M.Phil. in Mathematical Finance

Numerical Methods in Finance 2020

Tutorial 1

Problems

1. Implement a Linear Congruential Generator in a Matlab function called `LCG` that takes as input the variables a, c, m, x_1 and n. The function should generate a sequence of length n using

$$x_{i+1} = (ax_i + c) \bmod m$$

and then return the uniformly distributed variates computed using $\frac{1}{m}[x_1, x_2, \ldots, x_n]$.

Now, in a separate Matlab script, perform the following tasks:

- Use calls to `LCG` to generate two lists of 10,000 uniformly distributed numbers using the following parameters:
  - $a = 16807$, $c = 0$ and $m = 2^{31} - 1$, and
  - $a = 2^{16} + 3$, $c = 0$ and $m = 2^{31}$.

  In both cases use an initial seed of $x_1 = 1$.
- Produce histograms of these two sets of numbers side by side in a separate window.
- For each set of numbers, produce a (2d) plot of the points $\{(u_1, u_2), (u_3, u_4), (u_5, u_6), \ldots\}$, where u_i are the elements of each list. Display the plots side by side in a separate window.
- For each set of numbers, produce a (3d) plot of the points $\{(u_1, u_2, u_3), (u_4, u_5, u_6), (u_7, u_8, u_9), \ldots\}$, where u_i are the elements of each list (exclude $u_{10,000}$). Diplay the plots side by side in a separate window.

Note: Setting $c = 0$ means that the generator is in fact a "minimal standard generator", a simplified version of a linear congruential generator. The second set of parameters used above is the same as those used in the infamous `RANDU` function implemented on IBM mainframes (see notes for more information), which shows sequential correlation.

2. The Weibull distribution is a continuous distribution used widely in the epidemiology, engineering and actuarial fields. It has a distribution function given by

$$F(x) = 1 - e^{-(x/\lambda)^k},$$

where $\lambda > 0$ is called the scale parameter and $k > 0$ is called the shape parameter.

In a Matlab program perform the following tasks:

- Use your `LCG` function to generate $n = 10,000$ uniformly distributed numbers using an initial seed of $x_1 = 1$ and the following parameters
  - $a = 1103515245$, $c = 12345$ and $m = 2^{32}$.
- Using the inverse transform method, convert these uniform numbers into Weibull distributed numbers for the Weibull parameters $\lambda = 0.44$ and $k = 7$. Use the small simplification that if U is $\mathscr{U}[0, 1)$, then so is $1 - U$. The first 5 numbers should be $[0.6851, 0.4597, 0.4020, 0.4039, 0.4422]$.
- Produce a histogram (using 20 bins) of the Weibull distributed numbers generated.
- Calculate the sample mean and standard deviation.