POLS 6481. Research Design and Quantitative Methods II

Lecture 9. Rescaling~~, Partialling, and Mediation~~

Readings: Wooldridge, *Introductory Econometrics 5e,* 2.4a + 6.1

Outline:

1. Effects of Rescaling a Regressor (x)
2. Effects of Rescaling the Regressand (y)
3. Beta Coefficients

1½, 2½ and 3½. Example: *The NYC Marathon*

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1. Effects of Rescaling a Regressor

Suppose I double the value of x… let $X = 2x$; then:

Effects on $\widehat{\boldsymbol{\beta}_1}$

$cov(X, y) = 2 \times cov(x, y)$ each "rectangle" has double the area

$var(X) = 4 \times var(x)$ each "square" has quadruple the area

The ratio $\frac{cov(X,y)}{var(X)} = \frac{1}{2} \cdot \frac{cov(x,y)}{var(x)}$: twice as much ΔX is needed to yield the same Δy

Effects on $\boldsymbol{se(\widehat{\beta_1})}$

The residual standard error ($\hat{\sigma}$) will be unchanged

The standard deviation of X doubles: $s_x = 2 \times s_x$, and therefore $\sqrt{n} \cdot s_x$ will double

The ratio $\frac{\hat{\sigma}}{\sqrt{n} \cdot s_X} = \frac{1}{2} \cdot \frac{\hat{\sigma}}{\sqrt{n} \cdot s_x}$

Effects on $\boldsymbol{t}$ statistic

The ratio $\frac{\widehat{\beta_1}}{se(\widehat{\beta_1})}$ will be unchanged

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1½. Example: *The NYC Marathon*

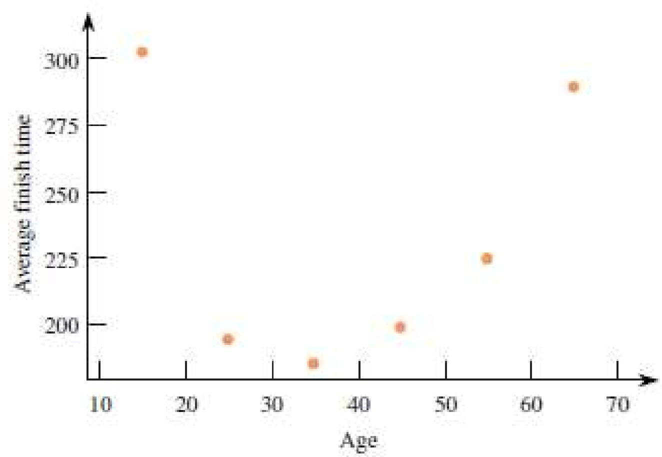
To investigate the effects of age on marathon finish times, the article "Master's Performance in the New York City Marathon" (*British Journal of Sports Medicine* [2004: 408-412]) gave the following data on average finishing time by age group for female participants in 1999:

The dependent variable is *y*, *Average.finish.time* (in minutes) by runners in 10-year age group

The independent variable is *x*, *Representative.age* for generic member of 10-year age group

The dataset is *NYCmarathon.csv*. The R script is *Lecture 9 NYCmarathon.R*.

Age Group	Representative Age	Average Finish Time
10–19	15	302.38
20–29	25	193.63
30–39	35	185.46
40–49	45	198.49
50–59	55	224.30
60–69	65	288.71



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1½. Example: *The NYC Marathon*

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The dependent variable is *y*, *Average.finish.time* (in minutes) by runners in 10-year age group

The independent variable is *x*, *Representative.age* for generic member of 10-year age group

The dataset is *NYCmarathon.csv*. The R script is *Lecture 9 scaling variables.R*.

The simple correlation of *x* and *y* is .04 including the 10–19 age group;
the simple correlation of *x* and *y* is .86 excluding the 10–19 age group.

A simple regression of *y* on *x* yields $\widetilde{\beta_1}$ = 2.29 & $se\left(\widetilde{\beta_1}\right)$ = .781 $\rightarrow t$ = 2.933

Suppose we transform the independent variable from years to decades, so that a 1-unit change in *x* refers to being a decade older.

A simple regression of *y* on $\frac{x}{10}$ yields $\widetilde{\beta_1}$ = 22.9 & $se\left(\widetilde{\beta_1}\right)$ = 7.81 $\rightarrow t$ = 2.933

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2. Effects of Rescaling the Regressand

Suppose I double the value of y … let $Y = 2y$; then:

Effects on $\widehat{\beta_1}$

$cov(x, Y) = 2 \times cov(x, y)$

$var(x)$ will be unchanged

The ratio $\frac{cov(x,Y)}{var(x)} = 2 \times \frac{cov(x,y)}{var(x)}$, implying the same Δx yields twice as much ΔY

Effects on $se(\widehat{\beta_1})$

The Residual standard error ($\hat{\sigma}$) will double

The standard deviation of x (s_x) will be unchanged, and therefore $\sqrt{n} \cdot s_x$ will be unchanged

The ratio $\frac{\hat{\sigma}}{\sqrt{n} \cdot s_x}$ will be doubled

Effects on t statistic

The ratio $\frac{\widehat{\beta_1}}{se(\widehat{\beta_1})}$ will be unchanged

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2½. Example: *The NYC Marathon*

To investigate the effects of age on marathon finish times, the article "Master's Performance in the New York City Marathon" (*British Journal of Sports Medicine* [2004: 408-412]) gave the following data on average finishing time by age group for female participants in 1999:

The dependent variable is *y*, *Average.finish.time* (in minutes) by runners in 10-year age group

The independent variable is *x*, *Representative.age* for generic member of 10-year age group

The dataset is *NYCmarathon.csv*. The R script is *Lecture 9 scaling variables.R*.

The simple correlation of *x* and *y* is .04 including the 10–19 age group;
the simple correlation of *x* and *y* is .86 excluding the 10–19 age group.

A simple regression of *y* on *x* yields $\widetilde{\beta_1} = 2.29$ & $se\left(\widetilde{\beta_1}\right) = .781$ $\rightarrow t = 2.933$

Suppose we transform the dependent variable from minutes to hours, so that a 1-unit change in *y* refers to the race taking an hour longer.

A simple regression of $\frac{y}{60}$ on *x* yields $\widetilde{\beta_1} = .038$ & $se\left(\widetilde{\beta_1}\right) = .013$ $\rightarrow t = 2.933$

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3. Standardized Coefficients

Regression coefficients indicate how *y* changes in response to a 1–unit change in *x*:

- sign indicates whether *y* increases (+) or decreases (–);
- magnitude indicates by how many units *y* changes.

Standardized coefficients indicate how *y* changes in response to a 1–*standard deviation* change in *x*:

- sign indicates whether *y* increases (+) or decreases (–);
- magnitude indicates by how many *standard deviations* *y* changes.

Characteristics of standardized coefficients:

1. in a simple regression, beta equals the correlation coefficient ($\widehat{beta}_j = \hat{\beta}_j \cdot \frac{s_y}{s_{x_j}}$)

2. just like correlation coefficients, betas will be between –1 and +1

Approach A. Standardize all variables prior to running model.
can use **Make.Z** function in **QuantPsyc** package

Approach B. Modify results afterward using **lm.beta*(model)*** function in **QuantPsyc** package