Ejercicio 1

Dados $\mathcal{F} = \{d, f, g\}$, donde d itene aridad 0, f aridad 2 y g aridad 3. ¿Cuáles de las siguientes cadenas son términos sobre $\mathcal{F}$?

- I. $g(d, d)$ ✗ g es ARIDAD 3.
- II. $f(X, g(Y, Z), d)$ ✗ f es ARIDAD 2
- III. $g(X, f(d, Z), d)$ ✓
- IV. $g(X, h(Y, Z), d)$ ✗ h no es símbolo de función.
- V. $f(f(g(d, X), f(g(d, X), Y, g(Y, d)), g(d, d)), g(f(d, d, X), d), Z)$ ✗ f es ARIDAD 2.

Ejercicio 3

Sea $\sigma = \exists X.P(Y, Z) \wedge \forall Y.\neg Q(Y, X) \vee P(Y, Z)$

I. Identificar todas las variables libres y ligadas.

II. Calcular $\sigma\{X := W\}$, $\sigma\{Y := W\}$, $\sigma\{Y := f(X)\}$ y $\sigma\{Z := g(Y, Z)\}$.

i) $\exists X . P(Y, Z) \wedge \forall Y . \neg Q(Y, X) \vee P(Y, Z)$

- LIGADA
- LIBRE

ii) Renombro las ligadas que se pueden confundir con libres. Vale por α-equivalencia.

a) $\exists X . P(Y, Z) \wedge \forall G . \neg Q(G, X) \vee P(G, Z) . \sigma\{X := W\}$

No hace nada, no existe X que está libre.

b) $\exists X . P(Y, Z) \wedge \forall G . \neg Q(G, X) \vee P(G, Z) . \sigma\{Y := W\}$

$\exists X . P(W, Z) \wedge \forall Y . \neg Q(Y, X) \vee P(Y, Z)$

c) $\exists X . P(Y, Z) \wedge \forall G . \neg Q(G, X) \vee P(G, Z) . \sigma\{Y := f(X)\}$

$\exists X . P(f(X), Z) \wedge \forall G . \neg Q(G, X) \vee P(G, Z) .$

d) $\exists X . P(Y, Z) \wedge \forall G . \neg Q(G, X) \vee P(G, Z) . \sigma\{Z := g(Y, Z)\}$ ¿TIENE SENTIDO? Sí. Es una OP. SINTÁCTICA, vale $Z := g(Y, Z)$.

$\exists X . P(Y, g(Y, Z)) \wedge \forall G . \neg Q(G, X) \vee P(G, g(Y, Z))$

Ejercicio 5 ★

Unir con flechas las expresiones que unifican entre sí (entre una fila y la otra). Para cada par unificable, exhibir el *mgu* ("most general unifier"). Asumir que a es una constante, X, Y, Z son variables, f y g son símbolos de función, y P y Q predicados.

$P(f(X))$	$P(\mathrm{a})$	$P(Y)$	$Q(X, f(Y))$	$Q(X, f(Z))$	$Q(X, f(\mathrm{a}))$	X	$f(X)$
$P(X)$	$P(f(\mathrm{a}))$	$P(g(Z))$	$Q(f(Y), X)$	$Q(f(Y), f(X))$	$Q(f(Y), Y)$	$f(f(c))$	$f(g(Y))$

1) $P(f(X))$ Con: $P(f(a))$

2) $P(a)$ Con: $P(X)$

3) $P(Y)$ Con: $P(X)$, $P(f(a))$, $P(g(Z))$

4) $Q(X, f(Y))$ Con: $Q(f(Y), X)$

5) $Q(X, f(Z))$ Con: $Q(f(Y), X)$, $Q(f(Y), f(X))$, $Q(f(Y), Y)$

6) $Q(X, f(a))$ Con: $Q(f(Y), f(X))$, $Q(f(Y), Y)$

7) X Con: $P(f(a))$, $P(g(Z))$, $f(f(c))$, $f(g(Y))$ ✓

8) $f(X)$ Con: $f(f(c))$, $f(g(Y))$

6) $f(x)$ Con: $f(f(a)), f(g(y))$

1) a) MGU$\{P(f(x)) \stackrel{?}{=} P(f(a))\}$

$\xrightarrow{DEC} \{f(x) \stackrel{?}{=} f(a)\}$

$\xrightarrow{DEC} \{x \stackrel{?}{=} a\}$

$\xrightarrow[\{x:=a\}]{ELim} \emptyset$

MGU: $\{x:=a\}$

2) MGU$\{P(a) \stackrel{?}{=} P(x)\}$

$\xrightarrow{DEC} \{a \stackrel{?}{=} x\}$

$\xrightarrow{SWAP} \{x \stackrel{?}{=} a\}$

$\xrightarrow[\{x:=a\}]{ELim} \emptyset$

MGU: $\{x:=a\}$

3)

a) MGU$\{P(y) \stackrel{?}{=} P(x)\}$

$\xrightarrow{DEC} \{y \stackrel{?}{=} x\}$

$\xrightarrow[\{y:=x\}]{ELim} \emptyset$

MGU: $\{y:=x\}$

b) MGU$\{P(y) \stackrel{?}{=} P(f(a))\}$

$\xrightarrow{DEC} \{y \stackrel{?}{=} f(a)\}$

$\xrightarrow[\{y:=f(a)\}]{ELim} \emptyset$

MGU: $\{y:=f(a)\}$

c) MGU$\{P(y) \stackrel{?}{=} P(g(z))\}$

$\xrightarrow{DEC} \{y \stackrel{?}{=} g(z)\}$

$\xrightarrow[\{y:=g(z)\}]{ELim} \emptyset$

$MGU: \{y := f(z)\}$

4)

a) $MGU\{Q(x, f(y)) \stackrel{?}{=} Q(f(y), x)\}$

$\xrightarrow{DEC} \{x \stackrel{?}{=} f(y), f(y) \stackrel{?}{=} x\}$

$\xrightarrow[\{x:=f(y)\}]{ELIM} \{f(y) \stackrel{?}{=} f(y)\}$

$\xrightarrow{DEC} \{y \stackrel{?}{=} y\}$

$\xrightarrow{DEC} \emptyset$

$MGU: \{x := f(y)\}$

5)

a) $MGU\{Q(x, f(z)) \stackrel{?}{=} Q(f(y), x)\}$

$\xrightarrow{DEC} \{x \stackrel{?}{=} f(y), f(z) \stackrel{?}{=} x\}$

$\xrightarrow[\{x:=f(y)\}]{ELIM} \{f(z) \stackrel{?}{=} f(y)\}$

$\xrightarrow{DEC} \{z \stackrel{?}{=} y\}$

$\xrightarrow[\{z:=y\}]{ELIM} \emptyset$

$MGU: S_2 \circ S_1 = \{z := y\} \circ \{x := f(y)\}$

$= \{z := y, x := f(z)\}$

b) $MGU\{Q(x, f(z)) \stackrel{?}{=} Q(f(y), f(x))\}$

$\xrightarrow{DEC} \{x \stackrel{?}{=} f(y), f(z) \stackrel{?}{=} f(x)\}$

$\xrightarrow{DEC} \{x \stackrel{?}{=} f(y), z \stackrel{?}{=} x\}$

$\xrightarrow[\{x:=f(y)\}]{ELIM} \{z \stackrel{?}{=} f(y)\}$

$\xrightarrow[\{z:=f(y)\}]{ELIM} \emptyset$

$MGU: S_2 \circ S_1 = \{z := f(y)\} \circ \{x := f(y)\}$

$= \{z := f(y), x := f(y)\}$ PASO IDEMPOTENTE

c) $MGU\{Q(X, f(Z)) \stackrel{?}{=} Q(f(Y), Y)\}$

$\xrightarrow{DEC} \{X \stackrel{?}{=} f(Y),\ f(Z) \stackrel{?}{=} Y\}$

$\xrightarrow{SWAP} \{X \stackrel{?}{=} f(Y),\ Y \stackrel{?}{=} f(Z)\}$

$\underset{\{Y:=f(Z)\}}{\xrightarrow{ELIM}} \{X \stackrel{?}{=} f(f(Z))\}$

$\underset{\{X:=f(f(Z))\}}{\xrightarrow{ELIM}} \emptyset$

$MGU: S_2 \circ S_1 = \{X := f(f(Z))\} \circ \{Y := f(Z)\}$

$= \{X := f(f(Z)),\ Y := f(Z)\}$ PASO IDEMPOTENTE.

