Hago $i \cdot z$, luego la imaginaria

1. Para los siguientes $z \in \mathbb{C}$, hallar $\mathrm{Re}(z)$, $\mathrm{Im}(z)$, $|z|$, $\mathrm{Re}(z^{-1})$ e $\mathrm{Im}(i \cdot z)$

i) $z = 5\,i(1+i)^4$

ii) $z = \underbrace{(\sqrt{2}+\sqrt{3}\,i)^2}_{x}\underbrace{(\overline{1-3\,i})}_{x^{1}}$ $\Rightarrow$ hi. el produc.

iii) $z = i^{17} + \frac{1}{2}\,i(1-i)^3$

iv) $z = \left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\,i\right)^{10}$

v) $z = \left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\,i\right)^{-1}$.

i) $z = 5 . i\underbrace{(1+i)^4}$

$$(1+i)^4 = (1+i)^2(1+i)^2 = (1^2+2i+i^2)\cdot(1^2+2i+i^2)$$
$$= (1+2i-1)\cdot(1+2i-1)$$
$$= (2i)\cdot(2i)$$
$$= (4i^2) = -4$$

$$5 . i(-4) = -20i$$

- $\mathrm{Re}(z) = 0$
- $\mathrm{Im}(z) = -20$
- $\mathrm{Im}(i.z) = -20i\cdot i = 20 = 0$
- $|z| = 20$
- $\mathrm{Re}(z^{-1}) = 0$

$$z^{-1} = \frac{\overline{z}}{|z|^2} = \frac{20i}{20} = i$$

b) $z = (\sqrt{2}+\sqrt{3}i)^2 \cdot \overline{(1-3i)}$

$(\sqrt{3})^2 \cdot i^2$

$$= \left((\sqrt{2})^2 + 2\sqrt{2}\sqrt{3}i + (\sqrt{3}i)^2 \right)(1+3i)$$

$$= (-1+2\sqrt{6}i)\cdot(1+3i)$$

$$= -1-3i+2\sqrt{6}i+6i^2\sqrt{6}$$

$$= -1-3i+2\sqrt{6}i-6\sqrt{6}$$

$$= -1-6\sqrt{6}+i(-3+2\sqrt{6})$$

- $Re(z) = -1-6\sqrt{6}$ · $Im(z) = -3+2\sqrt{6}$

$$|z| = \sqrt{(-1-6\sqrt{6})^2 + (-3+2\sqrt{6})^2} = \sqrt{250} = \underbrace{5\sqrt{10}}_{\text{¿Cómo paso a esto?}}$$

- $z^{-1} = \dfrac{\bar{z}}{|z|^2} = \dfrac{\overset{a}{(-1-6\sqrt{6})} \overset{-}{-} \overset{bi}{(-3+2\sqrt{6})}}{(5\sqrt{10})^2} = \dfrac{2-8\sqrt{6}}{250}$

$z = (\sqrt{2}+\sqrt{3}i)$ $\quad z = 1+3i$

$$|z| = \sqrt{(\cancel{\sqrt{}}2)^{\cancel{2}} + (\cancel{\sqrt{}}3)^{\cancel{2}}} = \sqrt{5}$$

$$Arg(z) = \arctan\left(\frac{\sqrt{3}}{\sqrt{2}}\right) \simeq 0.88RAD.$$

Como z está en primer cuadrante entonces 0.88RAD es el argumento.

Luego, $z^2 = (\sqrt{5})^2 \cdot \left(\cos(2\cdot 0.88RAD) + i\,sen(2.0.88RAD) \right)$

$$= 5\left(\cos(1.76RAD) + i\,sen(1.76RAD)\right)$$

Ahora veo z'

$$|z'| = \sqrt{(1)^2+(2)^2} = \sqrt{5}$$

$$\arg(z') = \arctan\left(\frac{2}{1}\right) \simeq 1.10\text{RAD}$$

Luego,

$$\underbrace{z \cdot z'}_{X} = 5\sqrt{5}\left(\cos(1.76\text{RAD}+1.10\text{RAD}) + i\,\text{sen}(1.76\text{RAD}+1.10\text{RAD})\right)$$

$$= 5\sqrt{5}\left(\cos(2.86\text{RAD}) + i\,\text{sen}(2.86\text{RAD})\right)$$

Luego,
- $\text{Re}(X) = 5\sqrt{5}\left(\cos(2.86\text{RAD})\right)$
- $\text{Im}(X) = 5\sqrt{5}\left(\text{sen}(2.86\text{RAD})\right)$
- $\text{Im}(i \cdot z) = (-11+3i) \cdot i = -11$

- $|z| = 5\sqrt{5}$
- $z^{-1} = \dfrac{\overline{z}}{|z|^2} = \dfrac{-11-3i}{125}$
- $\text{Re}(z^{-1}) = -\dfrac{11}{125}$

c) $z = i^{17} + \frac{1}{2}i(1-i)^3 \iff i + \frac{1}{2}i(1-i)^3$

$$(1-i)^3 = (1-i)^2(1-i) = (1^2 - 2i + \underbrace{(-i)^2}_{i^2})(1-i)$$
$$= (1-2i+i^2)(1-i)$$
$$= (-2i)(1-i)$$
$$= (-2i+2i^2)$$
$$= (-2i-2)$$

$$= i + \frac{1}{2}i\,(-2-2i)$$

$$= i - i - i^2$$

$$= i - i - (-1)$$

$$= 1$$

- $\operatorname{Re}(z) = 1$ • $\operatorname{Im}(z) = 0$ • $|z| = 1$ • $z^{-1} = \dfrac{\overline{z}}{|z|^2} = \dfrac{1}{1} = 1$

- $\operatorname{Re}(z^{-1}) = 1$ • $\operatorname{Im}(i\cdot z) = 1 \cdot i = i \Rightarrow \operatorname{Im}(i\cdot z) = 1$

iv) $z = \left(\underbrace{\frac{1}{\sqrt{2}}}_{\frac{\pi}{4}} + \underbrace{\frac{1}{\sqrt{2}}}_{\frac{\pi}{4}}\, i\right)^{10}$

θ	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
$\operatorname{sen}\theta$	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

Utilizo De De Moivre.

$$|z| = \sqrt{\left(\tfrac{1}{\sqrt{2}}\right)^2 + \left(\tfrac{1}{\sqrt{2}}\right)^2} = \sqrt{\tfrac{1}{2} + \tfrac{1}{2}} = 1$$

$$\operatorname{Arg}(z) = \frac{\pi}{4}$$

Luego, $z = 1.10\left(\cos\left(\frac{\pi}{4}\cdot 10\right) + i\operatorname{sen}\left(\frac{\pi}{4}\cdot 10\right)\right)$

$$= 10\left(\cos\left(\underbrace{\frac{5\pi}{2}}\right) + i\operatorname{sen}\left(\frac{5\pi}{2}\right)\right)$$

No es el arg principal por ser $> 2\pi$.

le resto $2k\pi$

$$= 10\left(\cos\left(\frac{\pi}{2}\right) + i\,\text{sen}\left(\frac{\pi}{2}\right)\right)$$

Luego, $\cdot\ Re(z) = 10 \cdot \cos\left(\frac{\pi}{2}\right) = 0$ $\quad \cdot\ Im(z) = 10 \cdot \text{sen}\left(\frac{\pi}{2}\right) = 10$

EN BINÓMICA: $z = 10i$

- $|z| = \sqrt{(0)^2 + (10)^2} = 10$ $\quad \cdot\ z^{-1} = \dfrac{\overline{z}}{|z|^2} = \dfrac{-10}{100} = \dfrac{-1}{10}$
- $\overline{z} = -10$
- $Im(i \cdot z) = 10i \cdot i = -10 \Rightarrow Im(i \cdot z) = 0$

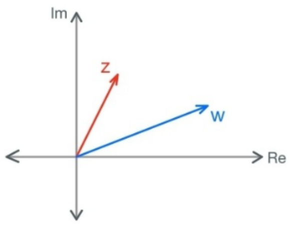
v) $\left(-\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)^{-1} = z^{-1} = \dfrac{\overline{z}}{|z|^2}$

$\overline{z} = -\frac{1}{2} - \frac{\sqrt{3}}{2} i \qquad |z| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$

$|z|^2 = 1$

Ent, $z^{-1} = \dfrac{-\frac{1}{2} - \frac{\sqrt{3}}{2} i}{1} = -\frac{1}{2} - \frac{\sqrt{3}}{2} i$

2. Dados los siguientes $z, w \in \mathbb{C}$ en el plano:



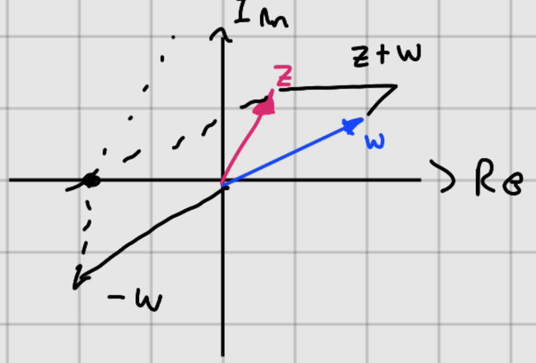
representar en un gráfico aproximado los números complejos de cada inciso

i) $z, w, z + w$ y $\underbrace{z - w}_{z + (-w)}$ $\qquad$ ii) $z, -z, 2z, \frac{1}{2}z, iz$ y $\overline{z}$ $\qquad$ iii) $z, w, |z|, |z + w|$ y $|\overline{w - z}|$.

