POLS 6481. Research Design and Quantitative Methods II

Lecture 26. Interactions I: Multiplying Variables

Readings: Wooldridge, *Introductory Econometrics 5e*, 7.2a + 7.4a
Kam & Franzese, p. 27–29, 54–56, 67–68, 85–86

Outline

1. A Single Dummy for a Single Dimension
2. Multiple Dummies for Multiple Dimensions
3. Multiple Dummies for Multiple Dimensions – Interaction Version
4. Gender & Party and Social Welfare Opinions

POLS 6481. Research Design and Quantitative Methods II
Lecture 26. Interactions I: Multiplying Variables
Readings: Wooldridge, *Introductory Econometrics 5e*, 7.2a + 7.4a

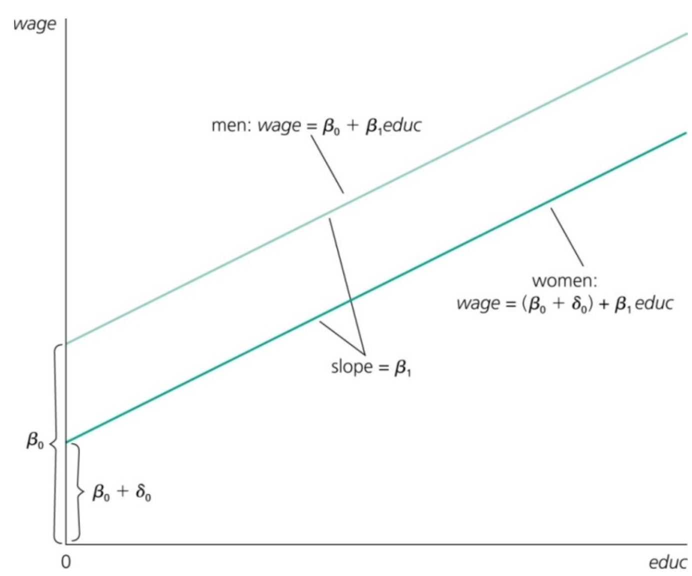
1. A Single Dummy for a Single Dimension

Combined model: $wage = \beta_0 + \delta_0 \cdot female + \beta_1 educ + u$

men only: $wage = \beta_0 + \beta_1 educ + u$

women only: $wage = (\beta_0 + \delta_0) + \beta_1 educ + u$

Intercept shift interpretation: $E(wage \mid female=1, educ = 0) = \beta_0 + \delta_0$
$E(wage \mid female=0, educ = 0) = \beta_0$



POLS 6481. Research Design and Quantitative Methods II
Lecture 26. Interactions I: Multiplying Variables
Readings: Wooldridge, *Introductory Econometrics 5e*, 7.2a + 7.4a

1. A Single Dummy for a Single Dimension

Combined model: $wage = \beta_0 + \delta_0 \cdot female + \beta_1 educ + u$

men only: $wage = \beta_0 + \beta_1 educ + u$

women only: $wage = (\beta_0 + \delta_0) + \beta_1 educ + u$

Treatment effect interpretation: $\delta_0 = \mathrm{E}(wage \mid female{=}1, educ) - \mathrm{E}(wage \mid female{=}0, educ)$

POLS 6481. Research Design and Quantitative Methods II
Lecture 26. Interactions I: Multiplying Variables
Readings: Wooldridge, *Introductory Econometrics 5e*, 7.2a + 7.4a

2. Multiple Dummies for Multiple Dimensions

New model

$$\begin{aligned} wage = \; & \beta_0 + \beta_1 educ + u \\ & + \gamma_0 \, married\ man \\ & + \delta_0 \, single\ woman \\ & + \lambda_0 \, married\ woman \end{aligned}$$

γ_0, δ_0, and λ_0 represent intercept shifts; each is relative to the omitted category (*single man*)

POLS 6481. Research Design and Quantitative Methods II
Lecture 26. Interactions I: Multiplying Variables
Readings: Wooldridge, *Introductory Econometrics 5e*, 7.2a + 7.4a

2. Multiple Dummies for Multiple Dimensions

for single men: $wage = \beta_0 + \beta_1 educ + u$

for married men: $wage = (\beta_0 + \gamma_0) + \beta_1 educ + u$

for single women: $wage = (\beta_0 + \delta_0) + \beta_1 educ + u$

for married women: $wage = (\beta_0 + \lambda_0) + \beta_1 educ + u$

Effect***s*** of gender:

Among unmarried: $E(wage \mid female=1, married = 0, educ = 0) = \beta_0 + \delta_0$
$E(wage \mid female=0, married = 0, educ = 0) = \beta_0$

$(+\ \delta_0)$

Among married: $E(wage \mid female=1, married = 1, educ = 0) = \beta_0 + \lambda_0$
$E(wage \mid female=0, married = 1, educ = 0) = \beta_0 + \gamma_0$

$(+\ \lambda_0 - \gamma_0)$

2. Multiple Dummies for Multiple Dimensions

for single men: $wage = \beta_0 + \beta_1 educ + u$

for married men: $wage = (\beta_0 + \gamma_0) + \beta_1 educ + u$

for single women: $wage = (\beta_0 + \delta_0) + \beta_1 educ + u$

for married women: $wage = (\beta_0 + \lambda_0) + \beta_1 educ + u$

Effect***s*** of marriage:

Among males:
$E(wage \mid female=0, married = 1, educ = 0) = \beta_0 + \gamma_0$
$E(wage \mid female=0, married = 0, educ = 0) = \beta_0$
$(+ \gamma_0)$

Among females:
$E(wage \mid female=1, married = 1, educ = 0) = \beta_0 + \lambda_0$
$E(wage \mid female=1, married = 0, educ = 0) = \beta_0 + \delta_0$
$(\lambda_0 - \delta_0)$

POLS 6481. Research Design and Quantitative Methods II
Lecture 26. Interactions I: Multiplying Variables
Readings: Wooldridge, *Introductory Econometrics 5e*, 7.2a + 7.4a

2. Multiple Dummies for Multiple Dimensions

Example 7.6 originally… (Wooldridge 235–236)

$$\widehat{log(wage)} = \underset{(.100)}{.321} + \underset{(.055)}{.213}\, marr.male - \underset{(.056)}{.110}\, sing.fem - \underset{(.058)}{.198}\, marr.fem + \underset{(.007)}{.079}\, educ + \ldots$$

E(*wage* | *female*=0, *married* = 0, *educ* = 0, …) = .321 [baseline]

E(*wage* | *female*=0, *married* = 1, *educ* = 0, …) = .534

E(*wage* | *female*=1, *married* = 0, *educ* = 0, …) = .211

E(*wage* | *female*=1, *married* = 1, *educ* = 0, …) = .123

Effects of marriage – for men: +.213 = .534 – .321
for women: –.088 = .123 – .211 (= –.198 – –.110)

Effects of gender – for singles: –.110 = .211 – .321
for marrieds: –.411 = .123 – .534 (= –.198 – .213)

Readings: Wooldridge, *Introductory Econometrics 5e*, 7.2a + 7.4a

3. Multiple Dummies for Multiple Dimensions – Interaction Version

Example 7.6 continues… (Wooldridge 240)

$$\widehat{log(wage)} = \underset{(.100)}{.321} + \underset{(.055)}{.213}\, married - \underset{(.056)}{.110}\, female - \underset{(.072)}{.301}\, marr{*}female + \underset{(.007)}{.079}\, educ + \ldots$$

E($wage$ | $female$=0, $married$ = 0, $educ$ = 0, …) = .321 [baseline]

E($wage$ | $female$=0, $married$ = 1, $educ$ = 0, …) = .534

E($wage$ | $female$=1, $married$ = 0, $educ$ = 0, …) = .211

E($wage$ | $female$=1, $married$ = 1, $educ$ = 0, …) = .123

Effects of marriage –	for men:	+.213	
	for women:	–.088	(= +.213 – .301)
Effects of gender –	for singles:	–.110	
	for marrieds:	–.411	(= –.110 – .301)

POLS 6481. Research Design and Quantitative Methods II
Lecture 26. Interactions I: Multiplying Variables
Readings: Wooldridge, *Introductory Econometrics 5e*, 7.2a + 7.4a

3. Multiple Dummies for Multiple Dimensions – Interaction Version

Combining coefficients – suppose single male (*sing.m)* is our baseline category

A. suppose we are using *marr.m*, *sing.f*, and *marr.f*

Effect of marriage for males is simply coefficient for *marr.m* $\left(\hat{\beta}_{marr.m}\right)$

... which has standard error:

$$\sqrt{var\left(\hat{\beta}_{marr.m}\right)}$$

Effect of marriage for females is difference of coefficients: $\left(\hat{\beta}_{marr.f} - \hat{\beta}_{sing.f}\right)$