6)

a) $MGU\{Q(X, f(a)) \stackrel{?}{=} Q(f(Y), f(X))\}$

$\xrightarrow{DEC} \{X \stackrel{?}{=} f(Y),\ f(a) \stackrel{?}{=} f(X)\}$

$\underset{\{X:=f(Y)\}}{\xrightarrow{ELIM}} \{f(a) \stackrel{?}{=} f(f(Y))\}$

$\xrightarrow{DEC} \{a \stackrel{?}{=} f(Y)\}$ CLASH. a es un símbolo función de aridad 0 dif a $f(Y)$.

b) $MGU\{Q(X, f(a)) \stackrel{?}{=} Q(f(Y), Y)\}$

$\xrightarrow{DEC} \{X \stackrel{?}{=} f(Y),\ f(a) \stackrel{?}{=} Y\}$

$\xrightarrow{SWAP} \{X \stackrel{?}{=} f(Y),\ Y \stackrel{?}{=} f(a)\}$

$\underset{\{Y:=f(a)\}}{\xrightarrow{ELIM}} \{X \stackrel{?}{=} f(f(a))\}$

$\underset{\{X:=f(f(a))\}}{\xrightarrow{ELIM}} \emptyset$

$MGU: \{X := f(f(a))\} \circ \{Y := f(a)\}$

$= \{X := f(f(a)),\ Y := f(a)\}$ PASO IDEMPOTENTE.

7)

a) $MGU\{X \stackrel{?}{=} P(f(a))\}$

$\underset{\{X:=P(f(a))\}}{\xrightarrow{ELIM}} \emptyset$

$MGU: \{X := P(f(a))\}$

b) $MGU\{X \overset{?}{=} P(f(Z))\}$

$\xrightarrow{ELIM} \emptyset$

$\{X := P(f(Z))\}$

$MGU: \{X := P(f(Z))\}$

c) $MGU\{X \overset{?}{=} f(f(c))\}$

$MGU: \{X := f(f(c))\}$

d) $MGU\{X \overset{?}{=} f(g(Y))\}$

$MGU: \{X := f(g(Y))\}$

8)

a) $MGU\{f(X) \overset{?}{=} f(f(c))\}$

$\xrightarrow{DEC} \{X \overset{?}{=} f(c)\}$

$\xrightarrow{ELIM} \emptyset$

$\{X := f(c)\}$

$MGU: \{X := f(c)\}$

b) $MGU\{f(X) \overset{?}{=} f(g(Y))\}$

$\xrightarrow{DEC} \{X \overset{?}{=} g(Y)\}$

$\xrightarrow{ELIM} \emptyset$

$\{X := g(Y)\}$

$MGU: \{X := g(Y)\}$

Ejercicio 6 ★

Determinar, para cada uno de los siguientes pares de términos de primer orden, si son unificables o no. En cada caso justificar su respuesta exhibiendo una secuencia exitosa o fallida (según el caso) del algoritmo de Martelli-Montanari. Asimismo, en caso de que los términos sean unificables indicar el *mgu* ("most general unifier"). Notación: X, Y, Z variables; a, b, c constantes; f, g símbolos de función.

I. $f(X, X, Y)$ y $f(a, b, Z)$
II. Y y $f(X)$
III. $f(g(c, Y), X))$ y $f(Z, g(Z, a))$
IV. $f(a)$ y $g(Y)$
V. $f(X)$ y X
VI. $g(X, Y)$ y $g(f(Y), f(X))$

i) $MGU\{f(X, X, Y) \overset{?}{=} f(a, b, Z)\}$

$\xrightarrow{DEC} \{X \overset{?}{=} a, X \overset{?}{=} b, Y \overset{?}{=} Z\}$

$\xrightarrow{ELIM} \{a \overset{?}{=} b, Y \overset{?}{=} Z\}$

$\{X := a\}$

Clash, $a \neq b$.

ii) MGU $\{Y \doteq f(X)\}$

MGU: $\{Y := f(X)\}$

iii) MGU $\{f(g(c,Y),X) \overset{?}{\doteq} f(Z, g(Z,a))\}$

$\xrightarrow{DEC} \{g(c,Y) \overset{?}{\doteq} Z,\ X \overset{?}{\doteq} g(Z,a)\}$

$\xrightarrow{SWAP} \{Z \overset{?}{\doteq} g(c,Y),\ X \overset{?}{\doteq} (Z,a)\}$

$\xrightarrow[\{Z := g(c,Y)\}]{ELIM} \{X \overset{?}{\doteq} (g(c,Y),a)\}$

$\xrightarrow[\{X := (g(c,Y),a)\}]{ELIM} \emptyset$

MGU: $\{X := (g(c,Y),a)\} \circ \{Z := g(c,Y)\}$

$= \{X := (g(c,Y),a),\ Z := g(c,Y)\}$ PASO IDEMPOTENTE.

iv) CLASH.

v) OCCURS-CHECK.

vi) MGU $\{g(X,Y) \overset{?}{\doteq} g(f(Y), f(X))\}$

$\xrightarrow{DEC} \{X \overset{?}{\doteq} f(Y),\ Y \overset{?}{\doteq} f(X)\}$

$\xrightarrow[\{X := f(Y)\}]{ELIM} \{Y \overset{?}{\doteq} f(f(Y))\}$

$\xrightarrow{O\text{-}C}$ FALLA POR OC-CHECK

Ejercicio 8 ★

Sean las constantes Nat y Bool y la función binaria → (representada como un operador infijo), determinar el resultado de aplicar el algoritmo MGU ("most general unifier") sobre las ecuaciones planteadas a continuación. En caso de tener éxito, mostrar la sustitución resultante.

I. MGU $\{\mathbf{T}_1 \to \mathbf{T}_2 \doteq \mathsf{Nat} \to \mathsf{Bool}\}$

II. MGU $\{\mathbf{T}_1 \to \mathbf{T}_2 \doteq \mathbf{T}_3\}$

III. MGU $\{\mathbf{T}_1 \to \mathbf{T}_2 \doteq \mathbf{T}_2\}$

IV. MGU $\{(\mathbf{T}_2 \to \mathbf{T}_1) \to \mathsf{Bool} \doteq \mathbf{T}_2 \to \mathbf{T}_3\}$

V. MGU $\{\mathbf{T}_2 \to \mathbf{T}_1 \to \mathsf{Bool} \doteq \mathbf{T}_2 \to \mathbf{T}_3\}$

VI. MGU $\{\mathbf{T}_1 \to \mathsf{Bool} \doteq \mathsf{Nat} \to \mathsf{Bool}, \mathbf{T}_1 \doteq \mathbf{T}_2 \to \mathbf{T}_3\}$

VII. MGU $\{\mathbf{T}_1 \to \mathsf{Bool} \doteq \mathsf{Nat} \to \mathsf{Bool}, \mathbf{T}_2 \doteq \mathbf{T}_1 \to \mathbf{T}_1\}$

VIII. MGU $\{\mathbf{T}_1 \to \mathbf{T}_2 \doteq \mathbf{T}_3 \to \mathbf{T}_4, \mathbf{T}_3 \doteq \mathbf{T}_2 \to \mathbf{T}_1\}$

i) MGU $\{T_1 \to T_2 \overset{?}{\doteq} NAT \to Bool\}$

$\xrightarrow{DEC} \{T_1 \overset{?}{\doteq} NAT,\ T_2 \overset{?}{\doteq} Bool\}$

$\xrightarrow[\{T_1 := NAT\}]{ELIM} \{T_2 \overset{?}{\doteq} Bool\}$

$\xrightarrow[\{T_2 := Bool\}]{ELIM} \emptyset$

MGU: $\{T_2 := Bool\} \circ \{T_1 := NAT\}$

$= \{T_2 := Bool,\ T_1 := NAT\}$ PASO IDEMPOTENTE.

