M.Phil. in Mathematical Finance

Numerical Methods in Finance II 2020

Tutorial 10 (The LIBOR Market Model)

Problems

1. In this problem you will reconstruct the zero coupon bond prices under the LIBOR Market Model as the expectation of discount factors at appropriate times.

   To start with, compute (input) bond prices $B(0, t_i)$ under the Vasicek model for times $0 = t_0, t_1, \dots, t_N = 40$ with $\Delta t = 2$ using the parameters

   $$r_0 = 7\%, \quad \alpha = 0.15, \quad b = 9\%, \quad \text{and} \quad \sigma_v = 2\%.$$

   Now calculate the corresponding LIBOR forward rates at inception.

   Using constant forward rate volatilities of $\sigma = 20\%$ for all maturities and a sample size of $n = 100,000$, implement the simple discrete LIBOR algorithm using $M = N$ and computing $B_i = \mathbb{E}[1/\bar{\beta}]$ at each time step. The B_i are the simulated bond prices with $B_0 = 1$ by definition.

   Graph these prices in comparison to the Vasicek prices used as input. In a separate graph, compare the input LIBOR rates and the LIBOR rates recovered from the simulated bonds.

   Hint: To keep implementation simple, just use $\delta_j = \Delta t$ for all time differences in the algorithm. Remember to initialise the seed as usual.

   Having implemented the simple algorithm, now augment your code to compute the same outputs using the Predictor-Corrector Algorithm.

2. Reuse your code from part one to price caplets with strikes of $K = 15\%$, for maturities $t_1, \dots, t_{N-1}$. Be careful to use the correct combination of rate and deflator (these have different times).

   In separate graphs (one for the standard algorithm and one for the predictor-corrector algorithm), compare the results obtained against the closed form values computed using the Black formula as per the notes. Include a three standard deviation boundary, either as a curve around the closed form solution or as error bars around the computed values (See MATLAB help on `errorbar`.)