M.Phil. in Mathematical Finance

Numerical Methods in Finance 2020

Tutorial 5

Problems

1. Implement the algorithm to generate Brownian bridges, found in the notes, as a Matlab function. You should produce vectorised code that computes multiple bridge paths in a single call.

   Hints: Think of the matrix W that you generate as having rows that are the individual Brownian bridge paths. Also, carefully consider how to handle indexing, since Matlab does not allow zero indexes (remember that $\boldsymbol{t}$ is indexed from 0 while $\boldsymbol{Z}$ is not). To check whether or not $\boldsymbol{Z} = []$ use the Matlab function `isempty` — see help for details.

   Now, in a separate program, use a call to your Brownian bridge function to generate the graphs in Figure 2.5 in the notes. In side-by-side plots, graph the Wiener paths and the stock price paths generated.

   To ensure that the same random numbers are used, initialize the seed using `rng(0);` and generate the random numbers required for the call to your function using the command `Z=randn(5,99);`.

2. Consider a stochastic process that follows the SDE

   $$dS_t = \mu S_t \, dt + \sigma S_t^{\alpha/2} \, dW_t,$$

   where $\mu > 0$, $\sigma > 0$ and $0 \leq \alpha < 2$ are constants. This process is known as the constant elasticity of variance (CEV) process and was first proposed by Cox in 1975 as a better process to model stock price paths than GBM. Implement that Euler-Maruyama and Milstein methods to generate a path with $N = 1000$ steps over the time period $t \in [0, 5]$ years, with the following parameters

   $$S_0 = 1, \qquad \mu = 0.5, \qquad \sigma = 0.4, \qquad \alpha = 1.2.$$

   Use the same set of random numbers for both methods. Now in three graphs, side by side, plot each process generated by both methods and the difference of the two processes. For debugging purposes, initialize your seed at the beginning of your program with the command `rng(0);`.

   What happens when you set $\alpha = 0$? Why?