ii) $MGU\{T_1 \to T_2 \overset{?}{=} T_3\}$

$\xrightarrow{\text{Swap}} \{T_3 \overset{?}{=} T_1 \to T_2\}$

$\xrightarrow{\text{Elim}} \emptyset$

$MGU: \{T_3 := T_1 \to T_2\}$

iii) $MGU\{T_1 \to T_2 \overset{?}{=} T_2\}$

$\xrightarrow{\text{Swap}} MGU\{T_2 \overset{?}{=} T_1 \to T_2\}$

$\xrightarrow{\text{OC}}$ FALLA POR OCCURS-CHECK ✓

iv) $MGU\{(T_2 \to T_1) \to Bool \overset{?}{=} T_2 \to T_3\}$

$\xrightarrow{\text{DEC}} \{T_2 \to T_1 \overset{?}{=} T_2,\ Bool \overset{?}{=} T_3\}$

$\xrightarrow{\text{Swap}} \{T_2 \overset{?}{=} T_2 \to T_1,\ Bool \overset{?}{=} T_3\}$

$\xrightarrow{\text{O-C}}$ FALLA POR OCCURS-CHECK

v) $MGU\{T_2 \to (T_1 \to Bool) \overset{?}{=} T_2 \to T_3\}$

$\xrightarrow{\text{DEC}} \{T_2 \overset{?}{=} T_2 \to T_3 \ldots\}$

$\xrightarrow{\text{O-C}}$ FALLA POR OCCURS-CHECK

vi) $MGU\{T_1 \to Bool \overset{?}{=} NAT \to Bool,\ T_1 \overset{?}{=} T_2 \to T_3\}$

$\xrightarrow{\text{DEC}} \{T_1 \overset{?}{=} NAT,\ Bool \overset{?}{=} Bool,\ T_1 \overset{?}{=} T_2 \to T_3\}$

$\xrightarrow{\text{DEC}} \{T_1 \overset{?}{=} NAT,\ T_1 \overset{?}{=} T_2 \to T_3\}$

$\xrightarrow[\{T_1 := NAT\}]{\text{Elim}} \{NAT \overset{?}{=} T_2 \to T_3\}$

$\neq$ $\quad \to (a.b) \neq NAT.$

$\xrightarrow{\text{CLASH}}$ FALLA POR CLASH

vii) $MGU\{T_1 \to Bool \overset{?}{=} NAT \to Bool,\ T_2 \overset{?}{=} T_1 \to T_1\}$

$\xrightarrow{\text{DEC}} \{T_1 \overset{?}{=} NAT,\ Bool \overset{?}{=} Bool,\ T_2 \overset{?}{=} T_1 \to T_1\}$

$\xrightarrow{\text{DEC}} \{T_1 \overset{?}{=} NAT,\ T_2 \overset{?}{=} T_1 \to T_1\}$

$\xrightarrow[\{T_1 := NAT\}]{\text{Elim}} \{T_2 \overset{?}{=} NAT \to NAT\}$

$\xrightarrow[\{T_2 := NAT \to NAT\}]{\text{Elim}} \emptyset$

$MGU: \{T_2 := NAT \to NAT\} \circ \{T_1 := NAT\}$

$: \{T_2 := NAT \to NAT,\ T_1 := NAT\}$

viii) $\text{Mgu}\{T_1 \to T_2 \overset{?}{=} T_3 \to T_4,\ T_3 \overset{?}{=} T_2 \to T_1\}$

$\xrightarrow{\text{DEC}} \{T_1 \overset{?}{=} T_3,\ T_2 \overset{?}{=} T_4,\ T_3 \overset{?}{=} T_2 \to T_1\}$

$\xrightarrow{\text{ELIM}} \{T_2 \overset{?}{=} T_4,\ T_3 \overset{?}{=} T_2 \to T_3\}$

$\{T_1 := T_3\}$

$\xrightarrow{\text{O-C}}$ FALLA POR OCCURS - CHECK

Ejercicio 9 ★

Demostrar en deducción natural que vale $\vdash \sigma$ para cada una de las siguientes fórmulas, **sin usar principios de razonamiento clásicos**, salvo que se indique lo contrario:

I. Intercambio ($\forall$): $\forall X.\forall Y.P(X,Y) \iff \forall Y.\forall X.P(X,Y)$.

II. Intercambio ($\exists$): $\exists X.\exists Y.P(X,Y) \iff \exists Y.\exists X.P(X,Y)$.

III. Intercambio ($\exists/\forall$): $\exists X.\forall Y.P(X,Y) \implies \forall Y.\exists X.P(X,Y)$.

IV. Universal implica existencial: $\forall X.P(X) \implies \exists X.P(X)$.

V. Diagonal ($\forall$): $\forall X.\forall Y.P(X,Y) \implies \forall X.P(X,X)$.

VI. Diagonal ($\exists$): $\exists X.P(X,X) \implies \exists X.\exists Y.P(X,Y)$.

VII. de Morgan (I): $\neg\exists X.P(X) \iff \forall X.\neg P(X)$.

VIII. de Morgan (II): $\neg\forall X.P(X) \iff \exists X.\neg P(X)$.
Para la dirección $\Rightarrow$ es necesario usar principios de razonamiento clásicos.

IX. Universal/conjunción: $\forall X.(P(X) \wedge Q(X)) \iff (\forall X.P(X) \wedge \forall X.Q(X))$.

X. Universal/disyunción: $\forall X.(P(X) \vee \sigma) \iff (\forall X.P(X)) \vee \sigma$, asumiendo que $X \notin \mathsf{fv}(\sigma)$.
Para la dirección $\Rightarrow$ es necesario usar principios de razonamiento clásicos.

XI. Existencial/disyunción: $\exists X.(P(X) \vee Q(X)) \iff (\exists X.P(X) \vee \exists X.Q(X))$.

XII. Existencial/conjunción: $\exists X.(P(X) \wedge \sigma) \iff (\exists X.P(X) \wedge \sigma)$, asumiendo que $X \notin \mathsf{fv}(\sigma)$.

i)

$\Rightarrow$)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \forall x.\forall y.P(x,y)}\text{Ax}}{\Gamma \vdash \forall y.P(x,y)}\{x:=x\}}{\Gamma \vdash P(x,y)}\forall e\ \{y:=y\}}{\Gamma = \forall x.\forall y.P(x,y) \vdash \forall x.P(x,y)}\forall i}{\forall x.\forall y.P(x,y) \vdash \forall y.\forall x.P(x,y)}\forall i}{\vdash \forall x.\forall y.P(x,y) \Rightarrow \forall y.\forall x.P(x,y)}\Rightarrow i$$

$\Leftarrow$)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma \vdash \forall y.\forall x.P(x,y)}\text{Ax}}{\Gamma \vdash \forall x.P(x,y)}\{y:=y\}}{\Gamma \vdash P(x,y)}\forall e\ \{x:=x\}}{\Gamma \vdash \forall y.P(x,y)}\forall i}{\Gamma = \forall y.\forall x.P(x,y) \vdash \forall x.\forall y.P(x,y)}\forall i}{\vdash \forall y.\forall x.P(x,y) \Rightarrow \forall x.\forall y.P(x,y)}\Rightarrow i$$

ii)

$\Rightarrow$)

=>)

$$\frac{}{\Gamma, P(x_0,y_0) \vdash P(x_0,y_0)}\ Ax \quad \{x := x_0\}$$

$$\frac{\Gamma, P(x_0,y_0) \vdash P(x_0,y_0)}{\Gamma, P(x_0,y_0) \vdash \exists x . P(x,y_0)}\ \exists_i \quad \{y := y_0\}$$

$$\frac{\Gamma, P(x_0,y_0) \vdash \exists x . P(x,y_0)}{\Gamma, P(x_0,y_0) \vdash \exists y . \exists x . P(x,y)}\ \exists_i$$

$$\frac{\dfrac{}{\Gamma \vdash \exists x . \exists y . P(x,y)}\ Ax \qquad \Gamma, P(x_0,y_0) \vdash \exists y . \exists x . P(x,y)}{\Gamma = \exists x . \exists y . P(x,y) \ \vdash \ \exists y . \exists x . P(x,y)}\ \exists e$$

$$\frac{\Gamma = \exists x . \exists y . P(x,y) \ \vdash \ \exists y . \exists x . P(x,y)}{\vdash \exists x . \exists y . P(x,y) \Rightarrow \exists y . \exists x . P(x,y)}\ \Rightarrow_i$$

