M.Phil. in Mathematical Finance

Numerical Methods in Finance II 2020

Tutorial 1 (Finite-difference Methods)

Problems

1. Implement the Theta Method for the diffusion (heat) equation using the following parameters:

$$N^- = -12, \quad N^+ = 12, \quad M = 20, \quad T = 0.8, \quad \delta_x = \pi/N^+, \quad \delta_\tau = T/M, \quad \theta = 0.5.$$

   Use the initial condition

$$u_0(x) = |\sin(x)|$$

   and the boundary conditions

$$u_{-\infty}(x,\tau) = \tau \qquad \text{and} \qquad u_\infty(x,\tau) = \tau.$$

   Hint: Ensure that the matrixes $\mathbf{C}$ and $\mathbf{D}$ are of size $(N^+ - N^- - 1) \times (N^+ - N^- - 1)$. Set up your solution as an $(N^+ - N^- - 1) \times (M+1)$ matrix and initialize the first column using the initial condition. Now iterate using the matrix formulation in the notes to compute subsequent columns of the solution. In anticipation of question 2, please ensure that you set up anonymous functions for $u_0(x)$, $u_{-\infty}(x,\tau)$ and $u_\infty(x,\tau)$ (called `u0`, `um` and `up`, respectively).

   Provide a surface plot of the result using the correct time and space scales. What happens when you use $\theta = 0$? Why?

2. By implementing the transformation functions in the notes as anonymous functions, modify your code in Question 1 to price a binary call option with payout (initial condition) $\mathbb{I}_{\{S_T > K\}}$, using the following option related parameters

$$\sigma = 40\%, \quad r = 6\%, \quad K = 50, \quad T = 1,$$

   and mesh related parameters

$$N^- = -100, \quad N^+ = 20, \quad M = 35, \quad \delta_x = 0.06, \quad \delta_\tau = \frac{\sigma^2 T}{2M}, \quad \theta = 0.5.$$

   Think carefully about what your boundary conditions should be. In particular, set up anonymous functions for $V_T(S)$, $V^0(S,t)$, $V^\infty(S,t)$, $f_s(x)$, $f_t(\tau)$ and $f(V,x,\tau)$ (called `VT`, `V0`, `Vinf`, `fs`, `ft` and `f`, respectively). Then define `u0`, `um` and `up` in terms of these functions. **Hint:** The boundary that specifies the far in the money values (V^∞) should be the guaranteed payout multiplied by an appropriate discount factor to ensure no arbitrage.

   Provide a surface plot of the diffusion solution. In a second plot, transform the diffusion solution back into S and t coordinates and plot a pricing surface. **Hint:** To make your transformation efficient, use the `meshgrid` command.

   Finally, in a third plot, compare the transformed solution at time $t = 0$ to the analytical formula value by plotting the difference between the two. The analytical solution is given by

$$V(S,0) = e^{-rT}\Phi(d),$$

   where

$$d = \frac{\log(S/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}.$$

3. (Homework) Modify your code in Question 2 to price various other options for which there is a Black-Scholes Solution — make sure you know how to derive the boundary conditions needed for consistent solutions.