

Geometric Sequences

Geometric sequences are of the form $a_n = a_1 \cdot r^{n-1}$, where r is called the *common ratio*.

Exercises

1. Given that a geometric sequence has the first term $a_1 = -2$ and the 4th term $a_4 = -54$, what is the index of the term -6 ?

The formula for a_n is $a_n = b \cdot r^{n-1}$. Since $a_1 = b$, we have:

$$\begin{aligned}a_4 &= -2 \cdot r^3 \\-54 &= -2 \cdot r^3 \\27 &= r^3 \\r &= \sqrt[3]{27} \\r &= 3\end{aligned}$$

So the formula is $a_n = -2 \cdot 3^{n-1}$. Now we are looking for the index. We substitute -6 for a_n :

$$\begin{aligned}-6 &= -2 \cdot 3^{n-1} \\3 &= 3^{n-1} \\n &= 2\end{aligned}$$

2. First term: $a_1 = 81$, common ratio $r = \frac{1}{3}$, and $a_n = 1$. Find n .

$$\begin{aligned}1 &= 81 \cdot \left(\frac{1}{3}\right)^{n-1} \\\frac{1}{81} &= \left(\frac{1}{3}\right)^{n-1} \\\left(\frac{1}{3}\right)^4 &= \left(\frac{1}{3}\right)^{n-1} \\4 &= n - 1 \\n &= 5\end{aligned}$$

3. Given $a_3 = 36$, $r = 3$, what is the index of the term 108?

First let's find the formula:

$$\begin{aligned}36 &= a_1 \cdot 3^{3-1} \\36 &= a_1 \cdot 9 \\a_1 &= 4\end{aligned}$$

So $a_n = 4 \cdot 3^{n-1}$. Now,

$$\begin{aligned}108 &= 4 \cdot 3^{n-1} \\27 &= 3^{n-1} \\3^3 &= 3^{n-1} \\3 &= n - 1 \\n &= 4\end{aligned}$$