<=)

$$\frac{}{\Gamma, P(x_0,y_0) \vdash P(x_0,y_0)}\ Ax \quad \{y := y_0\}$$

$$\frac{\Gamma, P(x_0,y_0) \vdash P(x_0,y_0)}{\Gamma, P(x_0,y_0) \vdash \exists y . P(x_0,y)}\ \exists_i \quad \{x := x_0\}$$

$$\frac{\Gamma, P(x_0,y_0) \vdash \exists y . P(x_0,y)}{\Gamma, P(x_0,y_0) \vdash \exists x . \exists y . P(x,y)}\ \exists_i$$

$$\frac{\dfrac{}{\Gamma \vdash \exists y . \exists x . P(x,y)}\ Ax \qquad \Gamma, P(x_0,y_0) \vdash \exists x . \exists y . P(x,y)}{\Gamma = \exists y . \exists x . P(x,y) \vdash \exists x . \exists y . P(x,y)}\ \exists e$$

$$\frac{\Gamma = \exists y . \exists x . P(x,y) \vdash \exists x . \exists y . P(x,y)}{\vdash \exists y . \exists x . P(x,y) \Rightarrow \exists x . \exists y . P(x,y)}\ \Rightarrow_i$$

iii)

$$\frac{}{\Gamma' \vdash \forall y . P(x,y)}\ Ax$$

$$\frac{\Gamma' \vdash \forall y . P(x,y)}{\Gamma' \vdash P(x,y)}\ \forall_i \quad \{y := y\}$$

$$\frac{\Gamma' \vdash P(x,y)}{\Gamma' \vdash \exists x . P(x,y)}\ \exists_i$$

$$\frac{\Gamma' \vdash \exists x . P(x,y)}{\Gamma' = \Gamma, \forall y . P(x,y) \vdash \forall y . \exists x . P(x,y)}\ \forall_i$$

$$\frac{\dfrac{}{\Gamma \vdash \exists x . \forall y . P(x,y)}\ Ax \qquad \Gamma' = \Gamma, \forall y . P(x,y) \vdash \forall y . \exists x . P(x,y)}{\Gamma = \exists x . \forall y . P(x,y) \ \vdash \ \forall y . \exists x . P(x,y)}$$

$$\frac{\Gamma = \exists x . \forall y . P(x,y) \ \vdash \ \forall y . \exists x . P(x,y)}{\vdash \exists x . \forall y . P(x,y) \Rightarrow \forall y . \exists x . P(x,y)}\ \Rightarrow_i$$

iv)

$$\frac{}{\forall x . P(x) \vdash \forall x . P(x)}\ Ax$$

$$\frac{\forall x . P(x) \vdash \forall x . P(x)}{\forall x . P(x) \vdash P(x)}\ \forall_E$$

$$\frac{\forall x . P(x) \vdash P(x)}{\forall x . P(x) \vdash \exists x . P(x)}\ \exists_i$$

$$\frac{\forall x . P(x) \vdash \exists x . P(x)}{\vdash \forall x . P(x) \Rightarrow \exists x . P(x)}\ \Rightarrow_i$$

v)

$$\frac{}{\Gamma \vdash \forall x . \forall y . P(x,y)}\ Ax$$

$$\frac{\Gamma \vdash \forall x . \forall y . P(x,y)}{\Gamma \vdash \forall y . P(x,y)}\ \forall e$$

$$\frac{\Gamma \vdash \forall y . P(x,y)}{\quad}\ \forall e$$

$$\Gamma \vdash P(x,y)$$
$\forall_i$
$$\Gamma = \forall x . \forall y . P(x,y) \vdash \forall x . P(x,y)$$
$\Rightarrow_i$ $y = x.$
$$\forall x . \forall y . P(x,y) \Rightarrow \forall x . P(x,x)$$

V:)

Ax
$$\Gamma' \vdash P(x_0, y_0)$$
$$\Gamma' \vdash \exists y . P(x_0, y)$$
Ax $\exists_i$
$$\Gamma \vdash \exists x . P(x,x) \quad \Gamma' = \Gamma, P(x_0,y_0) \vdash \exists x . \exists y . P(x,y)$$
$\exists_e$
$$\Gamma = \exists x . P(x,x) \vdash \exists x . \exists y . P(x,y)$$
$\Rightarrow_i$
$$\vdash \exists x . P(x,x) \Rightarrow \exists x . \exists y . P(x,y)$$

Vii)

$\Rightarrow$)

Ax
$$\Gamma' \vdash P(x)^{\{x := x\}}$$
Ax $\exists_i$
$$\Gamma' \vdash \neg \exists x . P(x) \quad \Gamma' \vdash \exists x . P(x)$$
$$\Gamma' = \Gamma, P(x) \vdash \bot$$
$\neg_i$
$$\Gamma = \neg \exists x . P(x) \vdash \neg P(x)$$
$\forall_e$
$$\neg \exists x . P(x) \vdash \forall x . \neg P(x)$$
$\Rightarrow_i$
$$\vdash \neg \exists x . P(x) \Rightarrow \forall x . \neg P(x)$$

No hay ningún x donde P(x) sea T $\vdash$ Para todo x, P(x) es FALSO

$\Leftarrow$)

Ax Ax Ax
$$\Gamma \vdash \forall x . \neg P(x)^{\{x := x_0\}} \qquad \Gamma' \vdash \exists x . P(x) \quad \Gamma', P(x_0) \vdash P(x_0)$$
asumo x part
$\exists_e$
$$\Gamma' \vdash \neg P(x_0) \qquad \Gamma' \vdash P(x_0)$$
$\neg_e$
$\neg\exists$ nadie de x pant.
$$\Gamma' = \forall x . \neg P(x), \exists x . P(x) \vdash \bot$$
$\neg_i$
$$\forall x . \neg P(x) \vdash \neg \exists x . P(x)$$
$\Rightarrow_i$
$$\forall x . \neg P(x) \Rightarrow \neg \exists x . P(x)$$

Para todo x, P(x) es F. (F)

No Existe x tal que P(x) v (F)

Ax.
$$\Gamma, \neg P(x) \vdash \neg P(x)$$
Ax $\exists_i$

V...)

$$\Gamma, \neg P(x) \vdash \neg \exists x . \neg P(x) \qquad \Gamma, \neg P(x) \vdash \exists x . \neg P(x)$$

viii) =>)

$\Gamma, \neg P(x) \vdash \neg\exists x.\neg P(x) \qquad \Gamma, \neg P(x) \vdash \exists x.\neg P(x)$

$\Gamma, \neg P(x) \vdash \bot$ PBC

$\Gamma \vdash P(x)$ $\forall_i$

Ax ; Ax

$\Gamma \vdash \neg\forall x.P(x) \qquad \Gamma \vdash \forall x.P(x)$ $\neg e$

$\Gamma = \neg\forall x.P(x), \neg\exists x.\neg P(x) \vdash \bot$ PBC

$\neg\forall x.P(x) \vdash \exists x.\neg P(x)$ $=>_i$

$\vdash \neg\forall x.P(x) => \exists x.\neg P(x)$

<=)

Ax ; Ax ; Ax

$\Gamma \vdash \exists x.\neg P(x) \qquad \Gamma, \neg P(x_0) \vdash \neg P(x_0) \qquad \Gamma \vdash \forall x.P(x)$ $\{x:=x_0\}$ $\forall e$

$\Gamma \vdash \neg P(x_0) \qquad \Gamma \vdash P(x_0)$ $\neg e$

$\Gamma = \exists x.\neg P(x), \forall x.P(x) \vdash \bot$ $\neg i$

$\exists x.\neg P(x) \vdash \neg\forall x.P(x)$ $=>_i$

$\vdash \exists x.\neg P(x) => \neg\forall x.P(x)$

Como introducción variable nueva, agarro instancia específica

ix) =>)

Ax ; Ax

$\Gamma \vdash \forall x.(P(x) \wedge Q(x))$ $\forall e$ $\qquad \Gamma \vdash \forall x.(P(x) \wedge Q(x))$ $\forall e$

$\Gamma \vdash P(x) \wedge Q(x)$ $\wedge e_1$ $\qquad \Gamma \vdash P(x) \wedge Q(x)$ $\{x:=x\}$ $\wedge e_2$

