2021

MA7008 Financial Mathematics

Portfolio Analysis & Optimization

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I. PORTFOLIO OVERVIEW

A diverse portfolio mitigates risks by reducing capital exposure from concentrated factors. Prior to optimization, a portfolio ought to contain a mixture of equities from varying market sectors compounded by different sizes which is generally described using its market capitalization. The magnitude of a security defines its volatility and predictability which provides an investor the perspective of potential risk as well as opportunity.

The portfolio to be constructed contains securities based in North America from sectors that include: Public Utilities, Energy, Technology, and Finance. The discrepancies in market capitalization for this portfolio are done according to the following ranges:

- Mega Cap: Securities with a market capitalization over $200 Billion.
- Large Cap: Securities with a market capitalization within the range of $10 Billion to $200 Billion.
- Mid Cap: Securities with a market capitalization within the range of $2 Billion to $10 Billion.

The portfolio is constructed with 1 mega-cap security, 2 Large-cap securities, and 2 mid-cap securities. Over 5 years from 2015 to 2019, the companies selected for this portfolio include:

1. *Cisco Systems (**CSCO**)*

Cisco is a global supplier of networking products. Within the five years of analysis, the companies activities include:

- Change in management to focus and prioritize software development over hardware
- Strategic investment in cloud hosting services
- Partnership with Hyundai to develop next-generation, hyper-connected cars
- Launch of an internet gateway to provide a service to reduce risks via internet access
- Acquisition of an AI-Driven business intelligence startup

The robust corporate activity coupled with technological advancement resulted in the mega-cap stock yield an annualized return of 14.96%.

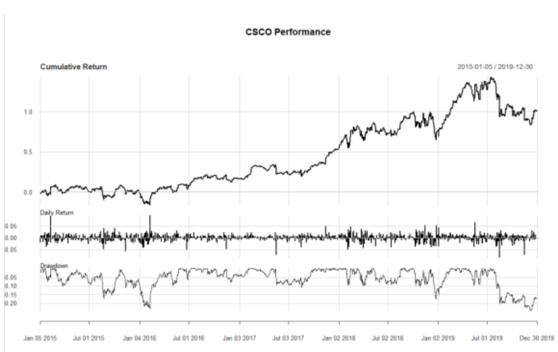


Figure 1. Cumulative Return, Daily Return, and Drawdown of Cisco System.

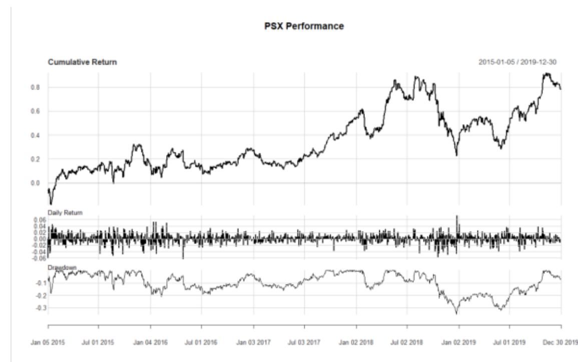
CSCO Performance:

- Cummulative Return: 100.34%
- Max Drawdown: 24.46%
- Annualized Return: 14.96%
- Annualized Volatility: 22.56%

2. *Phillips 66 (**PSX**)*

Phillips 66 is a North American energy company primarily focused on natural gas. The adoption of a more climate-friendly policy resulted in Phillps 66 landing a beneficial position due to the increased demand for low-carbon fossil fuels in North America. In addition to a benefit due to a policy change, Phillips 66 sold natural gas pipelines worth $1.01 billion in cash and stocks within the period of analysis.

The annualized return of the large-cap stock is 12.30%.



Cumulative Return, Daily Return, and Drawdown of Phillips 66.

PSX Performance:

- Cummulative Return: 78.30%
- Max Drawdown: 35.37%
- Annualized Return: 12.30%
- Annualized Volatility: 23.46%

3. *Lam Research (**LRCX**)*

Lam Research is a company that manufactures and designs semiconductors used in integrated circuits. The most prominent milestones during the period of analysis include:

- New research laboratory in Fremont
- 10nm technology node
- Acquisition of Coventor
- 64 layers 3D NAND

The commercial and corporate milestones resulted in Lam Research's leveraged position within the industry and an annualized return of 32.10% over 5 years.

