M.Phil. in Mathematical Finance

Numerical Methods in Finance 2020

Assignment 1

Instructions

- You are required to submit one Matlab function, named `PriceFadeIn`. Please use this exact name. Do not submit any other files; all your code should be included in `PriceFadeIn` — if you wish to use other functions, use subfunctions.
- Submission is by email to `tom@analytical.co.za` or `Thomas.McWalter@gmail.com`.
- The due date for the assignment is 12 noon on Monday the 15th of June — marks will be deducted for late submission (5% for every half hour, or part thereof, that the assignment is late). The arrival time stamp of the email will be used to decide if submission is late or not. Absolutely no excuses accepted — so, rather submit early.
- Absolutely no coding together or copying is allowed. If such activity is detected, all individuals involved will receive a mark of zero and disciplinary action will be taken.
- Marks (33%) are allocated for speed and efficiency — The run-times of the code (which produce the correct results, see below) will be ranked. The mark (out of 33) assigned for speed will be calculated as $1.5(23 - n)$ where n is the ranking (e.g. if the code was ranked 3rd fastest then 30 marks will be allocated out of 33).
- Marks (10%) are allocated for accuracy — The top 20 most accurate submissions will be ranked and awarded a mark between 10 and 0.5. Accuracy will be computed as the sum of absolute differences between results and analytical solution over all the values tested.

Fade-in Options

The discounted payoff of a fade-in option, with strike K, is given by

$$e^{-rT}\max(\eta(S_T - K), 0)\frac{1}{N}\sum_{i=1}^{N}\mathbb{I}_{\{L \leq S_{t_i} \leq U\}}$$

where $\eta = 1$ for a call, and $\eta = -1$ for a put. The payoff is weighted by the number of times the stock path appears in the range $[L, U]$, where the N fixing times are given by $0 < t_1, < t_2 < \cdots < t_N < T$.

Write a function called `PriceFadeIn` that takes three arguments `eta`, `sigma` and `K` in that order and produces a single Monte Carlo price for the option. Any Monte Carlo method based on psuedo-random numbers using `randn` or `rand` may be used including any combination of variance reduction techniques you think are appropriate. No random seed should be set in your function. The other parameters used are assumed fixed and given by

$$S_0 = 100, \qquad L = 70, \qquad U = 130, \qquad r = 10\%, \qquad \text{and} \qquad T = 1.$$

There are $N = 25$ fixing times with $t_i = i\Delta t$ where $\Delta t = T/(N + 1)$.

The prices produced by your function must be accurate to the nearest 5 cents with 99.0% certainty (Recall a three standard deviation bound provides 99.7% certainty). Thus, your function will be deemed to have produced an inaccurate result if the absolute difference between its result and the analytical result (given below) is greater than 0.05. In 1000 calls to your function, your function should not produce an inaccurate result more than 10 times, regardless of the seed used. If it does produce more than 10 inaccurate results, your function will not be eligible for a time or accuracy ranking. For the purposes of testing, you can assume that $0.1 \leq \sigma \leq 0.3$ and $80 \leq K \leq 120$, and that the function will be called to price both calls and puts.

The analytical price of the fade-in option is given by

$$\frac{1}{N}\sum_{i=1}^{N}\eta\Big(S_0[M(-d_{5,i},\eta d_1,\rho_i)-M(-d_{3,i},\eta d_1,\rho_i)]-Ke^{-rT}[M(-d_{6,i},\eta d_2,\rho_i)-M(-d_{4,i},\eta d_2,\rho_i)]\Big)$$

where $\rho_i=-\eta\sqrt{t_i}/\sqrt{T}$ and

$$\begin{aligned}
d_1 &= \frac{\log S_0/K+(r+\frac{1}{2}\sigma^2)T}{\sigma\sqrt{T}}, & d_2 &= d_1-\sigma\sqrt{T},\\
d_{3,i} &= \frac{\log S_0/L+(r+\frac{1}{2}\sigma^2)t_i}{\sigma\sqrt{t_i}}, & d_{4,i} &= d_{3,i}-\sigma\sqrt{t_i},\\
d_{5,i} &= \frac{\log S0/U+(r+\frac{1}{2}\sigma^2)t_i}{\sigma\sqrt{t_i}}, & d_{6,i} &= d_{5,i}-\sigma\sqrt{t_i}.
\end{aligned}$$

Here, $M(\cdot,\cdot,\cdot)$ is the bivariate normal CDF and may be coded in Matlab as

```
M=@(x,y,rho) mvncdf([x y],0,[1 rho; rho 1]);
```

To confirm that you have coded the analytical value correctly, a few prices for a fade-in call options are presented in Table 1.

Please note, the analytical solution is provided for your ability to test your function only. The function that you submit may not use it for producing results.

	$\sigma = 0.1$	$\sigma = 0.2$	$\sigma = 0.3$
$K = 80$	27.1977	23.9985	21.0406
$K = 100$	10.0383	10.4947	10.6009
$K = 120$	1.0781	3.1283	4.5905

Table 1: Some example values for a fade-in call option ($\eta = 1$).