Ejercicio 5 ★

Demostrar en deducción natural que las siguientes fórmulas son teoremas **sin usar principios de razonamiento clásicos** salvo que se indique lo contrario. Recordemos que una fórmula σ es un teorema si y sólo si vale $\vdash \sigma$:

- I. *Modus ponens* relativizado: $(\rho \Rightarrow \sigma \Rightarrow \tau) \Rightarrow (\rho \Rightarrow \sigma) \Rightarrow \rho \Rightarrow \tau$
- II. Reducción al absurdo: $(\rho \Rightarrow \bot) \Rightarrow \neg\rho$
- III. Introducción de la doble negación: $\rho \Rightarrow \neg\neg\rho$
- IV. Eliminación de la triple negación: $\neg\neg\neg\rho \Rightarrow \neg\rho$
- V. Contraposición: $(\rho \Rightarrow \sigma) \Rightarrow (\neg\sigma \Rightarrow \neg\rho)$
- VI. Adjunción: $((\rho \wedge \sigma) \Rightarrow \tau) \Leftrightarrow (\rho \Rightarrow \sigma \Rightarrow \tau)$
- VII. de Morgan (I): $\neg(\rho \vee \sigma) \Leftrightarrow (\neg\rho \wedge \neg\sigma)$
- VIII. de Morgan (II): $\neg(\rho \wedge \sigma) \Leftrightarrow (\neg\rho \vee \neg\sigma)$. Para la dirección $\Rightarrow$ es necesario usar principios de razonamiento clásicos.
- IX. Conmutatividad ($\wedge$): $(\rho \wedge \sigma) \Rightarrow (\sigma \wedge \rho)$
- X. Asociatividad ($\wedge$): $((\rho \wedge \sigma) \wedge \tau) \Leftrightarrow (\rho \wedge (\sigma \wedge \tau))$
- XI. Conmutatividad ($\vee$): $(\rho \vee \sigma) \Rightarrow (\sigma \vee \rho)$
- XII. Asociatividad ($\vee$): $((\rho \vee \sigma) \vee \tau) \Leftrightarrow (\rho \vee (\sigma \vee \tau))$

¿Encuentra alguna relación entre teoremas de adjunción, asociatividad y conmutatividad con algunas de las propiedades demostradas en la práctica 2?

i)

$\Gamma = (P \to Q \to T), (P \to Q), P$

$$\dfrac{\dfrac{}{\Gamma \vdash (P \to Q \to T)}\text{Ax} \qquad \dfrac{}{\Gamma \vdash P \to Q}\text{Ax}}{(P \to Q \to T), (P \to Q), P \vdash T}\Rightarrow e$$

$$\dfrac{(P \to Q \to T), (P \to Q), P \vdash T}{(P \to Q \to T), (P \to Q) \vdash P \to T}\Rightarrow i$$

$$\dfrac{(P \to Q \to T), (P \to Q) \vdash P \to T}{(P \to Q \to T) \vdash (P \to Q) \to P \to T}\Rightarrow i$$

$$\dfrac{(P \to Q \to T) \vdash (P \to Q) \to P \to T}{\underbrace{(P \to Q \to T)}_{F_2} \to ((\underbrace{P \to Q}_{F_1}) \to (\underbrace{P}_{P_1} \to (\underbrace{T}_{\text{SALIDA}})))}\Rightarrow i$$

ii)

$$\dfrac{\dfrac{}{(P \to \bot) \vdash P \to \bot}\text{Ax} \qquad \dfrac{\dfrac{}{(P \to \bot), \bot \vdash \bot}\text{Ax}}{(P \to \bot) \vdash \neg\bot}\neg i}{(P \to \bot) \vdash \neg P}\text{MT}$$

$$\dfrac{(P \to \bot) \vdash \neg P}{\emptyset \vdash (P \to \bot) \Rightarrow \neg P}\Rightarrow i$$

✓

iii)

$$\dfrac{\dfrac{\dfrac{}{P \vdash P}\text{Ax}}{P \vdash \neg\neg P}\neg\neg i}{P \to \neg\neg P}\Rightarrow i$$

iv)

$$\dfrac{\dfrac{}{\neg\neg\neg P, P \vdash \neg\neg\neg P}\text{Ax} \qquad \dfrac{\dfrac{\dfrac{}{\neg\neg\neg P, P \vdash P}\text{Ax}}{\neg\neg\neg P, P \vdash \neg\neg P}\neg\neg i}{\neg\neg\neg P, P \vdash \neg\neg\neg\neg P}\neg\neg i}{\neg\neg\neg P, P \vdash \bot}\neg e$$

$$\neg i$$

$$\dfrac{\neg\neg\neg P \vdash \neg P}{\neg\neg\neg P \Rightarrow \neg P} \Rightarrow i$$

V)

$$\dfrac{\dfrac{}{P \Rightarrow G, \neg G, P \vdash P \Rightarrow G}\,Ax \quad \dfrac{}{P \Rightarrow G, \neg G, P \vdash P}\,Ax}{P \Rightarrow G, \neg G, P \vdash G} \Rightarrow e \qquad \text{T: } G$$

$$\dfrac{P \Rightarrow G, \neg G, P \vdash G}{P \Rightarrow G, \neg G, P \vdash \neg\neg G} \neg\neg i$$

$$\dfrac{\dfrac{}{P \Rightarrow G, \neg G, P \vdash \neg G}\,Ax \quad P \Rightarrow G, \neg G, P \vdash \neg\neg G}{P \Rightarrow G, \neg G, P \vdash \bot} \neg e \qquad \text{T: } \neg G$$

$$\dfrac{P \Rightarrow G, \neg G, P \vdash \bot}{P \Rightarrow G, \neg G \vdash \neg P} \neg i$$

$$\dfrac{P \Rightarrow G, \neg G \vdash \neg P}{P \Rightarrow G \vdash \neg G \Rightarrow \neg P} \Rightarrow i$$

$$\dfrac{P \Rightarrow G \vdash \neg G \Rightarrow \neg P}{(P \Rightarrow G) \Rightarrow (\neg G \Rightarrow \neg P)} \Rightarrow i$$

Vi)