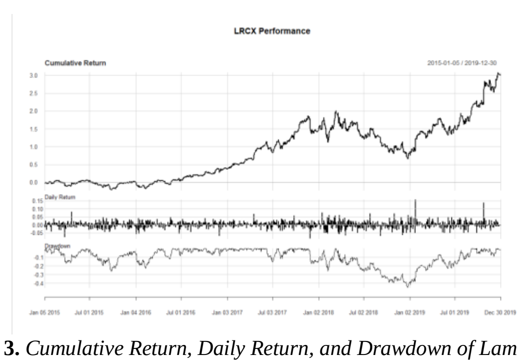


Figure 3. *Cumulative Return, Daily Return, and Drawdown of Lam Research.*

LRXC Performance:

- Cumulative Return: 300.53%
- Max Drawdown: 45.04%
- Annualized Return: 32.10%
- Annualized Volatility: 32.55%

4. *T.Rowe Price (**TROW**)*

T.Rowe Price is a global investment management firm that offers funds, advisory services, and account management. The investment firm increased its assets under management from $400 Billion to $1.6Trillion between 2010 and 2020. The growth of the assets under management during the analysis period resulted in an annualized return of 10.86% for the large-cap stock.

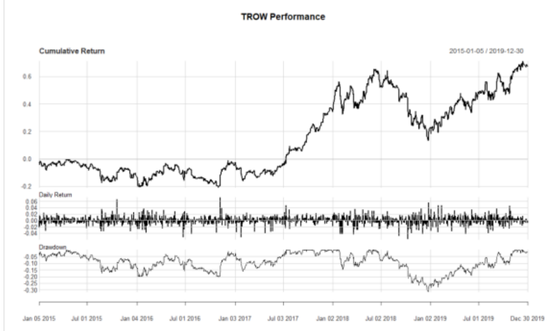


Figure 4. *Cumulative Return, Daily Return, and Drawdown of T. Rowe Price.*

TROW Performance:

- Cummulative Return: 67.17%
- Max Drawdown: 31.39%
- Annualized Return: 10.86%
- Annualized Volatility: 21.82%

5. *Brook Automation (**BRKS**)*

A mid-cap company that indulges in the provision of automation, vacuum, and instrumentation equipment for markets such as semiconductor manufacturing and technology device manufacturing.

Brooks automation made acquisitions of three companies within the period of analysis which includes: Biostorage Technologies, 4titude, and GENEWIZ. Brook Automation yielded an annualized return of 30.19%.

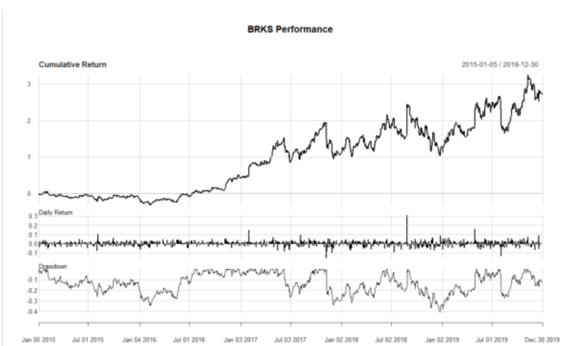


Figure 5. *Cumulative Return, Daily Return, and Drawdown of Brooks Automation.*

BRKS Performance:

- Cummulative Return: 272.39%
- Max Drawdown: 41.00%
- Annualized Return: 30.19%
- Annualized Volatility: 41.91%

Correlation between asset returns illustrates similarities or discrepancies between returns hence describing whether assets benefit from similar trends. The correlation matrix is useful to highlight the diversification of risk within a portfolio and whether assets are heavily concentrated on a singular factor.

