M.Phil. in Mathematical Finance

Numerical Methods in Finance II 2020

Tutorial 8 (Short Rate Models)

Problems

1. In this problem, you will compute the price of a call option on a bond using Monte Carlo and compare it to the closed form formula under the Vasicek model. Although it is not necessary to do so, to simplify things slightly, we will make the assumption that bond prices can be computed in closed form.

   Consider the payoff
   $$f = (B(T,S) - K)^+$$
   in terms of a bond price $B(T,S)$, with $0 < T < S$.

   Using the following parameters
   $$r(0) = 0.07, \quad \alpha = 0.1, \quad b = 0.09, \quad \sigma = 0.025, \quad K = 0.8, \quad T = 1 \quad \text{and} \quad S = 2,$$
   simulate joint realisations of the short rate (using equations (5), (6) & (14)) and corresponding discount factors (using equations (13) & (15)) for time T. For each short rate realisation, compute the corresponding bond price (using the two formulae after (12)). Then compute the expected value of the payoff in terms of the bond prices using the realised discount factors, i.e. compute
   $$C = \mathbb{E}\left[\frac{1}{\beta(T)}(B(T,S) - K)^+\right].$$
   Plot the Monte Carlo prices, with the usual three standard deviation error bounds, as a function of sample size in the range $n = 1000, 2000, \ldots, 50000$. Compare these results against the closed form solution given by
   $$C = B(0,S)\Phi(d_1) - KB(0,T)\Phi(d_2),$$
   where
   $$d_1 = \frac{\log\left(\frac{B(0,S)}{B(0,T)K}\right) + \frac{\bar{\sigma}^2}{2}}{\bar{\sigma}} \quad \text{and} \quad d_2 = d_1 - \bar{\sigma}$$
   with
   $$\bar{\sigma} = \sigma\sqrt{\frac{1 - e^{-2\alpha T}}{2\alpha}}A(T,S).$$
   To ensure comparable results, set your seed using `rng(0)` and generate your random numbers using `Z=randn(n,2)`.

2. In this problem, you will generate the graphs for CIR sample paths and bond prices as per the examples in the notes. Using the parameters
   $$r(0) = 0.07, \quad \alpha = 0.15, \quad b = 0.09 \quad \text{and} \quad \sigma = 0.07,$$
   simulate $n = 50000$ sample paths for the CIR short rate at annual intervals over 20 years. Plot the first 20 of these sample paths along with the mean of all the paths. In a separate figure, plot the zero coupon bond prices using simple and trapezoidal quadrature of these rates as compared to the closed form bond prices. Also show the yields as computed from these bond prices.

   Hint: you will need to use a loop that steps through time, otherwise everything else should be vectorised. To ensure comparable results, set your seed using `rng(0)`.

3. (Homework) Extend the code in problem one to a two-period Monte Carlo model so that you can price the bond option without the need to use the bond pricing formula. How does this affect the accuracy of the solution?