$\Rightarrow$)

$$\dfrac{\dfrac{}{((P \land G) \Rightarrow T), P, G \vdash P}\,Ax \quad \dfrac{}{((P \land G) \Rightarrow T), P, G \vdash G}\,Ax}{((P \land G) \Rightarrow T), P, G \vdash P \land G} \land i$$

$$\dfrac{\dfrac{}{((P \land G) \Rightarrow T), P, G \vdash (P \land G) \Rightarrow T}\,Ax \quad ((P \land G) \Rightarrow T), P, G \vdash P \land G}{((P \land G) \Rightarrow T), P, G \vdash T} \Rightarrow e \qquad \text{T: } (P \land G)$$

$$\dfrac{((P \land G) \Rightarrow T), P, G \vdash T}{((P \land G) \Rightarrow T), P \vdash G \Rightarrow T} \Rightarrow i$$

$$\dfrac{((P \land G) \Rightarrow T), P \vdash G \Rightarrow T}{((P \land G) \Rightarrow T) \vdash (P \Rightarrow (G \Rightarrow T))} \Rightarrow i$$

$$\dfrac{((P \land G) \Rightarrow T) \vdash (P \Rightarrow (G \Rightarrow T))}{((P \land G) \Rightarrow T) \Rightarrow (P \Rightarrow G \Rightarrow T)} \Rightarrow i$$

$\Leftarrow$)

$$\dfrac{\dfrac{}{(P \Rightarrow G \Rightarrow T), P \land G, P \vdash P \land G}\,Ax}{(P \Rightarrow G \Rightarrow T), P \land G, P \vdash G} \land e_2$$

$$\dfrac{(P \Rightarrow G \Rightarrow T), P \land G, P \vdash G}{(P \Rightarrow G \Rightarrow T), P \land G \vdash P \Rightarrow G} \Rightarrow i$$

$$\dfrac{\dfrac{}{(P \Rightarrow G \Rightarrow T), P \land G \vdash P \Rightarrow G \Rightarrow T}\,Ax \quad (P \Rightarrow G \Rightarrow T), P \land G \vdash P \Rightarrow G}{(P \Rightarrow G \Rightarrow T), P \land G \vdash T} \Rightarrow e$$

$$\dfrac{(P \Rightarrow G \Rightarrow T), P \land G \vdash T}{(P \Rightarrow G \Rightarrow T) \vdash ((P \land G) \Rightarrow T)} \Rightarrow i$$

$$\dfrac{(P \Rightarrow G \Rightarrow T) \vdash ((P \land G) \Rightarrow T)}{(P \Rightarrow G \Rightarrow T) \Rightarrow ((P \land G) \Rightarrow T)} \Rightarrow i$$

Vii)

$\Rightarrow$)

$$\dfrac{\dfrac{}{\neg(P \lor G), P \vdash P}\,Ax}{\neg(P \lor G), P \vdash (P \lor G)} \lor i_1 \qquad \neg\neg i$$

$$\dfrac{\dfrac{}{\neg(P \lor G), G \vdash G}\,Ax}{\neg(P \lor G), G \vdash P \lor G} \lor i_2 \qquad \neg\neg i$$

$$\dfrac{\neg(P\vee Q), P \vdash \neg(P\vee Q) \qquad \neg(P\vee Q), P \vdash \neg\neg(P\vee Q)}{\neg(P\vee Q), P \vdash \bot}\ \neg e \qquad \dfrac{\neg(P\vee Q), Q \vdash \neg(P\vee Q) \qquad \neg(P\vee Q), Q \vdash \neg\neg(P\vee Q)}{\neg(P\vee Q), Q \vdash \bot}\ \neg e$$

$$\dfrac{\neg(P\vee Q), P \vdash \bot}{\neg(P\vee Q) \vdash \neg P}\ \neg i \qquad \dfrac{\neg(P\vee Q), Q \vdash \bot}{\neg(P\vee Q) \vdash \neg Q}\ \neg i$$

$$\dfrac{\neg(P\vee Q) \vdash \neg P \qquad \neg(P\vee Q) \vdash \neg Q}{\neg(P\vee Q) \vdash \neg P\wedge\neg Q}\ \wedge i$$

$$\dfrac{\neg(P\vee Q) \vdash \neg P\wedge\neg Q}{\neg(P\vee Q) \Rightarrow (\neg P\wedge\neg Q)}\ \Rightarrow i$$

$\Leftarrow)$

$$\dfrac{}{\Gamma', P \vdash P}\ Ax \qquad \dfrac{\dfrac{}{\Gamma', P \vdash \neg P\wedge\neg Q}\ Ax}{\Gamma', P \vdash \neg P}\ \wedge e_1 \qquad \dfrac{}{\Gamma', Q \vdash Q}\ Ax \qquad \dfrac{\dfrac{}{\Gamma', Q \vdash \neg P\wedge\neg Q}\ Ax}{\Gamma', Q \vdash \neg Q}\ \wedge e_2$$

$$\dfrac{\Gamma', P \vdash P \qquad \Gamma', P \vdash \neg P}{\Gamma', P \vdash \bot} \qquad \dfrac{\Gamma', Q \vdash Q \qquad \Gamma', Q \vdash \neg Q}{\Gamma', Q \vdash \bot}\ \neg e$$

$$\dfrac{\dfrac{}{\Gamma' \vdash P\vee Q}\ Ax \qquad \Gamma', P \vdash \bot \qquad \Gamma', Q \vdash \bot}{\Gamma' = (\neg P\wedge\neg Q), (P\vee Q) \vdash \bot}\ \vee e$$

$$\dfrac{\Gamma' = (\neg P\wedge\neg Q), (P\vee Q) \vdash \bot}{(\neg P\wedge\neg Q) \vdash \neg(P\vee Q)}\ \neg i$$

$$\dfrac{(\neg P\wedge\neg Q) \vdash \neg(P\vee Q)}{\emptyset \vdash (\neg P\wedge\neg Q) \Rightarrow \neg(P\vee Q)}\ \Rightarrow i$$

viii)

$\Rightarrow)$

$$\dfrac{\dfrac{}{\neg(P\wedge Q), (\neg P\vee\neg Q), (P\wedge Q) \vdash (P\wedge Q)}\ Ax \qquad \dfrac{}{\neg(P\wedge Q), (\neg P\vee\neg Q), (P\wedge Q) \vdash \neg(P\wedge Q)}\ Ax}{\neg(P\wedge Q), (\neg P\vee\neg Q), (P\wedge Q) \vdash \bot}\ \neg e$$

$$\dfrac{\neg(P\wedge Q), (\neg P\vee\neg Q), (P\wedge Q) \vdash \bot}{\neg(P\wedge Q), \neg(\neg P\vee\neg Q) \vdash P\wedge Q}\ PBC$$

-> tengo $\neg(P\wedge Q)$. Busco negar $P\wedge Q$ para $\neg e$.