$(a + bi) - (a - bi)$

PREG

i)

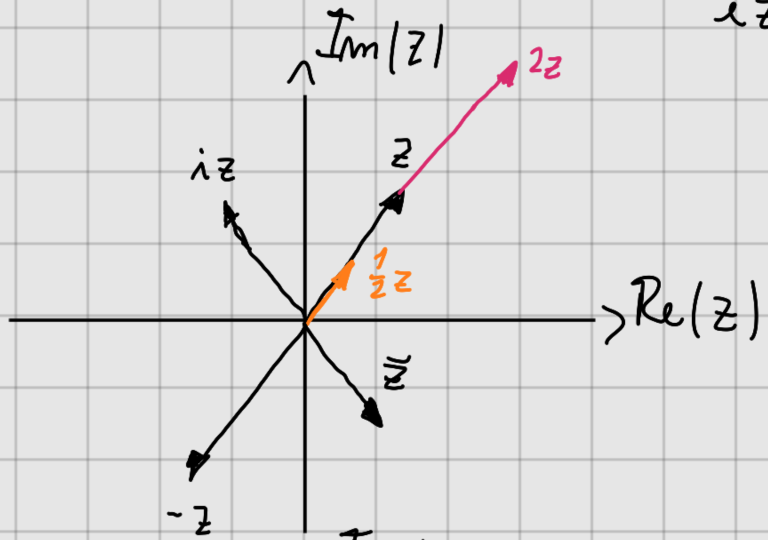


$Z = a+bi$

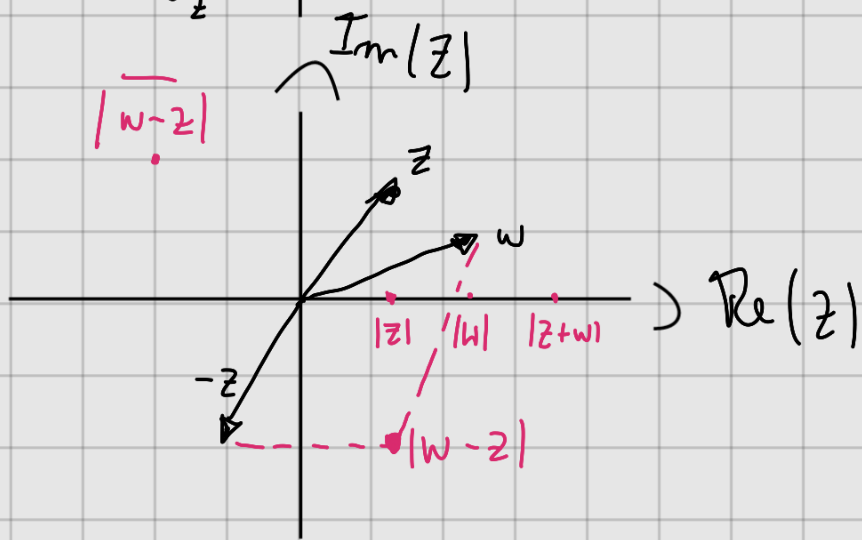
$iz = i(a+bi)$

$= ai - b = -b + ai$

ii)



iii)



3. Hallar todos los números complejos z tales que

i) $z^2 = -36$ ii) $z^2 = i$ iii) $z^2 = 7 + 24i$ iv) $z^2 + 15 - 8i = 0$

Llamo w a z

$z^2 = -36 \quad w^2 = z = -36 \quad w^2 = (x+yi)^2 = x^2 + 2xyi - y^2$

$= \underbrace{x^2 - y^2}_{Re(z)} + \underbrace{2xyi}_{Im(z)}$

$\left(x^2 - y^2 = -36 \right.$

$$\begin{cases} x^2 - y^2 = -36 \\ 2xy = 0 \end{cases}$$

TRUCO: Agrego tercera ecuación del módulo

$$|w|^2 = |w^2| = |z|$$

$$\left(\sqrt{x^2+y^2}\right)^2 = x^2+y^2 = \sqrt{(-36)^2+(0)^2} = 36$$

Luego,
$$\begin{cases} x^2 - y^2 = -36 \\ 2xy = 0 \\ x^2+y^2 = 36 \end{cases}$$

Por (3) $x^2 = 36 - y^2$

En (1) $(36-y^2) - y^2 = -36$

$$-2y^2 = -72$$

$$2y^2 = 72$$

$$y^2 = 36$$

$$\boxed{y = \pm 6}$$

Luego, en (3) $x^2 = 36 - (\pm 6)^2 \Rightarrow x^2 = 0$

Luego, las raíces son:

$w_0 = (0 + 6i)$ y $w_1 = (0 - 6i)$

3. Hallar todos los números complejos z tales que

i) $z^2 = -36$ ii) $z^2 = i$ iii) $z^2 = 7 + 24i$ iv) $z^2 + 15 - 8i = 0$

$z^2 = i$, Llamo z a w, $w^2 = i$, $w^2 = z = i$

$$w^2 = (x + yi)^2 = x^2 - y^2 + 2xyi$$

$$\begin{cases} x^2 - y^2 = 0 \\ 2xy = 1 \\ x^2 + y^2 = 1 \end{cases}$$

Por (3) $x^2 = 1 - y^2$

En (1) $(1 - y^2) - y^2 = 0 \Rightarrow 1 - 2y^2 = 0$

$$-2y^2 = -1$$

$$y^2 = \frac{1}{2}$$

$$y = \pm\sqrt{\frac{1}{2}}$$

En (3) $x^2 = 1 - \left(\sqrt{\frac{1}{2}}\right)^2$

$$x^2 = \frac{1}{2} \Rightarrow x = \pm\sqrt{\frac{1}{2}}$$

Luego, los pares son 4.

$w_0 = \left(\sqrt{\frac{1}{2}} + \sqrt{\frac{1}{2}}\, i\right)$ $w_1 = \left(\sqrt{\frac{1}{2}} - \sqrt{\frac{1}{2}}\, i\right)$

$$w_0 = \left(\sqrt{\tfrac{1}{2}} + \sqrt{\tfrac{1}{2}}i\right) \quad w_1 = \left(\sqrt{\tfrac{1}{2}} \ldots i\right)$$

$$w_2 = \left(-\sqrt{\tfrac{1}{2}} + \sqrt{\tfrac{1}{2}}i\right) \quad w_3 = \left(-\sqrt{\tfrac{1}{2}} - \sqrt{\tfrac{3}{2}}i\right)$$

NO fuera $2xy = 1$.

$$\sqrt{\tfrac{1}{2}} + \sqrt{\tfrac{1}{2}}i \Rightarrow 2\left(\sqrt{\tfrac{1}{2}}\right)\cdot\left(\sqrt{\tfrac{1}{2}}\right) = 1 \quad \checkmark$$

$$\sqrt{\tfrac{1}{2}} - \sqrt{\tfrac{1}{2}}i \Rightarrow 2\left(\sqrt{\tfrac{1}{2}}\right)\cdot\left(-\sqrt{\tfrac{1}{2}}\right) = -1 \quad \times$$

$$-\sqrt{\tfrac{1}{2}} + \sqrt{\tfrac{1}{2}}i \Rightarrow 2\left(-\sqrt{\tfrac{1}{2}}\right)\left(\sqrt{\tfrac{1}{2}}\right) = -1 \quad \times$$

$$-\sqrt{\tfrac{1}{2}} - \sqrt{\tfrac{1}{2}}i \Rightarrow 2\left(-\sqrt{\tfrac{1}{2}}\right)\left(-\sqrt{\tfrac{1}{2}}\right) = 1 \quad \checkmark$$

Luego, solo valen

$$\left(\sqrt{\tfrac{1}{2}} + \sqrt{\tfrac{1}{2}}i\right) \text{ y } \left(-\sqrt{\tfrac{1}{2}} - \sqrt{\tfrac{1}{2}}i\right)$$

c) $z^2 = w = 7 + 24i$

$$z^2 = (x+yi)^2 = x^2 + 2xyi + (yi)^2$$

$$= x^2 + 2xyi + y^2 i^2$$

$$= x^2 - y^2 + 2xyi$$

Luego, como sé que $|w^2| = |w|^2 = |z|$ → ¿por qué? [illegible] el cuadrado!

$$|w|^2 = \left(\sqrt{7^2 + 24^2}\right) = 25$$

Luego,
$$\begin{cases} x^2 - y^2 = 7 \\ 2xy = 24 \\ x^2 + y^2 = 25 \end{cases}$$

Por (3) y (1)

(3) $x^2 = 25 - y^2$

(1) $(25 - y^2) - y^2 = 7$

$$25 - 2y^2 = 7$$
$$-2y^2 = 7 - 25$$
$$-2y^2 = -18$$
$$2y^2 = 18$$
$$y^2 = 9$$
$$y = \pm 3$$

DEBERÍA PROBAR CON − 3 y + 3?

Θ lado

$x^2 = 25 - (3)^2$

Vuelvo a (3) $x^2 = 25 - (3)$

$$x^2 = 16$$

$$x = \pm 4$$

al final

cuales dos

¿soluciones valen?

CREERÍA solo con +3

y al final notaré.

(les cambió el

signo en los

cosos)

Ahora, luego (2)

$2xy = 24$

x POSIBLES: ± 4 ; y POSIBLES: ± 3

$x = +4 \wedge y = +3$ vale.

$x = -4 \wedge y = -3$ vale.