$\Gamma \vdash P(x)$ $\forall_i$ $\qquad \Gamma \vdash Q(x)$ $\forall_i$

$\Gamma \vdash \forall x.P(x) \qquad \Gamma \vdash \forall x.Q(x)$ $\wedge_i$

$\Gamma = \forall x.(P(x) \wedge Q(x)) \vdash \forall x.P(x) \wedge \forall x.Q(x)$ $=>_i$

$\vdash \forall x.(P(x) \wedge Q(x)) => (\forall x.P(x) \wedge \forall x.Q(x))$

<=)

Ax ; Ax

$\Gamma \vdash \forall x.P(x) \wedge \forall x.Q(x)$ $\wedge e_1$ $\qquad \Gamma \vdash \forall x.P(x) \wedge \forall x.Q(x)$ $\wedge e_2$

$\Gamma \vdash \forall x.P(x)$ $\forall e$ $\qquad \Gamma \vdash \forall x.Q(x)$ $\forall e$

$\Gamma \vdash P(x) \qquad \Gamma \vdash Q(x)$ $\wedge_i$

$\Gamma = (\forall x.P(x) \wedge \forall x.Q(x)) \vdash P(x) \wedge Q(x)$ $\forall_i$

$$\frac{(\forall x \cdot P(x) \land \forall x \cdot Q(x)) \vdash \forall x \cdot (P(x) \land Q(x))}{\vdash (\forall x \cdot P(x) \land \forall x \cdot Q(x)) \Rightarrow \forall x \cdot (P(x) \land Q(x))} \Rightarrow_i$$

X) $\Rightarrow$)

$$\frac{\forall x \cdot (P(x) \lor \tau) \vdash (\forall x \cdot P(x)) \lor \tau}{\vdash \forall x \cdot (P(x) \lor \tau) \Rightarrow (\forall x \cdot P(x)) \lor \tau} \Rightarrow_i$$

$\Leftarrow$)

$$\frac{\Gamma' \vdash \forall x \cdot P(x)}{\Gamma' \vdash P(x)} \forall e \qquad \frac{}{\Gamma', \forall x \cdot P(x)} Ax$$

$$\frac{\Gamma' \vdash P(x)}{\Gamma' \vdash P(x) \lor \sigma} \lor_{i1} \qquad \frac{\frac{}{\Gamma, \sigma \vdash \sigma} Ax}{\Gamma, \sigma \vdash P(x) \lor \sigma} \lor_{i2}$$

$$\frac{\frac{}{\Gamma \vdash (\forall x \cdot P(x)) \lor \sigma} Ax \quad \frac{\Gamma' \vdash P(x) \lor \sigma}{\Gamma' = \Gamma, (\forall x \cdot P(x)) \vdash \forall x (P(x) \lor \sigma)} \forall_i \quad \frac{\Gamma, \sigma \vdash P(x) \lor \sigma}{\Gamma, \sigma \vdash \forall x (P(x) \lor \sigma)} \forall e}{\Gamma = (\forall x \cdot P(x)) \lor \sigma \vdash \forall x (P(x) \lor \sigma)} \lor_e$$

$$\frac{\Gamma = (\forall x \cdot P(x)) \lor \sigma \vdash \forall x (P(x) \lor \sigma)}{\vdash (\forall x \cdot P(x)) \lor \sigma \Rightarrow \forall x \cdot (P(x) \lor \sigma)} \Rightarrow_i$$

Xi) $\Rightarrow$)

$$\frac{\frac{}{\Gamma' \vdash \exists x \cdot P(x)} Ax \quad \frac{}{\Gamma, P(x_0) \vdash P(x_0)} Ax}{\Gamma'' \vdash P(x_0)} \exists e \qquad \frac{\frac{}{\Gamma''' \vdash \exists x \cdot Q(x)} Ax \quad \frac{}{\Gamma''', Q(x_0) \vdash Q(x_0)} Ax}{\Gamma''' \vdash Q(x_0)} \exists e$$

$$\frac{\Gamma'' \vdash P(x_0)}{\Gamma'' \vdash P(x_0) \lor Q(x_0)} \lor_{i1} \qquad \frac{\Gamma''' \vdash Q(x_0)}{\Gamma''' \vdash P(x_0) \lor Q(x_0)} \lor_{i2}$$

$$\frac{\frac{}{\Gamma \vdash \exists x \cdot P(x) \lor \exists x \cdot Q(x)} Ax \quad \frac{\Gamma'' \vdash P(x_0) \lor Q(x_0)}{\Gamma'' = \Gamma, \exists x \cdot P(x) \vdash \exists x \cdot (P(x) \lor Q(x))} \exists_i \quad \frac{\Gamma''' \vdash P(x_0) \lor Q(x_0)}{\Gamma''' = \Gamma, \exists x \cdot Q(x) \vdash \exists x \cdot (P(x) \lor Q(x))} \exists_i}{\Gamma = \exists x \cdot P(x) \lor \exists x \cdot Q(x) \vdash \exists x \cdot (P(x) \lor Q(x))} \lor_e$$

$$\frac{\Gamma = \exists x \cdot P(x) \lor \exists x \cdot Q(x) \vdash \exists x \cdot (P(x) \lor Q(x))}{\vdash \exists x \cdot P(x) \lor \exists x \cdot Q(x) \Rightarrow \exists x \cdot (P(x) \lor Q(x))} \Rightarrow_i$$

Existe un algun X que hace V. Vale para P o x

Metes lo que aparece del $\exists$ al contexto. Desp te fijas cual es V (por casos)

$$\frac{\frac{}{\Gamma'', P(x_0) \vdash P(x_0)} Ax}{\Gamma'', P(x_0) \vdash \exists x \cdot P(x)} \exists_i \qquad \frac{\frac{}{\Gamma'', Q(x_0) \vdash Q(x_0)} Ax}{\Gamma'', Q(x_0) \vdash \exists x \cdot Q(x)} \exists_i$$

Ax $\lor_{i2}$ $\lor_{i2}$

$$\dfrac{\dfrac{\Gamma \vdash \exists x.(P(x) \vee Q(x)) \qquad \dfrac{\Gamma''' \vdash P(x_0) \vee Q(x_0) \qquad \Gamma, P(x_0) \vdash \tau \qquad \Gamma''', Q(x_0) \vdash \tau}{\Gamma'' = \Gamma, P(x_0) \vee Q(x_0) \vdash \overbrace{\exists x.P(x) \vee \exists x.Q(x)}^{\tau}}\ \vee e}{\Gamma = \exists x.(P(x) \vee Q(x)) \vdash \exists x.P(x) \vee \exists x.Q(x)}\ \exists e}{\vdash \exists x.(P(x) \vee Q(x)) \Rightarrow \exists x.P(x) \vee \exists x.Q(x)}\ \Rightarrow i$$

Ve no.

Ejercicio 10 ★

Demostrar en deducción natural: $(\forall X.\forall Y.R(X, f(Y))) \Rightarrow (\forall X.R(X, f(f(X))))$.

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\forall x.\forall y.R(x, f(y)) \vdash \forall x.\forall y.R(x, f(y))}\ Ax \quad \{x := x\}}{\forall x.\forall y.R(x, f(y)) \vdash \forall y.R(x, f(y))}\ \forall e \quad \{y := f(x)\}}{\forall x.\forall y.R(x, f(y)) \vdash R(x, f(f(x)))}\ \forall e \quad x \notin fv(\Gamma)}{\forall x.\forall y.R(x, f(y)) \vdash \forall x.R(x, f(f(x)))}\ \forall i}{\vdash (\forall x.\forall y.R(x, f(y))) \Rightarrow (\forall x.R(x, f(f(x))))}\ \Rightarrow i$$

Ejercicio 16

Dar derivaciones en DN de las siguientes fórmulas.

