POLS 6481. Research Design and Quantitative Methods II

Lecture 25. Binary Independent Variables II: Different Slopes for Different Groups

Readings: Wooldridge, *Introductory Econometrics 5e*, 6.2b + 7.4b

Kam & Franzese, p. 1–26, 43–54, 60–67, 78–85

Outline

1. Simple and Complex Questions

2. Adding Variables and/or Estimating Sub-Sample Regressions

3. Interpreting Coefficients in Various Specifications

4. Interactions of One Binary and One Continuous Independent Variable
   A. Introduce Example
   B. Marginal Effects
   C. Variances, Standard Errors, Hypothesis Tests
   D. The Logic of Conditional Coefficients

5. The Chow Test

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1. Simple and Complex Questions

- simple or 'first generation' — effect of x on y
- complex or 'second generation' — effect of x on y depends on z

- under what conditions?

- moderated by… (*not* mediated by…); constrained by…; magnified by…; mitigated by…; etc.

- see fn. 4, p. 10: *interactive*
- (see fn. 2, p. 3: *multiplicative* and *intervening* have been subsumed)

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2. Adding Variables and/or Estimating Sub-Sample Regressions

Five modeling strategies:

Linear pooled $y = \beta_0 + \beta_x x + e$

Linear additive $y = \beta_0 + \beta_x x + \beta_d d + e$

Linear interactive *without* a constituent term $y = \beta_0 + \beta_x x + \beta_{xd} xd + e$

Linear interactive with all constituent terms $y = \beta_0 + \beta_x x + \beta_d d + \beta_{xd} xd + e$

Linear, separate equations

$$y_{d=0} = \beta_{0,d=0} + \beta_{x,d=0} x + e_{d=0}$$
$$y_{d=1} = \beta_{0,d=1} + \beta_{x,d=1} x + e_{d=1}$$

- Draw pictures of fitted line (or *lines*) in each framework
- How do we interpret β_x and β_d and/or β_{xd} (if present) in each framework
- Draw marginal effects curves $\frac{\Delta y}{\Delta x}$ and confidence intervals around them.

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3. Interpreting coefficients in Various Specifications

Contrast pooled… $y = \beta_0 + \beta_x x + e$
… and additive $y = \beta_0 + \beta_x x + \beta_d d + e$
β_d is an intercept shift! Fitted curves are two parallel lines.
- for $d = 0$, intercept equals β_0 & slope equals β_x
- for $d = 1$, intercept equals $\beta_0 + \beta_d$ & slope equals β_x

Contrast pooled… $y = \beta_0 + \beta_x x + e$
… and interactive *without* all constituent terms $y = \beta_0 + \beta_x x + \beta_{xd} xd + e$
β_{xd} is a slope shift! Fitted curves have same intercept (β_0) but the trajectories deviate:
- for $d = 0$, intercept equals β_0 & slope equals β_x
- for $d = 1$, intercept equals β_0 & slope equals $\beta_x + \beta_{xd}$

Contrast pooled… $y = \beta_0 + \beta_x x + e$
… and interactive *with* all constituent terms $y = \beta_0 + \beta_x x + \beta_d d + \beta_{xd} xd + e$
β_d is an intercept shift and β_{xd} is a slope shift! Fitted curves are two entirely different lines:
- for $d = 0$, intercept equals β_0 & slope equals β_x
- for $d = 1$, intercept equals $\beta_0 + \beta_d$ & slope equals $\beta_x + \beta_{xd}$

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4. Interactions of One Binary and One Continuous Independent Variable

A. Example of number of presidential candidates depending on number of groups in society

Dataset: candidates.dta
R script: Lecture 25 candidates.R
Sample: N = 16 presidential democracies

Dependent variable: Effective number of candidates $\in$ [1.956 , 5.689]; mean = 3.156

Independent variable: Number of groups $\in$ [1 , 2.756]; mean = 1.578 (s.d. = 0.630)

Binary **moderator** variable: Runoff election system $\in$ {0,1}

Model:

$$\hat{y} = \underset{(.650)}{4.303} - \underset{(.407)}{0.979} \cdot Groups \quad \text{if } Runoff = 0$$

$$\hat{y} = \underset{(1.262)}{1.812} + \underset{(.708)}{1.026} \cdot Groups \quad \text{if } Runoff = 1$$

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4. Interactions of One Binary and One Continuous Independent Variable

Kam and Franzese p. 19

TABLE 1. OLS Regression Results, *Number of Presidential Candidates*

	Coefficient (standard error) *p*-Value
Ethnic Groups	−0.979
	(0.770)
	0.228
Runoff	−2.491
	(1.561)
	0.136
Ethnic Groups × *Runoff*	2.005
	(0.941)
	0.054
Intercept	4.303
	(1.229)
	0.004
N (degrees of freedom)	16 (12)
Adjusted R^2	0.203
$P > F$	0.132

Note: Cell entries are the estimated coefficient, with standard error in parentheses, and two-sided *p*-level (probability $|T| > t$) referring to the null hypothesis that $\beta = 0$ in italics.

