Tutorial 3 (American Options)

## Problems

1. By implementing new boundary and early exercise conditions, modify your solution for Question 2 of Tutorial 2 to price an American put option.

   Use the following option related parameters

   $$\sigma = 40\%, \quad r = 6\%, \quad K = 40 \quad \text{and} \quad T = 1$$

   and the following mesh related parameters

   $$S_{\max} = 160, \quad S_{\min} = 0, \quad M = 100, \quad N = 80 \quad \text{and} \quad \theta = 0.5.$$

   Use a convergence tolerance of 1e-5 and an over-relaxation parameter $\omega = 1.55$. As before, graph the solution and display the at-the-money price of the option at inception.

2. For the same option related parameters as in Question 1 and $S_0 = K$, implement the least squares Monte Carlo algorithm. Ensure that you use $N = 50$ updates for your Monte Carlo paths (i.e. 50 exercise times, including terminal time) and that for $n = 50000$ you generate $2n$ paths using both a standard and antithetic sample. Initialise your seed as usual with `rng(0);` and generate the normal random variates using the command `Z=randn(N,n);`. For the regression function, use only the first 3 Laguerre polynomials.

   **Hint:** The tricky part of the algorithm is to find a way to implement step 7. To do this, in step 5, create a logical index which identifies the paths for which early exercise is greater than zero. You will obviously use this index in step 6 to create the vector $Y$ (and $X$). You can then update the vector $Y$ with early exercise values where appropriate. Finally, write the vector $Y$ back into your vector $V$ using the index you created in step 5.

   Plot the sample used for regression and the regressed early exercise values for the time step just before expiry of the option (Hint: use an if statement in your `for` loop).

3. (Homework) For both questions 1 and 2 above, can you think of a way to determine and plot the early exercise boundaries implied by the methods.