$$\dfrac{\neg(P\wedge Q), \neg(\neg P\vee\neg Q) \vdash P\wedge Q}{\neg(P\wedge Q), \neg(\neg P\vee\neg Q) \vdash \neg\neg(P\wedge Q)}\ \neg\neg i$$

$$\dfrac{\dfrac{}{\neg(P\wedge Q), \neg(\neg P\vee\neg Q) \vdash \neg(P\wedge Q)}\ Ax \qquad \neg(P\wedge Q), \neg(\neg P\vee\neg Q) \vdash \neg\neg(P\wedge Q)}{\neg(P\wedge Q), \neg(\neg P\vee\neg Q) \vdash \bot}\ \neg e$$

$$\dfrac{\neg(P\wedge Q), \neg(\neg P\vee\neg Q) \vdash \bot}{\neg(P\wedge Q) \vdash (\neg P\vee\neg Q)}\ \neg i$$

$$\dfrac{\neg(P\wedge Q) \vdash (\neg P\vee\neg Q)}{\neg(P\wedge Q) \Rightarrow (\neg P\vee\neg Q)}\ \Rightarrow i$$

$\Leftarrow)$

$$\dfrac{\dfrac{}{(\neg P\vee\neg Q), (P\wedge Q), \neg(\neg P\vee\neg Q) \vdash (\neg P\vee\neg Q)}\ Ax \qquad \dfrac{}{(\neg P\vee\neg Q), (P\wedge Q), \neg(\neg P\vee\neg Q) \vdash \neg(\neg P\vee\neg Q)}\ Ax}{(\neg P\vee\neg Q), (P\wedge Q), \neg(\neg P\vee\neg Q) \vdash \bot}\ \neg i$$

$$\dfrac{(\neg P\vee\neg Q), (P\wedge Q), \neg(\neg P\vee\neg Q) \vdash \bot}{(\neg P\vee\neg Q), (P\wedge Q) \vdash \neg(\neg P\vee\neg Q)}\ PBC.$$

$$\dfrac{(\neg P\vee\neg Q), (P\wedge Q) \vdash (\neg P\vee\neg Q) \qquad (\neg P\vee\neg Q), (P\wedge Q) \vdash \neg(\neg P\vee\neg Q)}{(\neg P\vee\neg Q), (P\wedge Q) \vdash \bot}\ \neg e$$

$$\dfrac{(\neg P\vee\neg Q), (P\wedge Q) \vdash \bot}{(\neg P\vee\neg Q) \vdash \neg(P\wedge Q)}\ \neg i$$

$$\dfrac{(\neg P\vee\neg Q) \vdash \neg(P\wedge Q)}{(\neg P\vee\neg Q) \Rightarrow \neg(P\wedge Q)}\ \Rightarrow i$$

$(\neg P \lor \neg G) \implies \neg(P \land G)$

ix)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(P\land G)\vdash P\land G}\,\text{Ax}}{(P\land G)\vdash G}\,\land e_2 \qquad \dfrac{\dfrac{}{(P\land G)\vdash P\land G}\,\text{Ax}}{(P\land G)\vdash P}\,\land e_1}{(P\land G)\vdash G\land P}\,\land i}{(P\land G)\implies(G\land P)}\,\implies i$$

x)

$\implies$)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{((P\land G)\land T)\vdash(P\land G)\land T}\,\text{Ax}}{((P\land G)\land T)\vdash P\land G}\,\land e_2}{((P\land G)\land T)\vdash P}\,\land e_1 \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{((P\land G)\land T)\vdash(P\land G)\land T}\,\text{Ax}}{((P\land G)\land T)\vdash(P\land G)}\,\land e_1}{((P\land G)\land T)\vdash G}\,\land e_2 \qquad \dfrac{\dfrac{}{((P\land G)\land T)\vdash(P\land G)\land T}\,\text{Ax}}{((P\land G)\land T)\vdash T}\,\land e_2}{((P\land G)\land T)\vdash(G\land T)}}{((P\land G)\land T)\vdash(P\land(G\land T))}\,\land i}{((P\land G)\land T)\implies(P\land(G\land T))}\,\implies i$$

$\Leftarrow$)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P\land(G\land T)\vdash P\land(G\land T)}\,\text{Ax}}{P\land(G\land T)\vdash P}\,\land e_1 \qquad \dfrac{\dfrac{\dfrac{}{P\land(G\land T)\vdash P\land(G\land T)}\,\text{Ax}}{P\land(G\land T)\vdash G\land T}\,\land e_2}{P\land(G\land T)\vdash G}\,\land e_1}{P\land(G\land T)\vdash P\land G}\,\land i \qquad \dfrac{\dfrac{\dfrac{}{P\land(G\land T)\vdash P\land(G\land T)}\,\text{Ax}}{P\land(G\land T)\vdash(G\land T)}\,\land e_2}{P\land(G\land T)\vdash T}\,\land e_2}{P\land(G\land T)\vdash((P\land G)\land T)}\,\land i}{P\land(G\land T)\implies((P\land G)\land T)}\,\implies i$$

xi)

$$\dfrac{\dfrac{\dfrac{}{(P\lor G)\vdash P\lor G}\,\text{Ax} \qquad \dfrac{\dfrac{}{(P\lor G),P\vdash P}\,\text{Ax}}{(P\lor G),P\vdash G\lor P}\,\lor i_2 \qquad \dfrac{\dfrac{}{(P\lor G),G\vdash G}\,\text{Ax}}{(P\lor G),G\vdash G\lor P}\,\lor i_1}{(P\lor G)\vdash(G\lor P)}\,\lor e}{(P\lor G)\implies(G\lor P)}\,\implies i$$

xii)

$\Rightarrow)$

$$\dfrac{\dfrac{}{((p\vee\sigma)\vee\tau) \vdash (p\vee(\sigma\vee\tau))}\,\text{Ax} \qquad \dfrac{\dfrac{}{((p\vee\sigma)\vee\tau), p \vdash p}\,\text{Ax}}{((p\vee\sigma)\vee\tau), p \vdash (p\vee(\sigma\vee\tau))}\,\vee i_1 \qquad \dfrac{\dfrac{}{((p\vee\sigma)\vee\tau), (\sigma\vee\tau) \vdash (\sigma\vee\tau)}\,\text{Ax}}{((p\vee\sigma)\vee\tau), (\sigma\vee\tau) \vdash (p\vee(\sigma\vee\tau))}\,\vee i_2}{\dfrac{((p\vee\sigma)\vee\tau) \vdash (p\vee(\sigma\vee\tau))}{\emptyset \vdash ((p\vee\sigma)\vee\tau) \Rightarrow (p\vee(\sigma\vee\tau))}\,\Rightarrow i}\,\vee e$$

$\tau = p$

$\sigma = (\sigma\vee\tau)$

$\rho = (p\vee(\sigma\vee\tau))$

$\Leftarrow)$

$$\dfrac{\dfrac{}{p\vee(\sigma\vee\tau) \vdash (p\vee\sigma)\vee\tau}}{\emptyset \vdash p\vee(\sigma\vee\tau) \Rightarrow ((p\vee\sigma)\vee\tau)}\,\Rightarrow i$$