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4. Interactions of One Binary and One Continuous Independent Variable

Model: $\hat{y} = 4.303 - .979 \cdot Groups - 2.491 \cdot Runoff + 2.005\ Groups \cdot Runoff$

B. Marginal Effects…

… of Changing Number of Ethnic Groups $\partial\hat{y}/\partial G = -.979 + 2.005 \cdot Runoff$

$= -.979$ if $Runoff = 0$ *

$= 1.026$ if $Runoff = 1$ *

C. Variance of Marginal Effect: $V(\partial\hat{y}/\partial G) = V(\hat{\beta}_G) + Runoff^2 \cdot V(\hat{\beta}_{G\times R}) + 2 \cdot Runoff \cdot C(\hat{\beta}_G, \hat{\beta}_{G\times R})$

Standard Errors: $s.e.(\partial\hat{y}/\partial G) = \sqrt{V(\partial\hat{y}/\partial G)}$

Hypothesis Test<u>s</u>: $t = \dfrac{\partial\hat{y}/\partial G}{s.e.(\partial\hat{y}/\partial G)}$

*Note: compare these to the coefficients for sub-samples two slides earlier.

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4. Interactions of One Binary and One Continuous Independent Variable

TABLE 13. Estimated Variance-Covariance Matrix of Coefficient Estimates, Predicting *Number of Presidential Candidates*

	Groups	*Runoff*	*Groups* × *Runoff*	Intercept
Groups	0.593			
Runoff	0.900	2.435		
Groups × *Runoff*	−0.593	−1.377	0.885	
Intercept	−0.900	−1.509	0.900	1.509

C. Variance of Marginal Effect: $V(\partial\hat{y}/\partial G) = V(\hat{\beta}_G) + \ Runoff^2 \cdot V(\hat{\beta}_{G\times R}) + 2 \cdot Runoff \cdot C(\hat{\beta}_G, \hat{\beta}_{G\times R})$

$= .593 \ + Runoff^2 \cdot .885 \ + \ 2 \cdot Runoff \cdot (-.593)$

$= .593$ if *Runoff* = 0 (no runoff)

$= .292$ if *Runoff* = 1 (runoff)

Standard Errors $\quad s.e.(\partial\hat{y}/\partial G) = \sqrt{V(\partial\hat{y}/\partial G)}$ = .770 if *Runoff* = 0 *

= .540 if *Runoff* = 1 *

*Note: compare these to the standard errors for sub-samples three slides earlier.

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4. Interactions of One Binary and One Continuous Independent Variable

TABLE 14. Hypothesis Tests of whether *Groups* Affects *Number of Presidential Candidates*

	$\partial\hat{y}/\partial G$	*s.e.* $(\partial\hat{y}/\partial G)$	*t*-Statistic	One-Tailed *p*-Value H_0: $\beta_C + \beta_{GR} Runoff \leq 0$	One-Tailed *p*-Value H_0: $\beta_C + \beta_{GR} Runoff \geq 0$	90% Confidence Interval
Runoff = 0	−0.979	0.770	−1.271	0.886	0.114	[−2.352, 0.394]
Runoff = 1	1.026	0.540	1.902	0.041	0.959	[0.064, 1.988]

