Ejercicio 5 ★

Demostrar en deducción natural que las siguientes fórmulas son teoremas **sin usar principios de razonamiento clásicos** salvo que se indique lo contrario. Recordemos que una fórmula σ es un teorema si y sólo si vale $\vdash \sigma$:

- I. *Modus ponens* relativizado: $(\rho \Rightarrow \sigma \Rightarrow \tau) \Rightarrow (\rho \Rightarrow \sigma) \Rightarrow \rho \Rightarrow \tau$
- II. Reducción al absurdo: $(\rho \Rightarrow \bot) \Rightarrow \neg\rho$
- III. Introducción de la doble negación: $\rho \Rightarrow \neg\neg\rho$
- IV. Eliminación de la triple negación: $\neg\neg\neg\rho \Rightarrow \neg\rho$
- V. Contraposición: $(\rho \Rightarrow \sigma) \Rightarrow (\neg\sigma \Rightarrow \neg\rho)$
- VI. Adjunción: $((\rho \wedge \sigma) \Rightarrow \tau) \Leftrightarrow (\rho \Rightarrow \sigma \Rightarrow \tau)$
- VII. de Morgan (I): $\neg(\rho \vee \sigma) \Leftrightarrow (\neg\rho \wedge \neg\sigma)$
- VIII. de Morgan (II): $\neg(\rho \wedge \sigma) \Leftrightarrow (\neg\rho \vee \neg\sigma)$. Para la dirección $\Rightarrow$ es necesario usar principios de razonamiento clásicos.
- IX. Conmutatividad ($\wedge$): $(\rho \wedge \sigma) \Rightarrow (\sigma \wedge \rho)$
- X. Asociatividad ($\wedge$): $((\rho \wedge \sigma) \wedge \tau) \Leftrightarrow (\rho \wedge (\sigma \wedge \tau))$
- XI. Conmutatividad ($\vee$): $(\rho \vee \sigma) \Rightarrow (\sigma \vee \rho)$
- XII. Asociatividad ($\vee$): $((\rho \vee \sigma) \vee \tau) \Leftrightarrow (\rho \vee (\sigma \vee \tau))$

¿Encuentra alguna relación entre teoremas de adjunción, asociatividad y conmutatividad con algunas de las propiedades demostradas en la práctica 2?

i)

$\Gamma = (P \to Q \to T), (P \to Q), P$

$$\dfrac{\dfrac{}{\Gamma \vdash (P \to Q \to T)}\,Ax \qquad \dfrac{}{\Gamma \vdash P \to Q}\,Ax}{(P \to Q \to T), (P \to Q), P \vdash T}\,\Rightarrow e$$

$$\dfrac{(P \to Q \to T), (P \to Q), P \vdash T}{(P \to Q \to T), (P \to Q) \vdash P \to T}\,\Rightarrow i$$

$$\dfrac{(P \to Q \to T), (P \to Q) \vdash P \to T}{(P \to Q \to T) \vdash (P \to Q) \to P \to T}\,\Rightarrow i$$

$$\dfrac{(P \to Q \to T) \vdash (P \to Q) \to P \to T}{\underbrace{(P \to Q \to T)}_{F2} \to ((\underbrace{P \to Q}_{F1}) \to (\underbrace{P}_{P1} \to (\underbrace{T}_{SALIDA})))}\,\Rightarrow i$$

ii)

$$\dfrac{\dfrac{}{(P \to \bot) \vdash P \to \bot}\,Ax \qquad \dfrac{\dfrac{}{(P \to \bot), \bot \vdash \bot}\,Ax}{(P \to \bot) \vdash \neg\bot}\,\neg i}{(P \to \bot) \vdash \neg P}\,MT$$

$$\dfrac{(P \to \bot) \vdash \neg P}{\emptyset \vdash (P \to \bot) \Rightarrow \neg P}\,\Rightarrow i$$

iii)

$$\dfrac{\dfrac{\dfrac{}{P \vdash P}\,Ax}{P \vdash \neg\neg P}\,\neg\neg i}{P \to \neg\neg P}\,\Rightarrow i$$

iv)

$$\dfrac{\dfrac{}{\neg\neg\neg P, P \vdash \neg\neg\neg P}\,Ax \qquad \dfrac{\dfrac{\dfrac{}{\neg\neg\neg P, P \vdash P}\,Ax}{\neg\neg\neg P, P \vdash \neg\neg P}\,\neg\neg i}{\neg\neg\neg P, P \vdash \neg\neg\neg\neg P}\,\neg\neg i}{\neg\neg\neg P, P \vdash \bot}\,\neg e$$

$\neg i$

$$\frac{\neg\neg\neg P \vdash \neg P}{\neg\neg\neg P \Rightarrow \neg P}\Rightarrow i$$

V)

$$\frac{}{P\Rightarrow G, \neg G, P \vdash P \Rightarrow G}Ax \qquad \frac{}{P\Rightarrow G, \neg G, P \vdash P}Ax$$

$$\frac{P\Rightarrow G, \neg G, P \vdash P \Rightarrow G \qquad P\Rightarrow G, \neg G, P \vdash P}{P\Rightarrow G, \neg G, P \vdash G}\Rightarrow e \qquad T{:}G$$

$$\frac{P\Rightarrow G, \neg G, P \vdash G}{P\Rightarrow G, \neg G, P \vdash \neg\neg G}\neg\neg i$$

$$\frac{}{P\Rightarrow G, \neg G, P \vdash \neg G}Ax$$

$$\frac{P\Rightarrow G, \neg G, P \vdash \neg G \qquad P\Rightarrow G, \neg G, P \vdash \neg\neg G}{P\Rightarrow G, \neg G, P \vdash \bot}\neg e \qquad T{:}\neg G$$

$$\frac{P\Rightarrow G, \neg G, P \vdash \bot}{P\Rightarrow G, \neg G \vdash \neg P}\neg i$$

$$\frac{P\Rightarrow G, \neg G \vdash \neg P}{P\Rightarrow G \vdash \neg G \Rightarrow \neg P}\Rightarrow i$$

$$\frac{P\Rightarrow G \vdash \neg G \Rightarrow \neg P}{(P \Rightarrow G) \Rightarrow (\neg G \Rightarrow \neg P)}\Rightarrow i$$

Vi)

