What's the Point of Stats?

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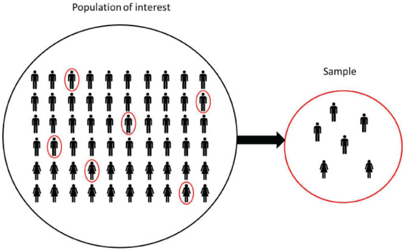
September 12, 2021

Why might we care about correlation?

- Correlation is not causation
- *But* it helps us look in the right place. It's where all science starts
- We want to understand how correlation works so we know exactly how useful it is $\implies$ we should revise some ST102

Populations and Samples

We want to know how the world works, but we won't observe everyone $\rightarrow$ the problem of dumb luck



Names: Everyone in the world is the **Population**; our data is a **Sample**; the thing we are trying to estimate is called a **Parameter**

What is statistics *really*?

Statistics really has two main areas:

- **Estimation**: given our sample, what is our best guess for the population parameter?
  $\rightarrow$ e.g. what is the average height of 20-year olds in the UK?
- **Inference (Hypothesis Testing)**: given our sample, how sure can we be that something is not true in the population?
  $\rightarrow$ e.g. is the average height of 20-year olds in the UK is above 170cm?

Both are fundamental to understanding correlation so we'll cover them here

Estimation I

'An estimator is just a function from data to the parameter space'. Ordinary Least Squares (OLS) is one type of estimator

Specifically:

$$y_i = \alpha + \beta x_i + \varepsilon_i$$

The Data:	(x_i, y_i) for each person i
The Parameters:	(α, β)
The Key Problem:	Different Data $\implies$ Different Estimates

We denote the estimates[1] by hats: e.g. $\hat{\alpha}$ is the estimate for α

[1] An *estimator* is the function itself, the (point) *estimate* is the value of the function for a particular set of data

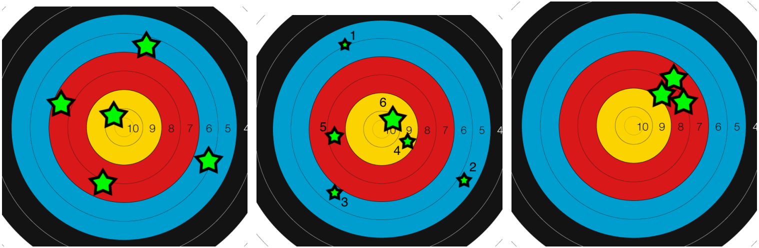
Estimation II

What properties might we like our estimator to have?

- **Unbiased**: on average it gives the true value
- **Consistent**: as our sample gets bigger, our estimate gets arbitrarily good eventually
- **Efficient**: our estimator has a small variance

In all statistics, we have to make some assumptions about the population to find properties of our estimator

Illustration



(a) Unbiasedness (b) Consistency (c) Efficiency

Figure: Three Key Properties

For consistency, labels $1, \ldots, 6$ are the order of the shots

Why OLS?

Under some assumptions, OLS is unbiased, consistent, and has the smallest variance of **any** (linear) estimator

1. Our model ($y_i = \alpha + \beta x_i + \varepsilon_i$) actually is how the world works
2. Not all x_i are the same
3. No matter what x_i is, what we don't know is random noise ($E(\varepsilon_i|x) = 0$)
4. No matter what x_i is, what we don't know has the same magnitude on average ($Var(\varepsilon_i|x)$ is the same for all i)
5. What we don't know is for person i is correlated with what we don't know for person j ($Cov(\varepsilon_i, \varepsilon_j|x) = 0$)

We'll return to these issues when we talk about issues of using OLS in the real world

Multivariate Regression I

The thing written here is called a ‘bivariate regression model’:

$$y_i = \alpha + \beta x_i + \varepsilon_i$$

If you imagine, we can extend this:

$$y_i = \alpha + \beta_1 x_{1i} + \beta_2 x_{2i} + ... + \beta_n x_{ni} + \varepsilon_i$$

Where the y_i is called the **outcome** variable and the x_{ji} are called the **explanatory** variables. A **variable** is just a piece of data. A **coefficient** is the thing in front of a variable

For example: $\text{Wage}_i = \alpha + \beta_1 \text{Education Level}_i + \beta_2 \text{Gender}_i + \varepsilon_i$

Multivariate Regression II

What do the numbers actually mean?