Ejercicio 6 ★

Demostrar en deducción natural que vale $\vdash \sigma$ para cada una de las siguientes fórmulas. Para estas fórmulas es imprescindible **usar lógica clásica**:

- I. Absurdo clásico: $(\neg\tau \Rightarrow \bot) \Rightarrow \tau$
- II. Ley de Peirce: $((\tau \Rightarrow \rho) \Rightarrow \tau) \Rightarrow \tau$
- III. Tercero excluido: $\tau \vee \neg\tau$
- IV. Consecuencia milagrosa: $(\neg\tau \Rightarrow \tau) \Rightarrow \tau$
- V. Contraposición clásica: $(\neg\rho \Rightarrow \neg\tau) \Rightarrow (\tau \Rightarrow \rho)$
- VI. Análisis de casos: $(\tau \Rightarrow \rho) \Rightarrow (\neg\tau \Rightarrow \rho) \Rightarrow \rho$
- VII. Implicación vs. disyunción: $(\tau \Rightarrow \rho) \Leftrightarrow (\neg\tau \vee \rho)$

i)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(\neg\tau\Rightarrow\bot), \neg\tau \vdash \neg\tau\Rightarrow\bot}\,\text{Ax} \qquad \dfrac{}{(\neg\tau\Rightarrow\bot), \neg\tau \vdash \neg\tau}\,\text{Ax}}{(\neg\tau\Rightarrow\bot), \neg\tau \vdash \bot}\,\Rightarrow e \ (\tau : \bot)}{(\neg\tau\Rightarrow\bot) \vdash \tau}\,\text{PBC}}{\ }\,\Rightarrow i$$

solo hago aparecer $\Rightarrow$ con $\Rightarrow e$.

$$(\neg T \Rightarrow \bot) \Rightarrow T$$

ii)

$$\dfrac{\dfrac{}{\Gamma \vdash T \vee \neg T}\,\text{Lem} \qquad \dfrac{}{\Gamma, T \vdash T}\,\text{Ax} \qquad \dfrac{}{\Gamma, \neg T \vdash T}}{\dfrac{\Gamma, ((T \Rightarrow P) \Rightarrow T) \vdash T}{((T \Rightarrow P) \Rightarrow T) \Rightarrow T}\,\Rightarrow_i}$$

iii)

$$\dfrac{\dfrac{\dfrac{}{\neg T \vdash \neg T}\,\text{Ax} \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\neg T, T \vdash \neg T}\,\text{Ax} \qquad \dfrac{}{\neg T, T \vdash T}\,\text{ax}}{\neg T, T \vdash \bot}\,\neg e \ (T = T)}{\neg T \vdash T}\,\text{PBC}}{\neg T \vdash \neg\neg T}\,\neg\neg i}{\dfrac{\neg T \vdash \bot}{\vdash T}\,\text{PBC}}\,\neg e \ (T = \neg T)}{\vdash (T \vee \neg T)}\,\vee i_1$$

iv)

$$\dfrac{\dfrac{\dfrac{}{(\neg T \Rightarrow T), \neg T \vdash \neg T \Rightarrow T}\,\text{Ax} \qquad \dfrac{}{(\neg T \Rightarrow T), \neg T \vdash \neg T}\,\text{Ax}}{(\neg T \Rightarrow T), \neg T \vdash T}\,\Rightarrow_e \qquad \dfrac{\dfrac{\dfrac{}{(\neg T \Rightarrow T), \neg T, T \vdash \neg T \qquad (\neg T \Rightarrow T), \neg T, T \vdash T}\,\text{Ax}}{(\neg T \Rightarrow T), \neg T, T \vdash \bot}\,\neg e}{(\neg T \Rightarrow T), \neg T \vdash \neg T}\,\text{PBC}}{\dfrac{\dfrac{(\neg T \Rightarrow T), \neg T \vdash \bot}{(\neg T \Rightarrow T) \vdash T}\,\text{PBC}}{(\neg T \Rightarrow T) \Rightarrow T}\,\Rightarrow_i}\,\neg e$$

v)

$$
\dfrac{
  \dfrac{
    \dfrac{}{(\neg P \Rightarrow \neg T), T, \neg P \vdash T}\;Ax
    \qquad
    \dfrac{
      \dfrac{}{(\neg P \Rightarrow \neg T), T, \neg P \vdash (\neg P \Rightarrow \neg T)}\;Ax
      \quad
      \dfrac{}{(\neg P \Rightarrow \neg T), T, \neg P \vdash \neg P}\;Ax
    }{(\neg P \Rightarrow \neg T), T, \neg P \vdash \neg T}\;\Rightarrow e
  }{
    \dfrac{(\neg P \Rightarrow \neg T), T, \neg P \vdash \bot}{
      \dfrac{(\neg P \Rightarrow \neg T), T \vdash P}{(\neg P \Rightarrow \neg T) \vdash T \Rightarrow P}\;\Rightarrow i
    }\;PBC
  }\;\neg e
}{(\neg P \Rightarrow \neg T) \Rightarrow (T \Rightarrow P)}\;\Rightarrow i
$$

PBC -) P dAn USAN $(\neg P \Rightarrow \neg T)$ & $\dots \vdash \neg P$.

Vii)

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{}{(T \Rightarrow P), (\neg T \Rightarrow P), \neg P \vdash \neg P}\;Ax
      \qquad
      \dfrac{
        \dfrac{
          \dfrac{}{(T \Rightarrow P), (\neg T \Rightarrow P), \neg P \vdash T \Rightarrow P}\;Ax
          \quad
          \dfrac{
            \dfrac{
              \dfrac{}{(T \Rightarrow P), (\neg T \Rightarrow P), \neg P, \neg T \vdash \neg P}\;Ax
              \quad
              \dfrac{
                \dfrac{}{(T \Rightarrow P), (\neg T \Rightarrow P), \neg P, \neg T \vdash \neg T \Rightarrow P}\;Ax
                \quad
                \dfrac{}{(T \Rightarrow P), (\neg T \Rightarrow P), \neg P, \neg T \vdash \neg T}\;Ax
              }{(T \Rightarrow P), (\neg T \Rightarrow P), \neg P, \neg T \vdash P}\;\Rightarrow e
            }{(T \Rightarrow P), (\neg T \Rightarrow P), \neg P, \neg T \vdash \bot}\;\neg e
          }{(T \Rightarrow P), (\neg T \Rightarrow P), \neg P \vdash T}\;PBC
        }{(T \Rightarrow P), (\neg T \Rightarrow P), \neg P \vdash P}\;\Rightarrow e
      }{(T \Rightarrow P), (\neg T \Rightarrow P), \neg P \vdash \neg\neg P}\;\neg\neg i
    }{(T \Rightarrow P), (\neg T \Rightarrow P), \neg P \vdash \bot}\;\neg e
  }{
    \dfrac{(T \Rightarrow P), (\neg T \Rightarrow P) \vdash P}{(T \Rightarrow P) \vdash (\neg T \Rightarrow P) \Rightarrow P}\;\Rightarrow i
  }\;PBC
}{(T \Rightarrow P) \Rightarrow ((\neg T \Rightarrow P) \Rightarrow P)}\;\Rightarrow i
$$

