Quantum computation as a new scheme for problem-solving

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March 8th, 2024

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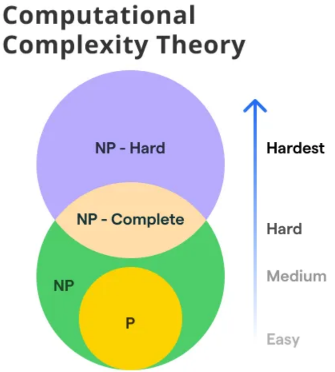
Computational Complexity

- **P-class** problems: decision problems in polynomial resources.
- **NP-class** problems: decision problems whose proof is verifiable in polynomial resources.
- **NP-Complete**: "Generalizations" of NP problems.
- **NP-Hard**: which are at least as hard as the hardest problems in NP



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Consider the integer factorization problem... It is suspected to be neither **P** nor **NP**-complete.

Computational Complexity

In: Proceedings, 35th Annual Symposium on Foundations of Computer Science,
Santa Fe, NM, November 20–22, 1994, IEEE Computer Society Press, pp. 124–134.

Algorithms for Quantum Computation:
Discrete Logarithms and Factoring

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Brief History of Quantum Mechanics

Stage: Late 19th. Century Physics:

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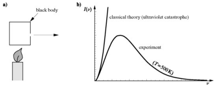

Figure: Classical Mechanics fails to describe blackbody radiation

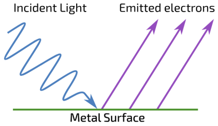

Figure: Electrons are only ejected when there is *enough* light hitting the surface, contrary to classical predictions.

History of Quantum Mechanics

- Max Planck proposed the idea of quantized energy levels to explain blackbody radiation.

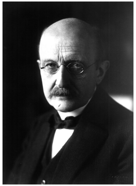

Figure: Max Planck (1858-1947)

History of Quantum Mechanics

- Max Planck proposed the idea of quantized energy levels to explain blackbody radiation.
- Albert Einstein applied the concept of quanta to explain the photoelectric effect, introducing the dual nature of light, suggesting both wave and particle characteristics (superposition)

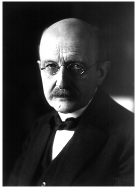

Figure: Max-Planck (1858-1947)

Figure: Einstein in 1905

History of Quantum Mechanics

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From then on, physicists expanded QM by exploring

Motivation

- What would happen if storing and computation devices became ever so small that quantum effects start to become noticeable?

Drawing Paralellisms

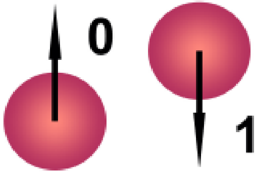
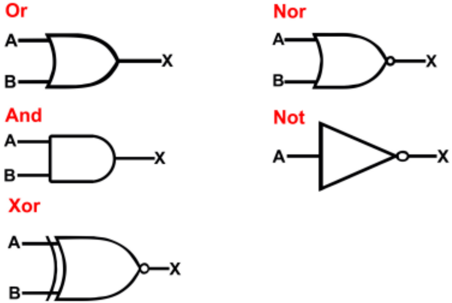


Figure: The classical bit

Drawing Paralellisms



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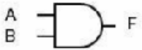

Inputs		Output
A	B	F
0	0	0
1	0	0
0	1	0
1	1	1

OR

Inputs		Output
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Figure: Caption

Drawing Paralellisms

Classical Gates can be thought of as functions.

Drawing Paralellisms

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For example:

$$\wedge : \{0,1\}^2 \longrightarrow \{0,1\} \text{ with}$$

$$\wedge(0,0) = 0$$

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$$\wedge(1,0) = 0$$

$$\wedge(1,1) = 1$$

Figure: Caption

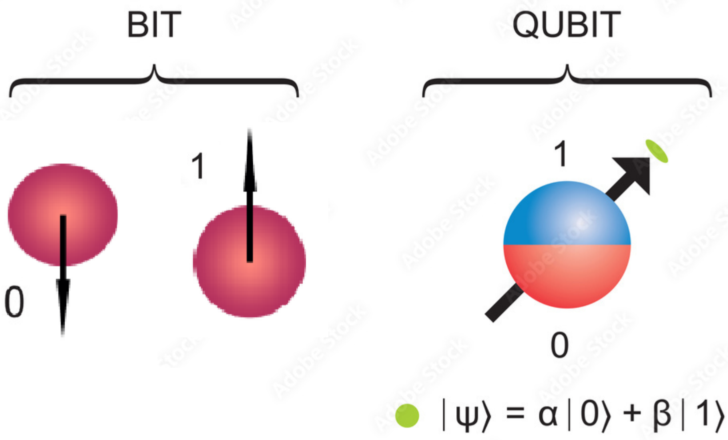
Drawing Paralellisms

An full circuit can then be thought of as a general function

$$F : \{0,1\}^n \to \{0,1\}^m.$$

The Quantum-Bit

Let us present the Quantum Bit or (QuBit)



The Qubit

A question arises...

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- this leads to the measurement process.

Measuring a Qubit

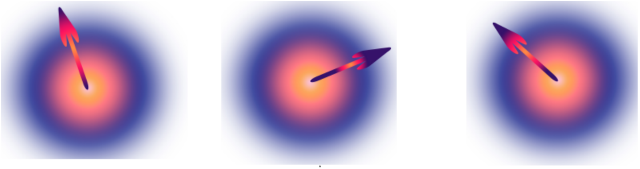

Figure: System of qubits at initial time $t = 0$.