... which has standard error:

$$\sqrt{var\left(\hat{\beta}_{marr.f}\right) + var\left(\hat{\beta}_{sing.f}\right) - 2 \cdot cov\left(\hat{\beta}_{marr.f}, \hat{\beta}_{sing.f}\right)}$$

POLS 6481. Research Design and Quantitative Methods II
Lecture 26. Interactions I: Multiplying Variables
Readings: Wooldridge, *Introductory Econometrics 5e*, 7.2a + 7.4a

3. Multiple Dummies for Multiple Dimensions – Interaction Version

Combining coefficients – suppose single male (*sing.m)* is our baseline category

B. suppose we are using *married*, *female*, and *married*female*

Effect of marriage for males is simply coefficient for *married* $\left(\hat{\beta}_{married}\right)$

... which has standard error:

$$\sqrt{var\left(\hat{\beta}_{married}\right)}$$

Effect of marriage for females is sum of coefficients: $\left(\hat{\beta}_{married} + \hat{\beta}_{married*female}\right)$

... which has standard error:

$$\sqrt{var\left(\hat{\beta}_{married}\right) + var\left(\hat{\beta}_{married*female}\right) + 2 \cdot cov\left(\hat{\beta}_{married}, \hat{\beta}_{married*female}\right)}$$

3. Multiple Dummies for Multiple Dimensions – Interaction Version

Shortcut for finding or verifying calculations on previous page: change baseline group!

In A., replace *sing.f* with *sing.m*;
then coefficient for *marr.f* is effect of marriage for women

In B., replace *female* with *male* and *married*female* with *married*male*;
then coefficient for *married* is effect of marriage for women

POLS 6481. Research Design and Quantitative Methods II
Lecture 26. Interactions I: Multiplying Variables
Readings: Kam & Franzese, p. 27–29, 54–56, 67–68, 85–86

4. Gender & Party and Social Welfare Opinions

Data: *socialwelfare.dta*
Script: Lecture 26 socialwelfare.R

N = 1,077 respondents in 2004 American National Election Study

Dependent variable : socwel (index described in fn. 16; range [0,1])

The dependent variable is compiled from support for services and spending; government provision of jobs and a standard of living; and support for federal spending on welfare programs, social security, public schools, child care, and assistance to the poor, rescaled to range from zero (least supportive) to one (most supportive).

Readings: Kam & Franzese, p. 27–29, 54–56, 67–68, 85–86

4. Gender & Party and Social Welfare Opinions

Data: *socialwelfare.dta*
Script: Lecture 26 socialwelfare.R

N = 1,077 respondents in 2004 American National Election Study

Dependent variable : socwel (a.k.a *Opinion*)

Independent variables: Female {0,1}
Republican {0,1}

$$Opinion = \beta_0 + \beta_F Female + \beta_R Republican + \beta_{FR} Female \times Republican \tag{16}$$

POLS 6481. Research Design and Quantitative Methods II
Lecture 26. Interactions I: Multiplying Variables
Readings: Kam & Franzese, p. 27–29, 54–56, 67–68, 85–86

4. Gender & Party and Social Welfare Opinions

TABLE 3. OLS Regression Results, *Support for Social Welfare*

	Coefficient (standard error) *p*-Value
Female	−0.0031
	(0.0144)
	0.828
Republican	$\boxed{-0.2205}$
	(0.0155)
	0.000
Female × *Republican*	$\boxed{0.0837}$
	(0.0214)
	0.000
Intercept	0.7451
	(0.0110)
	0.000
N (*df*)	1,077 (1,073)
Adjusted R^2	0.223
$P > F$	0.000

Note: Cell entries are the estimated coefficient, with standard error in parentheses, and two-sided *p*-level (probability $|T| > t$) referring to the null hypothesis that $\beta = 0$ in italics.

