Problems

1. Implement the characteristic pricing method for geometric Brownian motion, to price a call option. Use the same parameters as in the notes, i.e, use stock parameters $S_0 = 50$, $\sigma = 40\%$, $T = 1$, $r = 6\%$ and $K = 50$ and algorithm specific parameters $u_{\mathsf{max}} = 30$ and $N = 100$. As in the notes, plot the integrands over the specified interval. Compare this value to the Black-Scholes value. Once you have this working, also price a put option. Note that for a put
$$P(K) = Ke^{-rT}(1 - P_2) - S_0(1 - P_1).$$

2. Write a function called `myFFT` which implements the Fast Fourier Transform algorithm as described in the notes. To test your function, apply it to the vector `x=[zeros(1,16) ones(1,32) zeros(1,16)]`. In two graphs, plot the real and imaginary components of the discrete Fourier transform of `x`.

3. (Homework) Reproduce the Heston call option pricing problem in the notes for both the Milstein and characteristic pricing methods. Use option related parameters $S_0 = 100$, $\nu_0 = 0.06$, $\kappa = 9$, $\theta = 0.06$, $\sigma = 0.5$, $\rho = -0.4$, $T = 0.5$, $r = 3\%$ and $K = 105$.

   For the Mistein approach use $N_{\mathsf{m}} = 10$ updates (intervals) over the time interval and, for the characteristic pricing method, use $N_{\mathsf{q}} = 100$ quadrature steps with an upper integration limit of $u_{\mathsf{max}} = 30$.

   Plot the Milstein Monte Carlo estimates as a function of sample size, for sample sizes in the range $n = 1000, 2000, \ldots, 50000$, with a three-standard-deviation bound around the characteristic price. Remember to initialize your seed.

4. (Homework) Reproduce the Heston smile/skew plots in the notes using the characteristic pricing method.