or H_1"

"Innocent until proven guilty"

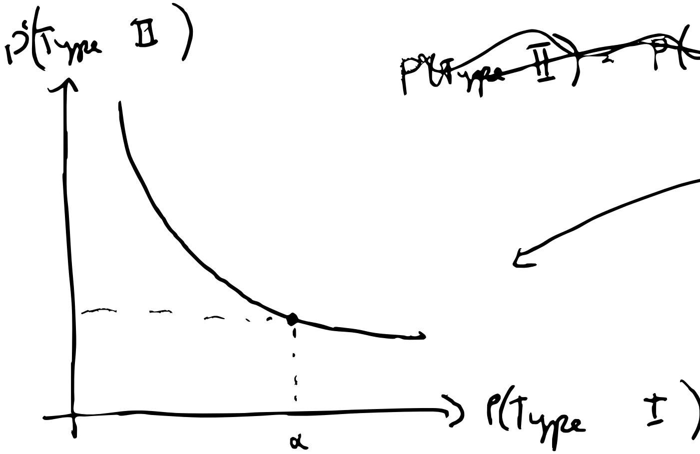
- Hear evidence
- Threshold in mind (reasonable doubt)

Under H_0 $T > t \Rightarrow$ reject H_0

High ~~Lower~~ threshold: (low α)

↳ Fewer innocents → jail (good) (Type I)

↳ Fewer guilty → jail (bad) (not Type II)



$= 0.05$

~~P(Type II) = P(~~

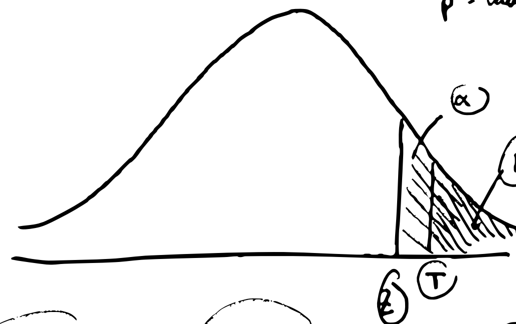
P(~~[illegible]~~ Free | Guilty) ≠ P(Jail | Innocent)

$\alpha = 1$

Choosing α

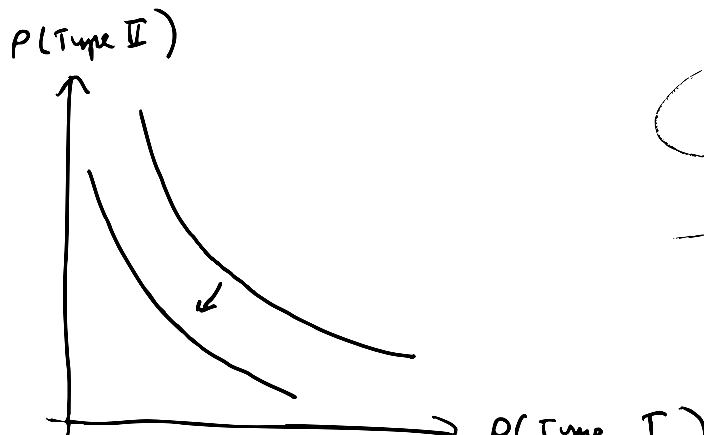
critical value

p-value

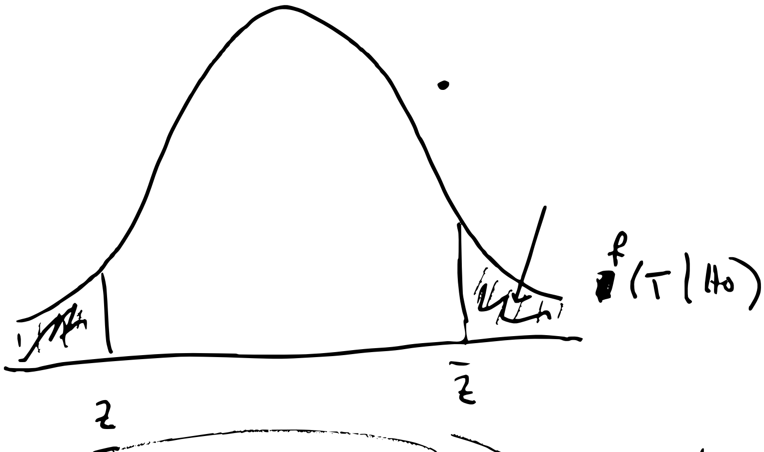


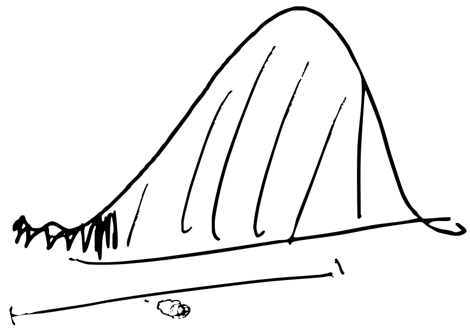
$T > z \iff \alpha > p$

C.V. p-value



- More data
- Better way of looking at the same





[Non-rejection region.]