Luego, los complejos son:

$Z_0 = 4 + 3i \quad Z_1 = -4 - 3i$

iv) $z^2 + 15 - 8i = 0$

$$z^2 = -15 + 8i$$

$$|z^2| = |z| = \left(\sqrt{x^2+y^2}\right)^2 = x^2+y^2$$

$$|-15+8i| = \sqrt{(-15)^2+(8)^2} = 17$$

$$z^2 = (x+yi)^2 = x^2-y^2+2xyi$$

Ent,
$$\begin{cases} x^2-y^2 = 15 & -15 \\ 2xy = -8 & 8 \\ x^2+y^2 = 17 \end{cases}$$

*1

PRUEBA CON SIGNOS ORIGINALES

(3) $x^2 = 17 - y^2$

(1) $(17-y^2) - y^2 = 15$

$17 - 2y^2 = 15$

$-2y^2 = 15 - 17$

$-2y^2 = -2$

$y^2 = 1$

$y = \pm 1$

(3) $x^2 = 17 - (1)^2$

$x = \sqrt{16}$

$x = \pm 4$

(3) $x^2 = 17 - y^2$

(1) $(17 - y^2) - y^2 = -15$

$17 - 2y^2 = -15$

$-2y^2 = -15 - 17$

$-2y^2 = -32$

$y^2 = 16$

$y = \pm 4$

(3) $x^2 = 17 - (4)^2$

$x = \pm 1$

(2) $2xy = -8$ $\quad * 1$

$x = \pm 4,\ y = \pm 1$ (Me dio al revés los signos)

Así $x = \pm 1$, $y = \pm 4$

$2xy = 8$

$x = 4 \wedge y = 1$ sirve

$x = -4 \wedge y = -1$ sirve

Luego, $z_0 = 4 + i \wedge z_1 = -4 - i$

4. Calcular los módulos y los argumentos de los siguientes números complejos

i) $(2+2i)(\sqrt{3}-i)$ ii) $(-1+\sqrt{3}\,i)^5$ iii) $(-1+\sqrt{3}\,i)^{-5}$ iv) $\dfrac{1+\sqrt{3}\,i}{1-i}$.

i) $z = (\overbrace{2+2i}^{x})(\overbrace{\sqrt{3}-i}^{x'})$

$|x| = \sqrt{(2)^2 + (2)^2} = \sqrt{8}$

$\arg(x) = \arctan(1) = 0.78\,\text{RAD} = \frac{\pi}{4}$

1 está en el argumento principal dado que está entre $[0, 2\pi]$

$$|x'| = \sqrt{(\sqrt{3})^2 + (-1)^2} = \sqrt{3+1} = 2$$

$$\arctan\left(\frac{1}{\sqrt{3}}\right) = 0.52 \text{ RAD.}$$

Este No es el arg principal para el complejo x'

está en el 4TO CUADRANTE.

Entonces, $\arg(x') = 2\pi - \overbrace{0.52 \text{ RAD}}^{\frac{\pi}{6}} = 2\pi - \frac{\pi}{6}$

$$= \frac{12\pi - \pi}{6} = \frac{11\pi}{6}$$

y este es el argumento principal pues está entre $[0, 2\pi]$

Luego, paso a polar x y x'.

$$x = \sqrt{8}\left(\cos\left(\frac{\pi}{4}\right) + i \operatorname{sen}\left(\frac{\pi}{4}\right)\right)$$

$$x' = 2\left(\cos\left(\frac{11\pi}{6}\right) + i \operatorname{sen}\left(\frac{11\pi}{6}\right)\right)$$

Luego, por multiplicación de complejos

$$x \cdot x' = 2\sqrt{8}\left(\cos\left(\frac{\pi}{4} + \frac{11\pi}{6}\right) + i \operatorname{sen}\left(\frac{\pi}{4} + \frac{11\pi}{6}\right)\right)$$

$$= 2\sqrt{8}\left(\cos\left(\frac{25\pi}{12}\right) + i \operatorname{sen}\left(\frac{25\pi}{12}\right)\right)$$

Como $\frac{25\pi}{12} > 2\pi \Rightarrow \arg(x \cdot x') = \frac{25\pi}{12} - 2\pi$

$= 0.26\text{RAD}$

$\|$

$\frac{\pi}{12}$

Luego, el complejo z es: $\underbrace{2\sqrt{8}}_{|z|}\left(\cos\left(\underbrace{\frac{\pi}{12}}_{\arg(z)}\right) + i\,\text{sen}\left(\frac{\pi}{12}\right)\right)$

ii) $z = (\underbrace{-1+\sqrt{3}i}_{x})^5 = x^5$

$$|x| = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2 \checkmark$$

$$\arg(x) = \arctan\left(\sqrt{3}\right) = 1.047\text{RAD} = \frac{\pi}{3}$$

Pero como x está en el 2DO CUADRANTE:

$$\arg(x) = \pi - \frac{\pi}{3} = \frac{3\pi - \pi}{3} = \frac{2\pi}{3} \checkmark$$

Luego, por Th De MOIVRE: MULT: $|z|^n(\cos(\theta \cdot n) \cdots)$

$$x^5 = 2^5\left(\cos\left(\frac{2\pi}{3} \cdot 5\right) + i\,\text{sen}\left(\frac{2\pi}{3} \cdot 5\right)\right)$$

$$= 2^5\left(\cos\left(\frac{10\pi}{3}\right) + i\,\text{sen}\left(\frac{10\pi}{3}\right)\right)$$

Luego, como $\frac{10\pi}{3} > 2\pi \Rightarrow \arg(x) = \frac{10\pi}{3} - 2\pi$

$= \frac{10\pi - 6\pi}{\ldots}$

$$3$$

$$= \frac{4\pi}{3}$$

$$\text{Entonces, } z = \underbrace{2^5}_{|z|} \left(\cos\left(\underbrace{\frac{4\pi}{3}}_{\arg(z)}\right) + i\,\text{sen}\left(\frac{4\pi}{3}\right)\right)$$

$$(z^{-1})^5 = \frac{\bar{z}}{|z|^2} \text{ si } z \neq 0$$

iii) $z = \left(-1 + \sqrt{3}\,i\right)^{-5}$

$$|z| = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$$

$$|z|^{-5} \text{ y } (\cos(\theta \cdot -5))$$

$$\arg(z) = \arctan\left(\sqrt{3}\right) = \frac{\pi}{3}$$

Luego, $\arg(z) = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

$$\text{Ent, } z = 2^{-5}\left(\cos\left(\frac{2\pi}{3} \cdot -5\right) + i\,\text{sen}\left(\frac{2\pi}{3} \cdot -5\right)\right)$$

$$= \frac{1}{32}\left(\cos\left(-\frac{10\pi}{3}\right)\right) \ldots = \underbrace{\frac{1}{32}}_{\text{módulo}}\left(\cos\left(\underbrace{\frac{2\pi}{3}}_{\arg}\right) + i\,\text{sen}\left(\frac{2\pi}{3}\right)\right)$$

iv) $z = \dfrac{\overbrace{1 + \sqrt{3}\,i}^{x}}{\underbrace{1 - i}_{x'}}$ De moivre sería mejor y divido módulos

$$|x| = \sqrt{(1)^2 + (\sqrt{3})^2} = 2$$

$$\arg(x) = \arctan\left(\sqrt{3}\right) = \frac{\pi}{3}$$

y Como x está en 1er CUADRANTE y $\frac{\pi}{3} < 2\pi$

es el arg principal

$|x'| = \sqrt{(1)^2+(-1)^2} = \sqrt{2}$

$Arg(x') = arctan(1) = \frac{\pi}{4}$

Pero como 1-i está en el 4º CUAD

$\Rightarrow 2\pi - \frac{\pi}{4} = \frac{8\pi - \pi}{4} = \frac{7\pi}{4}$

Luego, está en el argumento.

Así, $z = \frac{2}{\sqrt{2}}\left(\cos\left(\frac{\pi}{3} - \frac{7\pi}{4}\right) + i\,sen\left(\frac{\pi}{3} - \frac{7\pi}{4}\right)\right)$

$\frac{\pi}{3} - \frac{7\pi}{4} = \frac{4\pi - 3.7\pi}{12} = -\frac{17\pi}{12}$ pero

Como está es menor a 0, le sumo

$2k\pi$.

$Arg(z) = -\frac{17\pi}{12} + 2\pi = \frac{-17\pi + 24\pi}{12}$

$= \frac{7\pi}{}$

$= \frac{7\pi}{12}$

Ent, $Z = \frac{2}{\sqrt{2}} \left(\cos\left(\frac{7\pi}{12}\right) + i\,\text{sen}\left(\frac{7\pi}{12}\right) \right)$ ✓

SI ME PIDEN MOD y ARG uso FORMA TRIGONOM.

6. i) Determinar la forma binomial de $\left(\frac{1+\sqrt{3}\,i}{1-i}\right)^{17}$.

ii) Determinar la forma binomial de $(-1+\sqrt{3}\,i)^n$ para cada $n \in \mathbb{N}$.

i) $Z = \left(\frac{\overbrace{1+\sqrt{3}i}^{x}}{\underbrace{1-i}_{x'}} \right)^{17}$ => DIVIDO MÓDULOS y RESTO ÁNGULOS

Paso a polar

- $|x| = \sqrt{(1)^2 + (\sqrt{3})^2} = 2$ • $\arg(x) = \arctan\left|\sqrt{3}\right| = \frac{\pi}{3}$

Luego, como está en primer cuadrante es el

Arg principal de X.

$\cdot\,|x'| = \sqrt{(1)^2+(-1)^2} = \sqrt{2}$ $\quad \cdot\,\arg(x) = \arctan|1| = \frac{\pi}{4}$

Luego, el $\arg|x'| = 2\pi - \frac{\pi}{4} = \frac{8\pi-\pi}{4} = \frac{7\pi}{4}$

¿Van en algún orden? IMPORTA. ARRIBA IZQ, ABAJO DER

$$\text{Ent}, \quad \frac{X}{X'} = \underbrace{\frac{2}{\sqrt{2}}}_{}\overset{\sqrt{2}}{} \left(\cos\left(\frac{\pi}{3}-\frac{7\pi}{4}\right) + i\,\text{sen}\left(\frac{\pi}{3}-\frac{7\pi}{4}\right)\right)$$

No es el arg principal, pues es menor a 0

$$\text{Arg}\left(\frac{X}{X'}\right) = \frac{\pi}{3} - \frac{7\pi}{4} = \frac{4\pi-3.7\pi}{12} = -\frac{17\pi}{12} + 2\pi = \frac{-17\pi+12.2\pi}{12} = \frac{7\pi}{12} \;\; \arg\left(\frac{x}{x'}\right)$$

Luego, si quiero a la 17...

$$\Rightarrow \left(\sqrt{2}\right)^{17}\left(\cos\left(\frac{7\pi}{12}.17\right) + i\,\text{sen}\left(\frac{7\pi}{12}.17\right)\right)$$

Pero $\frac{7\pi}{12}.17 > 2\pi$, le resto $2k\pi$.

$$k=4 \Rightarrow \left(\frac{7\pi}{12}.17\right) - 8\pi = \frac{119\pi}{12} - 8\pi = \frac{119\pi-12.8\pi}{12} = \frac{23\pi}{12}$$

$$\text{Entonces}, \quad \left(\sqrt{2}\right)^{17}\left(\cos\left(\frac{23\pi}{12}\right) + i\,\text{sen}\left(\frac{23\pi}{12}\right)\right)$$

ii) $Z = (-1+\sqrt{3}i)^M$ ¿Qué valores de n puede tomar?