$\Rightarrow$)

$$\frac{}{((P\wedge G)\Rightarrow T), P, G \vdash P}Ax \qquad \frac{}{((P\wedge G)\Rightarrow T), P, G \vdash G}Ax$$

$$\frac{((P\wedge G)\Rightarrow T), P, G \vdash P \qquad ((P\wedge G)\Rightarrow T), P, G \vdash G}{((P\wedge G)\Rightarrow T), P, G \vdash P\wedge G}\wedge_i$$

$$\frac{}{((P\wedge G)\Rightarrow T), P, G \vdash (P\wedge G) \Rightarrow T}Ax$$

$$\frac{((P\wedge G)\Rightarrow T), P, G \vdash (P\wedge G) \Rightarrow T \qquad ((P\wedge G)\Rightarrow T), P, G \vdash P\wedge G}{((P\wedge G)\Rightarrow T), P, G \vdash T}\Rightarrow e \qquad T{:}(P\wedge G)$$

$$\frac{((P\wedge G)\Rightarrow T), P, G \vdash T}{((P\wedge G)\Rightarrow T), P \vdash G \Rightarrow T}\Rightarrow i$$

$$\frac{((P\wedge G)\Rightarrow T), P \vdash G \Rightarrow T}{((P\wedge G)\Rightarrow T) \vdash (P\Rightarrow (G \Rightarrow T))}\Rightarrow i$$

$$\frac{((P\wedge G)\Rightarrow T) \vdash (P\Rightarrow (G \Rightarrow T))}{((P\wedge G)\Rightarrow T) \Rightarrow (P\Rightarrow G \Rightarrow T)}\Rightarrow i$$

$\Leftarrow$)

$$\frac{}{(P\Rightarrow G\Rightarrow T), P\wedge G, P \vdash P\wedge G}Ax$$

$$\frac{(P\Rightarrow G\Rightarrow T), P\wedge G, P \vdash P\wedge G}{(P\Rightarrow G\Rightarrow T), P\wedge G, P \vdash G}\wedge e_2$$

$$\frac{(P\Rightarrow G\Rightarrow T), P\wedge G, P \vdash G}{(P\Rightarrow G\Rightarrow T), P\wedge G \vdash P\Rightarrow G}\Rightarrow i$$

$$\frac{}{(P\Rightarrow G\Rightarrow T), P\wedge G \vdash P\Rightarrow G\Rightarrow T}Ax$$

$$\frac{(P\Rightarrow G\Rightarrow T), P\wedge G \vdash P\Rightarrow G\Rightarrow T \qquad (P\Rightarrow G\Rightarrow T), P\wedge G \vdash P\Rightarrow G}{(P\Rightarrow G\Rightarrow T), P\wedge G \vdash T}\Rightarrow e$$

$$\frac{(P\Rightarrow G\Rightarrow T), P\wedge G \vdash T}{(P\Rightarrow G\Rightarrow T) \vdash ((P\wedge G)\Rightarrow T)}\Rightarrow i$$

$$\frac{(P\Rightarrow G\Rightarrow T) \vdash ((P\wedge G)\Rightarrow T)}{(P\Rightarrow G\Rightarrow T) \Rightarrow ((P\wedge G)\Rightarrow T)}\Rightarrow i$$

Vii)

$\Rightarrow$)

$$\frac{}{\neg(P\vee G), P \vdash P}Ax \qquad \frac{}{\neg(P\vee G), G \vdash G}Ax$$

$$\frac{\neg(P\vee G), P \vdash P}{\neg(P\vee G), P \vdash (P\vee G)}\vee i_1 \qquad \frac{\neg(P\vee G), G \vdash G}{\neg(P\vee G), G \vdash P\vee G}\vee i_2$$

$\neg\neg i$ $\qquad$ $\neg\neg i$

$$\dfrac{\neg(P\vee G), P \vdash \neg(P\vee G) \qquad \neg(P\vee G), P \vdash \neg\neg(P\vee G)}{\neg(P\vee G), P \vdash \bot}\ \neg e \qquad \dfrac{\neg(P\vee G), G \vdash \neg(P\vee G) \qquad \neg(P\vee G), G \vdash \neg\neg(P\vee G)}{\neg(P\vee G), G \vdash \bot}\ \neg e$$

$$\dfrac{\neg(P\vee G), P \vdash \bot}{\neg(P\vee G) \vdash \neg P}\ \neg i \qquad \dfrac{\neg(P\vee G), G \vdash \bot}{\neg(P\vee G) \vdash \neg G}\ \neg i$$

$$\dfrac{\neg(P\vee G) \vdash \neg P \qquad \neg(P\vee G) \vdash \neg G}{\neg(P\vee G) \vdash \neg P \wedge \neg G}\ \wedge i$$

$$\dfrac{\neg(P\vee G) \vdash \neg P \wedge \neg G}{\neg(P\vee G) \Rightarrow (\neg P \wedge \neg G)}\ \Rightarrow i$$

$\Leftarrow)$

$$\dfrac{}{\Gamma', P \vdash P}\ Ax \qquad \dfrac{\dfrac{}{\Gamma', P \vdash \neg P \wedge \neg G}\ Ax}{\Gamma', P \vdash \neg P}\ \wedge e_1 \qquad \dfrac{}{\Gamma', G \vdash G}\ Ax \qquad \dfrac{\dfrac{}{\Gamma', G \vdash \neg P \wedge \neg G}\ Ax}{\Gamma', G \vdash \neg G}\ \wedge e_2$$

$$\dfrac{\Gamma', P \vdash P \qquad \Gamma', P \vdash \neg P}{\Gamma', P \vdash \bot} \qquad \dfrac{\Gamma', G \vdash G \qquad \Gamma', G \vdash \neg G}{\Gamma', G \vdash \bot}\ \neg e$$

$$\dfrac{\dfrac{}{\Gamma' \vdash P\vee G}\ Ax \qquad \Gamma', P \vdash \bot \qquad \Gamma', G \vdash \bot}{\Gamma' = (\neg P \wedge \neg G), (P\vee G) \vdash \bot}\ \vee e$$

$$\dfrac{\Gamma' = (\neg P \wedge \neg G), (P\vee G) \vdash \bot}{(\neg P \wedge \neg G) \vdash \neg(P\vee G)}\ \neg i$$

$$\dfrac{(\neg P \wedge \neg G) \vdash \neg(P\vee G)}{\emptyset \vdash (\neg P \wedge \neg G) \Rightarrow \neg(P\vee G)}\ \Rightarrow i$$

viii)

$\Rightarrow)$

$$\dfrac{\dfrac{}{\neg(P\wedge G), (\neg P \vee \neg G), (P\wedge G) \vdash (P\wedge G)}\ Ax \qquad \dfrac{}{\neg(P\wedge G), (\neg P \vee \neg G), (P\wedge G) \vdash \neg(P\wedge G)}\ Ax}{\neg(P\wedge G), (\neg P \vee \neg G), (P\wedge G) \vdash \bot}\ \neg e$$

$$\dfrac{\neg(P\wedge G), (\neg P \vee \neg G), (P\wedge G) \vdash \bot}{\neg(P\wedge G), \neg(\neg P \vee \neg G) \vdash P \wedge G}\ PBC$$