Vij)

$\Rightarrow$)

$$
\dfrac{
  \dfrac{
    \dfrac{}{(T \Rightarrow P) \vdash (T \Rightarrow P)}\;Ax
    \qquad
    \dfrac{
      \dfrac{
        \begin{matrix}(T \Rightarrow P), \neg T, \neg P \vdash \bot \\ (T \Rightarrow P), \neg T \vdash P\end{matrix}
        \quad
        \dfrac{(T \Rightarrow P), \neg T, P \vdash \bot}{(T \Rightarrow P), \neg T \vdash \neg P}
      }{(T \Rightarrow P), \neg T \vdash \bot}
    }{(T \Rightarrow P) \vdash T}
  }{(T \Rightarrow P) \vdash P}\;\Rightarrow e
}{}\;\vee e_2
$$

$(A \Rightarrow B), \neg A \vdash \bot$

$$\dfrac{(\tau\Rightarrow P)\vdash(\neg\tau\vee P)}{((\tau\Rightarrow P)\Rightarrow(\neg\tau\vee P))}\Rightarrow_i$$

(⇐)

$$\dfrac{\dfrac{}{(\neg\tau\vee P),\tau,\neg\tau,\neg P\vdash\tau}Ax \qquad \dfrac{}{(\neg\tau\vee P),\tau,\neg\tau,\neg P\vdash\neg\tau}Ax}{(\neg\tau\vee P),\tau,\neg\tau,\neg P\vdash\bot}\neg e$$

$$\dfrac{\dfrac{}{(\neg\tau\vee P),\tau\vdash(\neg\tau\vee P)}Ax \qquad \dfrac{(\neg\tau\vee P),\tau,\neg\tau,\neg P\vdash\bot}{(\neg\tau\vee P),\tau,\neg\tau\vdash P}PBC \qquad (\neg\tau\vee P),\tau,P\vdash P}{\dfrac{(\neg\tau\vee P),\tau\vdash P}{\dfrac{(\neg\tau\vee P)\vdash\tau\Rightarrow P}{(\neg\tau\vee P)\Rightarrow(\tau\Rightarrow P)}\Rightarrow i}\Rightarrow i}\vee e$$

$\tau:\neg\tau$
$\sigma:P$

$$\dfrac{\Gamma,\tau\vdash\tau}{}\text{ax}$$

$$\dfrac{\Gamma\vdash\tau\quad\Gamma\vdash\sigma}{\Gamma\vdash\tau\wedge\sigma}\wedge_i \qquad \dfrac{\Gamma\vdash\tau\wedge\sigma}{\Gamma\vdash\tau}\wedge_{e_1} \qquad \dfrac{\Gamma\vdash\tau\wedge\sigma}{\Gamma\vdash\sigma}\wedge_{e_2}$$

$$\dfrac{\Gamma,\tau\vdash\sigma}{\Gamma\vdash\tau\Rightarrow\sigma}\Rightarrow_i \qquad \dfrac{\Gamma\vdash\tau\Rightarrow\sigma\quad\Gamma\vdash\tau}{\Gamma\vdash\sigma}\Rightarrow_e$$

$$\dfrac{\Gamma\vdash\tau}{\Gamma\vdash\tau\vee\sigma}\vee_{i_1} \qquad \dfrac{\Gamma\vdash\sigma}{\Gamma\vdash\tau\vee\sigma}\vee_{i_2} \qquad \dfrac{\Gamma\vdash\tau\vee\sigma\quad\Gamma,\tau\vdash\rho\quad\Gamma,\sigma\vdash\rho}{\Gamma\vdash\rho}\vee_e$$

$$\dfrac{\Gamma,\tau\vdash\bot}{\Gamma\vdash\neg\tau}\neg_i \qquad \dfrac{\Gamma\vdash\tau\quad\Gamma\vdash\neg\tau}{\Gamma\vdash\bot}\neg_e \qquad \dfrac{\Gamma\vdash\bot}{\Gamma\vdash\tau}\bot_e$$

Lógica intuicionista

Lógica clásica $\quad \dfrac{\Gamma\vdash\neg\neg\tau}{\Gamma\vdash\tau}\neg\neg_e$

Ejercicio 9

Probar los siguientes teoremas:

- I. $((P\Rightarrow Q)\Rightarrow Q)\Rightarrow((Q\Rightarrow P)\Rightarrow P)$
- II. $(P\Rightarrow Q)\Rightarrow((\neg P\Rightarrow Q)\Rightarrow Q)$

Reglas intuicionistas

$$\dfrac{\Gamma\vdash\tau}{\Gamma\vdash\neg\neg\tau}\neg\neg_i \qquad \dfrac{\Gamma\vdash\tau\Rightarrow\sigma\quad\Gamma\vdash\neg\sigma}{\Gamma\vdash\neg\tau}\text{MT}$$

Reglas clásicas

$$\dfrac{\Gamma,\neg\tau\vdash\bot}{\Gamma\vdash\tau}\text{PBC} \qquad \dfrac{}{\Gamma\vdash\tau\vee\neg\tau}\text{LEM}$$

i)

$$\dfrac{\dfrac{}{\Gamma,P\vdash P}Ax \qquad \dfrac{}{\Gamma,P\vdash\neg P}Ax}{\dfrac{\Gamma,P\vdash\bot}{\Gamma,P\vdash Q}\bot e}\neg e$$

$$\dfrac{\dfrac{}{\Gamma\vdash(P\Rightarrow Q)\Rightarrow Q}Ax \qquad \dfrac{\Gamma,P\vdash Q}{\Gamma\vdash(P\Rightarrow Q)}\Rightarrow i}{\underbrace{((P\Rightarrow Q)\Rightarrow Q),(Q\Rightarrow P),\neg P}_{\Gamma}\vdash P}\Rightarrow e$$