Supongo que la n es un número

$|z| = \sqrt{\;\;}$

• $|z| = \sqrt{(-1)^2 + (\sqrt{3})^2} = 2$

• $\arg(z) = \arctan\left(\sqrt{3}\right) = \frac{\pi}{3}$

Pero como está en el 2DO CUADRANTE

$$\pi - \frac{\pi}{3} = \frac{3\pi - \pi}{3} = \frac{2\pi}{3}$$

Luego,

$$z^n \Rightarrow 2^n \cdot \left(\cos\left(\frac{2\pi}{3} \cdot n\right) + i \operatorname{sen}\left(\frac{2\pi}{3} \cdot n\right)\right)$$

Donde, n para no pasarse de 2π, puede ser:

$\rightarrow$ MAX, $n = 3$ ya $\arg(z) = 2\pi$ luego n debe ser menor

$n = 1$, $n = 2$, $n = 3$

para el arg debe ser $0 \leq \frac{2\pi}{3} \cdot n < 2\pi$

Luego, $n = 0 \lor n = 1 \lor n = 2$

si $n = 0 \Rightarrow 1$

si $n = 1 \Rightarrow 2\left(\cos\left(\frac{2\pi}{3}\right) + i \operatorname{sen}\left(\frac{2\pi}{3}\right)\right)$

si $n = 2 \Rightarrow \frac{2\pi}{3} \cdot 2 = \frac{4\pi}{3} \Rightarrow 4\left(\cos\left(\frac{4\pi}{3}\right) + i \operatorname{sen}\left(\frac{4\pi}{3}\right)\right)$

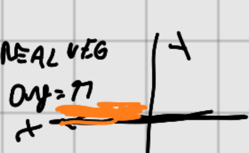
7. Hallar todos los $n \in \mathbb{N}$ tales que

i) $(\sqrt{3} - i)^n = 2^{n-1}(-1 + \sqrt{3}\,i)$.

No tiene parte Im y REAL NEG

Es decir $\arg = \pi$

ii) $(-\sqrt{3} + i)^n \left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$ es un número real negativo.



iii) $\arg\left((-1 + i)^{2n}\right) = \frac{\pi}{2}$ y $\arg\left((1 - \sqrt{3}i)^{n-1}\right) = \frac{2}{3}\pi$.

$\rightarrow 2^n$

i) $(\sqrt{3}-i)^m = 2^{m-1}(-1+\sqrt{3}\,i)$

$$2(\sqrt{3}-i)^m \overset{\cdot 2}{=} 2^m(-1+\sqrt{3}\,i)$$

$$\frac{(\sqrt{3}-i)^m}{2^m} \Rightarrow \left(\frac{\sqrt{3}-i}{2}\right)^m \overset{/2}{=} \underbrace{\left(\frac{-1+\sqrt{3}\,i}{2}\right)}_{x'}$$

Ahora, para que dos complejos sean iguales:

Módulo deben ser iguales

Arg principal debe ser el mismo.

$$|x| = \sqrt{\left(\frac{\sqrt{3}}{2}\right)^2 + \left(-\frac{1}{2}\right)^2} = \sqrt{\frac{3}{2}+\frac{1}{4}} = \sqrt{\frac{7}{4}}$$

$\arg(x) = \arctan\left(\frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}}\right) \approx 0.38 \text{ RAD} = \frac{\pi}{6}$ => no es arg principal

Luego, $2\pi - \frac{\pi}{6} = \frac{12\pi-\pi}{6} = \frac{11}{6}\pi$ => arg principal

$$|x'| = \sqrt{\left(-\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} = \sqrt{\frac{7}{4}}$$

-> NO FRACCIÓN PN=6.

Dividido

$\arg(x') = \arctan\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right) \approx \frac{\pi}{3} = \pi - \frac{\pi}{3} = \frac{2}{3}\pi$

Luego, $\left(\sqrt{\frac{7}{4}}\right)^m \cdot e^{i\frac{11\pi m}{6}} = \left(\sqrt{\frac{7}{4}}\right) \cdot e^{i\frac{2}{3}\pi}$

$x = x' \iff \begin{cases} |x| = |x|' \iff \sqrt{\frac{7}{4}} = \sqrt{\frac{7}{4}} \\ \arg(x) = \arg(x') + 2k\pi \text{ para } k \in \mathbb{Z} \end{cases}$

$$\frac{11\pi m}{6} = \frac{2}{3}\pi + 2k\pi$$

(DA IGUAL Q ARG VA IZQ O DER)

$$11\pi m = 6\left(2k\pi + \frac{2}{3}\pi\right)$$

$$11m \overset{/\pi}{=} 12k + 4$$

$$11m \equiv 4(12)$$

$$-m \equiv 4(12)$$

$$m \equiv 8(12) \Rightarrow m = 12k + 8$$

Luego, $x = x' \Leftrightarrow m \equiv 8(12)$

La idea es dejar la k con los números y del otro lado la m.

Lo repaso.

$$(\sqrt{3} - i)^m = 2^{m-1}(-1 + \sqrt{3}\, i)$$

Debo llevar esto a algo parecido.

El 2 debe estar en ambos lados

$$2(\sqrt{3} - i)^m = 2^m(-1 + \sqrt{3}\, i)$$

no tiene parte imaginaria

Con>0.

ii) $(-\sqrt{3}+i)^n \cdot \left(\frac{1}{2}+\frac{\sqrt{3}}{2}i\right)$ es un número real negativo

$z \cdot w$ los arg se suman *

$$\arg\Big(\underbrace{(-\sqrt{3}+i)}_{z}^{n}\cdot\underbrace{\left(\tfrac{1}{2}+\tfrac{\sqrt{3}}{2}i\right)}_{w}\Big)=\pi$$

$$\arg(z)=\arctan\left(\frac{1}{\sqrt{3}}\right)=\frac{\pi}{6}\implies$$

$$\therefore \arg(z)=\pi-\frac{\pi}{6}=\frac{6\pi-\pi}{6}=\frac{5\pi}{6}$$

$$\arg(w)=\arctan\left(\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}\right)=\arctan\left(\frac{2\sqrt{3}}{2}\right)=$$

$$\therefore \arg(w)=\frac{\pi}{3}$$

PROP

$$\arg(z^n) \; * + \arg(w)=\pi+2k\pi$$

$$n\cdot\arg(z)+\arg(w)=\pi+2k\pi$$

$$n\cdot\frac{5\pi}{6}+\frac{\pi}{3}=\pi+2k\pi$$

$$n\cdot\frac{5\pi}{6}=\pi-\frac{\pi}{3}+2k\pi$$

$$n\cdot\frac{5\pi}{6}=\frac{3\pi-\pi}{3}+2k\pi$$

$$M \cdot \frac{5\pi}{6} = \frac{2\pi}{3} + 2k\pi$$

$/\pi$

$$M \cdot \frac{5}{6} = \frac{2}{3} + 2K$$

$$M = \frac{\frac{2}{3} + 2K}{\frac{5}{6}}$$

$$M = \frac{6\left(\frac{2}{3} + 2K\right)}{5}$$

$$M = \frac{4 + 12K}{5}$$

Como m es un natural, $\frac{4+12K}{5}$ debe ser natural

$$5 \mid 4 + 12K \iff 4 + 12K \equiv 0 \ (5)$$

$$\iff 2K \equiv -4 \ (5) \overset{\cdot 3}{\iff} K \equiv 3 \ (5)$$

$$\Rightarrow K = 5q + 3$$

$$M = \frac{4 + 12(5q+3)}{5} \Rightarrow m = \frac{4 + 60q + 36}{5} = \frac{40 + 60q}{5}$$

$$= 8 + 12q$$

$$m \equiv 8 \ (12)$$

7. Hallar todos los $n \in \mathbb{N}$ tales que

i) $(\sqrt{3} - i)^n = 2^{n-1}(-1 + \sqrt{3}\, i)$.

ii) $(-\sqrt{3} + i)^n \left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$ es un número real negativo.

iii) $\arg\left((-1+i)^{2n}\right) = \frac{\pi}{2}$ y $\arg\left((1 - \sqrt{3}i)^{n-1}\right) = \frac{2}{3}\pi$.

Hallar el $n \in$ cuales:

iii) $\arg\left((-1+i)^{2n}\right) = \frac{\pi}{2}$ y $\arg\left((1-\sqrt{3}i)^{n-1}\right) = \frac{2}{3}\pi$

$\arg(-1+i) = \arctan\left(1\right) = \frac{\pi}{4}$, 2do cuad $\Rightarrow \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

luego, $\arg(-1+i) = \frac{3\pi}{4}$ pero $\arg(-1+i)^{2n} = \frac{3\pi}{4} \cdot 2n$

① $\frac{3\pi}{4} \cdot 2n = \frac{\pi}{2}$

$\arg(1-\sqrt{3}i) = \arctan\left(\sqrt{3}\right) = \frac{\pi}{3}$, 4to cuad $\Rightarrow 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$

luego, $\arg(1-\sqrt{3}i) = \frac{5\pi}{3}$ pero $\arg(1-\sqrt{3}i)^{n-1} = \frac{5\pi}{3} \cdot n-1$

② $\frac{5\pi}{3} \cdot (n-1) = \frac{2}{3}\pi$

① $\frac{3\pi}{\cancel{4}_2} \cdot \cancel{2}n = \frac{\pi}{2}$ $\quad /\pi \Rightarrow \frac{3}{2} \cdot n = \frac{1}{2} \overset{\cdot 2}{\Rightarrow} 3n = 1$

$\Rightarrow n = \frac{1}{3}$ ABS. $n \notin \mathbb{N}$.