$\to$ tengo $\neg(P\wedge G)$. Busco negar $P\wedge G$ para $\neg e$.

$$\dfrac{\neg(P\wedge G), \neg(\neg P \vee \neg G) \vdash P \wedge G}{\neg(P\wedge G), \neg(\neg P \vee \neg G) \vdash \neg\neg(P\wedge G)}\ \neg\neg i$$

$$\dfrac{\dfrac{}{\neg(P\wedge G), \neg(\neg P \vee \neg G) \vdash \neg(P\wedge G)}\ Ax \qquad \neg(P\wedge G), \neg(\neg P \vee \neg G) \vdash \neg\neg(P\wedge G)}{\neg(P\wedge G), \neg(\neg P \vee \neg G) \vdash \bot}\ \neg e$$

$$\dfrac{\neg(P\wedge G), \neg(\neg P \vee \neg G) \vdash \bot}{\neg(P\wedge G) \vdash (\neg P \vee \neg G)}\ \neg i$$

$$\dfrac{\neg(P\wedge G) \vdash (\neg P \vee \neg G)}{\neg(P\wedge G) \Rightarrow (\neg P \vee \neg G)}\ \Rightarrow i$$

$\Leftarrow)$

$$\dfrac{\dfrac{}{(\neg P \vee \neg G), (P\wedge G), \neg(\neg P \vee \neg G) \vdash (\neg P \vee \neg G)}\ Ax \qquad \dfrac{}{(\neg P \vee \neg G), (P\wedge G), \neg(\neg P \vee \neg G) \vdash \neg(\neg P \vee \neg G)}\ Ax}{(\neg P \vee \neg G), (P\wedge G), \neg(\neg P \vee \neg G) \vdash \bot}\ \neg i$$

$$\dfrac{(\neg P \vee \neg G), (P\wedge G), \neg(\neg P \vee \neg G) \vdash \bot}{(\neg P \vee \neg G), (P\wedge G) \vdash \neg(\neg P \vee \neg G)}\ PBC.$$

$$\dfrac{(\neg P \vee \neg G), (P\wedge G) \vdash (\neg P \vee \neg G) \qquad (\neg P \vee \neg G), (P\wedge G) \vdash \neg(\neg P \vee \neg G)}{(\neg P \vee \neg G), (P\wedge G) \vdash \bot}\ \neg e$$

$$\dfrac{(\neg P \vee \neg G), (P\wedge G) \vdash \bot}{(\neg P \vee \neg G) \vdash \neg(P\wedge G)}\ \neg i$$

$$\dfrac{(\neg P \vee \neg G) \vdash \neg(P\wedge G)}{(\neg P \vee \neg G) \Rightarrow \neg(P\wedge G)}\ \Rightarrow i$$

$(\neg P \vee \neg G) \Rightarrow \neg(P\wedge G)$

ix)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(P\wedge G)\vdash P\wedge G}\,Ax}{(P\wedge G)\vdash G}\,\wedge e_2 \qquad \dfrac{\dfrac{}{(P\wedge G)\vdash P\wedge G}\,Ax}{(P\wedge G)\vdash P}\,\wedge e_1}{(P\wedge G)\vdash G\wedge P}\,\wedge i}{(P\wedge G)\Rightarrow(G\wedge P)}\,\Rightarrow i$$

x)

$\Rightarrow$)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{((P\wedge G)\wedge T)\vdash(P\wedge G)\wedge T}\,Ax}{((P\wedge G)\wedge T)\vdash P\wedge G}\,\wedge e_2}{((P\wedge G)\wedge T)\vdash P}\,\wedge e_1 \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{((P\wedge G)\wedge T)\vdash(P\wedge G)\wedge T}\,Ax}{((P\wedge G)\wedge T)\vdash(P\wedge G)}\,\wedge e_1}{((P\wedge G)\wedge T)\vdash G}\,\wedge e_2 \qquad \dfrac{\dfrac{}{((P\wedge G)\wedge T)\vdash(P\wedge G)\wedge T}\,Ax}{((P\wedge G)\wedge T)\vdash T}\,\wedge e_2}{((P\wedge G)\wedge T)\vdash(G\wedge T)}}{((P\wedge G)\wedge T)\vdash(P\wedge(G\wedge T))}\,\wedge i}{((P\wedge G)\wedge T)\Rightarrow(P\wedge(G\wedge T))}\,\Rightarrow i$$

$\Leftarrow$)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{P\wedge(G\wedge T)\vdash P\wedge(G\wedge T)}\,Ax}{P\wedge(G\wedge T)\vdash P}\,\wedge e_1 \qquad \dfrac{\dfrac{\dfrac{}{P\wedge(G\wedge T)\vdash P\wedge(G\wedge T)}\,Ax}{P\wedge(G\wedge T)\vdash G\wedge T}\,\wedge e_2}{P\wedge(G\wedge T)\vdash G}\,\wedge e_1}{P\wedge(G\wedge T)\vdash P\wedge G}\,\wedge i \qquad \dfrac{\dfrac{\dfrac{}{P\wedge(G\wedge T)\vdash P\wedge(G\wedge T)}\,Ax}{P\wedge(G\wedge T)\vdash(G\wedge T)}\,\wedge e_2}{P\wedge(G\wedge T)\vdash T}\,\wedge e_2}{P\wedge(G\wedge T)\vdash((P\wedge G)\wedge T)}\,\wedge i}{P\wedge(G\wedge T)\Rightarrow((P\wedge G)\wedge T)}\,\Rightarrow i$$

xi)

$$\dfrac{\dfrac{\dfrac{}{(P\vee G)\vdash P\vee G}\,Ax \qquad \dfrac{\dfrac{}{(P\vee G),P\vdash P}\,Ax}{(P\vee G),P\vdash G\vee P}\,\vee i_2 \qquad \dfrac{\dfrac{}{(P\vee G),G\vdash G}\,Ax}{(P\vee G),G\vdash G\vee P}\,\vee i_1}{(P\vee G)\vdash(G\vee P)}\,\vee e}{(P\vee G)\Rightarrow(G\vee P)}\,\Rightarrow i$$

xii)