W

LR → Bayesian

LM

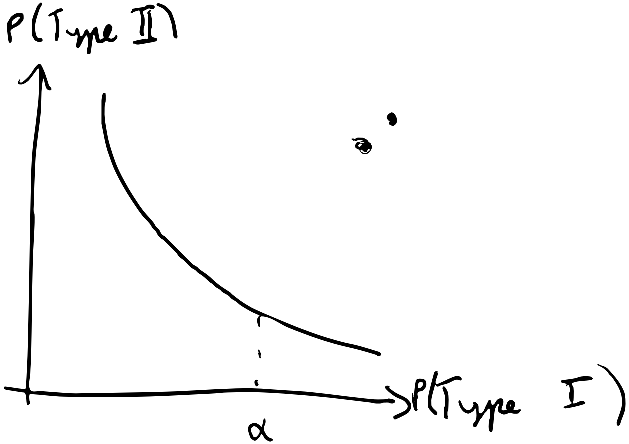
	Guilty	Innocent
Jail		
Free		

	H_1	H_0
reject H_0		
don't		

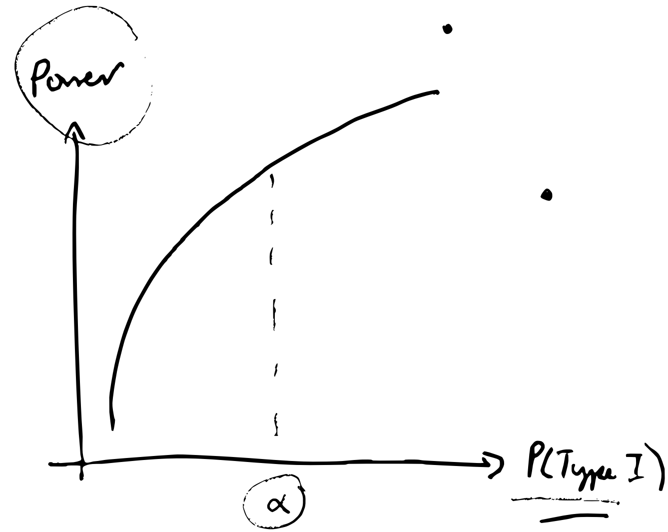
$P(\text{Type I}) = P(\text{jail} \mid \text{innocent})$

Power

$$\text{Power} = 1 - P(\text{Type II})$$



$\Leftrightarrow$



$X_i \sim N(\mu, \sigma^2)$

$\mathbb{E}X > \gamma$

$H_0: \mu = \gamma \qquad H_1: \mu > \gamma$

$H_1: \mu \neq \gamma$

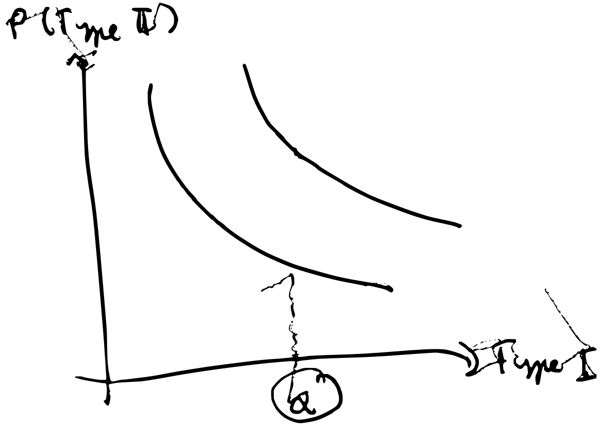
$H_0: \mu = \gamma$

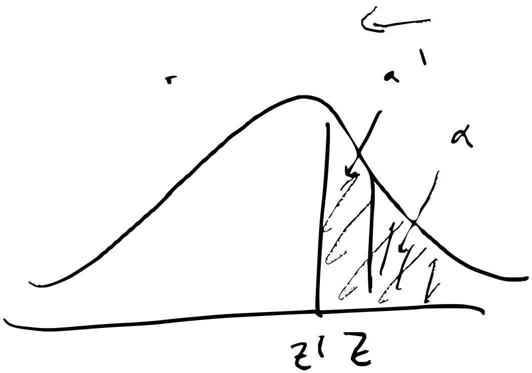
$$\boxed{\begin{aligned} H_0&: \mu \geq \gamma \\ H_1&: \mu < \gamma \end{aligned}}$$

e) $H_0: \mu > \gamma \qquad H_1: \mu = \gamma$

$$P(T = t \mid \mu > \gamma) = \left[\frac{P(T = t \cap \mu > \gamma)}{P(\mu > \gamma)}\right]$$