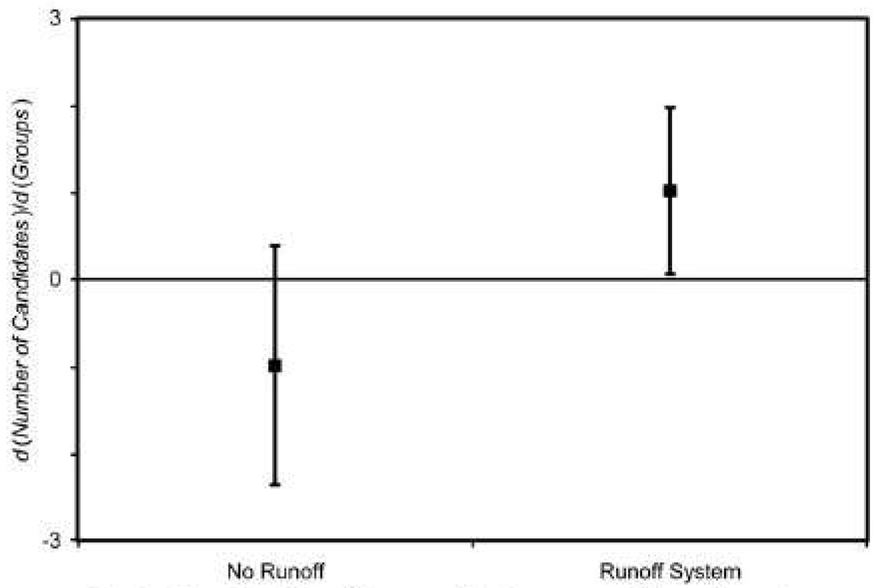

Fig. 6. Marginal effect of *Groups*, with 90 percent confidence intervals

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4. Interactions of One Binary and One Continuous Independent Variable

Model: $\hat{y} = 4.303 - .979 \cdot Groups - 2.491 \cdot Runoff + 2.005\, Groups \cdot Runoff$

B. Marginal Effect…

… of Changing Electoral System $\partial\hat{y}/\partial R = -2.491 + 2.005 \cdot Groups$

When Groups = 1: $\partial\hat{y}/\partial R = -2.491 + 2.005 \times 1 = -0.486$

When Groups = 1.5: $\partial\hat{y}/\partial R = -2.491 + 2.005 \times 1.5 = 0.517$

When Groups = 2: $\partial\hat{y}/\partial R = -2.491 + 2.005 \times 2 = 1.520$

When Groups = 2.5: $\partial\hat{y}/\partial R = -2.491 + 2.005 \times 2.5 = 2.522$

When Groups = 3: $\partial\hat{y}/\partial R = -2.491 + 2.005 \times 3 = 3.525$

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4. Interactions of One Binary and One Continuous Independent Variable

Model: $\hat{y} = 4.303 - .979 \cdot Groups - 2.491 \cdot Runoff + 2.005\, Groups \cdot Runoff$

B. Marginal Effect…

… of Changing Electoral System $\partial\hat{y}/\partial R = -2.491 + 2.005 \cdot Groups$

TABLE 2. Predicted *Number of Presidential Candidates*

	Runoff = 0	*Runoff* = 1	
Groups = 1	3.324	2.838	−0.486
Groups = 1.5	2.835	3.351	= 0.517
Groups = 2	2.345	3.865	1.520
Groups = 2.5	1.855	4.378	= 2.522
Groups = 3	1.366	4.891	3.525

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4. Interactions of One Binary and One Continuous Independent Variable

TABLE 13. Estimated Variance-Covariance Matrix of Coefficient Estimates, Predicting *Number of Presidential Candidates*

	Groups	*Runoff*	*Groups* × *Runoff*	Intercept
Groups	0.593			
Runoff	0.900	2.435		
Groups × *Runoff*	−0.593	−1.377	0.885	
Intercept	−0.900	−1.509	0.900	1.509

C. Variance of Marginal Effect: $V(\partial\hat{y}/\partial R) = V\left(\hat{\beta}_R\right) + \; Groups^2 \cdot V\left(\hat{\beta}_{G\times R}\right) + 2 \cdot Groups \cdot C\left(\hat{\beta}_R, \hat{\beta}_{G\times R}\right)$

$= 2.435 + Groups^2 \cdot .885 + 2 \cdot Groups \cdot (-1.377)$

$= 0.566$ if $Groups = 1$

$= 0.467$ if $Groups = 2$

= …

Standard Errors $s.e.(\partial\hat{y}/\partial R) = \sqrt{V(\partial\hat{y}/\partial G)}$

= 0.752 if $G = 1$

= 0.682 if $G = 2$

= …

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4. Interactions of One Binary and One Continuous Independent Variable

TABLE 15. Hypothesis Tests of whether *Runoff* Affects *Number of Presidential Candidates*

	$\partial\hat{y}/\partial R$	*s.e.* $(\partial\hat{y}/\partial R)$	*t*-Statistic	One-Tailed *p*-Value H_0: $\beta_R + \beta_{GR} Groups \leq 0$	One-Tailed *p*-Value H_0: $\beta_R + \beta_{GR} Groups \geq 0$	90% Confidence Interval
Groups = 1	−0.486	0.752	−0.646	0.735	0.265	[−1.826, 0.854]
Groups = 1.5	0.517	0.542	0.954	0.180	0.820	[−0.449, 1.483]
Groups = 2	1.520	0.682	2.229	0.023	0.977	[0.305, 2.735]
Groups = 2.5	2.522	1.038	2.430	0.016	0.984	[0.672, 4.373]
Groups = 3	3.525	1.461	2.413	0.016	0.984	[0.922, 6.128]