$\Rightarrow)$

$$\dfrac{\dfrac{}{((\rho\vee\sigma)\vee\tau)\vdash(\rho\vee(\sigma\vee\tau))}\text{Ax}\qquad \dfrac{\dfrac{}{((\rho\vee\sigma)\vee\tau),\rho\vdash\rho}\text{Ax}}{((\rho\vee\sigma)\vee\tau),\rho\vdash(\rho\vee(\sigma\vee\tau))}\vee i_1\qquad \dfrac{\dfrac{}{((\rho\vee\sigma)\vee\tau),(\sigma\vee\tau)\vdash(\sigma\vee\tau)}\text{Ax}}{((\rho\vee\sigma)\vee\tau),(\sigma\vee\tau)\vdash(\rho\vee(\sigma\vee\tau))}\vee i_2}{\dfrac{((\rho\vee\sigma)\vee\tau)\vdash(\rho\vee(\sigma\vee\tau))}{\emptyset\vdash((\rho\vee\sigma)\vee\tau)\Rightarrow(\rho\vee(\sigma\vee\tau))}\Rightarrow i}\vee e$$

$\tau=\rho$

$\sigma=(\sigma\vee\tau)$

$\rho=(\rho\vee(\sigma\vee\tau))$

$\Leftarrow)$

$$\dfrac{\dfrac{}{\rho\vee(\sigma\vee\tau)\vdash\rho\vee(\sigma\vee\tau)}\text{Ax}\qquad \dfrac{\dfrac{\dfrac{}{\Gamma'\vdash\rho}\text{Ax}}{\Gamma'\vdash(\rho\vee\sigma)}\vee i_1}{\Gamma'=\rho\vee(\sigma\vee\tau),\rho\vdash(\rho\vee\sigma)\vee\tau}\vee i_1\qquad \dfrac{\dfrac{}{\Gamma''\vdash\sigma\vee\tau}\text{Ax}\quad \dfrac{\dfrac{\dfrac{}{\Gamma'',\sigma\vdash\sigma}\text{Ax}}{\Gamma'',\sigma\vdash\rho\vee\sigma}\vee i_2}{\Gamma'',\sigma\vdash(\rho\vee\sigma)\vee\tau}\vee i_1\quad \dfrac{\dfrac{}{\Gamma'',\tau\vdash\tau}\text{Ax}}{\Gamma'',\tau\vdash(\rho\vee\sigma)\vee\tau}\vee i_2}{\Gamma''=\rho\vee(\sigma\vee\tau),(\sigma\vee\tau)\vdash(\rho\vee\sigma)\vee\tau}\vee e}{\dfrac{\rho\vee(\sigma\vee\tau)\vdash(\rho\vee\sigma)\vee\tau}{\emptyset\vdash\rho\vee(\sigma\vee\tau)\Rightarrow((\rho\vee\sigma)\vee\tau)}\Rightarrow i}\vee e$$

o sale σ o sale τ

Ejercicio 6 ★

Demostrar en deducción natural que vale $\vdash\sigma$ para cada una de las siguientes fórmulas. Para estas fórmulas es imprescindible **usar lógica clásica**:

- I. Absurdo clásico: $(\neg\tau\Rightarrow\bot)\Rightarrow\tau$
- II. Ley de Peirce: $((\tau\Rightarrow\rho)\Rightarrow\tau)\Rightarrow\tau$
- III. Tercero excluido: $\tau\vee\neg\tau$
- IV. Consecuencia milagrosa: $(\neg\tau\Rightarrow\tau)\Rightarrow\tau$
- V. Contraposición clásica: $(\neg\rho\Rightarrow\neg\tau)\Rightarrow(\tau\Rightarrow\rho)$
- VI. Análisis de casos: $(\tau\Rightarrow\rho)\Rightarrow(\neg\tau\Rightarrow\rho)\Rightarrow\rho$
- VII. Implicación vs. disyunción: $(\tau\Rightarrow\rho)\Leftrightarrow(\neg\tau\vee\rho)$

i)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(\neg\tau\Rightarrow\bot),\neg\tau\vdash\neg\tau\Rightarrow\bot}\text{Ax}\qquad\dfrac{}{(\neg\tau\Rightarrow\bot),\neg\tau\vdash\neg\tau}\text{Ax}}{(\neg\tau\Rightarrow\bot),\neg\tau\vdash\bot}\Rightarrow e}{(\neg\tau\Rightarrow\bot)\vdash\tau}\text{PBC}}{(\neg\tau\Rightarrow\bot)\Rightarrow\tau}\Rightarrow i$$

$(\tau:\bot)$

solo luego aparecen $\Rightarrow$ con $\Rightarrow e$.

ii)

$$\dfrac{\dfrac{}{\Gamma''\vdash(\tau\Rightarrow\rho)\Rightarrow\tau}\text{Ax}\qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma'',\tau\vdash\tau}\text{Ax}\quad\dfrac{}{\Gamma'',\tau\vdash\neg\tau}\text{Ax}}{\Gamma'',\tau\vdash\bot}\neg e}{\Gamma'',\tau\vdash\rho}\bot e}{\Gamma''\vdash\tau\Rightarrow\rho}\Rightarrow i}{\Gamma''=\Gamma',\neg\tau\vdash\tau}\Rightarrow e$$

$(\tau\Rightarrow\rho)\Rightarrow\tau,\neg\tau$ Contradicción

$$\dfrac{\dfrac{\dfrac{}{\Gamma'=\tau\vee\neg\tau}\text{Ax}\qquad\dfrac{}{\Gamma',\tau\vdash\tau}\text{Ax}\qquad\Gamma''=\Gamma',\neg\tau\vdash\tau}{\Gamma'=((\tau\Rightarrow\rho)\Rightarrow\tau)\vdash\tau}}{((\tau\Rightarrow\rho)\Rightarrow\tau)\Rightarrow\tau}\Rightarrow i$$

iii)

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\neg T \vdash \neg T$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\neg T, T \vdash \neg T$}
\AxiomC{}
\RightLabel{ax}
\UnaryInfC{$\neg T, T \vdash T$}
\RightLabel{$\neg e$ $(T = T)$}
\BinaryInfC{$\neg T, T \vdash \bot$}
\RightLabel{PBC}
\UnaryInfC{$\neg T \vdash T$}
\RightLabel{$\neg\neg i$}
\UnaryInfC{$\neg T \vdash \neg\neg T$}
\RightLabel{$\neg e$ $(T = \neg T)$}
\BinaryInfC{$\neg T \vdash \bot$}
\RightLabel{PBC}
\UnaryInfC{$\vdash T$}
\RightLabel{$\vee i_1$}
\UnaryInfC{$\vdash (T \vee \neg T)$}
\end{prooftree}
$$

iv)