$$\dfrac{((P\Rightarrow Q)\Rightarrow Q),(Q\Rightarrow P),\neg P\vdash P \qquad \dfrac{}{((P\Rightarrow Q)\Rightarrow Q),(Q\Rightarrow P),\neg P\vdash\neg P}Ax}{\dfrac{((P\Rightarrow Q)\Rightarrow Q),(Q\Rightarrow P),\neg P\vdash\bot}{\dfrac{((P\Rightarrow Q)\Rightarrow Q),(Q\Rightarrow P)\vdash P}{\dfrac{((P\Rightarrow Q)\Rightarrow Q)\vdash(Q\Rightarrow P)\Rightarrow P}{\emptyset\vdash((P\Rightarrow Q)\Rightarrow Q)\Rightarrow((Q\Rightarrow P)\Rightarrow P)}\Rightarrow i}\Rightarrow i}PBC}\neg e$$

ii)

$$\dfrac{\dfrac{}{\Gamma,\neg P\vdash\neg Q}Ax \qquad \dfrac{\dfrac{}{\Gamma,\neg P\vdash\neg P\Rightarrow Q}Ax \qquad \dfrac{}{\Gamma,\neg P\vdash\neg P}Ax}{\Gamma,\neg P\vdash Q}}{\dfrac{\Gamma,\neg P\vdash\bot}{\Gamma\vdash P}PBC}\neg e$$

$$\dfrac{\dfrac{\dfrac{}{\Gamma\vdash P\Rightarrow Q}Ax \qquad \Gamma\vdash P}{\Gamma\vdash Q} \qquad \dfrac{}{\Gamma\vdash\neg Q}Ax}{\Gamma:\ P\Rightarrow Q,\ \neg P\Rightarrow Q,\ \neg Q\vdash\bot}\neg e$$

Lo q busqué

$\Gamma\vdash Q \qquad \dfrac{}{\Gamma\vdash\neg Q}Ax$

$\Gamma\vdash\neg P\Rightarrow Q$ y $\Gamma\vdash\neg P$

$$\dfrac{\dfrac{\dfrac{\qquad}{P\Rightarrow Q,\ \neg P\Rightarrow Q \vdash Q}\ \text{PBC}}{(P\Rightarrow Q) \vdash (\neg P\Rightarrow Q)\Rightarrow Q}\Rightarrow i}{\emptyset \vdash (P\Rightarrow Q)\Rightarrow((\neg P\Rightarrow Q)\Rightarrow Q)}\Rightarrow i$$

Ejercicio 10

Demostrar las siguientes tautologías utilizando deducción natural.

- I. $(P \Rightarrow (P \Rightarrow Q)) \Rightarrow (P \Rightarrow Q)$
- II. $(R \Rightarrow \neg Q) \Rightarrow ((R \wedge Q) \Rightarrow P)$
- III. $((P \Rightarrow Q) \Rightarrow (R \Rightarrow \neg Q)) \Rightarrow \neg(R \wedge Q)$

i)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash P\Rightarrow(P\Rightarrow Q)}\text{Ax} \qquad \dfrac{}{\Gamma\vdash P}\text{Ax}}{\Gamma\vdash P\Rightarrow Q}\Rightarrow e \qquad \dfrac{}{\Gamma\vdash P}\text{Ax}}{\Gamma = P\Rightarrow(P\Rightarrow Q),\ P \vdash Q}\Rightarrow e}{P\Rightarrow(P\Rightarrow Q)\vdash (P\Rightarrow Q)}\Rightarrow i}{\emptyset\vdash (P\Rightarrow(P\Rightarrow Q))\Rightarrow(P\Rightarrow Q)}\Rightarrow i$$

ii)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma,\neg R\vdash R\wedge Q}\text{Ax}}{\Gamma,\neg R\vdash R}\wedge e_1 \qquad \dfrac{}{\Gamma,\neg R\vdash\neg R}\text{Ax}}{\Gamma,\neg R\vdash\bot}\neg e}{\Gamma\vdash R}\text{PBC} \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma,R\vdash R\wedge Q}\text{Ax}}{\Gamma,R\vdash Q}\wedge e_2 \qquad \dfrac{\dfrac{}{\Gamma,R\vdash R\Rightarrow\neg Q}\text{Ax} \quad \dfrac{}{\Gamma,R\vdash R}\text{Ax}}{\Gamma,R\vdash\neg Q}\Rightarrow e}{\Gamma,R\vdash\bot}\neg e}{\Gamma\vdash\neg R}\neg i}{\dfrac{\Gamma = (R\Rightarrow\neg Q),(R\wedge Q),\neg P\vdash\bot}{(R\Rightarrow\neg Q),(R\wedge Q)\vdash P}\text{PBC}}}{\dfrac{(R\Rightarrow\neg Q)\vdash(R\wedge Q)\Rightarrow P}{\emptyset\vdash(R\Rightarrow\neg Q)\Rightarrow((R\wedge Q)\Rightarrow P)}\Rightarrow i}\Rightarrow i$$

Estrategia

$$\dfrac{R\Rightarrow\neg Q \quad R}{\neg Q} \qquad \underline{Q}$$

iii)

$$\dfrac{\dfrac{\dfrac{}{\Gamma\vdash R\wedge Q}\text{Ax}}{\Gamma\vdash R}\wedge e_1 \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma,R\vdash R\wedge Q}\text{Ax}}{\Gamma,R\vdash Q}\wedge e_2 \qquad \dfrac{\dfrac{\dfrac{}{\Gamma,R\vdash(P\Rightarrow Q)\Rightarrow(R\Rightarrow\neg Q)}\text{Ax} \quad \dfrac{\dfrac{\dfrac{}{\Gamma,R,P\vdash R\wedge Q}\text{Ax}}{\Gamma,R,P\vdash Q}\wedge e_2}{\Gamma,R\vdash(P\Rightarrow Q)}\Rightarrow i}{\Gamma,R\vdash R\Rightarrow\neg Q}\Rightarrow e \qquad \dfrac{}{\Gamma,R\vdash R}\text{Ax}}{\Gamma,R\vdash\neg Q}\Rightarrow e}{\Gamma,R\vdash\bot}\neg e}{\Gamma\vdash\neg R}\neg i}{\dfrac{\Gamma = ((P\Rightarrow Q)\Rightarrow(R\Rightarrow\neg Q)),(R\wedge Q)\vdash\bot}{((P\Rightarrow Q)\Rightarrow(R\Rightarrow\neg Q))\vdash\neg(R\wedge Q)}\neg i}\neg e \ \Rightarrow i$$

$$(P\Rightarrow Q)\Rightarrow\ldots \qquad \dfrac{\dfrac{R\wedge Q\ \checkmark}{Q}}{P\Rightarrow Q}\Rightarrow i$$

$$\emptyset \vdash ((P \Rightarrow Q) \Rightarrow (R \Rightarrow \neg Q)) \Rightarrow \neg(R \wedge Q)$$