Readings: Kam & Franzese, p. 27–29, 54–56, 67–68, 85–86

4. Gender & Party and Social Welfare Opinions

Marginal Effects – easiest to start with predicted values, then calculate deltas:

TABLE 4. Predicted *Support for Social Welfare*

	Democrats (*Republican* = 0)	*Republicans* (*Republican* = 1)
Males (*Female* = 0)	0.745	0.525
Females (*Female* = 1)	0.742	0.605

Compare down columns:

	$\partial\hat{y}/\partial F$	*s.e.* ($\partial\hat{y}/\partial F$)	*t*-Statistic
Republican = 0	−0.003	0.0144	−0.218
Republican = 1	0.081	0.0158	5.109

Compare across rows:

	$\partial\hat{y}/\partial R$	*s.e.* ($\partial\hat{y}/\partial R$)	*t*-Statistic
Female = 0	−0.220	0.0155	−14.18
Female = 1	−0.137	0.0147	−9.33

Readings: Kam & Franzese, p. 27–29, 54–56, 67–68, 85–86

4. Gender & Party and Social Welfare Opinions

Model:

$$y = \gamma_0 + \beta_x x + \beta_z z + \beta_{xz} xz + \varepsilon \qquad (2)$$

Marginal Effects:

$$\partial y/\partial x = \beta_x + \beta_{xz} z \qquad (9)$$

$$\partial y/\partial z = \beta_z + \beta_{xz} x \qquad (10)$$

Variances

$$V(\partial \hat{y}/\partial x) = V(\hat{\beta}_x) + z^2 V(\hat{\beta}_{xz}) + 2zC(\hat{\beta}_x, \hat{\beta}_{xz}) \qquad (26)$$

$$V(\partial \hat{y}/\partial z) = V\left(\hat{\beta}_z\right) + x^2 V\left(\hat{\beta}_{xz}\right) + 2xC\left(\hat{\beta}_z, \hat{\beta}_{xz}\right)$$

Standard Errors:

$$s.e.(\partial \hat{y}/\partial x) = \sqrt{V(\partial \hat{y}/\partial x)}$$

$$s.e.(\partial \hat{y}/\partial z) = \sqrt{V(\partial \hat{y}/\partial z)}$$

Hypothesis Tests:

$$t = \frac{\partial \hat{y}/\partial x}{s.e.(\partial \hat{y}/\partial x)}$$

$$t = \frac{\partial \hat{y}/\partial z}{s.e.(\partial \hat{y}/\partial z)}$$

Readings: Kam & Franzese, p. 27–29, 54–56, 67–68, 85–86

4. Gender & Party and Social Welfare Opinions

Model: $SW = .7451 - .0031\, F - .2205\, R + .0837\, FR$

Marginal Effects:

$\partial SW / \partial F = -\ .0031 + .0837\, R$ = – .0031 among Democrats
= + .0806 among Republicans

$\partial SW / \partial R = -\ .2205 + .0837\, F$ = – .2205 among males
= – .1368 among females

POLS 6481. Research Design and Quantitative Methods II
Lecture 26. Interactions I: Multiplying Variables
Readings: Kam & Franzese, p. 27–29, 54–56, 67–68, 85–86

4. Gender & Party and Social Welfare Opinions

TABLE 26. OLS Regression Results, *Support for Social Welfare*, Pooled and Split Samples

	Pooled Sample Coefficient (standard error) *p*-Value	Males Only Coefficient (standard error) *p*-Value	Females Only Coefficient (standard error) *p*-Value
Female	−0.0031	—	—
	(0.0144)		
	0.828		
Republican	−0.2205	−0.2205	−0.1368
	(0.0155)	(0.0154)	(0.0148)
	0.000	*0.000*	*0.000*
Female × *Republican*	0.0837	—	—
	(0.0214)		
	0.000		
Intercept	0.7451	0.7451	0.7420
	(0.0110)	(0.0109)	(0.0094)
	0.000	*0.000*	*0.000*
N (df)	1,077 (1,073)	498 (496)	579 (577)
Adjusted R^2	0.223	0.290	0.128
$P > F$	0.000	0.000	0.000

Note: Cell entries are the estimated coefficient, with standard error in parentheses, and two-sided p-level (probability $|T| > t$) referring to the null hypothesis that $\beta = 0$ in italics.

POLS 6481. Research Design and Quantitative Methods II
Lecture 26. Interactions I: Multiplying Variables
Readings: Kam & Franzese, p. 27–29, 54–56, 67–68, 85–86

4. Gender & Party and Social Welfare Opinions

TABLE 16. Estimated Variance-Covariance Matrix of Coefficient Estimates, Predicting *Support for Social Welfare*

	Female	*Republican*	*Female* × *Republican*	Intercept
Female	0.00021			
Republican	0.00012	0.00024		
Female × *Republican*	−0.00021	−0.00024	0.00046	
Intercept	−0.00012	−0.00012	0.00012	0.00012

