M.Phil. in Mathematical Finance

Numerical Methods in Finance 2020

Tutorial 4

Problems

1. Consider a pair of correlated stocks modeled by geometric Brownian motions with the following parameters: $S_0^{(1)} = 100$, $S_0^{(2)} = 90$, $\mu_1 = 0.2$, $\mu_2 = 0.3$, $\sigma_1 = 0.25$, $\sigma_2 = 0.45$, $\rho = 0.9$. We will now perform an analysis similar to that done in Examples 2.2. and 2.3. in the notes.

   (a) Fix Matlab's seed using the command `rng(0);` and generate a matrix of $2 \times 1{,}000{,}000$ standard normal random numbers using the command `Z=randn(2,1000000);`.

   (b) Set up the correlation matrix and obtain its Cholesky decomposition. Use the Cholesky decomposition and `Z` to compute a matrix `X` whose columns are pairs of correlated standard normal numbers.

   (c) Use `X` to simulate a million realisations of the two correlated stock prices at a time $t = 0.25$ years in the future.

   (d) Side by side, produce two histograms (with 50 bins each) of the simulated prices of the two stocks.

   (e) In a separate figure, produce a plot of the realised prices of the first stock against the second stock (remember to use `'.'` markers).

   (f) In a separate figure, produce a bi-variate histogram of the two stocks (use `hist3` with a 50 by 50 grid).

   (g) Compute the sample mean and variance for each stock and compare with the theoretical mean and variance.

2. Generate a sample path, using $N = 1{,}000$ equal intervals over a year, i.e. $t \in [0, 1]$ and $\Delta t = 1/N$, for each of three correlated geometric Brownian motion stock price paths, with the following parameters:

$$\overline{S}_0 = \begin{bmatrix} 70 \\ 100 \\ 90 \end{bmatrix}, \qquad \overline{\mu} = \begin{bmatrix} 0.4 \\ 0.1 \\ 0.12 \end{bmatrix}, \qquad \overline{\sigma} = \begin{bmatrix} 0.4 \\ 0.22 \\ 0.25 \end{bmatrix}, \qquad \Sigma = \begin{bmatrix} 1 & 0.3 & 0.95 \\ 0.3 & 1 & 0.55 \\ 0.95 & 0.55 & 1 \end{bmatrix},$$

   where $\overline{S}_0$, $\overline{\mu}$ and $\overline{\sigma}$ are the initial values, drifts, and volatilities for the three stocks and Σ is a correlation matrix. Graph the three stock price paths. For easy debugging, fix Matlab's seed using the command `rng(0);` and generate the matrix of standard normal random numbers required using the command `Z=randn(3,N);`

   Hint: For the kth single stock, with initial value $S_0^{(k)}$, the value of the stock $S_i^{(k)}$ may be simulated at time $t = i\Delta t$ in the future using

$$\begin{aligned} S_i^{(k)} &= S_0^{(k)} \prod_{j=1}^{i} \left[e^{\left(\mu_k - \frac{1}{2}\sigma_k^2\right)\Delta t + \sigma_k \sqrt{\Delta t} X_j^{(k)}} \right] \\ &= S_0^{(k)} \exp\left(\sum_{j=1}^{i} \left[\left(\mu_k - \tfrac{1}{2}\sigma_k^2\right) \Delta t + \sigma_k \sqrt{\Delta t} X_j^{(k)} \right] \right), \end{aligned}$$

   where the $X_j^{(k)}$ is the jth element of a vector containing standard normal random numbers corresponding to the random increments for a single GBM path (with drift μ_k and volatility σ_k). Of course, the $X_j^{(k)}$ for this GBM path must be correlated with the $X_j^{(l)}$ used to generate the GBM path for stock $l \neq m$ — as in the first question. The last expression above lends itself to being implemented using a `cumsum` command.

   Now, using the stock price paths generated, calculate the log returns. Use the Matlab function `corrcoef` on the log returns to compute the sample correlation matrix. Is it as accurate as you would expect?

   (Challenge: implement the above so that all the stock prices are computed using one line of Matlab code. Hint: you can define your vectors with the correct dimensions so that everything can be done with element-wise operations.)