Ejercicio 11

Probar que los siguientes secuentes son válidos sin usar principios de razonamiento clásicos:

- I. $(P \wedge Q) \wedge R, S \wedge T \vdash Q \wedge S$
- II. $(P \wedge Q) \wedge R \vdash P \wedge (Q \wedge R)$
- III. $P \Rightarrow (P \Rightarrow Q), P \vdash Q$
- IV. $Q \Rightarrow (P \Rightarrow R), \neg R, Q \vdash \neg P$
- V. $\vdash (P \wedge Q) \Rightarrow P$
- VI. $P \Rightarrow \neg Q, Q \vdash \neg P$
- VII. $P \Rightarrow Q \vdash (P \wedge R) \Rightarrow (Q \wedge R)$
- VIII. $Q \Rightarrow R \vdash (P \vee Q) \Rightarrow (P \vee R)$
- IX. $(P \vee Q) \vee R \vdash P \vee (Q \vee R)$
- X. $P \wedge (Q \vee R) \vdash (P \wedge Q) \vee (P \wedge R)$
- XI. $(P \wedge Q) \vee (P \wedge R) \vdash P \wedge (Q \vee R)$
- XII. $\neg P \vee Q \vdash P \Rightarrow Q$
- XIII. $P \Rightarrow Q, P \Rightarrow \neg Q \vdash \neg P$
- XIV. $P \Rightarrow (Q \Rightarrow R), P, \neg R \vdash \neg Q$

i)

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash (P \wedge Q) \wedge R}\,Ax}{\Gamma \vdash P \wedge Q}\,\wedge e_1}{\Gamma \vdash Q}\,\wedge e_2 \qquad \dfrac{\dfrac{}{\Gamma \vdash S \wedge T}\,Ax}{\Gamma \vdash S}\,\wedge e_1$$

$$\dfrac{\Gamma \vdash Q \qquad \Gamma \vdash S}{\Gamma = (P \wedge Q) \wedge R,\ S \wedge T \vdash Q \wedge S}\,\wedge i$$

ii)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash (P \wedge Q) \wedge R}\,Ax}{\Gamma \vdash P \wedge Q}\,\wedge e_1}{\Gamma \vdash P}\,\wedge e_1 \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash (P \wedge Q) \wedge R}\,Ax}{\Gamma \vdash (P \wedge Q)}\,\wedge e_1}{\Gamma \vdash Q}\,\wedge e_2 \qquad \dfrac{\dfrac{}{\Gamma \vdash (P \wedge Q) \wedge R}\,Ax}{\Gamma \vdash R}\,\wedge e_2}{\Gamma \vdash Q \wedge R}\,\wedge i}{\Gamma = (P \wedge Q) \wedge R \vdash P \wedge (Q \wedge R)}\,\wedge i$$

iii)

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash P \Rightarrow (P \Rightarrow Q)}\,Ax \qquad \dfrac{}{\Gamma \vdash P}\,Ax}{\Gamma \vdash P \Rightarrow Q}\,\Rightarrow e \qquad \dfrac{}{\Gamma \vdash P}\,Ax}{\Gamma = P \Rightarrow (P \Rightarrow Q), P \vdash Q}\,\Rightarrow e$$

iv)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma, P \vdash Q \Rightarrow (P \Rightarrow R)}\,Ax \qquad \dfrac{}{\Gamma, P \vdash Q}\,Ax}{\Gamma, P \vdash P \Rightarrow R}\,\Rightarrow e \qquad \dfrac{}{\Gamma, P \vdash P}\,Ax}{\Gamma, P \vdash R}\,\Rightarrow e \qquad \dfrac{}{\Gamma, P \vdash \neg R}\,Ax}{\Gamma, P \vdash \bot}\,\neg e}{\;}\,\neg i$$

$\Gamma = Q \Rightarrow (P \Rightarrow R), \neg R, Q \vdash \neg P$

$\Gamma \vdash T \Rightarrow G$ $\qquad$ $\Gamma \vdash T$ $\qquad$ ESTRATEGIA

v)

$$\dfrac{\dfrac{\dfrac{}{P\land Q \vdash P\land Q}\,Ax}{P\land Q \vdash P}\,\land e_1}{\emptyset \vdash (P\land Q) \Rightarrow P}\,\Rightarrow i$$

vi)

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash Q}\,Ax \qquad \dfrac{\dfrac{}{\Gamma \vdash P \Rightarrow \neg Q}\,Ax \quad \dfrac{}{\Gamma \vdash P}\,Ax}{\Gamma \vdash \neg Q}\,\Rightarrow e}{\Gamma = P \Rightarrow \neg Q, Q, P \vdash \bot}\,\neg e}{P \Rightarrow \neg Q, Q \vdash \neg P}\,\neg i$$

vii)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash P \Rightarrow Q}\,Ax \qquad \dfrac{\dfrac{}{\Gamma \vdash P \land R}\,Ax}{\Gamma \vdash P}\,\land e_1}{\Gamma \vdash Q}\,\Rightarrow e \qquad \dfrac{\dfrac{}{\Gamma \vdash P \land R}\,Ax}{\Gamma \vdash R}\,\land e_2}{\Gamma = P \Rightarrow Q, P \land R \vdash Q \land R}\,\land i}{P \Rightarrow Q \vdash (P \land R) \Rightarrow (Q \land R)}\,\Rightarrow i$$

viii)

"O BIEN P V, O BIEN Q V"

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash P \lor Q}\,Ax \qquad \dfrac{\dfrac{}{\Gamma, P \vdash P}\,Ax}{\Gamma, P \vdash P \lor R}\,\lor i_1 \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, Q \vdash Q \Rightarrow R}\,Ax \quad \dfrac{}{\Gamma, Q \vdash Q}\,Ax}{\Gamma, Q \vdash R}\,\Rightarrow e}{\Gamma, Q \vdash P \lor R}\,\lor i_2}{\Gamma = (Q \Rightarrow R), (P \lor Q) \vdash P \lor R}\,\lor e}{Q \Rightarrow R \vdash (P \lor Q) \Rightarrow (P \lor R)}\,\Rightarrow i$$

ix)

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma \vdash (P\vee Q)\vee R$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma \vdash P\vee Q$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma, P \vdash P$}
\RightLabel{$\vee_{e1}$}
\UnaryInfC{$\Gamma, P \vdash P\vee(Q\vee R)$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma, Q \vdash Q$}
\RightLabel{$\vee_{e1}$}
\UnaryInfC{$\Gamma, Q \vdash Q\vee R$}
\TrinaryInfC{$\Gamma, (P\vee Q) \vdash P\vee(Q\vee R)$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma, R \vdash R$}
\RightLabel{$\vee_{i2\times 2}$}
\UnaryInfC{$\Gamma, R \vdash P\vee(Q\vee R)$}
\TrinaryInfC{$(P\vee Q)\vee R \vdash P\vee(Q\vee R)$}
\end{prooftree}
$$

