Length-k Clique Decoding for Surface Codes

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Introduction

Some problems can be solved much faster with quantum algorithms as opposed to classical algorithms:

Problem	Classical	Quantum
Factoring (Encryption)	$O(2^n)$ for n digits	$O(n^3)$
Unsorted Data Search	$O(n)$ for n numbers	$O(\sqrt{n})$
Particle Simulation	$O(2^n)$ for n atoms	$O(n^c)$
Linear Systems	$O(2^n)$ for n digits	$O(n)$

However, current quantum computers are too error-prone to implement these algorithms. Errors are caused by *noise*.

Quantum error correction reduces the effects of noise by detecting and correcting errors. The *surface code* is the current most promising approach.

This investigation aims to improve error correction efficiency on the surface code using a length-k on-chip clique decoder to detect and correct error strings of up to length-k.

Research Questions:

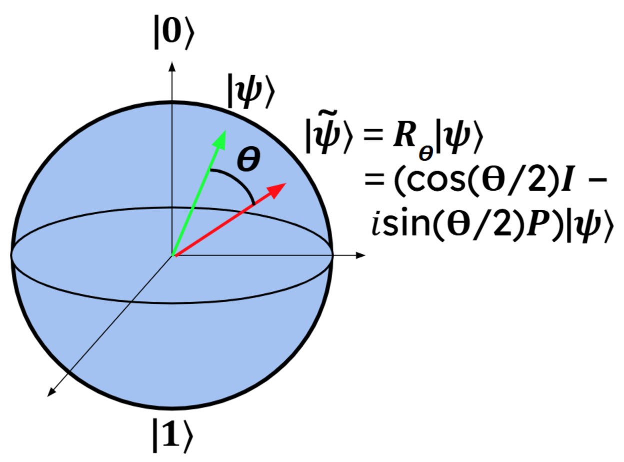
1. What is the error coverage for a length-k decoder on the surface code?
2. How can a length-k decoder be realistically implemented?

Classical vs. Quantum Computer

	Classical Computer	Quantum Computer
Fundamental Unit	Bit	Qubit
State	0 OR 1	$\lvert\psi\rangle = \alpha\lvert 0\rangle + \beta\lvert 1\rangle$ $\lvert 0\rangle$ AND $\lvert 1\rangle$
Measurement	• Discrete state • Deterministic measurement	• Probabilistic measurement • collapses superposition into basis state
Computing	• n bits = 1 n-bit state • Classical parallelism: 000 → f → f(000) 001 → f → f(001) 010 → f → f(010)	• n qubits = Up to 2^n states in superposition • Quantum Parallelism: $\alpha\lvert 000\rangle + \beta\lvert 001\rangle + \gamma\lvert 010\rangle$ → f → $\alpha'\lvert f(000)\rangle + \beta'\lvert f(001)\rangle + \gamma'\lvert f(010)\rangle$