Variances:

$$
\begin{aligned}
V(\partial\hat{y}/\partial F) &= V(\hat{\beta}_F) + R^2 \cdot V(\hat{\beta}_{FR}) + 2R \cdot C(\hat{\beta}_F, \hat{\beta}_{FR}) \\
&= .00021 + R^2 \cdot (.00046) + 2R \cdot (-.00021) \\
&= .00021 \text{ if } R = 0 \\
&= .00025 \text{ if } R = 1
\end{aligned}
$$

$$
\begin{aligned}
V(\partial\hat{y}/\partial R) &= V(\hat{\beta}_R) + F^2 \cdot V(\hat{\beta}_{FR}) + 2F \cdot C(\hat{\beta}_R, \hat{\beta}_{FR}) \\
&= .00024 + R^2 \cdot (.00046) + 2R \cdot (-.00024) \\
&= .00024 \text{ if } R = 0 \\
&= .00022 \text{ if } R = 1
\end{aligned}
$$

Standard Errors:

$\sqrt{.00021} = .0145$

$\sqrt{.00025} = .0158$

$\sqrt{.00024} = \boxed{.0155}$

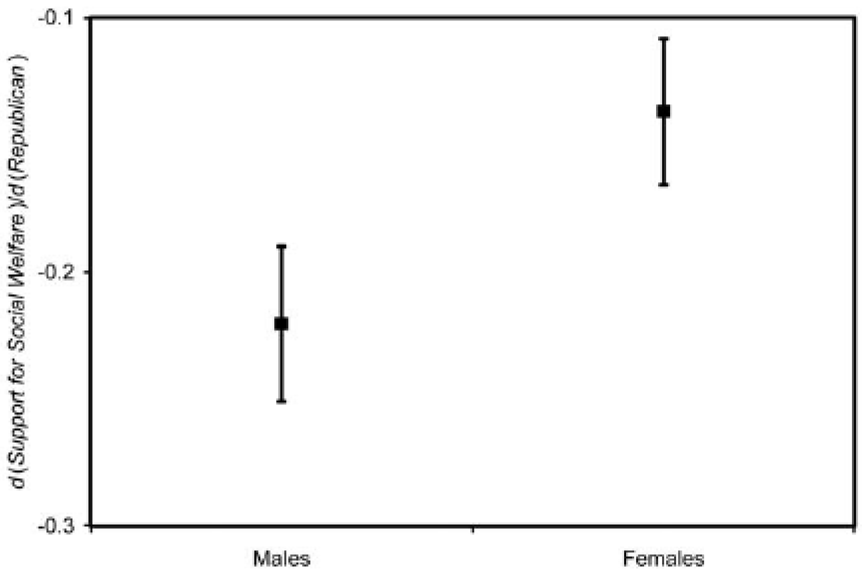
$\sqrt{.00022} = \boxed{.0148}$

Readings: Kam & Franzese, p. 27–29, 54–56, 67–68, 85–86

4. Gender & Party and Social Welfare Opinions

TABLE 18. Hypothesis Tests of whether *Republican* Affects *Support for Social Welfare*

	$\partial\hat{y}/\partial R$	s.e. ($\partial\hat{y}/\partial R$)	t-Statistic	One-Tailed p-Value H_0: $\beta_R + \beta_{FR}Female \leq 0$	One-Tailed p-Value H_0: $\beta_R + \beta_{FR}Female \geq 0$	95% Confidence Interval
Female = 0	−0.220	0.0155	−14.18	0.999	0.000	[−0.251, −0.190]
Female = 1	−0.137	0.0147	−9.33	0.999	0.000	[−0.166, −0.108]



POLS 6481. Research Design and Quantitative Methods II
Lecture 26. Interactions I: Multiplying Variables
Readings: Kam & Franzese, p. 27–29, 54–56, 67–68, 85–86

4. Gender & Party and Social Welfare Opinions

TABLE 17. Hypothesis Tests of whether *Female* Affects *Support for Social Welfare*

	$\partial\hat{y}/\partial F$	*s.e.* $(\partial\hat{y}/\partial F)$	*t*-Statistic	One-Tailed *p*-Value H_0: $\beta_F + \beta_{FR}Republican \leq 0$	One-Tailed *p*-Value H_0: $\beta_F + \beta_{FR}Republican \geq 0$	95% Confidence Interval
Republican = 0	−0.003	0.0144	−0.218	0.586	0.414	[−0.031, 0.025]
Republican = 1	0.081	0.0158	5.109	0.000	0.999	[0.050, 0.111]