When what we don't know is random, $E(\varepsilon_i|x) = 0$, so

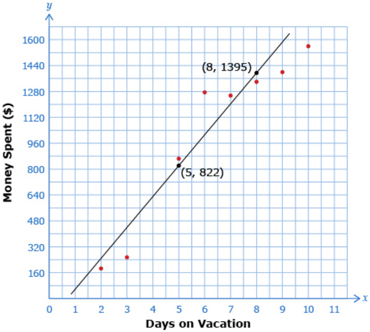
$$E(y|x) = \alpha + \beta_1 x_1 + \beta_2 x_2 + ... + \beta_n x_n$$

and therefore:

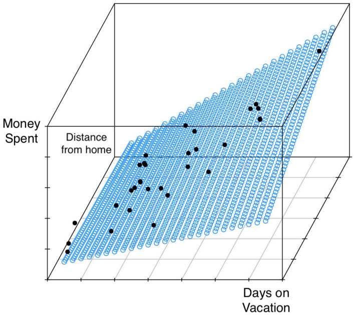
$$\beta_j = \frac{\partial E(y|x)}{\partial x_j}$$

Because of what partial derivatives mean: β_j is the change in the line of best fit in the direction x_j, holding others constant

Illustration



(a) Bivariate



(b) Multivariate

$$\text{Money Spent}_i = \alpha + \beta_1 \text{DOV}_i + \beta_2 \text{DFM}_i + \varepsilon_i$$

Multivariate Regression III

Because maths people like vectors, we can use the shorthand:

$$y_i = \beta' x_i + \varepsilon_i$$

where

$$\beta' = \begin{bmatrix} \alpha \ \beta_1 \ \dots \ \beta_n \end{bmatrix} \quad \text{and} \quad x_i = \begin{bmatrix} 1 \\ x_{1i} \\ \vdots \\ x_{ni} \end{bmatrix}$$

and talk about estimating β with $\hat{\beta}$

Hypothesis Testing I

As $\hat{\beta}$ is a random variable, it has a distribution, so we can do testing on it.

So what is a test?

Whenever you hear 'hypothesis', just think 'initial guess'. We aim to see if our estimate $\hat{\beta} \approx \beta_0$, our initial guess

- Make a *null hypothesis* H_0
- Assume that the null is true
- Work out the probability we saw what we just did, or more extreme
- If under our assumption it was really unlikely to see what we just saw, then our assumption was probably wrong!

Hypothesis Testing II (non-essential)

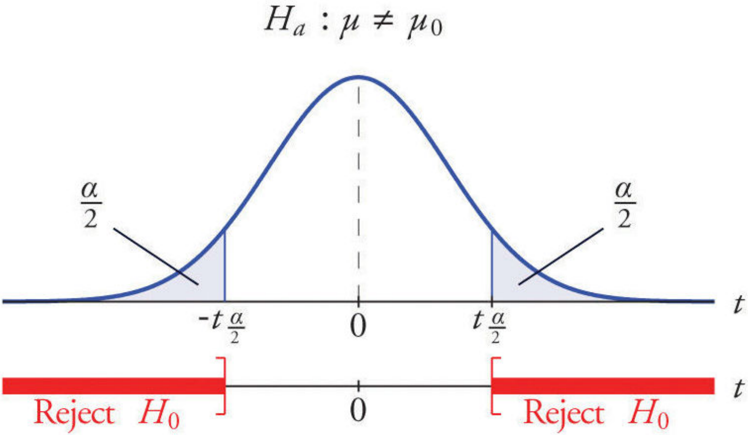
- For mathy/philosophy-types: this is like a generalised version of a proof by contradiction
- We never 'accept' a hypothesis.
  $\rightarrow$ To show why, say we have two possible null hypotheses:
  $H' : \beta_1 = 0$ and $H'' : \beta_1 = 0.000000001$
  $\rightarrow$ any test would probably reject both in very similar circumstances.
  $\rightarrow$ Accepting both would mean you think that β_1 takes two different values, which is philosophically sus
- We can do a test using its test statistic T or its p value
- Interpretation of a p value: assuming that the null is true, what is the probability that we saw what we just did, or something more 'extreme'