I. $(\forall X.P(X)) \Rightarrow P(a)$
II. $P(a) \Rightarrow \exists X.P(X)$
III. $(\forall X.\forall Y.(R(X,Y) \Rightarrow \neg R(Y,X))) \Rightarrow \forall X.\neg R(X,X)$
IV. $(\forall X.\forall Y.R(X,Y)) \Rightarrow \forall X.R(X,X)$
V. $(\exists X.P(X)) \Rightarrow (\forall Y.Q(Y)) \Rightarrow \forall X.\forall Y.(P(X) \Rightarrow Q(Y))$
VI. $(\forall X.(P(X) \Rightarrow Q(X))) \wedge (\exists X.P(X)) \Rightarrow \exists X.Q(X)$
VII. $(\neg\forall X.(P(X) \vee Q(X))) \Rightarrow \neg\forall X.P(X)$
VIII. $(\exists X.(P(X) \Rightarrow Q(X))) \Rightarrow (\forall X.P(X)) \Rightarrow \exists X.Q(X)$
IX. $(\forall X.(P(X) \Rightarrow Q(X))) \wedge (\neg\exists X.Q(X)) \Rightarrow \forall X.\neg P(X)$
X. $(\forall X.(\exists Y.R(Y,X) \Rightarrow P(X))) \Rightarrow (\exists X.\exists Y.R(X,Y)) \Rightarrow \exists X.P(X)$
XI. $(\exists X.(P(X) \vee Q(X))) \Rightarrow (\forall X.\neg Q(X)) \Rightarrow \exists X.P(X)$
XII. $(\neg\forall X.\exists Y.R(X,Y)) \Rightarrow \neg\forall X.R(X,X)$
XIII. $(\neg\exists X.\forall Y.R(Y,X) \Rightarrow \exists X.\exists Y.\neg R(X,Y))$
XIV. $\neg(\forall X.P(X) \wedge \exists X.\neg P(X))$
XV. $(\exists X.(R(X,X) \wedge P(X))) \Rightarrow \neg\forall X.(P(X) \Rightarrow \neg\exists Y.R(X,Y))$
XVI. $(\exists X.P(X) \Rightarrow \forall X.Q(X)) \Rightarrow \forall Y.(P(Y) \Rightarrow Q(Y))$
XVII. $\neg(\forall X.(P(X) \wedge Q(X))) \wedge \forall X.P(X) \Rightarrow \neg\forall X.Q(X)$
XVIII. $(\forall X.(R(X,X) \Rightarrow Q(X))) \wedge \exists X.\forall Y.R(X,Y) \Rightarrow \exists X.Q(X)$

i)

$$\dfrac{\dfrac{\dfrac{}{\forall x.P(x) \vdash \forall x.P(x)}\ Ax \quad \{x := a\}}{\forall x.P(x) \vdash P(a)}\ \forall e}{\vdash \forall x.P(x) \Rightarrow P(a)}\ \Rightarrow i$$

VALE PARA UNA a de TODOS los x

ii)

$$\dfrac{\dfrac{}{P(a) \vdash P(a)}\ Ax \quad \{x := a\}}{\quad}\ \exists i$$

[illegible]

$$\dfrac{P(a) \vdash \exists x . P(x)}{\vdash P(a) \Rightarrow \exists x . P(x)} \Rightarrow_i$$

Si $P(x)$ es VERDADERO, luego $x = a$

$\exists x . P(x) \rightsquigarrow$ sería algo de un "VALE UN x PARA $P(a)$"

$P(a)$ sería algo de un "SI VALÍA PARA x, EN PART PARA A" $\rightsquigarrow \forall x . P(x)$

iii)

$$\dfrac{\Gamma \vdash R(x_0,y_0) \Rightarrow \neg R(x_0,y_0) \qquad \Gamma \vdash R(x,y)}{\Gamma \vdash \neg R(x,y)} \Rightarrow_e$$

$$\dfrac{\Gamma \vdash \qquad \Gamma \vdash \neg R(x,y)}{\Gamma, R(x,x) \vdash \bot} \neg_e$$

$$\dfrac{\Gamma, R(x,x) \vdash \bot}{\Gamma \vdash \neg R(x,x)} PBC$$

$$\dfrac{\Gamma \vdash \neg R(x,x)}{\Gamma = \forall x . \forall y . (R(x,y) \Rightarrow \neg R(y,x)) \vdash \forall x . \neg R(x,x)} \forall_i$$

$$\dfrac{\Gamma = \forall x . \forall y . (R(x,y) \Rightarrow \neg R(y,x)) \vdash \forall x . \neg R(x,x)}{\vdash (\forall x . \forall y (R(x,y) \Rightarrow \neg R(y,x))) \Rightarrow \forall z . \neg R(x,x)} \Rightarrow_i$$

(MAYOR(3,2) $\Rightarrow \neg$ MAYOR(2,3)) $\Rightarrow \neg$ MAYOR(2,2)

iv)

$$\dfrac{}{\forall x . \forall y . R(x,y) \vdash \forall x . \forall y . R(x,y)} Ax$$

$$\dfrac{\forall x . \forall y . R(x,y) \vdash \forall x . \forall y . R(x,y) \quad \{x := x\}}{\forall x . \forall y . R(x,y) \vdash \forall y . R(x,y)} \forall_e$$

$$\dfrac{\forall x . \forall y . R(x,y) \vdash \forall y . R(x,y) \quad \{y := x\}}{\forall x . \forall y . R(x,y) \vdash R(x,x)} \forall_e$$

$$\dfrac{\forall x . \forall y . R(x,y) \vdash R(x,x)}{\forall x . \forall y . R(x,y) \vdash \forall x . R(x,x)} \forall_i$$

$$\dfrac{\forall x . \forall y . R(x,y) \vdash \forall x . R(x,x)}{\vdash (\forall x . \forall y . R(x,y)) \Rightarrow \forall x . R(x,x)} \Rightarrow_i$$

No sabía como exactamente agregar CUANTIFICADOR para DISTINGUIR x de Y.

v)

$$\dfrac{}{\Gamma, P(x) \vdash \forall y . Q(y)} Ax$$

$$\dfrac{\Gamma, P(x) \vdash \forall y . Q(y)}{\Gamma, P(x) \vdash Q(y)} \forall_e$$

$$\dfrac{\Gamma, P(x) \vdash Q(y)}{\Gamma \vdash P(x) \Rightarrow Q(y)} \Rightarrow_i$$

$$\dfrac{\Gamma \vdash P(x) \Rightarrow Q(y)}{\Gamma \vdash \forall y (P(x) \Rightarrow Q(y))} \forall_i$$

$$\dfrac{\Gamma \vdash \forall y (P(x) \Rightarrow Q(y))}{\Gamma = \exists x . P(x), \forall y . Q(y) \vdash \forall x . \forall y . (P(x) \Rightarrow Q(y))} \forall_i$$

$$\dfrac{\Gamma = \exists x . P(x), \forall y . Q(y) \vdash \forall x . \forall y . (P(x) \Rightarrow Q(y))}{\exists x . P(x) \vdash \forall y . Q(y) \Rightarrow \forall x . \forall y . (P(x) \Rightarrow Q(y))} \Rightarrow_i$$

$$\Rightarrow_i$$

$$(\exists x.P(x)) \Rightarrow ((\forall y.Q(y)) \Rightarrow \forall x.\forall y.(P(x) \Rightarrow Q(y)))$$

vi)

$$\overline{\Gamma \vdash (\forall x.P(x) \Rightarrow Q(x)) \wedge (\exists x.P(x))}\ \text{Ax}$$

$$\overline{\Gamma \vdash \forall x.P(x) \Rightarrow Q(x)}\ \wedge e_1$$

$$\overline{\Gamma \vdash P(x_0) \Rightarrow Q(x_0)}\ \forall e$$

$$\overline{\Gamma \vdash (\forall x.(P(x) \Rightarrow Q(x))) \wedge (\exists x.P(x))}\ \text{Ax}$$

$$\overline{\Gamma \vdash \exists x.P(x)}\ \wedge e_2 \qquad \overline{\Gamma, P(x_0) \vdash P(x_0)}\ \text{Ax}$$

$$\overline{\Gamma \vdash P(x_0)}\ \exists e$$

$$\overline{\Gamma \vdash Q(x_0)}\ \Rightarrow e$$

$$\overline{\Gamma = (\forall x.(P(x) \Rightarrow Q(x))) \wedge (\exists x.P(x)) \vdash \exists x.Q(x)}\ \exists i$$

$$\overline{\vdash (\forall x.(P(x) \Rightarrow Q(x))) \wedge (\exists x.P(x)) \Rightarrow \exists x.Q(x)}\ \Rightarrow i$$

vii)

$$\overline{\Gamma' \vdash \neg\forall x.(P(x) \vee Q(x))}\ \text{Ax}$$

$$\overline{\Gamma \vdash \forall x.P(x)}\ \text{Ax}$$

$$\overline{\Gamma' \vdash P(x)}\ \forall e$$

$$\overline{\Gamma' \vdash P(x) \vee Q(x)}\ \vee i_1$$

$$\overline{\Gamma' \vdash \forall x.P(x) \vee Q(x)}\ \forall i$$

$$\overline{\Gamma' = \neg\forall x.(P(x) \vee Q(x)), \forall x.P(x) \vdash \bot}\ \neg e$$

$$\overline{\neg\forall x.(P(x) \vee Q(x)) \vdash \neg\forall x.P(x)}\ \neg i$$

$$\overline{\vdash (\neg\forall x.(P(x) \vee Q(x))) \Rightarrow \neg\forall x.P(x)}\ \Rightarrow i$$