	CSCO	PSX	LRCX	TROW	BRKS
CSCO	1	0.418197	0.433455	0.517789	0.392571
PSX	0.418197	1	0.358283	0.49483	0.30228
LRCX	0.433455	0.358283	1	0.493232	0.570352
TROW	0.517789	0.49483	0.493232	1	0.382588
BRKS	0.392571	0.30228	0.570352	0.382588	1

Table 1. *Correlation Matrix of asset returns in the portfolio.*

The majority of asset returns have a correlation between 0.3 – 0.5 which is due to the general positive yield of all assets within the portfolio. The highest correlation between asset returns exists between Lam Research and Brook Automation (0.57) and can be attributed to both companies servicing the semiconductor manufacturing industry.

II. Portfolio Risk Optimization

Optimization of risk entails calculating the optimal weight/percentage allocating for each asset in the portfolio. The optimization process aims to minimize risk from more volatile assets in the portfolio to manage portfolio risk.

The solver function implemented to determine the portfolio risk is ROI. This function allows for portfolio optimization by solving the following problems which involve:

1. Maximizing Return subject to the sum of weights, individual asset weights, mean and standard deviation.
2. Minimizing Variance subject to the sum of weights, individual asset weights, mean and standard deviation.
3. Mean return maximization per unit standard deviation (Sharpe Ratio).

The constraints and objectives used for risk optimization include:

1. the limitation of the total sum of the percentage of investment for each asset as one (1),
2. limiting the individual asset weight to a maximum of 25% and a minimum of 10%
3. Objectives including the mean (Return) and standard deviation (Risk).

The optimization involving the use of constraints also included a specification to adjust weights according to a maximized Sharpe ratio, the weights of the portfolio are as calculated:

- Cisco Systems (CSCO) – 25.00%
- Phillips 66 (PSX) – 22.91%
- Lam Research (LRCX) – 25.00%
- T.Rowe Price (TROW) – 10.00%
- Brooks Automation (BRKS) – 17.09%

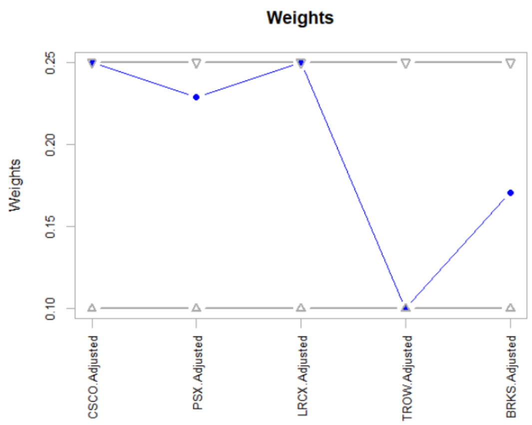


Figure 6. *Illustration of the percentage of investment in each asset in the portfolio.*

The result of portfolio optimization using the calculated weights for each asset is 22.18% in annualized returns and 22.92% in annualized volatility.

The Efficient Frontier Curve illustrates the minimum variance with the maximum expected return at a specified risk. The portfolio is seen to be efficient at points above the minimum-variance portfolio with the Capital Market Line existing tangentially to the parabola representing the Frontier Curve. Therefore, the portfolio is most feasible on the parabola.

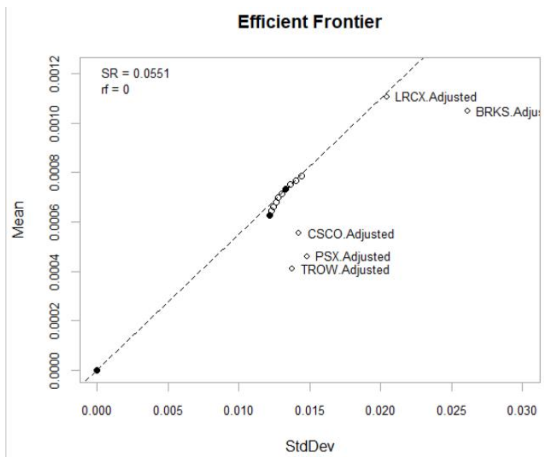


Figure 7. *Efficient Frontier Curve.*

III. Capital Market Line

The range of return and risk calculated during risked optimization and described through the efficiency curve can be further described using the Sharpe Ratio. The Sharpe Ratio is used to define the unit risk per return with a given risk-free return. A risk-free return of 1.5% yields the range:

	Return	Risk	Sharpe Ratio
1	0.626	1.218	0.513
2	0.644	1.231	0.522
3	0.662	1.245	0.530
4	0.679	1.260	0.538
5	0.697	1.277	0.545
6	0.715	1.302	0.548
7	0.733	1.331	0.550
8	0.751	1.364	0.549
9	0.768	1.400	0.548
10	0.786	1.440	0.545

Table 2. *Range of portfolio returns and volatilities with corresponding Sharpe Ratio.*

The Capital Market Line (CML) consists of risky assets and risk-free investments where the most efficient portfolio resides. The line estimates the portfolio return, R_p, with a given risk-free investment, R_f, and corresponding market volatility, σ_m, market return, R_m, and portfolio volatility, σ_p.

$$R_p = R_f + \left(\frac{R_m - R_f}{\sigma_m}\right)\sigma_p$$

Equation 1. *Equation of the Capital Market Line.*

The slope is the risk premium/Sharpe Ratio which has a value of 0.889 and with a given risk-free return of 1.5%, market volatility of 22.92%, and market return of 22.18%, the equation for portfolio returns is given as:

$$R_p = 0.015 + 0.889\,\sigma_p$$

Equation 2. *Equation of the Capital Market Line for the optimized Portfolio.*

The line represents the portfolio return with the consideration of the risk-free rate for borrowed funds required to purchase risky assets.

IV. Portfolio Risk Metrics

Risk metrics prescribe parameters that detail the potential hazards relative to a benchmark or industry standard. The two metrics used in this case study include Beta (β) and Value at Risk (VaR).

Beta is a coefficient used as a risk metric to describe the volatility of an asset or portfolio to a benchmark (which is commonly a market index). Beta, β, is the slope of the regression line which is fitted on the asset returns, R_i, against the benchmark returns, R_m.

$$R_i = \sigma + \beta R_m$$

Equation 3. *Regression Line of Asset returns against market returns.*

The consequence of an asset's Beta value depends on the value relative to 1.0. A Beta value of 1.0 implies the asset is not more or less risker than the market, a value greater than 1.0 means the asset is more volatile than the market and less than 1.0 indicates a relatively safe asset when compared to the market.

	CSCO	PSX	LRCX	TROW	BRKS
Beta: SPY	1.19	1.11	1.49	1.22	1.55

Table 3. *Beta coefficients of asset returns in the portfolio to the S&P500.*

BRKS is the most volatile asset in the portfolio with a beta value of 1.55 indicating the asset is 55% more volatile than the market while PSX has the least Beta value at 1.11 indicating volatility of 11% less than the market.

Value at Risk (**VaR**) is a quantity used to indicate the probability a percentage of capital allocated to an asset or portfolio can be lost within a specific period. The VaR (5%) estimated for the portfolio constructed, with the use of a gaussian distribution is **2.14%** annually. Therefore, a £1,000,000 capital allocation has a 5% probability of losing £21,355.22 annually.

Each asset had its contributions as stated in the table below:

Asset	**VaR (5%)**	**% Contribution**
CSCO	0.41%	19.14%
PSX	0.35%	16.55%
LRCX	0.67%	31.39%
TROW	0.15%	7.03%
BRKS	0.55%	25.89%

Table 4. *VaR (5%) of assets and corresponding contributions to portfolio VaR.*

The riskiest asset according to the VaR is LRCX with 0.67% which has a 31.39% contribution to the portfolio VaR. TROW is the safest asset in the constructed portfolio with 0.15% VaR and a 7.04% contribution to the portfolio VaR, however, TROW possessed the lowest asset allocation with only a 10% contribution to the portfolio.

V. Volatility Modelling

Volatility is an essential risk metric that provides insight into the elementary dynamics of market activity. Modeling volatility for analysis and prediction presents an opportunity to interpret the behavior of an asset.