Hypothesis Testing III

Figure: A bottle-filling machine

The inventor of the t-stat worked in a beer factory. The aim was to check if bottles were over-filled or under-filled

Illustration



Technical Note: we need to assume the distribution of $\hat{\beta}$. When small amounts of data assume t. When large amounts of data, Normal becomes an arbitrarily good approximation by the central limit theorem

Errors

I promise errors are a less confusing concept than you think!

$$\begin{aligned} \textit{Type I}: &\quad \hat{\beta}_j \text{ failed the test but it should have passed} \\ \textit{Type II}: &\quad \hat{\beta}_j \text{ passed the test but it should have failed} \end{aligned}$$

Note that:

- $P(\textit{Type I}) = \alpha$ (the significance level)
  $\rightarrow$ You get to pick this
- Power $= 1 - P(\textit{Type II})$

More power comes from either having more data, or using a more efficient estimator

Understanding Errors

A court case is effectively a hypothesis test:

- The jury have some beliefs about whether or not the defendant is guilty
- Someone is ‘innocent until proven guilty’, i.e. H_0 : Innocent and H_1 : Guilty
- The jury sees evidence and decides using the same principle as hypothesis testing: assuming that they are innocent, how likely is it that this evidence exists? Level of reasonable doubt $= \alpha$
- What are Type I and Type II errors here?
- If we reduce Type I (fewer innocents go to jail) then more guilty people go free (increase Type II), and vice versa
- How to solve? More evidence, or better inference from the evidence we have!

Testing in practice

What might we want to test?

- Whether a given $\beta_j = 0$ - This would mean that x_i wouldn't help to explain y_i (t test)
  $\rightarrow$ if we conclude $\beta_j \neq 0$ then we say that β_j is 'statistically significant'
- Whether *all* of the β are 0 This would mean that the model is useless (f test)
  $\rightarrow$ This is related to R^2: the percentage of variation in y_i that the model explains
  $\rightarrow$ This is Regression ANOVA from ST102
- Anything else depending on context!

Remember to realise the Type I / Type II tradeoffs! Usually pick $\alpha = 10\%^{*}, 5\%^{**}, 1\%^{***}$

Looking at stata

```
. reg price weight

      Source |       SS           df       MS      Number of obs   =        74
-------------+----------------------------------   F(1, 72)        =     29.42
       Model |   184233937         1   184233937   Prob > F        =    0.0000
    Residual |   450831459        72  6261548.04   R-squared       =    0.2901
-------------+----------------------------------   Adj R-squared   =    0.2802
       Total |   635065396        73  8699525.97   Root MSE        =    2502.3

------------------------------------------------------------------------------
       price |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      weight |   2.044063   .3768341     5.42   0.000     1.292857    2.795268
       _cons |  -6.707353    1174.43    -0.01   0.995     -2347.89    2334.475
------------------------------------------------------------------------------
```

Questions

- How do we rule out that a correlation is due to dumb luck?
- What is the most important property an estimator can have: unbiasedness, consistency, efficiency, something else?
- Why might OLS not be the best way to estimate a regression slope?
- Under which circumstances would you want to test a hypothesis using a high α? A low one?