* Como tengo $\neg\forall x.(P(x) \vee Q(x))$ y $\forall x.P(x)$ puedo llegar con $P(x)$ x [illegible] está [illegible] siempre ligada.

viii)

$$\overline{\Gamma'' \vdash \exists x.(P(x) \Rightarrow Q(x))}\ \text{Ax} \qquad \overline{\Gamma'', P(x_0) \Rightarrow Q(x_0) \vdash P(x_0) \Rightarrow Q(x_0)}\ \text{Ax}$$

$$\overline{\Gamma'' = \exists x.(P(x) \Rightarrow Q(x)), \forall x.P(x) \vdash P(x_0) \Rightarrow Q(x_0)}\ \exists e$$

$$\overline{\Gamma' \vdash \forall x.P(x)}\ \text{Ax}$$

$$\overline{\Gamma' = \exists x.(P(x) \Rightarrow Q(x)), \forall x.P(x) \vdash P(x_0)}\ \forall e$$

$$\overline{\exists x.(P(x) \Rightarrow Q(x)), \forall x.P(x) \vdash Q(x_0)}\ \Rightarrow e$$

$$\overline{\exists x.(P(x) \Rightarrow Q(x)), \forall x.P(x) \vdash \exists x.Q(x)}\ \exists i$$

$$\overline{\exists x.(P(x) \Rightarrow Q(x)) \vdash \forall x.P(x) \Rightarrow \exists x.Q(x)}\ \Rightarrow i$$

$$\overline{\vdash (\exists x.(P(x) \Rightarrow Q(x))) \Rightarrow ((\forall x.P(x)) \Rightarrow \exists x.Q(x))}\ \Rightarrow i$$

ix)

$$\Gamma \vdash \forall x \cdot (P(x) \Rightarrow Q(x)) \quad (Ax)$$

$$\Gamma \vdash P(x) \Rightarrow Q(x) \qquad \Gamma \vdash P(x) \quad (Ax)$$

$$\Gamma \vdash Q(x) \quad (\Rightarrow e)$$

$$\Gamma \vdash \exists x \cdot Q(x) \quad (\exists i)$$

$$\Gamma \vdash \forall x \cdot (P(x) \Rightarrow Q(x)) \wedge (\neg \exists x \cdot Q(x)) \quad (Ax)$$

$$\Gamma \vdash \neg \exists x \cdot Q(x) \quad (\wedge e_2)$$

$$\Gamma = \forall x \cdot (P(x) \Rightarrow Q(x)) \wedge (\neg \exists x \cdot Q(x)), P(x) \vdash \bot \quad (\neg e)$$

$$\forall x \cdot (P(x) \Rightarrow Q(x)) \wedge (\neg \exists x \cdot Q(x)) \vdash \neg P(x) \quad (\neg i)$$

$$\forall x (P(x) \Rightarrow Q(x)) \wedge (\neg \exists x \cdot Q(x)) \vdash \forall x \cdot \neg P(x) \quad (\forall i)$$

$$\vdash (\forall x \cdot (P(x) \Rightarrow Q(x))) \wedge (\underbrace{\neg \exists x \cdot Q(x)}_{F}) \Rightarrow \forall x \cdot \neg P(x) \quad (\Rightarrow i)$$

$V \Rightarrow F = F$

X)

$$\quad (\Rightarrow e)$$

$$\Gamma'', R(x_0, y_0) \vdash P(x_0) \quad (\Rightarrow i)$$

$$\Gamma \vdash \exists x \cdot R(y,x) \Rightarrow P(x) \quad (Ax) \qquad \Gamma'' = \Gamma, R(y_0, x_0) \Rightarrow P(x_0) \vdash R(x_0, y_0) \Rightarrow P(x_0)$$

$$\Gamma \vdash \exists x \cdot \exists y \cdot R(x,y) \quad (Ax) \qquad \Gamma, R(x_0, y_0) \vdash R(x_0, y_0) \quad (Ax)$$

$$\Gamma \vdash R(x_0, y_0) \Rightarrow P(x_0) \quad (\exists e)$$

-> ¿Esto es legal? Llamo a ambos x como x_0

$$\Gamma \vdash R(x_0, y_0) \quad (\exists e)$$

$$\Gamma = \forall x \cdot (\exists y \cdot R(y,x) \Rightarrow P(x)), \exists x \cdot \exists y \cdot R(x,y) \vdash P(x_0) \quad (\Rightarrow e)$$

$$\forall x \cdot (\exists y \cdot R(y,x) \Rightarrow P(x)), \exists x \cdot \exists y \cdot R(x,y) \vdash \exists x \cdot P(x) \quad (\exists i)$$

$$\forall x \cdot (\exists y \cdot R(y,x) \Rightarrow P(x)) \vdash \exists x \cdot \exists y \cdot R(x,y) \Rightarrow \exists x \cdot P(x) \quad (\Rightarrow i)$$

$$\vdash (\forall x \cdot (\exists y \cdot R(y,x) \Rightarrow P(x))) \Rightarrow ((\exists x \cdot \exists y \cdot R(x,y)) \Rightarrow \exists x \cdot P(x)) \quad (\Rightarrow i)$$

Xi)

$$\Gamma'', Q(x_0) \vdash \forall x \cdot \neg Q(x) \quad (Ax)$$

$$\Gamma'', Q(x_0) \vdash Q(x_0) \quad (Ax) \qquad \Gamma'', Q(x_0) \vdash \neg Q(x_0) \quad (\forall e)$$

$$\Gamma'', Q(x_0) \vdash \bot \quad (\neg e)$$

$$\Gamma'' \vdash P(x_0) \vee Q(x_0) \quad (Ax) \qquad \Gamma'', P(x_0) \vdash P(x_0) \quad (Ax) \qquad \Gamma'', Q(x_0) \vdash P(x_0) \quad (\bot e)$$

$$\Gamma'' = \Gamma', P(x_0) \vee Q(x_0) \vdash P(x_0) \quad (\vee e)$$

$$\Gamma' \vdash \exists x \cdot (P(x) \vee Q(x)) \quad (Ax)$$

$$\Gamma' \vdash P(x_0) \quad (\exists e)$$

$$\Gamma' = \exists x \cdot (P(x) \vee Q(x)), \forall x \cdot \neg Q(x) \vdash \exists x \cdot P(x) \quad (\exists i)$$

$$\exists x \cdot (P(x) \vee Q(x)) \vdash \forall x \cdot \neg Q(x) \Rightarrow \exists x \cdot P(x) \quad (\Rightarrow i)$$

$$\vdash (\exists x \cdot (P(x) \vee Q(x))) \Rightarrow (\forall x \cdot \neg Q(x) \Rightarrow \exists x \cdot P(x)) \quad (\Rightarrow i)$$

$\vdash (\exists x \cdot (P(x) \vee Q(x))) \Rightarrow (\forall x \cdot \neg Q(x)) \Rightarrow \exists x \cdot P(x)$

Como $Q(x)$ no vale nunca, luego P verdadero, y somos Q y $\neg Q$.

xii)