①∧② $\frac{5\pi}{3} \cdot (n-1) - \frac{2}{3}\pi = \left(\frac{3}{4}\pi \cdot 2n\right) - \frac{\pi}{2}$

$\frac{5\pi}{3}n - \frac{5\pi}{3} - \frac{2}{3}\pi = \frac{6}{4}\pi n - \frac{\pi}{2}$

$\frac{5\pi}{3}n - \frac{6}{4}\pi n = -\frac{\pi}{2} + \frac{7\pi}{3}$

$\frac{20\pi n - 18\pi n}{12} = \frac{-3\pi + 14\pi}{6}$

$\frac{2\pi n}{12} = \frac{11\pi}{6}$

$\frac{5\pi}{3} \cdot (n-1) = \frac{2}{3}\pi + 2k\pi$

$\frac{5\pi}{3}n - \frac{5\pi}{3} = \frac{2}{3}\pi + 2k\pi$

$\frac{5\pi}{3}n - \pi = 2k\pi$

$\frac{5n}{3} - 1 = 2k$

$$\frac{2m}{12} = \frac{11}{6}$$

$$2m = \left(\frac{11}{6}\right).12$$

$$2m = 22$$

$$\boxed{m = 11}$$

VERIF: $\frac{3\pi}{4} \cdot 2(11) = \frac{3}{4}\pi.22 \cong 51.8 \text{RAD} - 16\pi = \frac{\pi}{2}$ ✓

$\frac{5\pi}{3} \cdot (11-1) \cong 52.35 \text{RAD} - 16\pi = \frac{2\pi}{3}$ ✓

NO. Me piden todos los m

$$5m - 1 = 6k$$

$$\boxed{5m-1 \equiv ?(6) \iff -m \equiv 1(6) \iff m \equiv 5(6)} \; *$$

$$\frac{3\pi}{4} \cdot 2n = \frac{\pi}{2} + 2k\pi$$

$$\frac{3}{2}\pi \cdot m = \frac{\pi}{2} + 2k\pi$$

$$\frac{3}{2}m - \frac{1}{2} = 2k$$

$$3m - 1 = 4k$$

$$3m \equiv 1(4)$$

$$\Updownarrow$$

$$-m \equiv 1(4) \iff \boxed{m \equiv 3(4)} \; *$$

$$* \begin{cases} m \equiv 5(6) \\ m \equiv 3(4) \end{cases} \iff \begin{cases} m \equiv 5(3) \\ m \equiv 5(2) \\ m \equiv 3(4) \end{cases}$$

$$\iff \begin{cases} m \equiv 2(3) \\ m \equiv 1(2) \\ m \equiv 3(4) \end{cases} \Rightarrow 3 \equiv 1(2)?\ 1 \equiv 1(2) \Rightarrow \begin{cases} m \equiv 2(3) \\ m \equiv 3(4) \end{cases}$$

Por el TCR, como 4 y 3 son coprimos dos a dos $\exists$ solución es única y en módulo 12.

(S_1) $4y_1 \equiv 2(3) \iff y_1 \equiv 2(3) \Rightarrow x_1 = 8$

(S_2) $3y_2 \equiv 3(4) \iff -y_2 \equiv 3(4) \iff y_2 \equiv 1(4)$

$\Rightarrow x_2 = 3$

Luego, $m \equiv 11(12)$ o $m \equiv 5(6)$ $\quad m \equiv 5(6) \subseteq m \equiv 11(12)$

8. Hallar en cada caso las raíces n-ésimas de $z \in \mathbb{C}$:

i) $z = 8,\ n = 6$

ii) $z = -4,\ n = 3$

iii) $z = -1 + i,\ n = 7$

iv) $z = (2 - 2i)^{12},\ n = 6.$

$z^6 = 8$ $\qquad$ $z^m = r^m \left(\cos(m\alpha) + i \operatorname{sen}(m\alpha) \right)$

$$\begin{cases} \rho = \sqrt[m]{r} \\ \theta = \frac{\alpha}{m} + \frac{2\pi k}{m} \end{cases}$$

$|z| = 8$, $\rho = \sqrt[6]{8}$

$\operatorname{Arg}(z) = 0.$

$$\begin{cases} \rho = \sqrt[6]{8} \\ \theta = 0 + \frac{2k\pi}{6} \end{cases}$$

Busco los k tales que $\theta < 2\pi$ (max 6)

Luego, $0 \leq k < 6$

$$z_0 = \sqrt[6]{8} \cdot \left(\cos(0) + i \operatorname{sen}(0) \right)$$

$$z_1 = \sqrt[6]{8} \left(\cos\left(\frac{\pi}{3}\right) + i \operatorname{sen}\left(\frac{\pi}{3}\right) \right)$$

$$z_2 = \sqrt[6]{8} \left(\cos\left(\frac{2\pi}{3}\right) + i \operatorname{sen}\left(\frac{2\pi}{3}\right) \right)$$

$$z_3 = \sqrt[6]{8} \left(\cos(\pi) + i \operatorname{sen}(\pi) \right)$$

$$z_4 = \sqrt[6]{8} \left(\cos\left(\frac{4\pi}{3}\right) + i \operatorname{sen}\left(\frac{4\pi}{3}\right) \right)$$

$$z_5 = \sqrt[6]{8} \left(\cos\left(\frac{5\pi}{3}\right) + i \operatorname{sen}\left(\frac{5\pi}{3}\right) \right)$$

8. Hallar en cada caso las raíces n-ésimas de $z \in \mathbb{C}$:

i) $z = 8,\ n = 6$

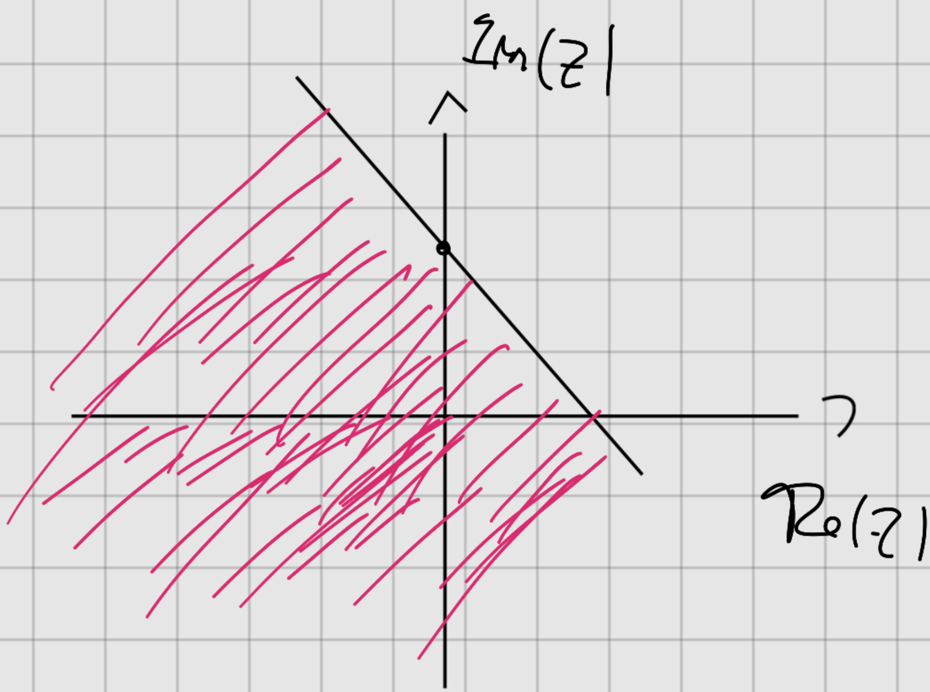
ii) $z = -4,\ n = 3$

iii) $z = -1 + i,\ n = 7$

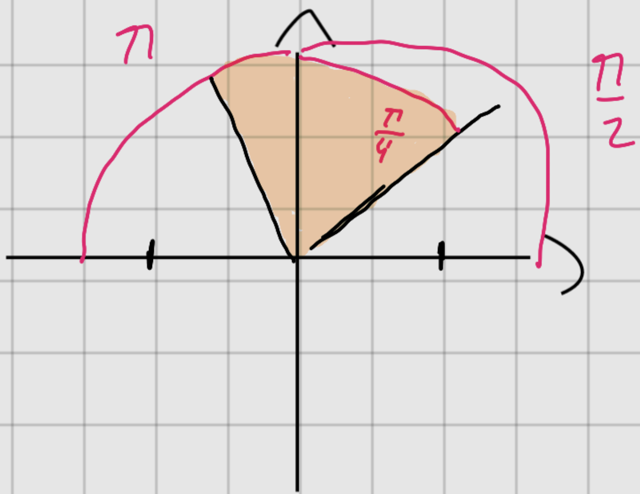
iv) $z = (2-2i)^{12},\ n = 6.$

i) $a + 5b = 8$

$$b = \frac{8-a}{5}$$

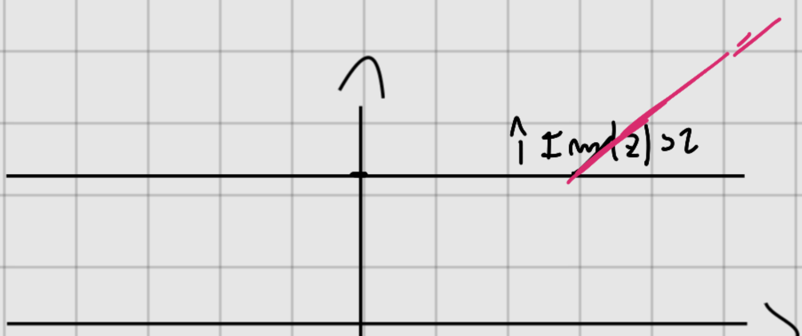


ii)



