POLS 6481. Research Design and Quantitative Methods II
Lecture 2. Simple Regression
Readings: Wooldridge, *Introductory Econometrics 5e*, 2.1 + 2.2 + 2.3a

Outline:

Prelude – PED Use in Baseball

1. Review of the Linear Simple Regression Model
2. Summation (Review Appendix A.1, A.2)
3. Definitions (Review Appendix B.3, B.4)

Interlude – PED Use in Baseball

4. Assumptions of the Linear Simple Regression Model

Postlude – PED Use in Baseball

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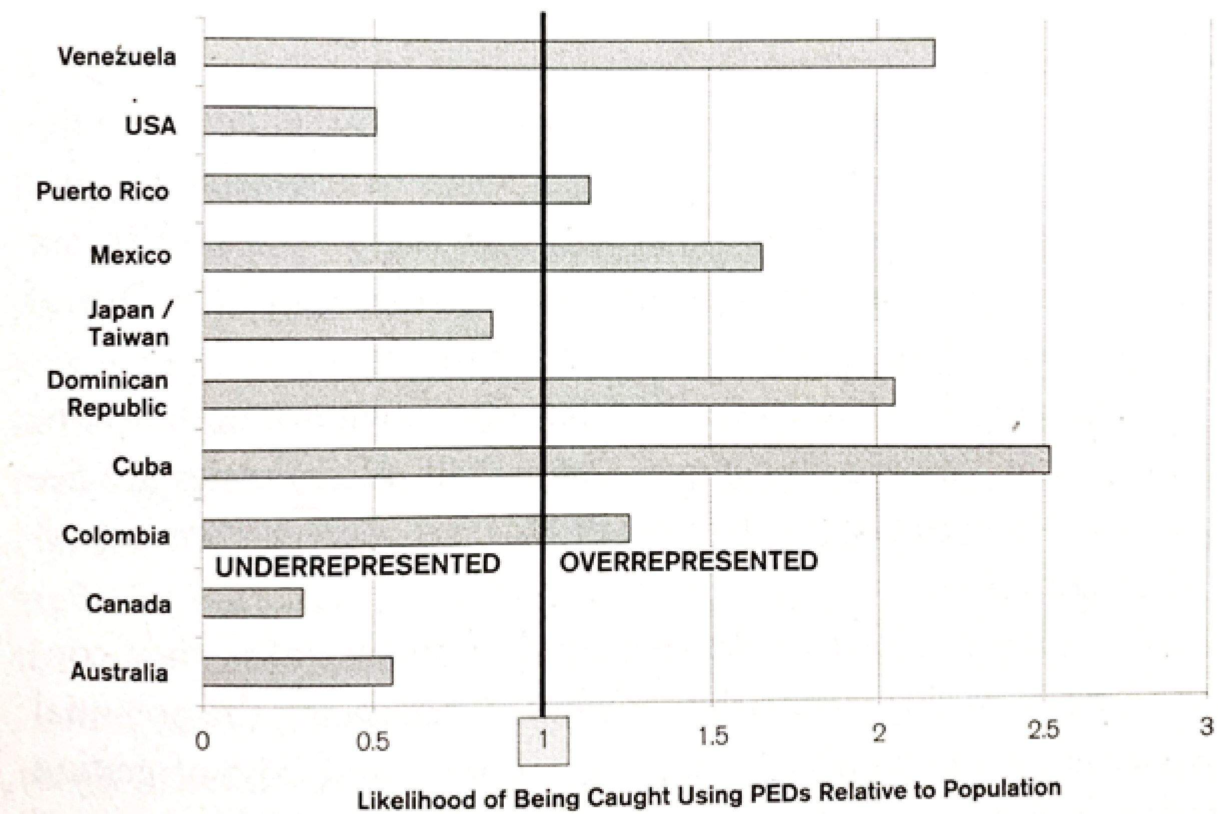
Prelude – PED Use in Baseball



Data for 1,520 U.S. Major League and 8,569 U.S. minor league players from 2005 to 2009.