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$(\neg T \Rightarrow T), \neg T \vdash \neg T \Rightarrow T$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$(\neg T \Rightarrow T), \neg T \vdash \neg T$}
\RightLabel{$\Rightarrow e$}
\BinaryInfC{$(\neg T \Rightarrow T), \neg T \vdash T$}
\AxiomC{$(\neg T \Rightarrow T), \neg T, T \vdash \neg T$}
\AxiomC{$(\neg T \Rightarrow T), \neg T, T \vdash T$}
\RightLabel{$\neg e$}
\BinaryInfC{$(\neg T \Rightarrow T), \neg T, T \vdash \bot$}
\RightLabel{PBC}
\UnaryInfC{$(\neg T \Rightarrow T), \neg T \vdash \neg T$}
\RightLabel{$\neg e$}
\BinaryInfC{$(\neg T \Rightarrow T), \neg T \vdash \bot$}
\RightLabel{PBC}
\UnaryInfC{$(\neg T \Rightarrow T) \vdash T$}
\RightLabel{$\Rightarrow i$}
\UnaryInfC{$(\neg T \Rightarrow T) \Rightarrow T$}
\end{prooftree}
$$

(Ax over both top sequents of the right branch.)

v)

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$(\neg P \Rightarrow \neg T), T, \neg P \vdash T$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$(\neg P \Rightarrow \neg T), T, \neg P \vdash (\neg P \Rightarrow \neg T)$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$(\neg P \Rightarrow \neg T), T, \neg P \vdash \neg P$}
\RightLabel{$\Rightarrow e$}
\BinaryInfC{$(\neg P \Rightarrow \neg T), T, \neg P \vdash \neg T$}
\RightLabel{$\neg e$}
\BinaryInfC{$(\neg P \Rightarrow \neg T), T, \neg P \vdash \bot$}
\RightLabel{PBC}
\UnaryInfC{$(\neg P \Rightarrow \neg T), T \vdash P$}
\RightLabel{$\Rightarrow i$}
\UnaryInfC{$(\neg P \Rightarrow \neg T) \vdash T \Rightarrow P$}
\RightLabel{$\Rightarrow i$}
\UnaryInfC{$(\neg P \Rightarrow \neg T) \Rightarrow (T \Rightarrow P)$}
\end{prooftree}
$$

→ Para usar $(\neg P \Rightarrow \neg T)$ g ... $\vdash \neg P$.

vii)

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$(T \Rightarrow P), (\neg T \Rightarrow P), \neg P, \neg T \vdash \neg P$}
\end{prooftree}
$$

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$(T \Rightarrow P), (\neg T \Rightarrow P), \neg P, \neg T \vdash \neg T \Rightarrow P$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$(T \Rightarrow P), (\neg T \Rightarrow P), \neg P, \neg T \vdash \neg T$}
\RightLabel{$\Rightarrow e$}
\BinaryInfC{$(T \Rightarrow P), (\neg T \Rightarrow P), \neg P, \neg T \vdash P$}
\end{prooftree}
$$

$$\dfrac{\dfrac{}{(\tau\Rightarrow P),(\neg\tau\Rightarrow P),\neg P\vdash \tau\Rightarrow P}\,Ax \qquad \dfrac{(\tau\Rightarrow P),(\neg\tau\Rightarrow P),\neg P,\neg\tau\vdash\bot}{(\tau\Rightarrow P),(\neg\tau\Rightarrow P),\neg P\vdash\tau}\,PBC}{(\tau\Rightarrow P),(\neg\tau\Rightarrow P),\neg P\vdash P}\Rightarrow_e$$

(the upper line above $(\tau\Rightarrow P),(\neg\tau\Rightarrow P),\neg P,\neg\tau\vdash\bot$ is labelled $\neg e$)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(\tau\Rightarrow P),(\neg\tau\Rightarrow P),\neg P\vdash\neg P}\,Ax \qquad \dfrac{(\tau\Rightarrow P),(\neg\tau\Rightarrow P),\neg P\vdash P}{(\tau\Rightarrow P),(\neg\tau\Rightarrow P),\neg P\vdash\neg\neg P}\,\neg\neg i}{(\tau\Rightarrow P),(\neg\tau\Rightarrow P),\neg P\vdash\bot}\,\neg e}{(\tau\Rightarrow P),(\neg\tau\Rightarrow P)\vdash P}\,PBC}{\dfrac{(\tau\Rightarrow P)\vdash(\neg\tau\Rightarrow P)\Rightarrow P}{(\tau\Rightarrow P)\Rightarrow((\neg\tau\Rightarrow P)\Rightarrow P)}\Rightarrow_i}\Rightarrow_i$$

Vii)

$\Rightarrow$)

$$\dfrac{\dfrac{\dfrac{}{(\tau\Rightarrow P)\vdash(\tau\Rightarrow P)}\,Ax \qquad \dfrac{\dfrac{\dfrac{(\tau\Rightarrow P),\neg\tau,\neg P\vdash\bot}{(\tau\Rightarrow P),\neg\tau\vdash P}\qquad\dfrac{(\tau\Rightarrow P),\neg\tau,P\vdash\bot}{(\tau\Rightarrow P),\neg\tau\vdash\neg P}}{(\tau\Rightarrow P),\neg\tau\vdash\bot}}{(\tau\Rightarrow P)\vdash\tau}}{(\tau\Rightarrow P)\vdash P}\Rightarrow_e$$

$$\dfrac{\dfrac{(\tau\Rightarrow P)\vdash P}{(\tau\Rightarrow P)\vdash(\neg\tau\vee P)}\,\vee e_2}{((\tau\Rightarrow P)\Rightarrow(\neg\tau\vee P))}\Rightarrow_i$$

($\Leftarrow$)

$$\dfrac{\dfrac{\dfrac{}{(\neg\tau\vee P),\tau,\neg\tau,\neg P\vdash\tau}\,Ax\qquad\dfrac{}{(\neg\tau\vee P),\tau,\neg\tau,\neg P\vdash\neg\tau}\,Ax}{(\neg\tau\vee P),\tau,\neg\tau,\neg P\vdash\bot}\,\neg e}{(\neg\tau\vee P),\tau,\neg\tau\vdash P}\,PBC$$

$$\dfrac{\dfrac{\dfrac{\dfrac{}{(\neg\tau\vee P),\tau\vdash(\neg\tau\vee P)}\,Ax\qquad(\neg\tau\vee P),\tau,\neg\tau\vdash P\qquad(\neg\tau\vee P),\tau,P\vdash P}{(\neg\tau\vee P),\tau\vdash P}\,\vee e}{(\neg\tau\vee P)\vdash\tau\Rightarrow P}\Rightarrow_i}{(\neg\tau\vee P)\Rightarrow(\tau\Rightarrow P)}\Rightarrow_i$$

