Rationalizing the denominator

In general, you want to avoid radicals in the denominator. To rationalize the denominator, multiply both the numerator and the denominator by the conjugate of the denominator.

$$\frac{1}{1+\sqrt{2}}$$

$$=\frac{1}{1+\sqrt{2}}\cdot\frac{1-\sqrt{2}}{1-\sqrt{2}}$$

$$=\frac{1-\sqrt{2}}{\left(1+\sqrt{2}\right)\left(1-\sqrt{2}\right)}$$

$$=\frac{1-\sqrt{2}}{1^2-\sqrt{2}^2}$$

$$=\frac{1-\sqrt{2}}{1-2}$$

$$=\frac{1-\sqrt{2}}{-1}$$

$$=-1+\sqrt{2}$$

Exercises

1.

$$\frac{1+\sqrt{8}}{\sqrt{9}-\sqrt{2}}$$

$$=\frac{\left(1+\sqrt{8}\right)\left(\sqrt{9}+\sqrt{2}\right)}{7}$$

$$=\frac{\sqrt{9}+3\sqrt{8}+\sqrt{2}+\sqrt{16}}{7}$$

$$=\frac{3+6\sqrt{2}+\sqrt{2}+4}{7}$$

$$=\frac{7+7\sqrt{2}}{7}$$

$$=1+\sqrt{2}$$