O vale P o vale Q. PERO P y Q

x)

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma \vdash P\wedge(Q\vee R)$}
\RightLabel{$\wedge_{e2}$}
\UnaryInfC{$\Gamma \vdash Q\vee R$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma' \vdash P\wedge(Q\vee R)$}
\RightLabel{$\wedge_{e1}$}
\UnaryInfC{$\Gamma' \vdash P$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma' \vdash Q$}
\RightLabel{$\wedge_i$}
\BinaryInfC{$\Gamma' = P\wedge(Q\vee R), Q \vdash (P\wedge Q)$}
\RightLabel{$\vee_{i1}$}
\UnaryInfC{$P\wedge(Q\vee R), Q \vdash (P\wedge Q)\vee(P\wedge R)$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma'' \vdash P\wedge(Q\vee R)$}
\RightLabel{$\wedge_{e1}$}
\UnaryInfC{$\Gamma'' \vdash P$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma'' \vdash R$}
\RightLabel{$\wedge_i$}
\BinaryInfC{$\Gamma'' \vdash P\wedge R$}
\RightLabel{$\vee_{i2}$}
\UnaryInfC{$\Gamma'' = P\wedge(Q\vee R), R \vdash (P\wedge Q)\vee(P\wedge R)$}
\RightLabel{$\vee_e$}
\TrinaryInfC{$P\wedge(Q\vee R) \vdash (P\wedge Q)\vee(P\wedge R)$}
\end{prooftree}
$$

Si valen P y Q. ME QUEDO CON $(P\wedge Q)$

SE Q VALEN P y R. ME QUEDO CON PR

P vale. Averiguo si vale Q o R.

xi)

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma \vdash (P\wedge Q)\vee(P\wedge R)$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma, (P\wedge Q) \vdash P\wedge Q$}
\RightLabel{$\wedge_{e1}$}
\UnaryInfC{$\Gamma, (P\wedge Q) \vdash P$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma, (P\wedge Q) \vdash (P\wedge Q)$}
\RightLabel{$\wedge_{e2}$}
\UnaryInfC{$\Gamma, (P\wedge Q) \vdash Q$}
\RightLabel{$\vee_{i1}$}
\UnaryInfC{$\Gamma, (P\wedge Q) \vdash Q\vee R$}
\RightLabel{$\wedge_i$}
\BinaryInfC{$\Gamma, (P\wedge Q) \vdash P\wedge(Q\vee R)$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma, (P\wedge R) \vdash (P\wedge R)$}
\RightLabel{$\wedge_{e1}$}
\UnaryInfC{$\Gamma, (P\wedge R) \vdash P$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma, (P\wedge R) \vdash (P\wedge R)$}
\RightLabel{$\wedge_{e2}$}
\UnaryInfC{$\Gamma, (P\wedge R) \vdash R$}
\RightLabel{$\vee_{i2}$}
\UnaryInfC{$\Gamma, (P\wedge R) \vdash (Q\vee R)$}
\RightLabel{$\wedge_i$}
\BinaryInfC{$\Gamma, (P\wedge R) \vdash P\wedge(Q\vee R)$}
\RightLabel{$\vee_e$}
\TrinaryInfC{$(P\wedge Q)\vee(P\wedge R) \vdash P\wedge(Q\vee R)$}
\end{prooftree}
$$

VALEN P y Q. TENGO P, BUSCO Q

VALEN P y R. TENGO P, BUSCO R.

P vale en ambos.

Debo ver si valen Q o R.

xii)

$$
\begin{prooftree}
\AxiomC{}
\LeftLabel{Ax}
\UnaryInfC{$\neg P\vee Q, P \vdash \neg P\vee Q$}
\AxiomC{}
\LeftLabel{Ax}
\UnaryInfC{$\Gamma' \vdash P$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma' \vdash \neg P$}
\RightLabel{$\neg e$}
\BinaryInfC{$\Gamma' \vdash \bot$}
\RightLabel{$\bot e$}
\UnaryInfC{$\Gamma' = \neg P\vee Q, P, \neg P \vdash Q$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\neg P\vee Q, P, Q \vdash Q$}
\TrinaryInfC{$\neg P\vee Q, P \vdash Q$}
\RightLabel{$\Rightarrow_i$}
\UnaryInfC{}
\end{prooftree}
$$

FUERZO BOT.

$\neg P \vee Q \vdash P \Rightarrow Q$

O vale $\neg P$ o Q. Necessito dos casos.

xiii)

$$\dfrac{\dfrac{\dfrac{}{\Gamma' \vdash P \Rightarrow Q}\,\text{Ax} \qquad \dfrac{}{\Gamma' \vdash P}\,\text{Ax}}{\Gamma' = P \Rightarrow Q,\ P \Rightarrow \neg Q,\ P \vdash Q}\,{\Rightarrow}e \qquad \dfrac{\dfrac{}{\Gamma'' \vdash P \Rightarrow \neg Q}\,\text{Ax} \qquad \dfrac{}{\Gamma'' \vdash P}\,\text{Ax}}{\Gamma'' = P \Rightarrow Q,\ P \Rightarrow \neg Q,\ P \vdash \neg Q}\,{\Rightarrow}e}{\dfrac{P \Rightarrow Q,\ P \Rightarrow \neg Q,\ P \vdash \bot}{P \Rightarrow Q,\ P \Rightarrow \neg Q \vdash \neg P}\,\neg i}\,\neg e$$

VALE P, BUSCO METER P A CONTEXTO

xiv)

$$\dfrac{\dfrac{\dfrac{}{\Gamma'' \vdash \neg R}\,\text{Ax} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma'' \vdash P \Rightarrow (Q \Rightarrow R)}\,\text{Ax} \qquad \dfrac{}{\Gamma'' \vdash P}\,\text{Ax}}{\Gamma'' \vdash (Q \Rightarrow R)}\,{\Rightarrow}e \qquad \dfrac{}{\Gamma'' \vdash Q}\,\text{Ax}}{\Gamma'' \vdash R}\,{\Rightarrow}e}{\Gamma'' = P \Rightarrow (Q \Rightarrow R),\ P,\ \neg R,\ Q \vdash \bot}\,\neg e}{\Gamma' = P \Rightarrow (Q \Rightarrow R),\ P,\ \neg R \vdash \neg Q}\,\neg i$$

VALE P

NO VALE R.

$(Q \Rightarrow R)$. necesito Q xq gracias a él habría R. $(Q = V)$