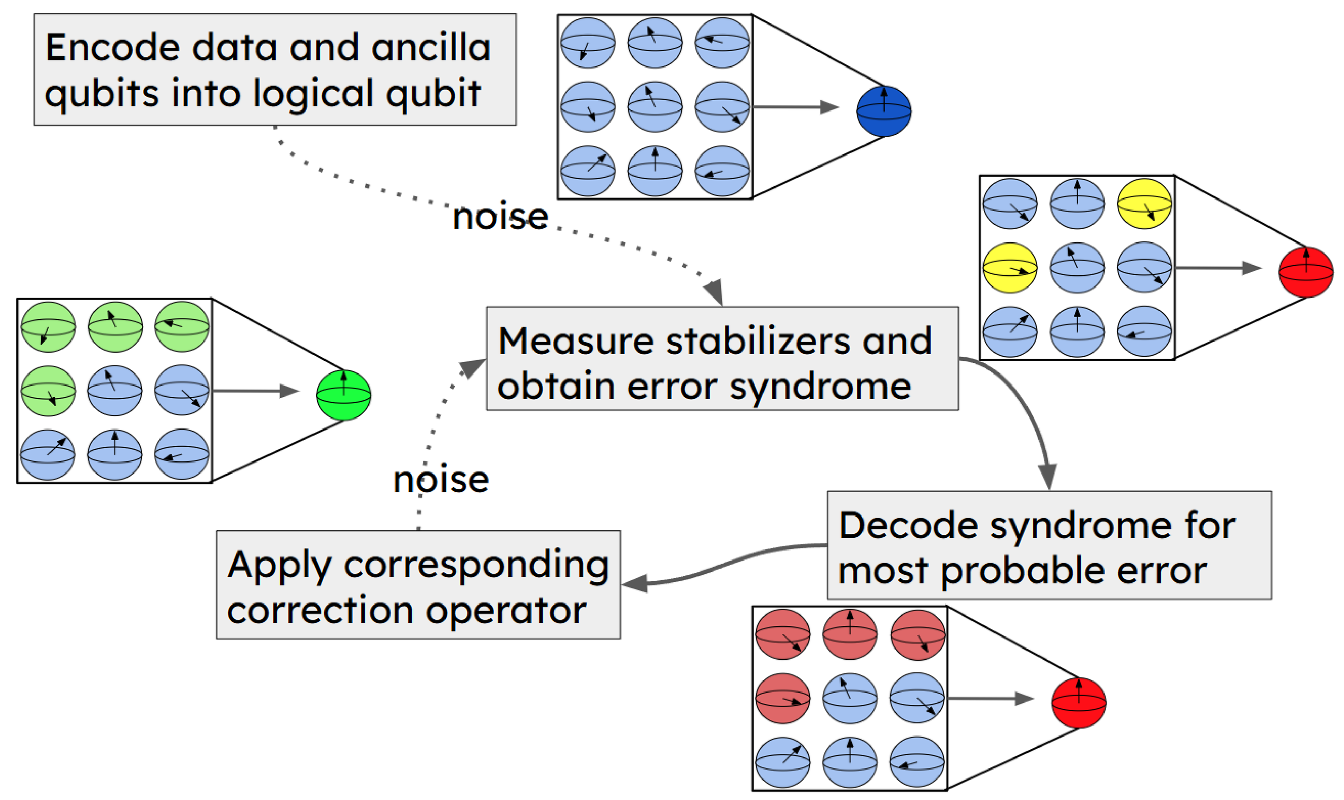
Quantum Error Correction

Decomposing continuous rotations into discrete errors:

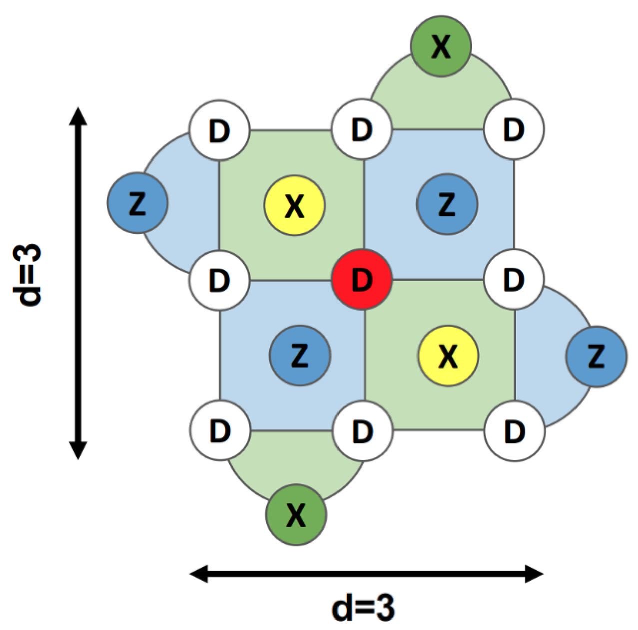


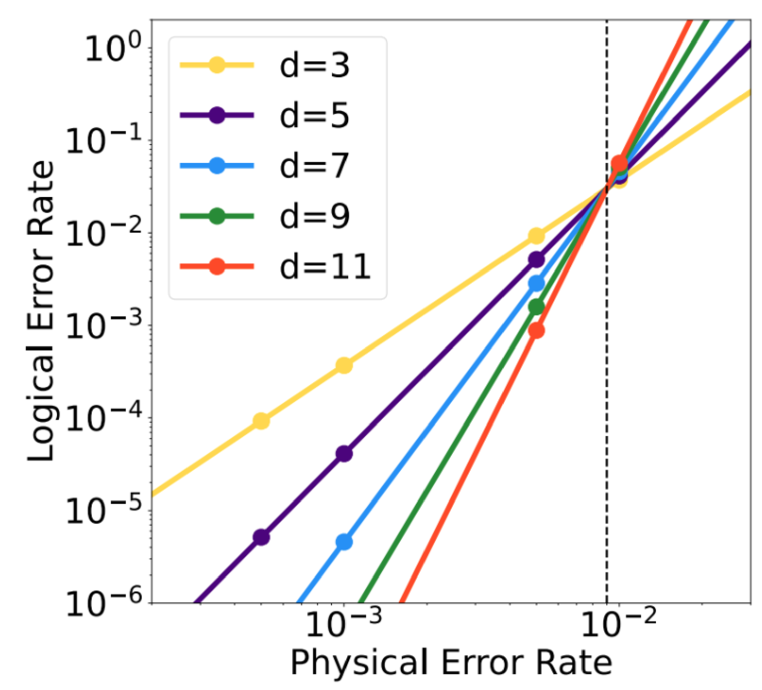
Pauli Error P	Effect on Quantum state	Example
I	No error	$I(\alpha\lvert 0\rangle+\beta\lvert 1\rangle) = \alpha\lvert 0\rangle+\beta\lvert 1\rangle$
X	Bit flip	$X(\alpha\lvert 0\rangle+\beta\lvert 1\rangle) = \alpha\lvert 1\rangle+\beta\lvert 0\rangle$
Z	Phase flip	$Z(\alpha\lvert 0\rangle+\beta\lvert 1\rangle) = \alpha\lvert 0\rangle-\beta\lvert 1\rangle$
$Y=ZX$	Bit + phase flip	$Y(\alpha\lvert 0\rangle+\beta\lvert 1\rangle) = \alpha\lvert 1\rangle-\beta\lvert 0\rangle$

Error correction cycle using stabilizer codes:

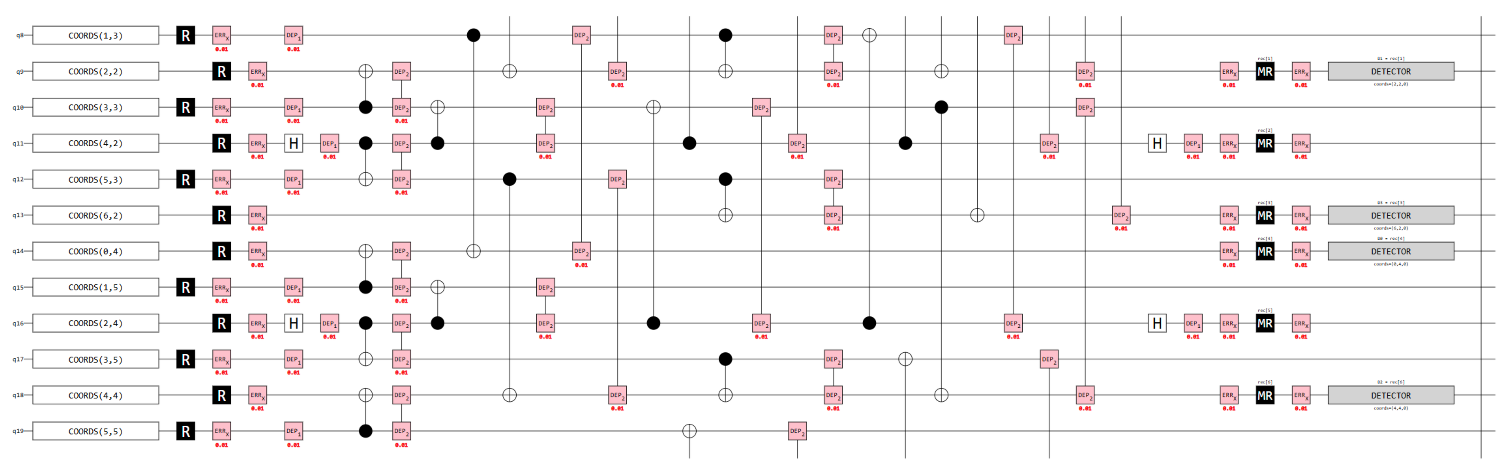


Why is the surface code promising?

