Tutorial 4 (Local Volatility)

Problems

1. Implement the closed form formula for both a CEV put and call option as an inline function. Since we will be computing finite difference estimates using these pricing formulae, make sure that your inline function takes strike and maturity as parameters.

   Recall that the central difference estimate for first and second partial derivatives of a function f in terms of a small perturbation δx are given as

   $$\frac{\partial f(\cdot,x)}{\partial x} = \frac{f(\cdot,x+\delta x) - f(\cdot,x-\delta x)}{2\delta x} \quad \text{and} \quad \frac{\partial^2 f(\cdot,x)}{\partial x^2} = \frac{f(\cdot,x+2\delta x) - 2f(\cdot,x) + f(\cdot,x-2\delta x)}{4\delta x^2}.$$

   Implement inline functions that compute the central difference estimates for the partial derivatives required to compute the Dupire formula using small perturbation parameters δT and δK.

   Now, as a function of strike $K \in [1, 2, \ldots, 80]$, compute the Dupire estimates for the local volatility using CEV call option prices for $K < 40$ and put option prices for $K \geq 40$ for the parameters

   $$S_0 = 40, \quad \sigma = 60\%, \quad \alpha = 0.85, \quad r = 6\%, \quad T = 2, \quad \delta T = 0.001 \quad \text{and} \quad \delta K = 0.001.$$

   Plot the closed form local volatility using the same strikes. In a separate graph plot the error in the Dupire estimate as a difference between these two and scale the y-axis using `ylim([-5e-5 5e-5])`.

2. As per the notes in Lectures 2 and 4, implement the Theta Finite-difference Scheme for a put option in the CEV model, using the following option related parameters

   $$\sigma = 40\%, \quad \alpha = 0.9, \quad r = 6\%, \quad K = 40, \quad T = 1,$$

   and using the following mesh related parameters

   $$S_{\min} = 0, \quad S_{\max} = 160, \quad M = 80, \quad N = 160 \quad \text{and} \quad \theta = 0.5,$$

   Provide a surface plot of the result (in terms of S and $\tau = T - t$). Also read off the price estimates for $S_0 = K$ (at-the-money) and $S_0 = 2 \times K$ (out-the-money).

   Finally, compute and plot the error in the finite difference values (the difference between the finite difference solution at $t = 0$ and the closed form value).

   Hint: Things simplify somewhat because the local volatility function for the CEV model is not time dependent.

3. (Homework) Modify the code in Question 2 to perform the same task for a call option.