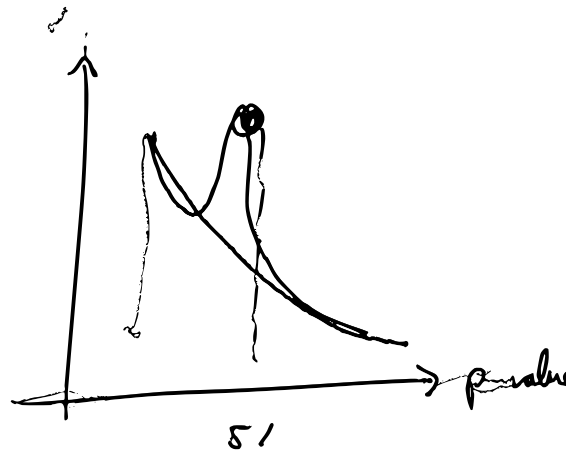
QUIZ 2





$$y_i = x_i'\beta + \varepsilon_i$$

$$\ln y_i = x_i'\beta + \varepsilon_i$$



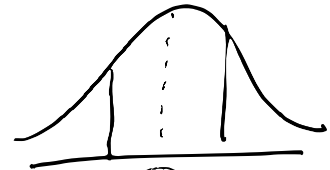
PS 4

$X \sim N(\mu, 1)$

$\boxed{X_1 - \mu \sim N(0,1)}$

$n = 100$

a) 95% CI for μ



X_1

$\bar{X} \xrightarrow{p} \mu$

$\cancel{\bar{X} \sim N(\mu, \tfrac{1}{100})}$

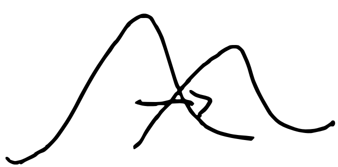
$\cancel{X \sim N(\mu, 1)}$

$\cancel{X - \mu \sim N(0,1)}$

$\bar{X} \sim N(\mu, \frac{1}{100}) \Rightarrow \bar{X} - \mu \sim N(0, \frac{1}{100})$

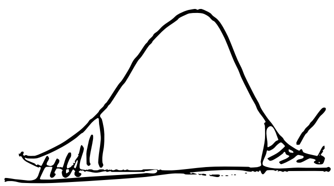
$\frac{\bar{X} - \mu}{1/10} \sim N(0,1)$

$\boxed{10(\bar{X} - \mu) \sim N(0,1)}$



$\cancel{P(-z < 10(\bar{x}-\mu}$

$$\boxed{z = 10(\bar{x}-\mu)}$$



$$\boxed{P(-1.96 \le z \le 1.96) = 0.95}$$

$$\Downarrow$$

$$P(-1.96 \le 10(\bar{x}-\mu) \le 1.96) = 0.95$$

$$P\left(-\frac{1.96}{10} \le \bar{x}-\mu \le \frac{1.96}{10}\right) = 0.95$$

$$P\left(\frac{1.96}{10} \ge \mu-\bar{x} \ge -\frac{1.96}{10}\right)$$

$$=$$

$$P\left(-\frac{1.96}{10} \le \mu-\bar{x} \le \frac{1.96}{10}\right)$$

$$\left[\bar{x}-\frac{1.96}{10}, \bar{x}+\frac{1.96}{10}\right]$$

$$0.95 = P\left(\bar{x}-\frac{1.96}{10} \le \mu \le \bar{x}+\frac{1.96}{10}\right)$$

b) $H_0: \mu = 0$ $\qquad \alpha = 5\%$ $\qquad$ reject $H_0 \iff \bar{X} \notin CI$

$H_1: \mu \neq 0$

$\bar{X} \sim N(0, \frac{1}{100})$

$$Z = \frac{\bar{X} - \mu}{sd} = \frac{\bar{X} - 0}{1/10} = 10\bar{X} \sim N(0,1)$$

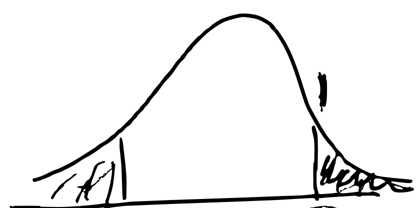
• reject $H_0 \iff 10\bar{x} > z_{97.5}$ or $10\bar{x} < z_{2.5}$