$z = a+bi \quad \therefore i = -ai - b$

iii)



5. Graficar en el plano complejo

i) $\{z \in \mathbb{C} \,/\, \mathrm{Re}(z) + 5\,\mathrm{Im}(z) \leq 8\}$.

ii) $\{z \in \mathbb{C} - \{0\} \,/\, |z| \geq 2 \text{ y } \frac{\pi}{4} \leq \arg(z) \leq \frac{2\pi}{3}\}$.

iii) $\{z \in \mathbb{C} - \{0\} \,/\, \mathrm{Im}(z) > 2 \text{ y } \arg(-i\,z) = \frac{\pi}{4}\}$.

iv) $\{z \in \mathbb{C} - \{0\} \,/\, \arg(z^4) = \arg\left((-1+i)\overline{z}^2\right)\}$.

ii) $z^3 = -4$

$|z| = 4$

$\mathrm{Arg}(-4) = \pi \Rightarrow \alpha = \pi$

Luego,

$$\begin{cases} r = \sqrt[3]{4} \\ \theta = \dfrac{\alpha + 2k\pi}{n} \end{cases}$$

Luego, pueden existir hasta z_2 soluciones

$z_0 = \sqrt[3]{4}\left(\cos\left(\frac{\pi}{3}\right) + i\,\mathrm{sen}\left(\frac{\pi}{3}\right)\right)$

$$z_1 = \sqrt[3]{4}\left(\cos(\pi) + i\,\text{sen}(\pi)\right)$$

$$z_2 = \sqrt[3]{4}\left(\cos\left(\frac{5\pi}{3}\right) + i\,\text{sen}\left(\frac{5\pi}{3}\right)\right)$$

Luego, listo con las raíces n-ésimas

$$z^7 = -1 + i$$

7 RAICES. $K \in \{0, 1, \dots, 6\}$

$$|z| = \sqrt{(1)^2 + (1)^2} = \sqrt{2}$$

PRIMER CUAD

$$\text{Arg}(z) = \left(1\right) = \frac{\pi}{4} \Rightarrow \pi - \frac{\pi}{4} = \frac{3\pi}{4} \rightarrow \alpha = \frac{3\pi}{4}$$

$$\begin{cases} r = \sqrt[7]{\sqrt{2}} = \sqrt[14]{2} \\ \theta = \frac{3\pi}{28} + \frac{2K\pi}{7} \end{cases}$$

$$z_0 = \sqrt[14]{2} \cdot \left(\cos\left(\frac{3\pi}{28}\right) + i\,\text{sen}\left(\frac{3\pi}{28}\right)\right)$$

$$z_1 = \sqrt[14]{2} \cdot \left(\cos\left(\frac{11}{28}\pi\right) + i\,\text{sen}\left(\frac{11\pi}{28}\right)\right)$$

En exponencial xq me cansé de polar.

$$z_2 = \sqrt[14]{2} \cdot e^{\frac{19}{28}\pi i} \qquad z_5 = \sqrt[14]{2} \cdot e^{\frac{43}{28}\pi i}$$

$$z_3 = \sqrt[14]{2} \cdot e^{\frac{27}{28}\pi i} \qquad z_6 = \sqrt[14]{2} \cdot e^{\frac{51}{28}\pi i}$$

$$z_4 = \sqrt[14]{2} \cdot e^{\frac{5}{4}\pi i}$$

iv) $z = (2-2i)^{12}$, $n=6$

$z^6 = (2-2i)^{12}$

$|z| = \sqrt{(2)^2+(-2)^2} = \sqrt{8}$

$\text{Arg}(z) = \text{arctan}(1) = \frac{\pi}{4} \Rightarrow 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

Luego, $\underbrace{(\cancel{\sqrt{8}})^{\cancel{12}}}_{8^6} \cdot \left(\cos\left(\frac{7\pi}{4} \cdot 12\right) + i\,\text{sen}\left(\frac{7\pi}{4} \cdot 12\right)\right)$

$8^6 \cdot (\cos(\underbrace{21\pi}_{\pi}) + i\,\text{sen}(\underbrace{21\pi}_{\pi}))$

Entonces, $z^6 = 8^6(\cos(\pi) + i\,\text{sen}(\pi))$

$$\begin{cases} r = \sqrt[6]{8^6} \\ \theta = \frac{\pi}{6} + \frac{2k\pi}{6} \end{cases}$$

$z_0 = 8 \cdot e^{\frac{\pi}{6}i}$ $\quad z_1 = 8 \cdot e^{\frac{\pi}{2}i}$

$z_2 = 8 \cdot e^{\frac{5}{6}\pi i}$ $\quad z_3 = 8 \cdot e^{\frac{7\pi}{6}i}$

$z_4 = 8 \cdot e^{\frac{3}{2}\pi i}$ $\quad z_5 = 8 \cdot e^{\frac{11\pi}{6}i}$

11. i) Calcular $w + \overline{w} + (w + w^2)^2 - w^{38}(1 - w^2)$ para cada $w \in G_7$.

ii) Calcular $w^{73} + \overline{w} \cdot w^9 + 8$ para cada $w \in G_3$.

iii) Calcular $1 + w^2 + w^{-2} + w^4 + w^{-4}$ para cada $w \in G_{10}$.

iv) Calcular $w^{14} + w^{-8} + \overline{w}^4 + \overline{w^{-3}}$ para cada $w \in G_5$.

w^{-1}

i) $w + \overline{w} + w^2 + 2w^3 + w^4 - w^{38} + w^{40}$ en G_7

$= w + w^6 + w^2 + 2w^3 + w^4 - w^3 + w^5$

$= w + w^2 + w^3 + w^4 + w^5 + w^6$

(Literalmente es para usar a sumatoria.

$$\sum_{m}^{k=0} w^k = \begin{cases} \frac{w^{m+1}-1}{w-1} & w \neq 1 \\ m+1 & w=1 \end{cases}$$

$$\sum_{K=1}^{6} w^K = \left(\sum_{K=0}^{6} w^K\right) - \underbrace{w^0}_{1} = \begin{cases} \frac{w^7-1}{w-1} - 1 = 0 & w \neq 1 \\ \underbrace{7}_{m+1} - 1 = 6 & w = 1 \end{cases}$$

se le resta el término q agrego.

Ej: $\sum_{K=1}^{6} w^K = w^1 + w^2 + w^3 \ldots$

$\sum_{K=0}^{6} w^0 + w^1 + w^2 + w^3$ ≠ por tengo w^0. Asi lo resto. La idea es que $\sum_{K=0} = \sum_{K=1}$

Cuando muevo hacia otros, resto los K que agregué

SUMA RAÍCES PRIMITIVAS DA 0, RAÍCES DA 1

ii) $w^{73} + \overline{w} \cdot w^9 + 8$ para cada $w \in G_3$

$\Rightarrow w + w^{-1} \cdot w^9 + 8$

$\Rightarrow w + w^2 \cdot \underbrace{w^9}_{w^3} + 8$

$\Rightarrow w + w^2 \cdot 1 + 8$

$\Rightarrow w^1 + w^2 + 8$

$$\sum_{K=1}^{2} w^K = \left(\sum_{K=0}^{2} w^K\right) - \overset{1}{w^0} = \begin{cases} \frac{w^3-1}{w-1} & \text{si } w \neq 1 \\ 2+1 & \text{si } w = 1 \end{cases}$$

$$= \begin{cases} 0 - 1 + 8 = 7 & \text{si } w \neq 1 \\ 3 - 1 + 8 = 10 & \text{si } w = 1 \end{cases}$$

iii) $1 + w^2 + w^8 + w^4 + w^6$ para cada $w \in G_{10} \Rightarrow$ tengo solo exp pares.

$$\sum_{k=0}^{4} \underbrace{(\omega^2)^k}_{\in G_5} = \begin{cases} \frac{(\omega^2)^5 - 1}{\omega^2 - 1} = 0 \checkmark & \omega^2 \neq 1 \\ 4+1 = 5 \checkmark & \omega^2 = 1 \end{cases}$$

$$\sum_{k=0}^{4} \tilde{\omega}^k = \begin{cases} \frac{\tilde{\omega}^5 - 1}{\tilde{\omega} - 1} = 0 & \tilde{\omega} \neq 1 \\ 5 & \tilde{\omega} = 1 \end{cases}$$

$\tilde{\omega} \in G_5$

$\omega \in G_{10}$

$\omega^{10} = 1$

$(\underbrace{\omega^2}_{\in G_5})^5 = \omega^{10} = 1$

iv) $w^4 + w^2 + \overline{w}^4 + \overline{w^{-3}}$ para $w \in G_5$

$$= w^4 + w^2 + (w^{-1})^4 + (w^{-3})^{-1}$$
$$= w + w^2 + w^3 + w^4$$

$$\sum_{k=1}^{4} w^k = \left(\sum_{k=0}^{4} w^k\right) - w^0 = \begin{cases} \frac{\overset{0}{w^5 - 1}}{w-1} & \text{si } w \neq 1 \\ 4+1 & \text{si } w = 1 \end{cases}$$

$\Downarrow$

$$\begin{cases} 0 - 1 = -1 \text{ si } w \neq 1 \checkmark \\ 5 - 1 = 4 \text{ si } w = 1 \checkmark \end{cases}$$

FALTAN: 10 y 12

$\to$ 5 soluciones

9. Hallar todos los $z \in \mathbb{C}$ tales que $3z^5 + 2|z|^5 + 32 = 0$.

$z = a + bi$ $\arg(z)$ está en $[0, 2\pi)$

$$3z^5 = -(\underbrace{2|z|^5 + 32}_{\in \mathbb{R}_{>0}})$$

$\arg(3z^5) = \pi$

$|z|^5 = |z^5|$

$$3z^5 + 2|z^5| + 32 = 0$$

$$w = z^5$$

$$3w + 2|w| + 32 = 0$$

$$\boxed{w = a + bi} \qquad |w| = \sqrt{a^2 + b^2}$$

$$3(a + bi) + 2\sqrt{a^2 + b^2} + 32 = 0$$

$$\underbrace{3a + 2\sqrt{a^2+b^2} + 32}_{Re(z)} + \underbrace{3bi}_{Im(z)} = 0 \qquad b = 0$$

$$w \qquad w = re^{2\pi i\theta}$$

$$\boxed{z^5 = w} \qquad z^5 = re^{2\pi i\theta}$$

$$z^5 = w$$

$$3a + 2\sqrt{a^2} + 32 = 0$$

Vemos que z NO tiene parte imaginaria, osea está sobre el eje x y en negativo.

$$3a + 2a + 32 = 0$$

$$5a + 32 = 0$$

$$\mathrm{Arg}(z) = \pi$$

$$5a = -32$$

$$a = -32/5$$

$$z^5 = -\frac{32}{5} + 0i$$

Tengo que buscar las raíces n-ésimas.