The method used to fit a volatility model is done using the AutoRegressive Conditional Heteroscedasticity (ARCH) and Generalized ARCH (GARCH). The concept of ARCH is to model the return R_t, according to past return R_{t-1}.

$$R_t = \varepsilon_t \sqrt{\alpha + \alpha_1 R_{t-1}^2}, \quad \sigma_t = \sqrt{\alpha + \alpha_1 R_{t-1}^2}$$
$$R_t = \varepsilon_t \sigma_t$$

Equation 4. *AutoRegressive Conditional Heteroscedasticity (ARCH) equation.*

The improvement of the ARCH model is done by accounting for past volatility as well as returns.

$$R_t = \varepsilon_t \sqrt{\alpha + \alpha_1 R_{t-1}^2 + \beta \sigma_{t-1}^2}, \quad \sigma_t = \sqrt{\alpha + \alpha_1 R_{t-1}^2 + \beta \sigma_{t-1}^2}$$
$$R_t = \varepsilon_t \sigma_t$$

Equation 5. *Generalized AutoRegressive Conditional Heteroscedasticity (GARCH) equation.*

Each model's return fits volatility through the variance σ^2 and the model coefficients (ϵ_t, α, α_1 and β). The variance σ_t is normally distributed N(0,1) but an inverse Gaussian distribution IG(0,1) is used to improve the shape and skew of the empirical density of residuals.

Cisco Systems CSCO is the asset in the portfolio whose return will be modeled using the ARCH/GARCH.

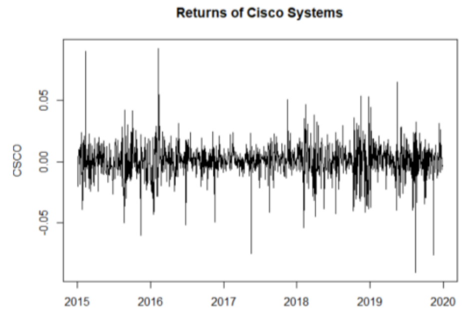


Figure 7. *Daily Returns of Cisco Systems CSCO.*

The following GARCH models are fitted:

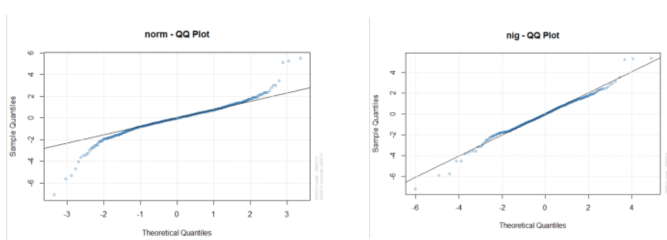
- GARCH(1,1) – Model with a 1-day lag for volatility and returns.
- GARCH(2,1) – Model with a 2-day lag for returns and 1-day lag for volatility.
- GARCH(0,2) – Model with a 2-day lag for volatility.
- GARCH(2,2) – Model with a 1-day lag for volatility and returns.

The parameters used for model selection are the Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) for each model. The preferred model based on the AIC is the GARCH(2,1) while the BIC estimates the GARCH(1,1) as the preferred model. A choice is made to select the best model based on the AIC.

S/N	Model	AIC	BIC
1	GARCH(1,1)	-5.96477	-5.94433
2	GARCH(2,1)	-5.96671	-5.94218
3	GARCH(2,2)	-5.96657	-5.93795
4	GARCH(0,2)	-5.87433	-5.8498

Table 5. *Akaike Information Criterion (AIC) and Bayesian Information Criterion (BIC) of models fitted to CSCO returns.*

Understanding the precision of the model is done using various diagnostic plots. Proper fitting of the probability distribution of the returns is essential to maintain accurate shape and location for model parameters. Analysis of the model theoretical quantiles illustrates the difference between the GARCH(2,1) with a fitted inverse gaussian distribution and a normal distribution.



(a) *(b)*

Figure 8. *Theoretical Quantiles of GARCH(2,1) model with a fitted normal distribution (a) and inverse Gaussian distribution (b).*

The inverse Gaussian distribution presents a more adequate fit due to the majority of data points residing on the optimal fitted line. Furthermore, the fitted distribution in figure 9 demonstrates how the shape is considered as the extra hyperparameter of the inverse Gaussian distribution ensures an appropriate fit.

