POLS 6481. Research Design and Quantitative Methods II
Lecture 21. Polynomial Functional Forms I: Coefficients and Marginal Effects
Readings: Wooldridge, *Introductory Econometrics 5e*, 6.2b + 9.1

Outline

1. Monomial Nonlinear Specifications (Dougherty, *Introduction to Econometrics*, ch. 4)
2. Polynomial Nonlinear Specifications
3. Example: Wages and experience
4. Combining Logarithms and Quadratics

Lecture 21 = *focus on the 'numerator'* (*coefficients, marginal effects*)

Lecture 22 = *focus on the 'denominator'* (*standard error*) *and 'ratio'* (*t statistic*)

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1. Monomial Nonlinear Specifications

Three functional forms discussed in "Nonlinear Models and Transformation of Variables"

1. Reciprocal (hyperbola) $y = \beta_0 + \beta_1 \frac{1}{x} + u \quad = \beta_0 + \beta_1 x^{-1} + u$
2. Radical $y = \beta_0 + \beta_1 \sqrt{x} + u \quad = \beta_0 + \beta_1 x^{.5} + u$
3. Exponential** $y = \beta_0 + \beta_1 \log(x) + u$

These three functional forms: manipulate the **right hand side** of the regression equation;
are linear in the parameters (β_0 and β_1);
have additive disturbance (u),** and therefore are best suited to situations where y is normally distributed.

(If time is available, take some first derivatives on this slide and the next slide…)

** Model 3 looks like a level-log model, but it can be rewritten: $e^{y} = f(x, u) = e^{\beta_0} \cdot x^{\beta_1} \cdot v$,
where $v = e^{u}$

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2. Polynomial Nonlinear Specifications

Three kinds of polynomial models for $y = f(x)$:

	Linear	Quadratic	Cubic*
equation	$y_i = \beta_0 + \beta_1 x_i + u_i$	$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + u_i$	$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \beta_3 x_i^3 + u_i$
graph	line	parabola (convex/concave)	S-curve (convex & concave)
effect of Δx	$\frac{\Delta y}{\Delta x} = \beta_1$ (constant)	$\frac{\Delta y}{\Delta x} = \beta_1 + 2\beta_2 x_i$ (conditional on x)	$\frac{\Delta y}{\Delta x} = \beta_1 + 2\beta_2 x_i + 3\beta_3 x_i^2$ (conditional on x)
vertices?	none	one at $x^* = -\frac{\beta_1}{2\beta_2}$	three
slope	positive if $\beta_1 > 0$	both positive and negative (equals 0 at vertex)	both positive and negative (equals 0 once, twice, thrice)

* the "cubic spline" models are cubic but not polynomial

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3. Example: Wages and experience

Linear regression model: $\widehat{wage} = \underset{(.26)}{5.37} + \underset{(.01)}{.03} \cdot exper$

Marginal effect: $\dfrac{\partial\, wage}{\partial\, exper} = .03$

Note that when Wooldridge uses WAGE1 dataset in chapters 2–4, he usually uses *log*(*wage*) as the dependent variable and/or uses *educ* as the key explanatory variable.

```
library(foreign); data <- read.dta("C:/WAGE1.dta")
linear <- lm(wage ~ exper, data); summary(linear)$coefficients
plot(data$exper, data$wage, pch=19, cex=.67); abline(linear, col="deepskyblue", lwd=3)
summary(linear)$r.squared

quad <- lm(wage ~ exper + expersq, data); summary(quad)$coefficients
summary(quad)$r.squared
```