$10\bar{x} > 1.96$

$\Rightarrow \bar{x} > \frac{1.96}{10}$

<u>OR</u> $\quad 10\bar{x} < -1.96$

$\bar{x} < \frac{-1.96}{10}$



$N(0,1)$

$-z_{2.5}$ $\qquad$ $z_{97.5}$

$\Phi(z_{97.5}) = 97.5$

1.96

reject H_0 $[\alpha]$

$\mu = 0$

$$\boxed{\bar{x} > \frac{1.96}{\sqrt{10}} \quad \text{or} \quad \bar{x} < -\frac{1.96}{\sqrt{10}}}$$

CI

$$\boxed{0 \notin \left[\bar{x} - \frac{1.96}{\sqrt{10}},\ \bar{x} + \frac{1.96}{\sqrt{10}}\right]}$$

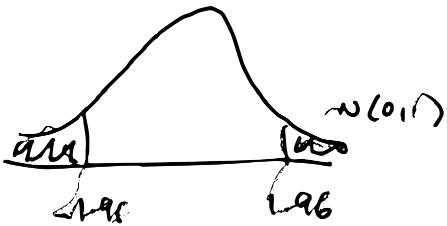
$P(\mu \in CI) = 0.95$

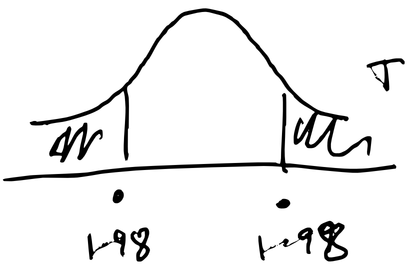
c) $\frac{\bar{X} - \mu}{sd/\sqrt{n}}$

$$\frac{\bar{X} - \mu}{\sqrt{S^2/n}} \sim t_{n-1}$$

$$\chi^2_n = \sum_{i=1}^{n} N(0,1)^2$$

c) $T = \dfrac{\bar{X}-\mu}{\sqrt{S^2/n}} \sim t_{n-1}$





$P(-1.98 \le T \le 1.98) = 0.95$

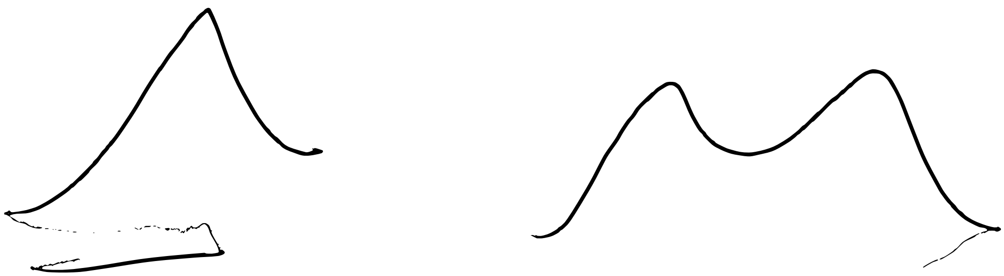
$= P\left(-1.98\sqrt{\frac{S^2}{n}} \le \bar{X}-\mu \le 1.98\sqrt{\frac{S^2}{n}}\right)$

$= P\left(-1.98\sqrt{\frac{s^2}{n}} \le \bar{x}-\mu \le 1.98\sqrt{\frac{s^2}{n}}\right)$

$1.98\sqrt{\frac{S^2}{n}} \ge \mu - \bar{X}$ $\qquad \mu - \bar{X} \ge -1.98\sqrt{\frac{S^2}{n}}$

$= P\left(-1.98\sqrt{\frac{S^2}{n}} \le \mu - \bar{X} \le 1.98\sqrt{\frac{S^2}{n}}\right)$

$$P\left(\bar{X} - 1.98\sqrt{\frac{s^2}{n}} \le \mu \le \bar{X} + 1.98\sqrt{\frac{s^2}{n}}\right) = 0.95$$



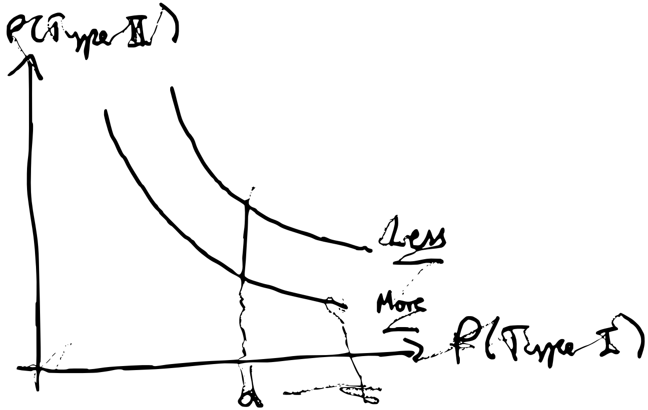
$H_0: \mu = \gamma$ $\qquad$ $H_1: \mu > \gamma$