$\tau:\neg\tau$

$\sigma:P$

$$\dfrac{}{\Gamma,\tau\vdash\tau}\,ax$$

$$\dfrac{\Gamma\vdash\tau\quad\Gamma\vdash\sigma}{\Gamma\vdash\tau\wedge\sigma}\wedge_i\qquad\dfrac{\Gamma\vdash\tau\wedge\sigma}{\Gamma\vdash\tau}\wedge_{e_1}\qquad\dfrac{\Gamma\vdash\tau\wedge\sigma}{\Gamma\vdash\sigma}\wedge_{e_2}$$

$$\dfrac{\Gamma,\tau\vdash\sigma}{\Gamma\vdash\tau\Rightarrow\sigma}\Rightarrow_i\qquad\dfrac{\Gamma\vdash\tau\Rightarrow\sigma\quad\Gamma\vdash\tau}{\Gamma\vdash\sigma}\Rightarrow_e$$

$$\dfrac{\Gamma\vdash\tau}{\Gamma\vdash\tau\vee\sigma}\vee_{i_1}\qquad\dfrac{\Gamma\vdash\sigma}{\Gamma\vdash\tau\vee\sigma}\vee_{i_2}\qquad\dfrac{\Gamma\vdash\tau\vee\sigma\quad\Gamma,\tau\vdash\rho\quad\Gamma,\sigma\vdash\rho}{\Gamma\vdash\rho}\vee_e$$

$$\dfrac{\Gamma, \tau \vdash \bot}{\Gamma \vdash \neg\tau}\ \neg_i \qquad \dfrac{\Gamma \vdash \tau \quad \Gamma \vdash \neg\tau}{\Gamma \vdash \bot}\ \neg_e \qquad \dfrac{\Gamma \vdash \bot}{\Gamma \vdash \tau}\ \bot_e$$

Lógica intuicionista

$$\dfrac{\Gamma \vdash \neg\neg\tau}{\Gamma \vdash \tau}\ \neg\neg_e$$

Lógica clásica

Ejercicio 9

Probar los siguientes teoremas:

- I. $((P \Rightarrow Q) \Rightarrow Q) \Rightarrow ((Q \Rightarrow P) \Rightarrow P)$
- II. $(P \Rightarrow Q) \Rightarrow ((\neg P \Rightarrow Q) \Rightarrow Q)$

i)

$$\dfrac{\dfrac{}{\Gamma, P \vdash P}\text{Ax} \quad \dfrac{}{\Gamma, P \vdash \neg P}\text{Ax}}{\Gamma, P \vdash \bot}\ \neg e$$

$$\dfrac{\Gamma, P \vdash \bot}{\Gamma, P \vdash Q}\ \bot e$$

$$\dfrac{\Gamma, P \vdash Q}{\Gamma \vdash (P \Rightarrow Q)}\ \Rightarrow i$$

$$\dfrac{\dfrac{}{\Gamma \vdash (P \Rightarrow Q) \Rightarrow Q}\text{Ax} \quad \Gamma \vdash (P \Rightarrow Q)}{\underbrace{((P \Rightarrow Q) \Rightarrow Q), (Q \Rightarrow P), \neg P}_{\Gamma} \vdash P}\ \Rightarrow e$$

$$\dfrac{((P \Rightarrow Q) \Rightarrow Q), (Q \Rightarrow P), \neg P \vdash P \quad \dfrac{}{((P \Rightarrow Q) \Rightarrow Q), (Q \Rightarrow P), \neg P \vdash \neg P}\text{Ax}}{((P \Rightarrow Q) \Rightarrow Q), (Q \Rightarrow P), \neg P \vdash \bot}\ \neg e$$

$$\dfrac{((P \Rightarrow Q) \Rightarrow Q), (Q \Rightarrow P), \neg P \vdash \bot}{((P \Rightarrow Q) \Rightarrow Q), (Q \Rightarrow P) \vdash P}\ \text{PBC}$$

$$\dfrac{((P \Rightarrow Q) \Rightarrow Q), (Q \Rightarrow P) \vdash P}{((P \Rightarrow Q) \Rightarrow Q) \vdash (Q \Rightarrow P) \Rightarrow P}\ \Rightarrow i$$

$$\dfrac{((P \Rightarrow Q) \Rightarrow Q) \vdash (Q \Rightarrow P) \Rightarrow P}{\emptyset \vdash ((P \Rightarrow Q) \Rightarrow Q) \Rightarrow ((Q \Rightarrow P) \Rightarrow P)}\ \Rightarrow i$$

ii)

$$\dfrac{\dfrac{}{\Gamma, \neg P \vdash \neg P \Rightarrow Q}\text{Ax} \quad \dfrac{}{\Gamma, \neg P \vdash \neg P}\text{Ax}}{\Gamma, \neg P \vdash Q}$$

$$\dfrac{\dfrac{}{\Gamma, \neg P \vdash \neg Q}\text{Ax} \quad \Gamma, \neg P \vdash Q}{\Gamma, \neg P \vdash \bot}\ \neg e$$

$$\dfrac{\Gamma, \neg P \vdash \bot}{\Gamma \vdash P}\ \text{PBC}$$

$$\dfrac{\dfrac{}{\Gamma \vdash P \Rightarrow Q}\text{Ax} \quad \Gamma \vdash P}{\Gamma \vdash Q}$$

$$\dfrac{\Gamma \vdash Q \quad \dfrac{}{\Gamma \vdash \neg Q}\text{Ax}}{\Gamma: P \Rightarrow Q, \neg P \Rightarrow Q, \neg Q \vdash \bot}\ \neg e$$

$$\dfrac{P \Rightarrow Q, \neg P \Rightarrow Q, \neg Q \vdash \bot}{P \Rightarrow Q, \neg P \Rightarrow Q \vdash Q}\ \text{PBC}$$

$$\dfrac{P \Rightarrow Q, \neg P \Rightarrow Q \vdash Q}{(P \Rightarrow Q) \vdash (\neg P \Rightarrow Q) \Rightarrow Q}\ \Rightarrow i$$

$$\dfrac{(P \Rightarrow Q) \vdash (\neg P \Rightarrow Q) \Rightarrow Q}{\emptyset \vdash (P \Rightarrow Q) \Rightarrow ((\neg P \Rightarrow Q) \Rightarrow Q)}\ \Rightarrow i$$

Lo q busqué

$\Gamma \vdash Q \qquad \dfrac{}{\Gamma \vdash \neg Q}\text{Ax}$

$\Gamma \vdash \neg P \Rightarrow Q \ , \ \Gamma \vdash \neg P$

Ejercicio 10

Demostrar las siguientes tautologías utilizando deducción natural.