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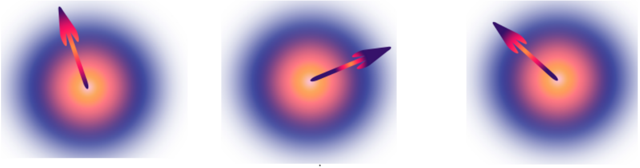

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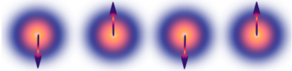
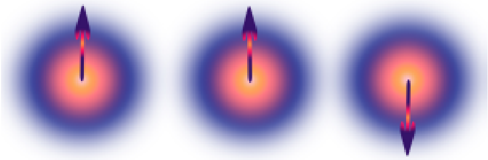


Figure: System of qubits measured at some later time $t > t_0$.

Measuring a Qubit

If we repeat the experiment, we might get...



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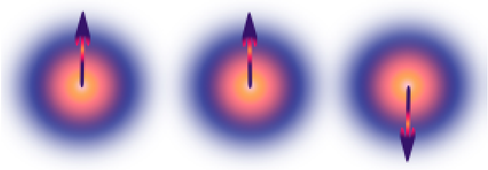

Figure: We might call this state $|\Psi\rangle = |\uparrow\uparrow\downarrow\rangle$

We can build a probability distribution $P_i(a = 0_i, 1_i) = |\langle a|\Psi\rangle|^2$.

Single Qubit Gates

$$X = \sigma_x = \mathrm{NOT} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix},$$

$$Y = \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix},$$

$$Z = \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}.$$

Two-Qubit Gates

Of special interest is the CNOT gate, given by

$$\mathrm{CNOT} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Classical and Quantum Computation

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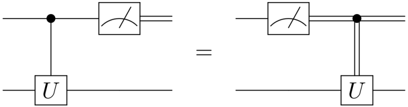

Figure: Deferred measurement

Classical and Quantum Computation

Now, we will study an application: Superdense coding

Secure Quantum Comunications: Superdense Coding

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Classical and Quantum Computation

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With quantum computation, we need only $\mathcal{O}(\sqrt{N})$ *operations* $\Rightarrow$ Grover's search Algorithm

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3. We apply an additional reflection.

   End(Repeat)

Grover: Initialization

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We initialize via the operation $|S\rangle = H^{\otimes n}$

Grove: Oracle (first reflection)

Let the oracle be:

$$U_\omega |x\rangle = \begin{cases} |x\rangle \text{ if } x \neq \omega \\ -|x\rangle \text{ if } x = \omega \end{cases}$$

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Then, the new state is

$$|\psi_1\rangle = U_\omega |S\rangle = U_\omega H^{\otimes} |x\rangle \,.$$

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$$|\psi_2\rangle = U_s\,|\psi_1\rangle = U_S U_\omega H^{\otimes}\,|x\rangle\,.$$

Grover: Repetition of these steps

Then, we repeat these steps s.t. the whole operation now reads

$$\left|\psi_i\right\rangle = (U_S U_{\omega)}\left|\psi_{i-1}\right\rangle ,$$

or, more succintly,

$$\left|\psi_{r(N)}\right\rangle = (U_S U_\omega)^{r(N)} H^{\otimes}\left|x\right\rangle ,$$

where $r(N) \approx \frac{\pi}{4}\sqrt{N}$, with N the total number of states.

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- Target state: $|\omega\rangle = |11\rangle$.
- For $N = 4$, $r(N) \approx 1.57 \approx 2$.

Qiskit: Example for $N = 4$

```
from qiskit import IBMQ, Aer, transpile, execute
from qiskit import QuantumCircuit, ClassicalRegister, QuantumRegister
from qiskit.providers.ibmq import least_busy
```

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n = 2
grover_circuit = QuantumCircuit(n)
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    for q in qubits:
        qc.h(q)
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```
grover_circuit = initalize_s(grover_circuit, [0,1])
grover_circuit.h([0,1])
grover_circuit.z([0,1])
grover_circuit.cz(0,1)
grover_circuit.h([0,1])
grover_circuit.draw()
```

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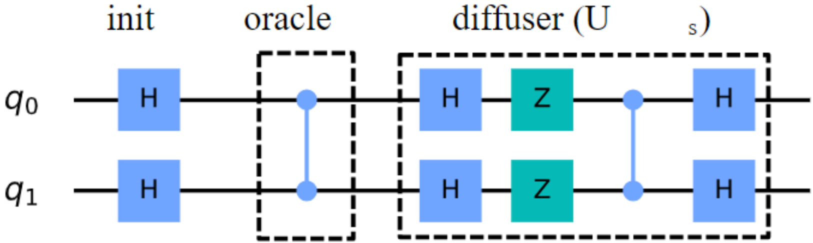

Qiskit: Example for $N = 4$

```
grover_circuit.measure_all()
qasm_sim = Aer.get_backend('qasm_simulator')
result = qasm_sim.run(grover_circuit).result()
counts = result.get_counts()
plot_histogram(counts)
```

Qiskit: Example for $N = 4$