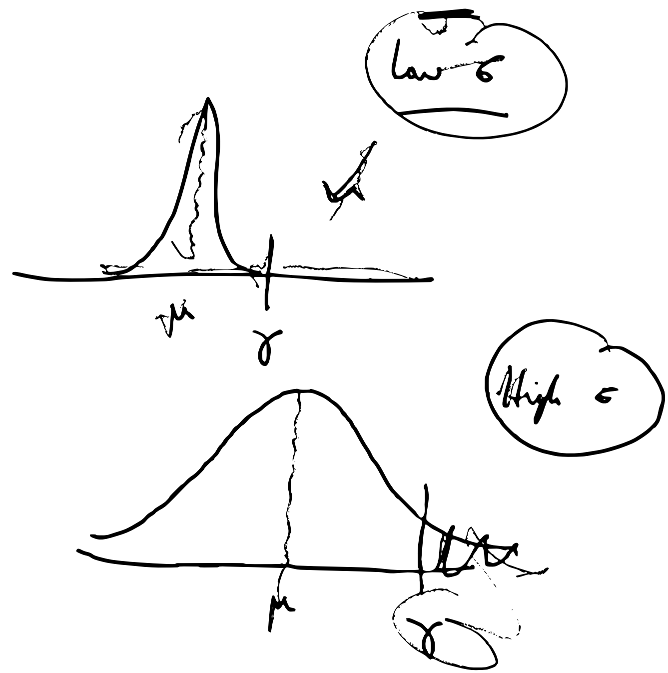
$\boxed{X_i \sim N(\mu, \sigma^2)}$ $\qquad$ $\bar{X} \sim N\left(\mu, \frac{\sigma^2}{n}\right)$

$$\boxed{P\left(\frac{\bar{X} - \mu}{\sigma/\sqrt{n}} > z_\alpha \;\middle|\; \mu > \gamma \right)}$$

Jail $\qquad$ Guilty

(fail test) $\qquad$ (H_1)





$\mu = \gamma_1 > \gamma$ $\quad\quad \nearrow \dfrac{\bar{X} - \gamma}{\sigma/\sqrt{n}} \quad\quad H_0: \mu = \gamma_0$

$\text{Power}(\gamma_1) = P\left(T > z_\alpha \mid \mu > \gamma_1\right)$

$$= \frac{\bar{X} - \gamma}{S} > z_\alpha \iff \underbrace{\frac{\bar{X} - \gamma_1}{S}}_{N(0,1)} > z_\alpha + \frac{\gamma - \gamma_1}{S}$$

$$P(\gamma_1) = 1 - \Phi\left(z_\alpha - \frac{\gamma_1 - \gamma}{\sigma/\sqrt{n}}\right)$$

$X \sim f(x) = \begin{cases} \theta x^{\theta-1} & 0 \le x \le 1 \\ 0 & \text{otw} \end{cases}$

pdf of Y

$Y = X^2 - 1$

- $Y = u(X)$
- $X = v(Y)$

$X^2 = Y + 1$

$X = \sqrt{Y+1}$

$-\sqrt{Y+1}$

$v(y) = \sqrt{y+1}$

$= (y+1)^{1/2}$

$\boxed{0 \le x \le 1}$

$g(y) = f(v(y)) \cdot |v'(y)| = \begin{cases} \theta \left[\sqrt{y+1}\right]^{\theta-1} \cdot \frac{1}{2}(y+1)^{-1/2} \\ 0 & \text{otw} \end{cases}$

$0 \le x \le 1 \iff 0 \le \sqrt{y+1} \le 1$

$x \ge 0$

$0 \le y+1 \le 1$

$\iff -1 \le y \le 0$

$g(y) = \theta (y+1)^{1/2 (\theta - 1)} \frac{1}{2} (y+1)^{-1/2}$ $(-1 \le y \le 0)$

$= \frac{\theta}{2} (y+1)^{\frac{1}{2}\theta - 1}$

$$g(y) = \begin{cases} \frac{\theta}{2} (y+1)^{\frac{1}{2}\theta - 1} & -1 \le y \le 0 \\ 0 & \text{otw} \end{cases}$$

$\square$