- I. $(P \Rightarrow (P \Rightarrow Q)) \Rightarrow (P \Rightarrow Q)$
- II. $(R \Rightarrow \neg Q) \Rightarrow ((R \wedge Q) \Rightarrow P)$
- III. $((P \Rightarrow Q) \Rightarrow (R \Rightarrow \neg Q)) \Rightarrow \neg(R \wedge Q)$

i)

$$\dfrac{\dfrac{}{\Gamma \vdash P \Rightarrow (P \Rightarrow Q)}\text{Ax} \quad \dfrac{}{\Gamma \vdash P}\text{Ax}}{\Gamma \vdash P \Rightarrow Q}\ \Rightarrow e \qquad \dfrac{}{\Gamma \vdash P}\text{Ax}$$

$$\Rightarrow e$$

$$\Gamma = P \Rightarrow (P \Rightarrow Q), P \vdash Q$$

$$\Rightarrow i$$

$$P \Rightarrow (P \Rightarrow Q) \vdash (P \Rightarrow Q)$$

$$\Rightarrow i$$

$$\emptyset \vdash (P \Rightarrow (P \Rightarrow Q)) \Rightarrow (P \Rightarrow Q)$$

ii)

$$\frac{\dfrac{\dfrac{}{\Gamma, \neg R \vdash R \wedge Q}\,Ax}{\Gamma, \neg R \vdash R}\wedge e_1 \qquad \dfrac{}{\Gamma, \neg R \vdash \neg R}\,Ax}{\Gamma, \neg R \vdash \bot}\neg e$$

$$\frac{\dfrac{\dfrac{}{\Gamma, R \vdash R \wedge Q}\,Ax}{\Gamma, R \vdash Q}\wedge e_2 \qquad \dfrac{\dfrac{}{\Gamma, R \vdash R \Rightarrow \neg Q}\,Ax \quad \dfrac{}{\Gamma, R \vdash R}\,Ax}{\Gamma, R \vdash \neg Q}\Rightarrow e}{\Gamma, R \vdash \bot}\neg e$$

$$\frac{\dfrac{\Gamma, \neg R \vdash \bot}{\Gamma \vdash R}\,PBC \qquad \dfrac{\Gamma, R \vdash \bot}{\Gamma \vdash \neg R}\,\neg i}{\Gamma = (R \Rightarrow \neg Q), (R \wedge Q), \neg P \vdash \bot}$$

$$\frac{\Gamma = (R \Rightarrow \neg Q), (R \wedge Q), \neg P \vdash \bot}{(R \Rightarrow \neg Q), (R \wedge Q) \vdash P}\,PBC$$

$$\Rightarrow i$$

$$(R \Rightarrow \neg Q) \vdash (R \wedge Q) \Rightarrow P$$

$$\Rightarrow i$$

$$\emptyset \vdash (R \Rightarrow \neg Q) \Rightarrow ((R \wedge Q) \Rightarrow P)$$

Estrategía

$$\frac{R \Rightarrow \neg Q \quad R}{\neg Q} \qquad \frac{Q}{}$$

iii)

$$\frac{\dfrac{}{\Gamma, R, P \vdash R \wedge Q}\,Ax}{\Gamma, R, P \vdash Q}\wedge e_2$$

$$\frac{\Gamma, R, P \vdash Q}{\Gamma, R \vdash (P \Rightarrow Q)}\Rightarrow i$$

$$\frac{\dfrac{}{\Gamma, R \vdash (P \Rightarrow Q) \Rightarrow (R \Rightarrow \neg Q)}\,Ax \qquad \Gamma, R \vdash (P \Rightarrow Q)}{\Gamma, R \vdash R \Rightarrow \neg Q}\Rightarrow e$$

$$\frac{\dfrac{\dfrac{}{\Gamma, R \vdash R \wedge Q}\,Ax}{\Gamma, R \vdash Q}\wedge e_2 \qquad \dfrac{\Gamma, R \vdash R \Rightarrow \neg Q \quad \dfrac{}{\Gamma, R \vdash R}\,Ax}{\Gamma, R \vdash \neg Q}\Rightarrow e}{\Gamma, R \vdash \bot}\neg e$$

$$\frac{\dfrac{\dfrac{}{\Gamma \vdash R \wedge Q}\,Ax}{\Gamma \vdash R}\wedge e_1 \qquad \dfrac{\Gamma, R \vdash \bot}{\Gamma \vdash \neg R}\neg i}{\Gamma = ((P \Rightarrow Q) \Rightarrow (R \Rightarrow \neg Q)), (R \wedge Q) \vdash \bot}\neg e$$

$$\frac{\Gamma = ((P \Rightarrow Q) \Rightarrow (R \Rightarrow \neg Q)), (R \wedge Q) \vdash \bot}{((P \Rightarrow Q) \Rightarrow (R \Rightarrow \neg Q)) \vdash \neg (R \wedge Q)}\neg i$$

$$\Rightarrow i$$

$$\emptyset \vdash ((P \Rightarrow Q) \Rightarrow (R \Rightarrow \neg Q)) \Rightarrow \neg (R \wedge Q)$$

$(P \Rightarrow Q) \Rightarrow \ldots$

$$\frac{\dfrac{R \wedge Q \ \checkmark}{Q}}{P \Rightarrow Q}\Rightarrow i$$

Ejercicio 11

Probar que los siguientes secuentes son válidos sin usar principios de razonamiento clásicos:

- I. $(P \wedge Q) \wedge R, S \wedge T \vdash Q \wedge S$
- II. $(P \wedge Q) \wedge R \vdash P \wedge (Q \wedge R)$
- III. $P \Rightarrow (P \Rightarrow Q), P \vdash Q$
- IV. $Q \Rightarrow (P \Rightarrow R), \neg R, Q \vdash \neg P$
- V. $\vdash (P \wedge Q) \Rightarrow P$
- VI. $P \Rightarrow \neg Q, Q \vdash \neg P$
- VII. $P \Rightarrow Q \vdash (P \wedge R) \Rightarrow (Q \wedge R)$
- VIII. $Q \Rightarrow R \vdash (P \vee Q) \Rightarrow (P \vee R)$
- IX. $(P \vee Q) \vee R \vdash P \vee (Q \vee R)$
- X. $P \wedge (Q \vee R) \vdash (P \wedge Q) \vee (P \wedge R)$
- XI. $(P \wedge Q) \vee (P \wedge R) \vdash P \wedge (Q \vee R)$
- XII. $\neg P \vee Q \vdash P \Rightarrow Q$
- XIII. $P \Rightarrow Q, P \Rightarrow \neg Q \vdash \neg P$
- XIV. $P \Rightarrow (Q \Rightarrow R), P, \neg R \vdash \neg Q$

i) $\qquad \dfrac{}{\quad}\,Ax$

$$\dfrac{\dfrac{\dfrac{\Gamma\vdash(P\wedge Q)\wedge R}{\Gamma\vdash P\wedge Q}\,\wedge e_1}{\Gamma\vdash Q}\,\wedge e_2 \qquad \dfrac{\dfrac{}{\Gamma\vdash S\wedge T}\,Ax}{\Gamma\vdash S}\,\wedge e_1}{}\,\wedge i$$

