M.Phil. in Mathematical Finance

Numerical Methods in Finance 2020

Tutorial 7

Problems

1. Consider an arithmetic average rate call option with the following payoff

$$f^{\text{arith}}(\boldsymbol{Z}) = \max\left(\frac{1}{N+1}\sum_{i=0}^{N} S_i(\boldsymbol{Z}) - K, 0\right),$$

where $S_i(\boldsymbol{Z})$ is a GBM stock price process generated at times ih, with time interval $h = T/N$, using $\boldsymbol{Z} \sim \mathscr{N}_N(0, I)$, N is the number of averaging intervals and K is the strike price.

Write a Matlab program that uses crude Monte Carlo to estimate the price of this option using the following parameters:

$$T = 1, \qquad S_0 = 100, \qquad \sigma = 45\%, \qquad r = 12\%, \qquad K = 50, \qquad N = 6,$$

as a function of sample size, for sample sizes in the range $n = 1000, 2000, \ldots, 50000$. Plot three-standard-deviation error bounds around each of these estimates (note, that these will be jagged lines as there is no analytical estimate).

Now, in a separate plot, reuse the Monte Carlo samples with the method of common random variables to produce forward-difference estimates of the delta for this option as a function of sample size. Use $\Delta S_0 = 0.1$.

On the same plot (using different markers—circles, say), use the method of pathwise derivatives to compute the delta for the option as a function of sample size. Plot three-standard-deviation error bounds for this delta.

For debugging purposes, initialize your seed at the beginning of your program with the Matlab command `rng(0);` and in the loop that cycles through sample size, generate the normal random numbers with the command `randn(N,n)`. As far as possible vectorize your code.

2. A cash-or-nothing call option has the payoff function $X\mathbb{I}_{\{S_T > K\}}$, where X is the amount of cash and K is the strike price. An analytical pricing formula for the option is given by

$$c = Xe^{-rT}\Phi(d) \qquad \text{where} \qquad d = \frac{\log(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}.$$

With parameters

$$T = 2, \qquad S_0 = 100, \qquad \sigma = 40\%, \qquad r = 10\%, \qquad K = 110, \qquad X = 10,$$

compute and plot crude Monte Carlo estimates for the option price as a function of sample sizes in the range $n = 1000, 2000, \ldots, 50000$. Compute and plot the analytical price and plot three-standard deviation bounds for the Monte Carlo estimates around the analytical price.

Now, augment your code to compute likelihood ratio estimates of the delta of the option. Ensure that you reuse the paths and normal random variates used to compute the price of the option. In a separate figure, plot the delta estimates as a function of sample size with three-standard deviation bounds in comparison to the analytical value give by

$$\Delta = \frac{Xe^{-rT}}{s_0\sigma\sqrt{T}}\phi(d),$$

where ϕ is the standard normal density function.

As usual, initialize your seed using the command `rng(0);`