M.Phil. in Mathematical Finance

Numerical Methods in Finance II 2020

Tutorial 7 (The FFT and Cosine Methods)

Problems

1. Write code to implement the FFT algorithm of Carr and Madan. Implement the quadrature approach that includes Simpson's rule weighting factors in Equation (6) from the notes. Use the following GBM stock parameters: $S_0 = 50$, $\sigma = 0.4$, $T = 1$ and $r = 6\%$. You should also use the following algorithm specific parameters: $N = 2^{10}$, $\delta_v = 0.25$ and $\alpha = 1.5$. Compute δ_k using the Nyquist relation and then, using this value, compute b.

   Now, produce the graph in the notes by plotting the values of C against strike for $0.002 \leq K \leq 100$ (Hint: use logical indexing to do this). Alongside this graph, plot the $\log_{10}$ absolute error between these prices and the corresponding Black-Scholes prices on a $\log_{10}$ scale for strike.

2. Using the parameters

   $$S_0 = 5, \quad \sigma = 0.4, \quad T = 2, \quad K = 6 \quad \text{and} \quad r = 6\%,$$

   price a call option using the Fourier cosine method of Fang and Oosterlee.

   Remember to compute the bounds of integration using the cumulants for GBM.

   Graph the price as a function of the number of terms in the expansion $N = 1, 2, \ldots, 50$. Alongside this, plot the $\log_{10}$ absolute error between these prices and the correct Black-Scholes price.

   In a second figure plot estimates of the Delta for the option and the corresponding $\log_{10}$ absolute error.

3. (Homework) Using the parameters

   $$S_0 = 7, \quad \sigma = 0.4, \quad T = 2, \quad K = 6, \quad X = 10 \quad \text{and} \quad r = 6\%,$$

   repeat the process above, but this time for a cash-or-nothing put option that pays X, and computing the Vega of the option instead of the Delta.

   The closed form price and Vega for the option are given as

   $$\begin{aligned} P &= Xe^{-rT}\Phi(-d_2) \\ \nu &= Xe^{-rT}\phi(-d_2)(\sqrt{T} + d_2/\sigma), \end{aligned}$$

   where

   $$d_2 = \frac{\log(S_0/K) + (r - \sigma^2/2)T}{\sigma\sqrt{T}}.$$