M.Phil. in Mathematical Finance

Numerical Methods in Finance 2020

Tutorial 9

Problems

1. For a standard put option, with parameters

$$T = 1, \qquad S_0 = 100, \qquad \sigma = 40\%, \qquad r = 10\%, \qquad K = 50,$$

compute and plot crude Monte Carlo estimates for the option price as a function of sample sizes in the range $n = 1000, 2000, \ldots, 50000$. Compute and plot the Black-Scholes price and plot three-standard deviation bounds for the Monte Carlo estimates around the Black-Scholes price:

$$\text{BSput} = e^{-rT} K \Phi(-d_2) - S_0 \Phi(-d_1)$$

with

$$d_1 = \frac{\log(S_0/K) + \left(r + \frac{1}{2}\sigma^2\right) T}{\sigma\sqrt{T}} \qquad \text{and} \qquad d_2 = d_1 - \sigma\sqrt{T}.$$

Now, use the distribution

$$v(x) := \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}(x-\mu)^2\right),$$

with $\mu = -2$, as an importance sampler to provide better Monte Carlo estimates. To generate variates from v, just add μ to the same normal random numbers used to generate the crude Monte Carlo estimate. Plot these estimates and corresponding three-standard deviation bounds.

For debugging purposes, initialize the seed at the beginning of your program with the Matlab command `rng(0);`.

2. (a) Generate $n = 3000$ standard normal variates using the Matlab `randn` function and plot them using a histogram, with 50 bins.

Now, write code to produce n variates from a stratified standard normal distribution with $d = 50$ equally probable strata. You can do this by generating n/d uniform variates from $\mathscr{U}\left[\frac{i-1}{d}, \frac{i}{d}\right]$ and using the inverse method to generate the normal variates for each of the $1 \le i \le d$ strata (use Matlab's `norminv` command). Next to the previous graph, plot the histogram of the stratified variates.

For debugging purposes, initialize the seed at the beginning of your program with the Matlab command `rng(0);`.

2. (b) For a standard call option, with parameters

$$T = 2, \qquad S_0 = 100, \qquad \sigma = 40\%, \qquad r = 10\%, \qquad K = 100,$$

compute and plot crude Monte Carlo estimates for the option price as a function of sample sizes in the range $n = 1000, 2000, \ldots, 50000$. Compute and plot the Black-Scholes price and plot three-standard deviation bounds for the Monte Carlo estimates around the Black-Scholes price:

$$\text{BScall} = S_0 \Phi(d_1) - e^{-rT} K \Phi(d_2).$$

Now, with $d = 50$ equally probable strata, compute and plot Monte Carlo estimates and three-standard deviation bounds using stratified normal samples for sample sizes in the range $n = 1000, 2000, \ldots, 50000$.

Hint: recall that the sample variance for the stratified Monte Carlo estimate is computed as the scaled sum of conditional sample variances (for each stratum). So, it is best to compute these variances as you compute the numbers for each stratum. Also, for debugging purposes, set the seed and generate random numbers in the same order as in part (a).