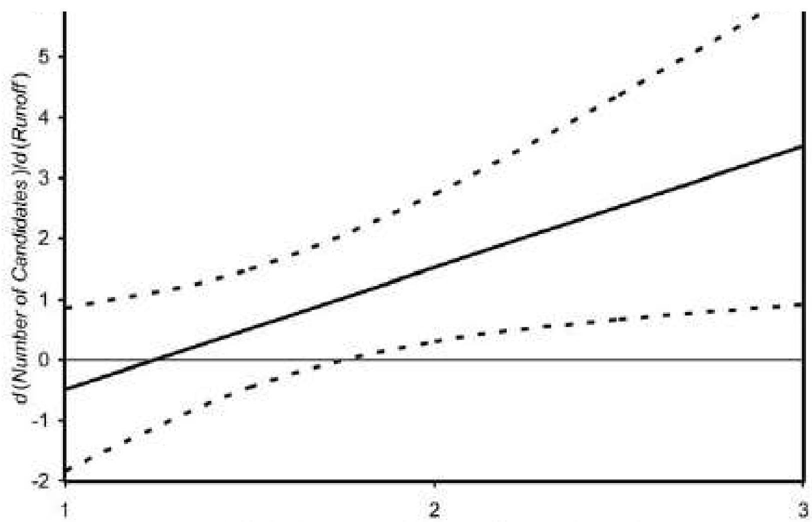

Fig. 4. Marginal effect of *Runoff*, with 90 percent confidence interval

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Readings: $\varnothing$

4. Interactions of One Binary and One Continuous Independent Variable

D. The Logic of Conditional Coefficients

If you have an interaction, say between variables x and z, then:

- the value of β_x is the marginal effect of x when $z = 0$;
- the value of β_z is the marginal effect of z when $x = 0$.

The same interpretations apply to the standard errors:

- the standard error of β_x is the standard error of the marginal effect of x when $z = 0$;
- the standard error of β_z is the standard error of the marginal effect of z when $x = 0$.

If you wish to get the marginal effect and standard error at some other point, you can

- change the baseline group (in case z is binary; e.g., change from *female* to *male*)
- subtract a constant from all values of x (in case x is continuous)

(At a minimum, you should change do this $\uparrow$ to check your math.)

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5. The Chow Test – *To Pool or Not to Pool?*

♦ as described on top half of p. 247 contrasts pooled model and separate equations model

Get SSR_P from: $y = \beta_0 + \beta_x x + e$

Get SSR_0 from: $y_{d=0} = \beta_{0,d=0} + \beta_{x,d=0} x + e_{d=0}$

Get SSR_1 from: $y_{d=1} = \beta_{0,d=1} + \beta_{x,d=1} x + e_{d=1}$

$$F = \frac{(SSR_P - (SSR_0 + SSR_1))/2}{(SSR_0 + SSR_1)/(n-4)}$$

SSR_P = 20.651 when: $\hat{y} = 2.497 + 0.418 \cdot Groups$

SSR_0 = 1.935 when: $\hat{y} = 4.303 - 0.979 \cdot Groups$ if $Runoff = 0$

SSR_1 = 11.898 when: $\hat{y} = 1.812 + 1.026 \cdot Groups$ if $Runoff = 1$

$$F = \frac{(6.818)/2}{(13.833)/(12)} = 2.957$$

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5. The Chow Test – *To Pool or Not to Pool?*

♠ as described on bottom half of p. 247 contrasts additive model (has common slope, but intercept shift for d = 1) and separate equations model

Get $SSR_{P'}$ from: $y = \beta_0 + \beta_x x + \beta_d d + e$

Get SSR_0 from: $y_{z=0} = \beta_{0,d=0} + \beta_{x,d=0} x + e_{d=0}$

Get SSR_1 from: $y_{z=1} = \beta_{0,d=1} + \beta_{x,d=1} x + e_{d=1}$

$$F = \frac{(SSR_{P'} - (SSR_0 + SSR_1))/1}{(SSR_0 + SSR_1)/(n-4)}$$

SSR_P = 19.073 when: $\hat{y} = 2.263 + 0.366 \cdot Groups + .631 \cdot Runoff$

SSR_0 = 1.935 when: $\hat{y} = 4.303 - 0.979 \cdot Groups$ if $Runoff = 0$

SSR_1 = 11.898 when: $\hat{y} = 1.812 + 1.026 \cdot Groups$ if $Runoff = 1$

$$F = \frac{(5.240)/1}{(13.833)/(12)} = 4.546$$