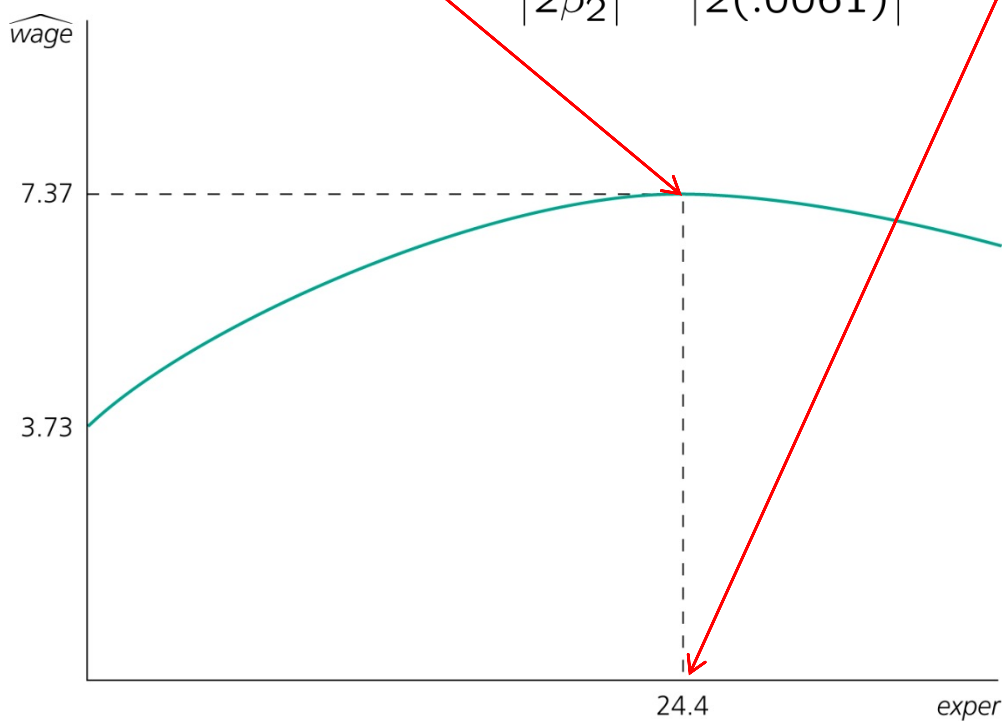
--> remember the F test for adding the squared-experience term

3. Example: Wages and experience

Quadratic regression model: $\underset{}{\widehat{wage}} = \underset{(.35)}{3.73} + \underset{(.041)}{.298}\ exper - \underset{(.0009)}{.0061}\ exper^2$

Marginal effect: $\dfrac{\partial wage}{\partial exper} = .298 - 2(.0061)exper$

Turning point: $x^* = \left|\dfrac{\widehat{\beta}_1}{2\widehat{\beta}_2}\right| = \left|\dfrac{.298}{2(.0061)}\right| \approx 24.4$



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4. Combining Logarithms and Quadratics

Rules for interpreting coefficients with logged dependent variables:

Log-level	$\log(y)$	x	$\%\,\Delta y = (100 \cdot \beta_1)\,\Delta x$

A one-unit change in x leads to *approximately* a $100 \cdot \beta_1$ percent change in y

Log-log	$\log(y)$	$\log(x)$	$\%\,\Delta y = \beta_1 \cdot \%\Delta x$

A one-percent change in x leads to a β_1 percent change in y

Rule for calculating marginal effects in quadratic models:

Model: $\log(y_i) = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + u_i$

Marginal Effects: $\dfrac{\Delta \log(y)}{\Delta x} = \beta_1 + 2\beta_2 x_i \quad \rightarrow \quad \dfrac{\%\,\Delta y}{\Delta x} = 100 \cdot (\beta_1 + 2\beta_2 x_i)$

Vertex: marginal effect = 0 when $x = -\dfrac{\beta_1}{2\beta_2}$

4. Combining Logarithms and Quadratics

Examples 4.5, 6.1, and 6.2 use HPRICE2 dataset; sample is 506 communities in Boston area.

price is community's median housing price; *rooms* is community's mean # rooms in houses.

nox is a measure of air pollution (nitrogen oxide); 1% ↑ in air pollution → .71% ↓ in price

$$\underset{}{\log(price)} = \underset{(.19)}{9.23} - \underset{(.066)}{.713}\, \log(nox) + \underset{(.019)}{.306}\, rooms \qquad [6.7]$$

$$\Rightarrow \quad \frac{\partial \log(price)}{\partial rooms} = \frac{\%\partial price}{\partial rooms} = +.306$$