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{AX}
\UnaryInfC{$\Gamma \vdash \neg \forall x \cdot \exists y \cdot R(x,y)$}
\AxiomC{}
\RightLabel{AX}
\UnaryInfC{$\Gamma \vdash \forall x \cdot R(x,x)$}
\RightLabel{$\forall e$ $\{x := y\}$}
\UnaryInfC{$\Gamma \vdash R(x,y)$}
\RightLabel{$\exists i$}
\UnaryInfC{$\Gamma \vdash \exists y \cdot R(x,y)$}
\RightLabel{$\forall i$}
\UnaryInfC{$\Gamma \vdash \forall x \cdot \exists y \cdot R(x,y)$}
\RightLabel{$\neg e$}
\BinaryInfC{$\Gamma = \neg \forall x \cdot \exists y \cdot R(x,y), \forall x \cdot R(x,x) \vdash \bot$}
\RightLabel{$\neg i$}
\UnaryInfC{$\neg \forall x \cdot \exists y \cdot R(x,y) \vdash \neg \forall x \cdot R(x,x)$}
\RightLabel{$\Rightarrow i$}
\UnaryInfC{$\vdash \neg \forall x \cdot \exists y \cdot R(x,y) \Rightarrow \neg \forall x \cdot R(x,x)$}
\end{prooftree}
$$

xiii)

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{AX}
\UnaryInfC{$\Gamma \vdash \neg \exists x \cdot \forall y \cdot R(y,x)$}
\AxiomC{}
\RightLabel{$\forall i$}
\UnaryInfC{$\Gamma \vdash \forall y \cdot R(y,y_0)$}
\RightLabel{$\exists i$}
\UnaryInfC{$\Gamma \vdash \exists x \cdot \forall y \cdot R(y,x)$}
\RightLabel{$\neg e$}
\BinaryInfC{$\Gamma = \neg \exists x \cdot \forall y \cdot R(y,x), R(x_0,y_0) \vdash \bot$}
\RightLabel{$\neg i$}
\UnaryInfC{$\neg \exists x \cdot \forall y \cdot R(y,x) \vdash \neg R(x_0,y_0)$}
\RightLabel{$\exists i$}
\UnaryInfC{$\neg \exists x \cdot \forall y \cdot R(y,x) \vdash \exists y \cdot \neg R(x_0,y)$}
\RightLabel{$\exists i$}
\UnaryInfC{$\neg \exists x \cdot \forall y \cdot R(y,x) \vdash \exists x \cdot \exists y \cdot \neg R(x,y)$}
\RightLabel{$\Rightarrow i$}
\UnaryInfC{$\vdash \neg \exists x \cdot \forall y \cdot R(y,x) \Rightarrow \exists x \cdot \exists y \cdot \neg R(x,y)$}
\end{prooftree}
$$

xiv)

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{AX}
\UnaryInfC{$\Gamma \vdash \forall x \cdot P(x) \wedge \exists x \cdot \neg P(x)$}
\RightLabel{$\wedge e_1$}
\UnaryInfC{$\Gamma \vdash \forall x \cdot P(x)$}
\RightLabel{$\forall e$}
\UnaryInfC{$\Gamma \vdash P(x_0)$}
\AxiomC{}
\RightLabel{AX}
\UnaryInfC{$\Gamma \vdash \forall x \cdot P(x) \wedge \exists x \cdot \neg P(x)$}
\RightLabel{$\wedge e_2$}
\UnaryInfC{$\Gamma \vdash \exists x \cdot \neg P(x)$}
\AxiomC{}
\RightLabel{AX}
\UnaryInfC{$\Gamma, \neg P(x_0) \vdash \neg P(x_0)$}
\BinaryInfC{$\Gamma \vdash \neg P(x_0)$}
\RightLabel{$\neg e$}
\BinaryInfC{$\Gamma = \forall x \cdot P(x) \wedge \exists x \cdot \neg P(x) \vdash \bot$}
\end{prooftree}
$$

$\vdash \neg(\forall x. P(x) \wedge \exists x. \neg P(x))$ $\quad \neg i$

XV)

$\neg e$

PBC

Ax

$\Gamma \vdash \exists x.(R(x,x) \wedge P(x)) \qquad \Gamma, R(x_0,x_0) \wedge P(x_0) \vdash \forall x.(P(x) \Rightarrow \neg \exists y. R(x,y))$ $\quad \exists e$

$\Gamma \vdash \forall x.(P(x) \Rightarrow \neg \exists y. R(x,y)) \qquad \Gamma \vdash \neg \forall x.(P(x) \Rightarrow \neg \exists y. R(x,y))$ Ax $\quad \neg e$

$\Gamma \vdash \exists x.(R(x,x) \wedge P(x)), \neg \forall x.(P(x) \Rightarrow \neg \exists y. R(x,y)) \vdash \bot$ $\quad$ PBC

$\exists x.(R(x,x) \wedge P(x)) \vdash \neg \forall x.(P(x) \Rightarrow \neg \exists y. R(x,y))$ $\quad \Rightarrow i$

$\vdash (\exists x.(R(x,x) \wedge P(x))) \Rightarrow \neg \forall x.(P(x) \Rightarrow \neg \exists y. R(x,y))$

- No es tan fácil de armar, necesitaría tener $R(x,x) \wedge P(x)$ como testigo
- fácil de armar
- Ambos tienen $P(x)$, puedo aprovechar el $\neg\forall$, Axioma y el otro $\forall$ desarmarlo, meter $P(x)$ al CONTEXTO y $\exists e$ con $P(x_0)$

XVI)

Ax

$\Gamma \vdash P(y)$ $\quad \exists i$

Ax

$\Gamma \vdash \exists x. P(x) \Rightarrow \forall x. Q(x) \qquad \Gamma \vdash \exists x. P(x)$ $\quad \Rightarrow e$

$\Gamma = \exists x. P(x) \Rightarrow \forall x. Q(x), P(y) \vdash \forall x. Q(x)$ $\quad \forall e$

$\exists x. P(x) \Rightarrow \forall x. Q(x), P(y) \vdash Q(y)$ $\quad \Rightarrow i$

$\exists x. P(x) \Rightarrow \forall x. Q(x) \vdash P(y) \Rightarrow Q(y)$ $\quad \forall i$

$\exists x. P(x) \Rightarrow \forall x. Q(x) \vdash \forall y.(P(y) \Rightarrow Q(y))$ $\quad \Rightarrow i$

$\vdash (\exists x. P(x) \Rightarrow \forall x. Q(x)) \Rightarrow \forall y.(P(y) \Rightarrow Q(y))$

lo hago aparecer con $\exists e$

lo hago ap con T=>6. (T) debería tener $P(x_0)$ en CONTEXTO o algo para hacer $\exists i$ y que sea AX

XVII)

AX

$\Gamma \vdash \neg(\forall x.(P(x) \wedge Q(x))) \wedge \forall x. P(x)$ $\quad \wedge e_2$

$\Gamma \vdash \forall x. P(x)$ $\quad \forall e$ $\qquad$ Ax $\quad \Gamma \vdash \forall x. Q(x)$ $\quad \forall e$

Ax $\quad \Gamma \vdash P(x) \qquad \Gamma \vdash Q(x)$

$$
\dfrac{\dfrac{\dfrac{\Gamma \vdash \neg(\forall x.(P(x) \wedge Q(x))) \wedge \forall x.P(x)}{\Gamma \vdash \neg(\forall x.(P(x) \wedge Q(x)))}\,\wedge e_1 \qquad \dfrac{\dfrac{}{\Gamma \vdash P(x) \wedge Q(x)}\,\wedge i}{\Gamma \vdash \forall x.(P(x) \wedge Q(x))}\,\forall i}{\dfrac{\Gamma = \neg(\forall x.(P(x) \wedge Q(x))) \wedge \forall x.P(x),\ \forall x.Q(x) \ \vdash \bot}{\neg(\forall x.(P(x) \wedge Q(x))) \wedge \forall x.P(x) \vdash \neg\forall x.Q(x)}\,PBC}\,\neg e}{\vdash \neg(\forall x.(P(x) \wedge Q(x))) \wedge \forall x.P(x) \Rightarrow \neg\forall x.Q(x)}\,\Rightarrow i
$$

- Hacer aparecer el $\wedge$ en una regla.
- No tengo info alguna de P, solo que vale. Si · es FALSO será por el Q(x).

El $\neg$ de · no es sencillo de armar, conviene hacer $\neg e$.