The power rule

Given a monomial $f(x) = x^n$, where n is a real number,

$f'(x) = nx^{n-1}$

The antiderivative

The antiderivative is the opposite of the derivative.

Infinitely many antiderivatives map to the same function.

For example, since the derivative of x^2 is $2x$, we have x^2 as an antiderivative of $2x$. But $x^2 + 2$, $x^2 + 1$, etc. are also antiderivatives of $2x$.

In general, the antiderivatives of $2x$ follow the pattern $x^2 + C$ where C is a constant.

We can also formulate that using the $\int$ symbol for integration:

$$\int 2x dx = x^2 + C$$

Where C is called the **constant of integration**.

These integrals are **indefinite integrals**.

The function being integrated (here $2x$) is called the **integrand**.

The power rule for integration

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

Exercises

1. Antiderivative of $\frac{1}{x^4}$?

$$\begin{aligned} \int \frac{1}{x^4} dx &= \int x^{-4} dx \\ &= \frac{x^{-4+1}}{-4+1} + C \\ &= \frac{x^{-3}}{-}3 + C \\ &= -\frac{1}{3}x^3 + C \end{aligned}$$

2. Antiderivative of $x^{\frac{5}{8}}$

$$\begin{aligned} \int x^{\frac{5}{8}} dx &= \frac{x^{\frac{5}{8}+1}}{\frac{5}{8}+1} + C \\ &= \frac{x^{\frac{13}{8}}}{\frac{13}{8}} + C \\ &= \frac{8x^{\frac{13}{8}}}{13} + C \end{aligned}$$

3. Antiderivative of $\sqrt[5]{x^7}$

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\begin{aligned}
\int x^{\frac{7}{5}} dx &= \frac{x^{7/5+1}}{7/5+1} + C \\
&= \frac{x^{12/5}}{12/5} \\
&= \frac{5x^{12/5}}{12}
\end{aligned}
$$