Problems

1a. Using bases 2, 3 and 5, generate a 3 dimensional Halton sequence of length 50001. In a single figure, plot the first one thousand points of each of the 2 dimensional Halton subsequences (i.e. dimensions (1,2), (1,3) and (2,3)) and the corresponding normal numbers generated by performing the inverse transform method (remember to exclude the first point (0) for the normal numbers).

1b. Consider a European basket put option, with payoff function

$$f(\mathbf{S}_T) = \max\left(K - \sum_i S_T^{(i)}, 0\right),$$

strike $K = 210$ and maturity $T = 1.5$ years, on three correlated stocks with the following parameters:

$$\mathbf{S}_0 = \begin{bmatrix} S_0^{(1)} \\ S_0^{(2)} \\ S_0^{(3)} \end{bmatrix} = \begin{bmatrix} 25 \\ 65 \\ 120 \end{bmatrix}, \qquad \begin{bmatrix} \sigma^{(1)} \\ \sigma^{(2)} \\ \sigma^{(3)} \end{bmatrix} = \begin{bmatrix} 0.4 \\ 0.25 \\ 0.30 \end{bmatrix}, \qquad \Sigma = \begin{bmatrix} 1 & 0.8 & 0.9 \\ 0.8 & 1 & 0.75 \\ 0.9 & 0.75 & 1 \end{bmatrix}, \qquad r = 10\%$$

where Σ is a correlation matrix. Compute and plot both crude and quasi-Monte Carlo estimates for the option price as a function of sample sizes in the range $n = 1000, 2000, \ldots, 50000$. Plot three-standard-deviation error bounds around these estimates (you can use the best estimate generated by the Quasi-Monte Carlo method (i.e., the estimate using 50000 samples) as an estimate for the real value of the option).

For debugging purposes, initialize your seed at the beginning of your program with the Matlab command `rng(0);`, and in the loop that cycles through sample size, generate the normal random numbers with the command `randn(3,n)`.

1c. Augment, your code in problem 1a & b to include estimates using a Hammersley sequence with bases 2 and 3. For comparison with the Halton sequence, produce the 2 dimensional subsequence plots.

2. Consider an additive (vanilla) cliquet call option (also called a ratchet option) with $N = 4$ equi-spaced start dates, which has a payoff function

$$\sum_{i=1}^{N} e^{-rih}(S_{ih} - S_{(i-1)h})^+,$$

where $h = T/N$, $T = 1$ is the terminal time and $r = 10\%$ is the risk free rate. This is a compound option which is essentially a sum of forward starting call options struck at the money. The analytical price for this option is given by

$$c = NS_0(\Phi(d_1) - e^{-rh}\Phi(d_2)),$$

where

$$d_1 = \frac{(r + \frac{1}{2}\sigma^2)h}{\sigma\sqrt{h}}, \qquad d_2 = d_1 - \sigma\sqrt{h},$$

and $S_0 = 100$ and $\sigma = 30\%$.

Plot quasi-Monte Carlo estimates as a function of sample sizes in the range $n = 1000, 2000, \ldots, 50000$, with stock price paths generated using the standard increments formula for geometric Brownian motion and inverse normal transformed values of the Hammersley set $H = \{(i/(n+1), \varphi_2(i), \varphi_3(i), \varphi_5(i)) | 1 \le i \le n\}$. Also plot the analytical value.

Now, compute and plot quasi-Monte Carlo estimates for the option price with Brownian bridge stock price paths generated using inverse normal transformed values of the permuted Hammersley set
$H' = \{(\varphi_3(i), \varphi_2(i), \varphi_5(i), i/(n+1)) | 1 \le i \le n\}$, where the last sequence in the set is used to generate the endpoints of the Brownian motions.