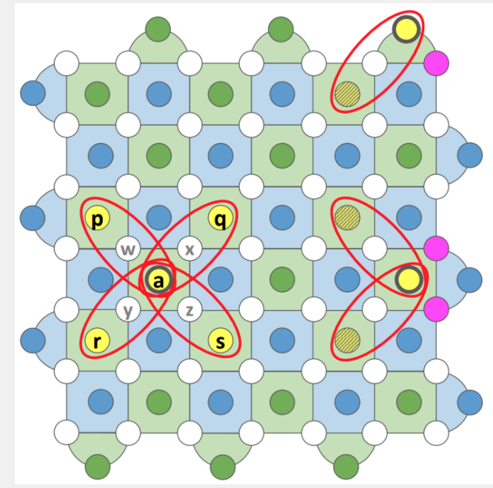




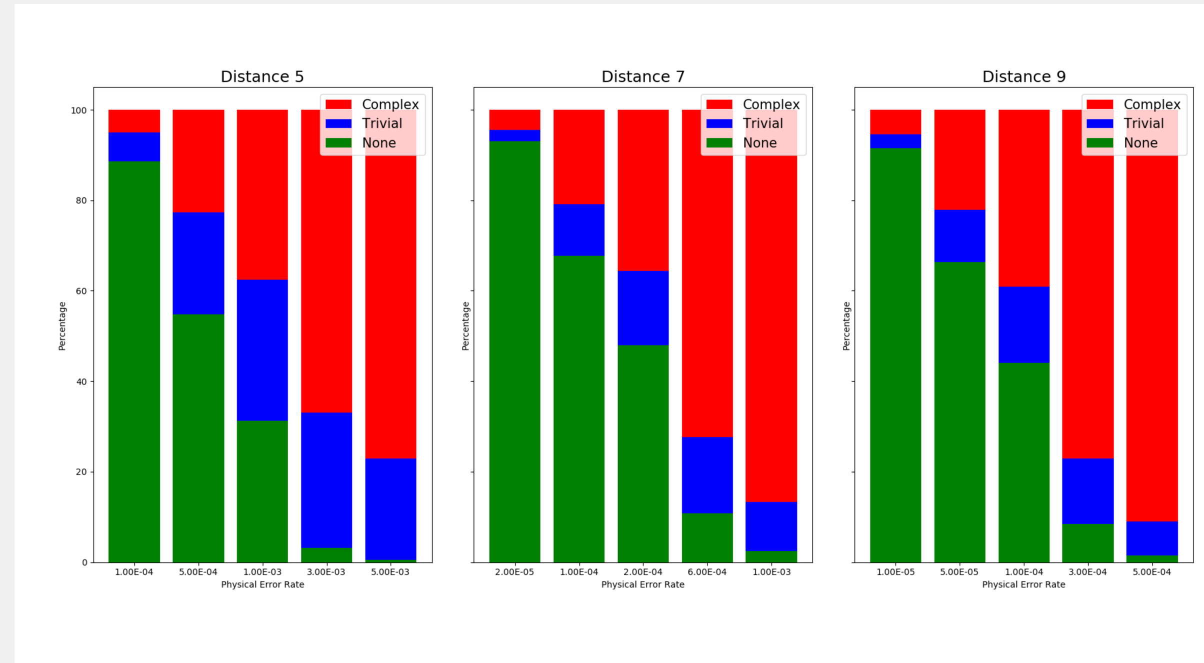
Circuit-level noise simulation with Stim: [1]



Length-1 Error Distribution Analysis [2]




```
[if(a == TRUE && !parity(p,q,r,s))] <any a>:
        COMPLEX_DECODE
else: #CLIQUE_DECODE
        if(a && p == TRUE): correct w
        if(a && q == TRUE): correct x
        if(a && r == TRUE): correct y
        if(a && s == TRUE): correct z
```



Conclusion & Future Research

A length-1 clique decoder is capable of handling a significant percentage of errors on-chip. This serves as a lower bound for a length-k clique decoder, which can correct error strings from length 1 to k. The results also suggest that lower distance surface codes have a higher percentage of trivial errors, likely due to more gates needed to encode and measure higher distance codes, thus increasing the probability of errors under a circuit-level noise model.

In future research, the actual error distribution may be compared to the predicted error distribution from syndrome measurement to analyze the performance of the decoding scheme in detecting errors accurately. One can also investigate the detection of length-k error strings on-chip to provide a better analysis on the error coverage of a length-k clique decoder.

References

[1] Craig Gidney.
Stim: a fast stabilizer circuit simulator, July 2021.

[2] Gokul Subramanian Ravi, Jonathan M. Baker, Arash Fayyazi, Sophia Fuhui Lin, Ali Javadi-Abhari, Massoud Pedram, and Frederic T. Chong.
Better than worst-case decoding for quantum error correction, 2022.

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