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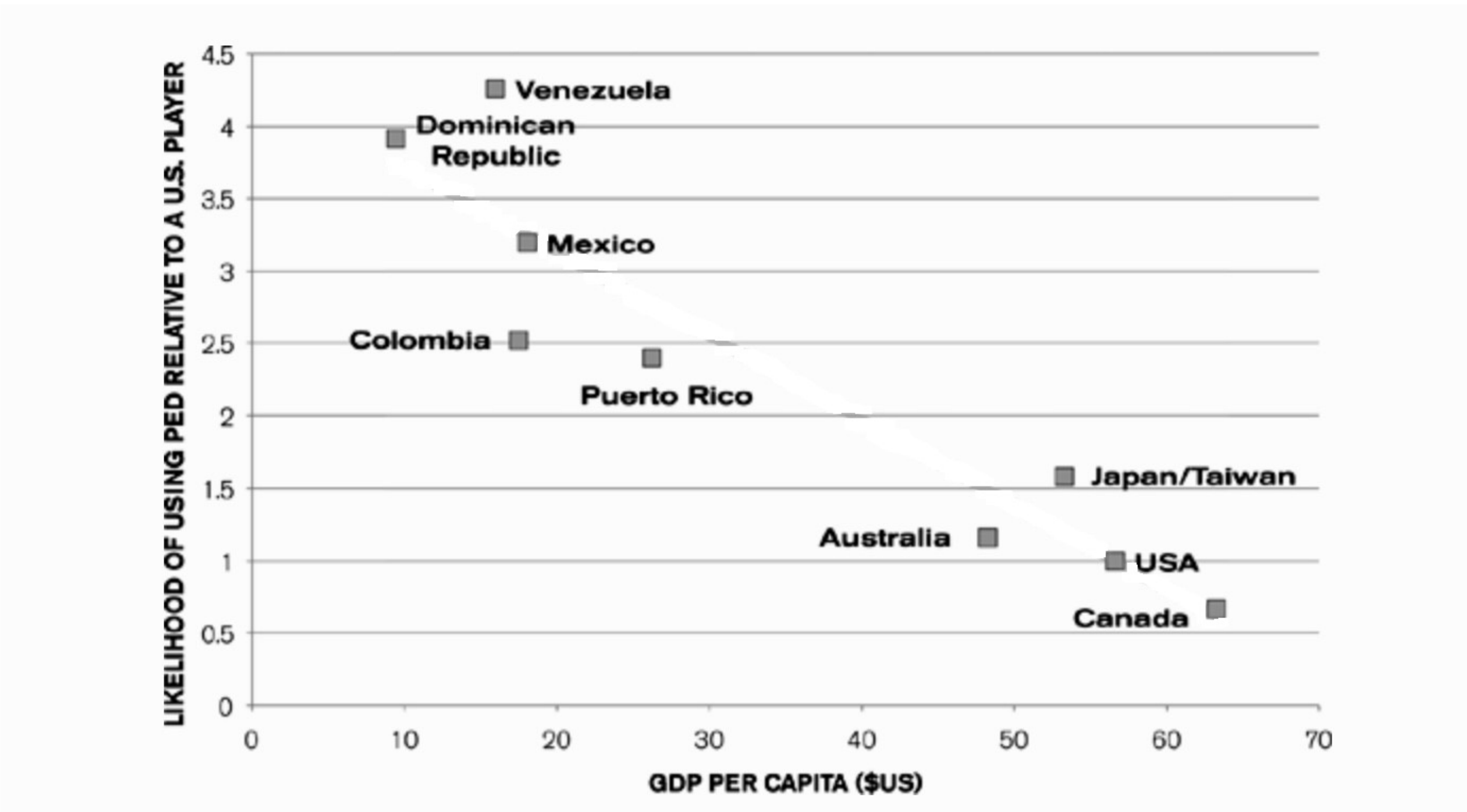

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1. Review of the Linear Simple Regression Model

"explained" variable	y	a.k.a. dependent variable, *regressand*
"explanatory" variable	x	a.k.a. independent variable, *regressor*
intercept parameter	β_0	$E(y \mid x = 0)$
slope parameter	β_1	$\dfrac{\Delta E(y\|x)}{\Delta x}$
Population regression function	$E(y \mid x) = \beta_0 + \beta_1 x$	*systematic component* [2.8]
disturbance	u	*stochastic component*
Simple linear regression model	$y = \beta_0 + \beta_1 x + u$	

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2. Summation (Review Appendix A.1)

For the following, assume summation is over $i = 1, \ldots, n$

[A.1] $\sum x_i = x_1 + x_2 + \cdots$

[A.5] $\frac{\sum x_i}{n} = \bar{x}$

[A.6] $\sum (x_i - \bar{x}) = 0$

[A.2] $\sum c = n \cdot c$

[A.3] $\sum c \cdot x_i = c \cdot \sum x_i$

[A.7] $\sum x_i \cdot (x_i - \bar{x}) = \sum (x_i - \bar{x})^2 = \sum (x_i)^2 - \frac{(\sum x_i)^2}{n}$