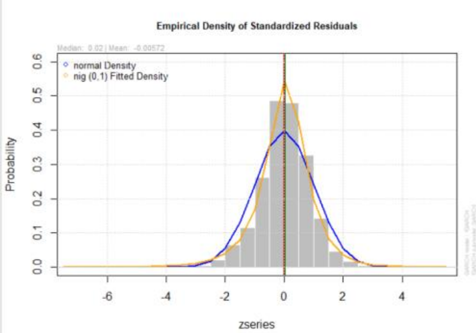
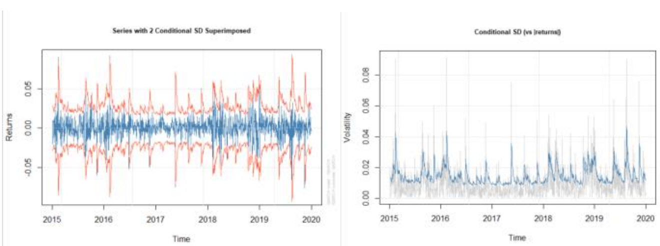


Figure 9. *Empirical Density of standardized residuals.*

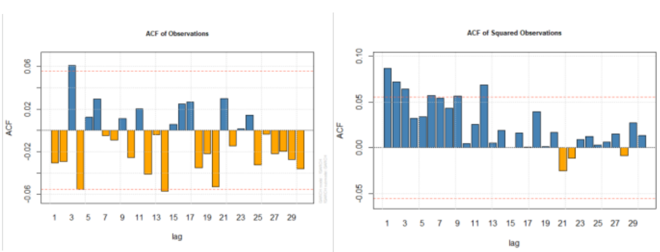
The conditional standard deviation is a true measure of uncertainty and is utilized in describing the extent to which returns, and volatility is predictable based on the model's fitted probability distribution. A superimposed 2 conditional standard deviation with returns in figure 10 describes how the majority of returns reside within that range. In addition, the absolute returns with the model volatility is deemed suitable as the model fits multiple spikes in volatility adequately.



(a) *(b)*

Figure 10. *Superimposed 2 Condition Standard deviation of CSCO Returns (a) Fitted Model Volatility and CSCO daily volatility (b).*

The auto-correlation function describes the relationship between present and past values of observations. The ACF of return from the fitted model illustrates a decent fit with most lags residing within the confidence band. Similarly, the squared observations produce a satisfactory output with the confidence band marginally respected as illustrated in figure 11.



(a) *(b)*

Figure 11. *ACF of fitted returns from GARCH(2,1) model (a) ACF of fitted squared returns from GARCH(2,1) model (b).*

The application of this model includes risk analysis and volatility forecasting. The fitted model predicts the volatility of CSCO for two days.

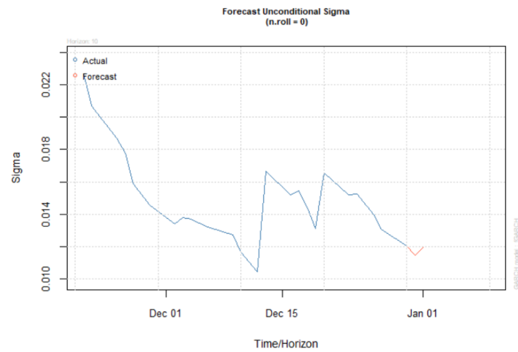


Figure 12. *Volatility Forecast using GARCH(2,1) model.*

VI. RECOMMENDATIONS

The portfolio constructed exposes capital invested to multiple sectors to diversify risk. Diversification is illustrated in the correlation matrix as asset returns from varying sectors are not highly correlated.

Asset allocation per risk-adjusted return ensures an optimal level of capital exposure. Consequently, returns are estimated with the minimum volatility from the constructed portfolio.

In comparison to the market benchmark, this portfolio remains relatively more volatile but can be attributed to the existence of a mid-cap asset (BRKS). Furthermore, the portfolio risk remains minimal as the probability of losing at most 2.14% annually is 5%.

In conclusion, this portfolio is highly recommended due to its stability in terms of diversification across multiple sectors and asset sizes.