Como $n=5$, ent., $0 \le k < 5$ es decir $k \in [0,4]$

Luego,

$$|z| = \sqrt{\left(-\frac{32}{5}\right)^2} = \frac{32}{5}$$

$$\begin{cases} r^5 = \frac{32}{5} \Rightarrow |z| = \sqrt[5]{\frac{32}{5}} \\ \\ \theta = \frac{\pi + 2k\pi}{5} \end{cases}$$

$k=0$, $z_0 = \sqrt[5]{\frac{32}{5}} \cdot e^{\frac{\pi}{5} i}$

$z_1 = \sqrt[5]{\frac{32}{5}} \cdot e^{\frac{3\pi}{5} i}$

$z_2 = \sqrt[5]{\frac{32}{5}} \cdot e^{\pi i}$

$z_3 = \sqrt[5]{\frac{32}{5}} \cdot e^{\frac{7\pi}{5} i}$

$z_4 = \sqrt[5]{\frac{32}{5}} \cdot e^{\frac{9\pi}{5} i}$

PREGUNTAR: si Z era un Complejo REAL

aparezca sería complejo, no? claro sí, pero pregunto xq las raíces reales me dan (la MAYORÍA) sería (MAG; ..

VERIF:

$$z_3 : \sqrt[5]{\frac{32}{5}}\left(\cos\left(\frac{7\pi}{5}\right) + i\,\text{sen}\left(\frac{7\pi}{5}\right)\right)$$

$\sqrt[m]{\ell}$ en CALC: $m\sqrt[x]{\ell}$

A BINÓMICA: $\frac{2}{5}\left(\cos\left(\frac{7\pi}{5}\right) + i\,\text{sen}\left(\frac{7\pi}{5}\right)\right)$

$= -0.12 - 0.95i$

$$3z^5 + 2\underbrace{|z^5|}_{|z|^5} + 32 = 0$$

$$3(-0.12 - 0.95i)^5 + 2(1)^5 + 32 = 0$$

$$3(-0.12 - 0.95i)^2 (-0.12 - 0.95i)^2 (-0.12 - 0.95i) + 34 = 0$$

No da. Algo hice MAL.

$$3z^5 = -(\underbrace{2|z|^5 + 32}_{\in \mathbb{R}_{>0}})$$

$\arg(3z^5) = \pi$

Por igualdad de nros complejos.

(MOIVRE)

RDo: $\arg(a\cdot b) = \arg(a) + \arg(b)$

$$\arg(3z^5) = \arg(-2|z|^5 - 32)$$

$$\arg(3) + 5\arg(z) = \arg(-2(|z|^5 - 16))$$

$$0 + 5\theta = \arg(-2) + \underbrace{\arg(|z|^5 - 16)}_{?}$$

$$5\theta = \pi + 2k\pi$$

$$\theta = \frac{\pi + 2k\pi}{5}$$

OK. llegué a lo mismo

Ahora veo el MODULO (siempre positivo)

$$\underbrace{|3||z|^5}_{|3\cdot|z|^5| = |3|\cdot||z|^5|} = -2|z|^5 + |-32|$$

$$\overset{|a\cdot b| = |a|\cdot|b|}{=} |-2\,\overset{\mathbb{R}}{|z|^5}| + 32$$

$$= |-2|\cdot||z|^5| + 32$$

$$= |-2|\cdot\overset{||a|| = a}{||z^5||} + 32$$

$$3\,z^5 = 2\cdot z^5 + 32$$

$$z^5 = 32$$

Luego, $\begin{cases} r^5 = 32 \Rightarrow r = \sqrt[5]{32} = 2 \\ \theta = \dfrac{\pi + 2k\pi}{5} \quad \text{con } k \in [0,4] \end{cases}$

$z_0 = 2 \cdot e^{\frac{\pi}{5} i}$

$z_1 = 2 \cdot e^{\frac{3\pi}{5} i}$

$z_2 = 2 \cdot e^{\pi i}$

$z_3 = 2 \cdot e^{\frac{7\pi}{5} i}$

$z_4 = 2 \cdot e^{\frac{9\pi}{5} i}$

VERIF.:

z_2 a Binom: $2(-1+0)$

$= -2$

Luego,

$3z^5 + 2|z^5| + 32 = 0$

$3(-2)^5 + 2|-2|^5 + 32 = 0$

$-96 + 64 + 32 = 0$ ✓

$$0 = 0 \checkmark$$

7. Hallar todos los $n \in \mathbb{N}$ tales que

i) $(\sqrt{3} - i)^n = 2^{n-1}(-1 + \sqrt{3}\,i)$.

ii) $(-\sqrt{3} + i)^n \left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$ es un número real negativo.

iii) $\arg\left((-1+i)^{2n}\right) = \frac{\pi}{2}$ y $\arg\left((1 - \sqrt{3}i)^{n-1}\right) = \frac{2}{3}\pi$.

Rehago

i) $(\sqrt{3} - i)^n = 2^{n-1}(-1 + \sqrt{3}\,i)$

Por igualdad de complejos:

Módulo

$$|(\sqrt{3} - i)^n| = |2^{n-1}(-1 + \sqrt{3}\,i)|$$

$$|(\sqrt{3} - i)|^n = |2^{n-1}| \cdot |-1 + \sqrt{3}\,i|$$

$$\left|\sqrt{(\sqrt{3})^2 + (-1)^2}\right|^n = |2|^{n-1} \cdot \sqrt{(-1)^2 + (\sqrt{3})^2}$$

$$2^n = 2^{n-1} \cdot 2$$

$$2^{n+1} = 2^n \cdot 2$$

$$\frac{2^{n+1}}{2^n} = 2$$

$$2 = 2$$

$$|z| = |w|$$

Arg:

$$\arg(\sqrt{3}-i)^n = \arg(\underbrace{2^{n-1}}_{2^0, 2^1 = 1 \arg = 0}) + \arg(-1+\sqrt{3}i)$$

$$\left(2\pi - \frac{\pi}{6}\right)^n = (n-1)\underbrace{(\arg(2))}_{0} + \left(\pi - \frac{\pi}{3}\right) + 2k\pi$$

$$n \cdot \left(\frac{11\pi}{6}\right) = 0 + \frac{2\pi}{3} + 2k\pi$$

$$n \cdot \frac{11}{6} \overset{/\pi}{=} \frac{2}{3} + 2k$$

$$n \cdot 11 = \left(\frac{2}{3} + 2k\right) \cdot 6$$

$$11n = 4 + 12k$$

$$11n \equiv 4 (12)$$

$$-n \equiv 4(12)$$

$$\Updownarrow$$

$$\boxed{n \equiv 8(12)}$$

$$\begin{cases} |z| = |w| \\ \\ \frac{2}{3}\pi = \frac{11}{6} n\pi + 2k\pi. \end{cases}$$

ii) $(-\sqrt{3}+i)^n \left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right)$

$$\arg(-\sqrt{3}+i)^n + \arg\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) = \pi$$

$$n \arg(-\sqrt{3}+i) + \arg\left(\frac{1}{2} + \frac{\sqrt{3}}{2} i\right) = \pi$$

$$n.\left(\pi - \frac{\pi}{6}\right) + \frac{\pi}{3} = \pi + 2k\pi$$

$$n\left(\frac{5}{6}\pi\right) + \frac{\pi}{3} = \pi + 2k\pi$$

$$n\left(\frac{5\pi}{6}\right) = \pi - \frac{\pi}{3} + 2k\pi$$

$$n\,\frac{5\pi}{6} = \frac{2\pi}{3} + 2k\pi$$

$$n\,5\pi = \left(\frac{2\pi}{3} + 2k\pi\right)6$$

$$n.5\pi = 4\pi + 12k\pi$$

$$5n = 4 + 12k$$

$5n \equiv 4\ (12) \overset{\cdot 5}{\Longleftrightarrow} n \equiv 8\ (12)$

Luego, los n que valen, son $n = 12k + 8$.