$\sum x_i \cdot (y_i - \bar{y}) = \sum (x_i - \bar{x}) \cdot (y_i - \bar{y})$

3. Definitions (Review Appendix B.3, B.4)

For the following, assume summation is over $i = 1, \ldots, n$

Mean $\quad E(x) = \frac{1}{n}\sum x_i \qquad E(y) = \frac{1}{n}\sum y_i$

Variance $\quad E[(x-\mu_x)^2] = \frac{1}{n}\sum(x_i-\bar{x})^2 \qquad E\left[\left(y-\mu_y\right)^2\right] = \frac{1}{n}\sum(y_i-\bar{y})^2$ [B.23]

Covariance $\quad E\left[(x-\mu_x)\cdot\left(y-\mu_y\right)\right] = \frac{1}{n}\sum(x_i-\bar{x})\cdot(y_i-\bar{y})$ [B.26]

Correlation $\quad r_{xy} = \frac{cov(x,y)}{\sqrt{var(x)\cdot var(y)}} = \frac{\sum(x_i-\bar{x})\cdot(y_i-\bar{y})}{\sqrt{\sum(x_i-\bar{x})^2\cdot\sum(y_i-\bar{y})^2}}$ [B.29]

Regression slope $\quad \beta_1 = \frac{cov(x,y)}{var(x)} = \frac{\sum(x_i-\bar{x})\cdot(y_i-\bar{y})}{\sum(x_i-\bar{x})^2}$ [2.19]

Regression intercept $\quad \beta_0 = \bar{y} - \beta_1\bar{x}$ [2.17]

3. Definitions, continued

Sample regression function $\hat{y} = \widehat{\beta_0} + \widehat{\beta_1} \cdot x$ [2.23]

Fitted values $\widehat{y_i} = \widehat{\beta_0} + \widehat{\beta_1} \cdot x_i$ for $i = 1, \ldots, n$ [2.20]

Residual $\widehat{u_i} = y_i - \widehat{y_i}$

$= y_i - \widehat{\beta_0} - \widehat{\beta_1} \cdot x_i$ [2.21]

Sum of squared residuals $\sum \widehat{u_i}^2 = \sum (y_i - \widehat{y_i})^2 = \sum \left(y_i - \widehat{\beta_0} - \widehat{\beta_1} \cdot x_i\right)^2$ [2.22]

4. Assumptions of the Linear Simple Regression Model

Assumption 1. Zero mean residual $\qquad E(\hat{u}) = 0 \qquad$ [2.10]

⟦ used to prove $\widehat{\beta_0} = \bar{y} - \beta_1 \bar{x}$; see whiteboard, p. 28 ⟧

① Assume $E(\hat{u}_i) = 0$; prove $\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$

define $\hat{u}_i = y_i - \hat{y} = y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i$

then $\frac{1}{n} \sum \hat{u}_i = 0 = \frac{1}{n} \sum (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)$

$$0 = \frac{1}{n} \sum y_i - \frac{1}{n} \sum \hat{\beta}_0 - \frac{1}{n} \sum \hat{\beta}_1 x_i$$

$$0 = \bar{y} - \frac{1}{n} \cdot n \hat{\beta}_0 - \hat{\beta}_1 \bar{x}$$

def. of $\bar{y}$ — rule that $\sum_{i=1}^{n} c = nc$ — def. of $\bar{x}$ and rule that $\sum c \cdot x_i = c \sum x_i$

$$0 = \bar{y} - \hat{\beta}_0 - \hat{\beta}_1 \bar{x}$$

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}$$

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4. Assumptions of the Linear Simple Regression Model

Assumption 2. Zero correlation of residual and regressor $E(\hat{u} \cdot x) = 0$ [2.11]

⟦ used to prove $\widehat{\beta_1} = \dfrac{cov(x,y)}{var(x)}$; see whiteboard, p. 28-29 ⟧