Increase rooms by 1: + 30.6% *price*

On p. 197 (in 5th ed) Wooldridge states that coefficient for *rooms* is + .255 when two control variables added: log(*dist*) is weighted distance of community from five employment centers; *stratio* is average student-teacher ratio of schools in community. See p. 132–3 (in 5th ed):

$$\log(price) = \underset{(.32)}{11.08} - \underset{(.117)}{.954}\, \log(nox) + \underset{(.019)}{.255}\, rooms - \underset{(.043)}{.134}\, \log(dist) - \underset{(.006)}{.052}\, stratio$$

$$\Rightarrow \quad \frac{\partial \log(price)}{\partial rooms} = \frac{\%\partial price}{\partial rooms} = +.255$$

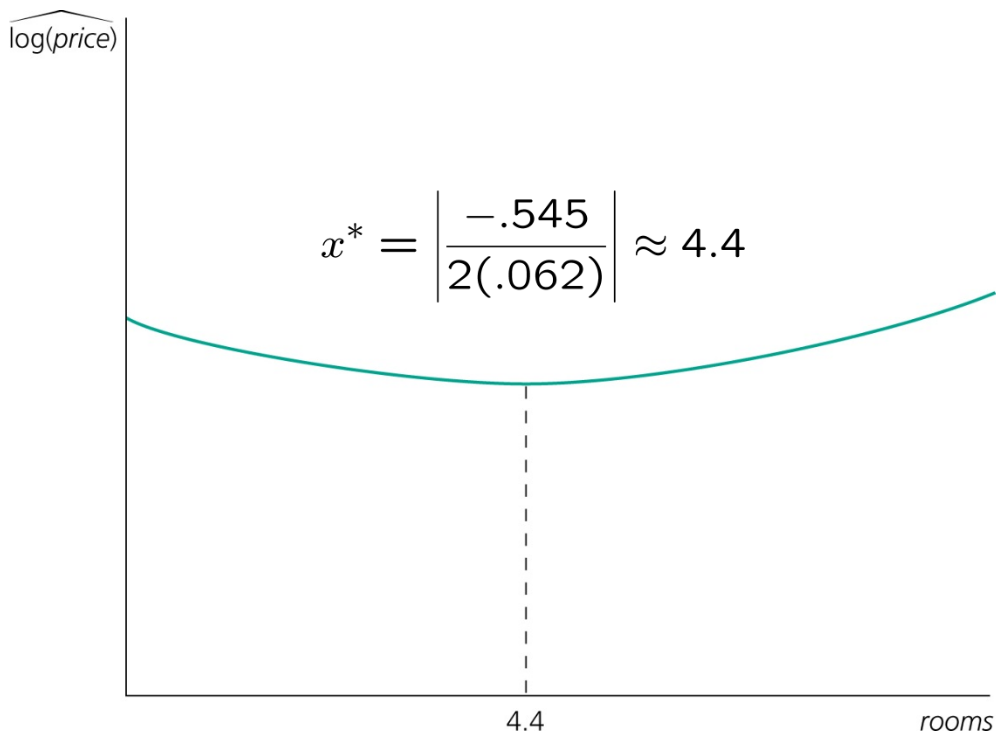
Increase rooms by 1: + 25.5% *price*

Readings: Wooldridge, *Introductory Econometrics 5e*, 6.2b + 9.1

4. Combining Logarithms and Quadratics

$$\widehat{\log}(price) = \underset{(.57)}{13.39} - \underset{(.115)}{.902}\ \log(nox) - \underset{(.043)}{.087}\ \log(dist) - \underset{(.165)}{.545}\ rooms + \underset{(.013)}{.062}\ rooms^2 - \underset{(.006)}{.048}\ stratio \qquad [6.14b]$$

$$\Rightarrow \quad \frac{\partial \log(price)}{\partial rooms} = \frac{\%\partial price}{\partial rooms} = -.545 + .124\, rooms$$



Increase # rooms from 5 to 6:

$$-.545 + .124(5) = +7.5\%\ price$$

Increase # rooms from 6 to 7:

$$-.545 + .124(6) = +19.9\%\ price$$

Increase # rooms from 7 to 8?

p. 198 (in 5th ed) gets silly: including a squared-logged term, for example