POLS 6481. Research Design and Quantitative Methods II Lecture 3. Properties of Simple Regression Estimates and Residuals Readings: source unknown

i	X_i	y_i	$(x_i - \overline{x})$	$(x_i - \overline{x})^2$	$(y_i - \overline{y})$	$(y_i - \overline{y})^2$	$(x_i-\overline{x})(y_i-\overline{y})$
1930	8.7	3					
1931	15.9	28					
1932	23.6	21					
1933	24.9	17					
1934	21.7	37					
1935	20.1	30					
1936	16.9	31					
1937	14.3	22					
1938	19.0	2					
1939	17.2	9					
Σ	182.3	200	$SST_x =$		$SST_y =$		
Σ/n	$\bar{x} = 18.2$	$\overline{y} = 20$					-
$\Sigma/(n-1)$			var(x) =		var(y) =		cov(x,y) =

Slope:
$$\widehat{\beta_1} = \frac{cov(x,y)}{var(x)} = - =$$
 Intercept: $\widehat{\beta_0} = \overline{y} - \widehat{\beta_1}\overline{x} =$ ____ = ___

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i	X _i	y_i	$(x_i - \overline{x})$	$(x_i - \overline{x})^2$	$(y_i - \overline{y})$	$(y_i - \overline{y})^2$	$(x_i - \overline{x})(y_i - \overline{y})$
1930	8.7	3	-9.5	90.25	-17	289	161.5
1931	15.9	28	-2.3	5.29	8	64	-18.4
1932	23.6	21	5.4	29.16	1	1	5.4
1933	24.9	17	6.7	44.89	-3	9	-20.1
1934	21.7	37	3.5	12.25	17	289	59.5
1935	20.1	30	1.9	3.61	10	100	19
1936	16.9	31	-1.3	1.69	11	121	-14.3
1937	14.3	22	-3.9	15.21	2	4	-7.8
1938	19.0	2	8.0	0.64	-18	324	-14.4
1939	17.2	9	-1.0	1.00	-11	121	11
Σ	182.3	200	$SST_x =$	203.99	$SST_y =$	1322	
Σ/n	$\bar{x} = 18.2$	$\overline{y} = 20$					
$\Sigma/(n-1)$			var(x) =	22.67	var(y) =	146.89	cov(x,y) = 20.16

Slope:
$$\widehat{\beta}_1 = \frac{cov(x,y)}{var(x)} = \frac{20.16}{22.67} = 0.889$$
 Intercept: $\widehat{\beta}_0 = \overline{y} - \widehat{\beta}_1 \overline{x} = 20 - 0.889 \cdot 18.23 = 3.79$

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Readings: source unknown

i	X _i	y_i	$\widehat{y_i} = \widehat{\beta_0} + \widehat{\beta_1} x_i$	$(y_i - \widehat{y_i}) = \widehat{u_i}$	$\widehat{u_i} \cdot x_i$	$\widehat{u_i}^2 = (y_i - \widehat{y_i})^2$
1930	8.7	3				
1931	15.9	28				
1932	23.6	21				
1933	24.9	17				
1934	21.7	37				
1935	20.1	30				
1936	16.9	31				
1937	14.3	22				
1938	19.0	2				
1939	17.2	9				
Σ	182.3	200		$\Sigma \widehat{u_i} = 0$	$\Sigma \widehat{u_i} \cdot x_i \approx 0$	SSR =
Σ/n	$\bar{x} = 18.23$	$\bar{y} = 20$	$\overline{\hat{y}} = 20$			

$$sigma: \ \widehat{\sigma} = \sqrt{\widehat{\sigma^2}} = \sqrt{\frac{SSR}{n-2}} = \underline{\qquad} \qquad \qquad se(\widehat{\beta_1}) = \frac{\widehat{\sigma}}{\sqrt{\sum (x_i - \bar{x})^2}} = \underline{\qquad} = \underline{\qquad}$$

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4. Example: *Unemployment and Political Violence*

i	X _i	y_i	$\widehat{y}_i = \widehat{\beta_0} + \widehat{\beta_1} x_i$	$(y_i - \widehat{y}_i) = \widehat{u}_i$	$\widehat{u_i} \cdot x_i$	$\widehat{u_i}^2 = (y_i - \widehat{y_i})^2$
1930	8.7	3	11.5	-8.5	-73.95	72.25
1931	15.9	28	17.9	10.1	160.59	102.01
1932	23.6	21	24.8	-3.8	-89.68	14.44
1933	24.9	17	25.9	-8.9	-221.61	79.21
1934	21.7	37	23.1	13.9	301.63	193.21
1935	20.1	30	21.7	8.3	166.83	68.89
1936	16.9	31	18.8	12.2	206.18	148.84
1937	14.3	22	16.5	5.5	78.65	30.25
1938	19.0	2	20.7	-18.7	-355.30	349.69
1939	17.2	9	19.1	-10.1	-173.72	102.01
Σ	182.3	200		$\Sigma \widehat{u_i} = 0$	$\Sigma \widehat{u_i} \cdot x_i \approx 0$	SSR = 1160.8
Σ/n	$\bar{x} = 18.23$	$\bar{y} = 20$	$\bar{\hat{y}} = 20$			

 $sigma: \ \widehat{\sigma} = \sqrt{\widehat{\sigma^2}} = \sqrt{\frac{SSR}{n-2}} = 12.04575$ $se(\widehat{\beta_1}) = \frac{\widehat{\sigma}}{\sqrt{\sum (x_i - \bar{x})^2}} = \frac{12.04575}{14.2825} = 0.8434$

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Readings: source unknown

```
Unrest <- read.table("C:/Unrest.txt", header=TRUE, quote="\"")
Unrest
covxy = cov(Unrest[2:3], use = "pairwise.complete.obs"); covxy
model <- lm(Riots~Unemp, data=Unrest)
summary(model)$coef
summary(model)$sigma
                                                              ⇐ fill in column 4 on previous slide
round(model$fitted, digits=1)
                                                              ⇐ fill in column 5 on previous slide
round(model$residuals, digits=1)
                                                              \Leftarrow check property \sum \hat{u_i} = 0
round(sum(model$residuals), digits=4)
                                                              \Leftarrow check property \sum \hat{u_i} x_i = 0
round(cor(model$residuals,Unrest$Unemp), digits=4)

        ← make figure on next page ...

plot(Unemp, Riots, pch = 16, ylim = c(0.40))
                                                                  with labels ...
text(Unemp, Riots, labels= Year, pos=3, cex=.6, xpd=TRUE)
                                                                  and fitted line ...
abline(lm(Riots~Unemp, data=Unrest), col= "red")
                                                                  and predicted values
points(Unemp, model$fitted, pch = 16, col= "red")
```

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4. Example: Unemployment and Political Violence

Check R results – estimate slope & intercept; compute fitted values and residuals; check properties; plot points and include regression line