② Assume $E(\hat{u} \cdot x) = 0$; prove that $\hat{\beta}_1 = \dfrac{cov(x,y)}{var(x)}$ =

$$\frac{1}{n} \Sigma u_i x_i = 0 = \frac{1}{n} \Sigma x_i (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)$$

substitute $\bar{y} - \hat{\beta}_1 \bar{x}$ for $\hat{\beta}_0$: $\frac{1}{n} \Sigma x_i (y_i - \bar{y} + \hat{\beta}_1 \bar{x} - \hat{\beta}_1 x_i)$

$$= \frac{1}{n} \Sigma x_i \left((y_i - \bar{y}) - \hat{\beta}_1 (x_i - \bar{x})\right)$$

$$= \frac{1}{n} \Sigma x_i (y_i - \bar{y}) - \hat{\beta}_1 \frac{1}{n} \Sigma x_i (x_i - \bar{x})$$

$$= \frac{1}{n} \Sigma (x_i - \bar{x})(y_i - \bar{y}) - \hat{\beta}_1 \cdot \frac{1}{n} \Sigma (x_i - \bar{x})^2$$

$$0 = cov(x,y) - \hat{\beta}_1 \, var(x)$$

$$\hat{\beta}_1 \, var(x) = cov(x,y)$$

$$\hat{\beta}_1 = \frac{cov(x,y)}{var(x)}$$

Interlude – PED Use in Baseball

Scorecasting, by T.J. Moskowitz & L.J. Wertheim (2011)

Dataset:	*PEDs in baseball.csv*
Sample:	National aggregate data (n = 9ish) from 2005 to 2009
Dependent variable:	Relative likelihood of PED use (1 = American ballplayer)
Independent variable:	Gross Domestic Product per capita (US $)

```
x <- c(48, 63, 17, 9, 53, 18, 26, 56, 16)
y <- c(1.2, 0.7, 2.5, 3.9, 1.6, 3.2, 2.4, 1.0, 4.2)
origin <- c("Australia", "Canada", "Colombia", "Dominican", "JapanTaiwan", "Mexico",
            "PuertoRico", "USA", "Venezuela")
plot(x, y, pch=16, xlim = c(0,70), ylim = c(0,4.5))
text(x, y, labels=origin, pos=1, xpd=TRUE)
```

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Interlude – PED Use in Baseball

i	x_i	y_i	$(x_i - \bar{x})$	$(x_i - \bar{x})^2$	$(y_i - \bar{y})$	$(y_i - \bar{y})^2$	$(x_i - \bar{x})(y_i - \bar{y})$
Australia	48	1.2					
Canada	63	0.7					
Colombia	17	2.5					
Dominican	9	3.9					
JapanTaiwan	53	1.6					
Mexico	18	3.2					
PuertoRico	26	2.4					
USA	56	1.0					
Venezuela	16	4.2					
Σ			$SST_x =$		$SST_y =$		
Σ/n	$\bar{x} =$	$\bar{y} =$	$var(x)=$		$var(y)=$		$cov(x,y)=$

Slope: $\widehat{\beta_1} = \frac{cov(x,y)}{var(x)} = \underline{\qquad\qquad} =$

Intercept: $\widehat{\beta_0} = \bar{y} - \widehat{\beta_1}\bar{x} =$

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Postlude – PED Use in Baseball

i	x_i	y_i	$\hat{y}_i = \widehat{\beta_0} + \widehat{\beta_1} x_i$	$\hat{u}_i = (y_i - \hat{y}_i)$	$\hat{u}_i \cdot x_i$	$\hat{u}_i^2 = (y_i - \hat{y}_i)^2$
Australia	48	1.2				
Canada	63	0.7				
Colombia	17	2.5				
Dominican	9	3.9				
JapanTaiwan	53	1.6				
Mexico	18	3.2				
PuertoRico	26	2.4				
USA	56	1.0				
Venezuela	16	4.2				
Σ			$\Sigma\hat{y}_i =$	$\Sigma\hat{u}_i =$	$\Sigma\hat{u}_i \cdot x_i =$	SSR =
Σ/n	$\bar{x} =$	$\bar{y} =$	$\bar{\hat{y}} =$			

$$sigma: \quad \sqrt{\widehat{\sigma^2}} = \sqrt{\frac{SSR}{n-2}} =$$

$$se(\widehat{\beta_1}) = \frac{\hat{\sigma}}{\sqrt{\sum(x_i - \bar{x})^2}} = - =$$

$$se(\widehat{\beta_0}) = \frac{\hat{\sigma}}{\sqrt{\sum(x_i - \bar{x})^2}} \cdot \sqrt{\frac{\sum x_i^2}{n}} =$$