→ PREG: lo mismo pero, le resto al toque el $2k\pi$

$$\frac{8\pi}{3} + 8k\pi$$

iii)

- $2n \arg(-1+i) = \frac{\pi}{2}$
- $n-1 \cdot \arg(1-\sqrt{3}i) = \frac{2\pi}{3}$

1) $\cancel{2}n \cdot \frac{3}{\cancel{4}}\pi = \frac{\pi}{2} + 2k\pi$

$$\frac{3\pi}{2} n = \frac{\pi}{2} + 2k\pi$$

$$\frac{3}{2} n \overset{\cdot 2}{=} \frac{1}{2} + 4k$$

$$3n = 1 + 4k$$

$$3m \equiv 1(4) \iff m \equiv 3(4)$$

2) $(m-1)\cdot\left(\frac{5}{3}\pi\right) = \frac{2\pi}{3} + 2k\pi$

$$(m-1)(5\pi) = \left(\frac{2\pi}{3} + 2k\pi\right)\cdot 3$$

$$(m-1)(5\pi) = \frac{6\pi}{3} + 6k\pi$$

$$(m-1)(5\pi) = 2\pi + 6k\pi$$

$$(m-1)(5) \overset{/\pi}{=} 2 + 6k$$

$$5m-5 = 2+6k$$

$$5m = 7+6k$$

$$5m \equiv 7(6) \iff -m \equiv 1(6)$$

$$\iff m \equiv 5(6)$$

Ahora,

$\begin{cases} m \equiv 5(6) \end{cases}$ ¿$5 \equiv 3(4)$? No. Con diff.

$$n \equiv 3(4)$$

$$\begin{cases} n \equiv 5(3) \\ n \equiv 5(2) \\ n \equiv 3(4) \end{cases} \iff \left.\begin{cases} n \equiv 2(3) \\ n \equiv 1(2) \\ n \equiv 3(4) \end{cases}\right] \; 3 \equiv 1(2)\text{? sí.}$$

$$\iff \begin{cases} n \equiv 2(3) \\ n \equiv 3(4) \end{cases}$$ Por TCR $\exists$ solución y es mod 12

S1: $4y_1 \equiv 2(3) \iff y_1 \equiv 2(3) \implies x_1 = 8$

S2: $3y_2 \equiv 3(4) \iff -y_2 \equiv 3(4) \iff y_2 \equiv 1(4)$

$\implies x_2 = 3$

$n \equiv 11(12)$ ✓

VERIF: $q = 0,\ n = 11$ $\neq$

$\cdot\ 2n \arg(-1+i) = \frac{\pi}{2} + 2k\pi$

$\cdot\ n = 1 \quad \arg(1-\sqrt{3}i) = 2\pi$

$$2(11)\cdot\frac{3}{4}\pi=\frac{\pi}{2}+2k\pi$$

12. i) Sea $w\in G_{36}$, $w^4\neq 1$. Calcular $\sum_{k=7}^{60} w^{4k}$.

ii) Sea $w\in G_{11}$, $w\neq 1$. Calcular $\mathrm{Re}\left(\sum_{k=0}^{60} w^k\right)$.

$w^{36}=1$, $w^4\neq 1$.

$$\sum_{k=0}^{6} w^{4k}$$

$$\sum_{k=7}^{60} w^{4k} = \underbrace{\sum_{k=7}^{60}(w^4)^k}_{*} \overset{*}{=} \sum_{k=0}^{60}(w^4)^k - \underbrace{w^{4^0}-w^{4^1}-w^{4^2}-w^{4^3}-w^{4^4}-w^{4^5}-w^{4^6}}$$

* Union w^{4k} MULTIPLOS de 4.

$$w^{28}+w^{32}+w^{36}+w^{40}$$

$$w^{36}=1$$

$$\underbrace{(w^4)}_{\in G_9}{}^{9}=w^{36}=1$$

$$* \sum_{k=0}^{60} (w^4)^k = \frac{(w^4)^{61} - 1}{w^4 - 1} = \frac{w^{244} - 1}{w^4 - 1} = \frac{w^{28} - 1}{w^4 - 1}$$

$$* \frac{(w^4)^7 - 1}{(w^4) - 1} = \frac{w^{28} - 1}{w^4 - 1}$$

$$\Rightarrow \frac{w^{28} - 1}{w^4 - 1} - \frac{w^{28} - 1}{w^4 - 1} = 0$$

ii)

12. i) Sea $w \in G_{36}$, $w^4 \neq 1$. Calcular $\sum_{k=7}^{60} w^{4k}$.

ii) Sea $w \in G_{11}$, $w \neq 1$. Calcular $\mathrm{Re}\left(\sum_{k=0}^{60} w^k\right)$.

$$\left(\sum_{k=0}^{60} w^k\right) = \frac{w^{61}-1}{w-1} = \frac{w^6-1}{w-1}$$

w es un complejo, $\overline{w}$ es el conjugado.

$$\frac{\overline{w}^6-1}{w-1} \Rightarrow \frac{(w^{-1})^6-1}{w-1} = \frac{w^5-1}{w-1}$$

$$Re(z) = \frac{\frac{w^6-1}{w-1} + \frac{\overline{w}^6-1}{\overline{w}-1}}{2}$$

$$\overline{w} = w^{-1} = w^{10}$$

$$\overline{w}^6 = (w^{-1})^6 = w^{-6} = w^5$$

$$\frac{w^6-1}{w-1} + \frac{w^5-1}{w^{10}-1} = \frac{(w^{10}-1)(w^6-1)+(w-1)(w^5-1)}{(w-1)(w^{10}-1)}$$

$$= (w^{10}-1)(w^6-1)+(w-1)(w^5-1)$$

$$\overline{}$$
$$2(w-1)(w^{10}-1)$$

$$= \frac{(w^{16}-w^{10}-w^{6}+1)+(w^{6}-w-w^{5}+1)}{2(w^{11}-w-w^{10}+1)}$$

$$= \frac{\cancel{w^{5}}-w^{10}-\cancel{w^{6}}+1+\cancel{w^{6}}-w-\cancel{w^{5}}+1}{2w^{11}-2w-2w^{10}+2}$$

$$= \frac{-w^{10}-w+2}{2w^{11}-2w-2w^{10}+2}$$

$$= \frac{-w^{10}-w+2}{2(w^{11}-w^{10}-w+1)}$$

$$= \frac{\cancel{-w^{10}-w+2}}{2(\cancel{1-w^{10}-w+1})}$$

$$= \frac{1}{2}$$

Repaso.

$$\operatorname{Re}(z) = \frac{z + \overline{z}}{2}$$

12. i) Sea $w \in G_{36}$, $w^4 \neq 1$. Calcular $\sum\limits_{k=7}^{60} w^{4k}$.

ii) Sea $w \in G_{11}$, $w \neq 1$. Calcular $\operatorname{Re}\left(\sum\limits_{k=0}^{60} w^k\right)$.

$$G_{11} \Rightarrow w^{11} = 1$$

$$\operatorname{Re}(z) = \frac{\sum\limits_{k=0}^{60} w^k + \sum\limits_{k=0}^{60} \overline{w}^k}{2}$$

$$\cdot \sum_{k=0}^{60} w^k = \frac{w^{61} - 1}{w - 1} = \frac{w^6 - 1}{w - 1}$$

$$\cdot \sum_{k=0}^{60} \overline{w}^k = \frac{\overline{w}^{61} - 1}{\overline{w} - 1} = \frac{(w^{-1})^{61} - 1}{w^{-1} - 1} = \frac{w^5 - 1}{w^{10} - 1}$$

Cioè, $\dfrac{w^{6}-1}{w-1} + \dfrac{w^{5}-1}{w^{10}-1}$ tutto / 2.

$$= \frac{(w^{10}-1)(w^{6}-1)+(w-1)(w^{5}-1)}{(w-1)(w^{10}-1)}$$

$$= \frac{\cancel{w^{5}} - w^{10} \cancel{- w^{6}} + 1 \cancel{+ w^{6}} - w \cancel{- w^{5}} + 1}{2(w^{11} - w - w^{10} + 1)}$$

$$= \frac{-w^{10} - w + 2}{2(w^{0} - w - w^{10} + 1)}$$

$$= \frac{\cancel{-w^{10} - w + 2}}{2(\cancel{-w^{10} - w + 2})} = \frac{1}{2}$$

E ci ricavo $Im(z)$?

$$Im(z) = \frac{z - \bar{z}}{2}$$

$$\frac{w^{6}-1}{} - \frac{w^{5}-1}{} \quad \text{tutto } /2.$$

$$w - 1 \qquad\qquad w^{10} - 1$$

$$= \frac{(w^{10}-1)(w^{6}-1) - (w-1)(w^{5}-1)}{(w-1)(w^{10}-1)}$$

$$= \frac{w^{16} - w^{10} - w^{6} + 1 - (w^{6} - w - w^{5} + 1)}{2(w-1)(w^{10}-1)}$$

$$= \frac{w^{5} - w^{10} - w^{6} + 1 - w^{6} + w + w^{5} - 1}{2(w^{11} - w - w^{10} + 1)}$$

$$= \frac{-w^{10} - 2w^{6} + 2w^{5} + w}{2(1 - w - w^{10} + 1)}$$

$$= \frac{-w^{10} - 2w^{6} + 2w^{5} + w}{2(-w^{10} - w + 2)}$$

11. i) Calcular $w + \overline{w} + (w + w^2)^2 - w^{38}(1 - w^2)$ para cada $w \in G_7$.

ii) Calcular $w^{73} + \overline{w} \cdot w^9 + 8$ para cada $w \in G_3$.

iii) Calcular $1 + w^2 + w^{-2} + w^4 + w^{-4}$ para cada $w \in G_{10}$.

iv) Calcular $w^{14} + w^{-8} + \overline{w}^4 + \overline{w^{-3}}$ para cada $w \in G_5$.

11) iii) $w \in G_{10}$

w^0

$1 + w^2 + w^8 + w^4 + w^6$

$w^0 + w^2 + w^4 + w^6 + w^8$

$w^{10} = (w^2)^5 \Rightarrow w^2 \in G_5$

llamo $z = w^2$

$1 + z + z^2 + z^3 + z^4$

$\hookrightarrow \sum_{j=0}^{4} z^j = \begin{cases} 0 \text{ si } w \neq 1 \\ 5 \text{ si } w = 1 \end{cases}$

Pero, $z = w^2$

ent,

$\begin{cases} 0 \text{ si } w \neq \pm 1 \end{cases}$

$\quad 5 \text{ si } w = \pm 1$

iv)

$\Rightarrow w^4 + w^2 + (w^4)^{-1} + (w^{-1})^{-3}$

$\Rightarrow w^4 + w^2 + w + w^3$

$$\sum_{j=1}^{4} w^j = \sum_{j=0}^{4} w^j - 1$$

$$= \begin{cases} \frac{w^5 - 1}{w - 1} - 1 = -1 \text{ si } w \neq 1 \\ 4 \quad \text{si } w = 1 \end{cases}$$