M.Phil. in Mathematical Finance

Numerical Methods in Finance II 2020

Tutorial 9 (The Heath Jarrow Morton Model)

Problems

1. In this problem you will reconstruct the zero coupon bond prices under the HJM model as the expectation of discount factors at appropriate times.

   To start with, compute bond prices $B(0, t_i)$ under the Vasicek model for times $0 = t_0, t_1, \ldots, t_N = 10$ with $\Delta t = 0.5$ using the parameters

   $$r_0 = 7\%, \quad \alpha = 0.15, \quad b = 9\%, \quad \text{and} \quad \sigma = 2\%.$$

   Now calibrate the discrete forward curve at inception $\hat{f}(t_0, t_i)$ $(i = 0, 1, \ldots, N-1)$ to these prices.

   Using a sample size of $n = 100,000$, implement the discrete HJM algorithm using $M = N$ and computing $H_i = \mathbb{E}[1/\beta]$ at each time step. The H_i are the simulated bond prices with $H_0 = 1$ by definition. To ensure you recover the Vasicek bond prices, use the HJM volatility function given by

   $$\hat{\sigma}(t_i, t_j) = \sigma e^{-\alpha(t_j - t_i)}.$$

   Graph these prices in comparison to the Vasicek prices used as input. In a separate graph, compare the input and simulated yields.

   Hints: To keep implementation simple, just use Δt for all time differences in the algorithm. Be very careful to distinguish between mathematical indices and Matlab indices. For steps 3.3.1–3.3.4, define a vector j of indices and use vectorized commands to compute σ_j, μ_j and to update f_j in terms of this vector of indices. As usual, set your seed, using `rng(0)`.

2. Reuse your code from part one to price a put option, with maturity $t_M = 3$ and strike $K = 98$, on a coupon bearing bond. The bond has par value 100 and pays coupons of 10% NACS.

   Use the same time discretisation as before, except that $t_N = 5$. This time compute the discretised forward rate values, $\hat{f}(t_0, t_i)$, using Vasicek bond prices with the following parameters

   $$r_0 = 10\%, \quad \alpha = 0.1, \quad b = 10\%, \quad \text{and} \quad \sigma = 2\%.$$

   Plot the Monte Carlo prices as a function of sample size $n = 1000, 2000, \ldots, 50000$. Also plot the closed form value of 0.87513 along with three-standard deviation bounds around this value.

   Hint: You will only need to evolve the forward curve to time t_M. The payoff of the option can then be computed using the cashflows (5 5 5 and 105) and the forward rates (zero coupon bond prices) at that time.