$\Gamma = (P\wedge Q)\wedge R,\ S\wedge T \vdash Q\wedge S$

ii)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma\vdash(P\wedge Q)\wedge R}\,Ax}{\Gamma\vdash P\wedge Q}\,\wedge e_1}{\Gamma\vdash P}\,\wedge e_1 \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma\vdash(P\wedge Q)\wedge R}\,Ax}{\Gamma\vdash(P\wedge Q)}\,\wedge e_1}{\Gamma\vdash Q}\,\wedge e_2 \qquad \dfrac{\dfrac{}{\Gamma\vdash(P\wedge Q)\wedge R}\,Ax}{\Gamma\vdash R}\,\wedge e_2}{\Gamma\vdash Q\wedge R}\,\wedge i}{}\,\wedge i$$

$\Gamma = (P\wedge Q)\wedge R \vdash P\wedge(Q\wedge R)$

iii)

$$\dfrac{\dfrac{\dfrac{}{\Gamma\vdash P\Rightarrow(P\Rightarrow Q)}\,Ax \qquad \dfrac{}{\Gamma\vdash P}\,Ax}{\Gamma\vdash P\Rightarrow Q}\,\Rightarrow e \qquad \dfrac{}{\Gamma\vdash P}\,Ax}{}\,\Rightarrow e$$

$\Gamma = P\Rightarrow(P\Rightarrow Q),\ P\vdash Q$

iv)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma,P\vdash Q\Rightarrow(P\Rightarrow R)}\,Ax \quad \dfrac{}{\Gamma,P\vdash Q}\,Ax}{\Gamma,P\vdash P\Rightarrow R}\,\Rightarrow e \qquad \dfrac{}{\Gamma,P\vdash P}\,Ax}{\Gamma,P\vdash R}\,\Rightarrow e \qquad \dfrac{}{\Gamma,P\vdash\neg R}\,Ax}{\Gamma,P\vdash\bot}\,\neg e}{}\,\neg i$$

$\Gamma = \underline{Q\Rightarrow}(P\Rightarrow R),\ \neg R,\ \underline{Q}\vdash\neg P$

$\Gamma\vdash T\Rightarrow G$ $\qquad$ $\Gamma\vdash T$ $\qquad$ ESTRATEGia

v)

$$\dfrac{\dfrac{\dfrac{}{P\wedge Q\vdash P\wedge Q}\,Ax}{P\wedge Q\vdash P}\,\wedge e_1}{\emptyset\vdash(P\wedge Q)\Rightarrow P}\,\Rightarrow i$$

vi)

$$\dfrac{\dfrac{}{\Gamma \vdash Q}\,Ax \qquad \dfrac{\dfrac{}{\Gamma \vdash P \Rightarrow \neg Q}\,Ax \qquad \dfrac{}{\Gamma \vdash P}\,Ax}{\Gamma \vdash \neg Q}\Rightarrow e}{\dfrac{\Gamma = P \Rightarrow \neg Q, Q, P \vdash \bot}{P \Rightarrow \neg Q, Q \vdash \neg P}\neg i}\neg e$$

vii)

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash P \Rightarrow Q}\,Ax \qquad \dfrac{\dfrac{}{\Gamma \vdash P \wedge R}\,Ax}{\Gamma \vdash P}\wedge e_1}{\Gamma \vdash Q}\Rightarrow e \qquad \dfrac{\dfrac{}{\Gamma \vdash P \wedge R}\,Ax}{\Gamma \vdash R}\wedge e_2}{\dfrac{\Gamma = P \Rightarrow Q,\ P \wedge R \vdash Q \wedge R}{P \Rightarrow Q \vdash (P \wedge R) \Rightarrow (Q \wedge R)}\Rightarrow i}\wedge_i$$

viii)

"O BIEN P V, O BIEN Q V"

$$\dfrac{\dfrac{\dfrac{}{\Gamma \vdash P \vee Q}\,Ax \qquad \dfrac{\dfrac{}{\Gamma, P \vdash P}\,Ax}{\Gamma, P \vdash P \vee R}\vee i_1 \qquad \dfrac{\dfrac{\dfrac{}{\Gamma, Q \vdash Q \Rightarrow R}\,Ax \qquad \dfrac{}{\Gamma, Q \vdash Q}\,Ax}{\Gamma, Q \vdash R}\Rightarrow e}{\Gamma, Q \vdash P \vee R}\vee i_2}{\Gamma = (Q \Rightarrow R), (P \vee Q) \vdash P \vee R}\vee e}{Q \Rightarrow R \vdash (P \vee Q) \Rightarrow (P \vee R)}\Rightarrow i \ ✓$$

ix)

$$\dfrac{\dfrac{}{\Gamma \vdash (P \vee Q) \vee R}\,Ax \qquad \dfrac{\dfrac{}{\Gamma \vdash P \vee Q}\,Ax \qquad \dfrac{\dfrac{}{\Gamma, P \vdash P}\,Ax}{\Gamma, P \vdash P \vee (Q \vee R)}\vee e_1 \qquad \dfrac{\dfrac{}{\Gamma, Q \vdash Q}\,Ax}{\Gamma, Q \vdash Q \vee R}\vee e_1}{\Gamma, (P \vee Q) \vdash P \vee (Q \vee R)} \qquad \dfrac{\dfrac{}{\Gamma, R \vdash R}\,Ax}{\Gamma, R \vdash P \vee (Q \vee R)}\vee i_{2 \times 2}}{(P \vee Q) \vee R \vdash P \vee (Q \vee R)}$$

o vale P o vale Q. pero P y Q

x)

$$\dfrac{}{\quad}\,Ax$$

$$
\dfrac{
\dfrac{\dfrac{}{\Gamma \vdash P\wedge(Q\vee R)}\,Ax}{\Gamma \vdash Q\vee R}\,\wedge e_2
\qquad
\dfrac{\dfrac{\dfrac{\Gamma'\vdash P\wedge(Q\vee R)}{\Gamma'\vdash P}\,\wedge e_1 \quad \dfrac{}{\Gamma'\vdash Q}\,Ax}{\Gamma' = P\wedge(Q\vee R), Q \vdash (P\wedge Q)}\,\wedge_i}{P\wedge(Q\vee R), Q \vdash (P\wedge Q)\vee(P\wedge R)}\,\vee i_1
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma''\vdash P\wedge(Q\vee R)}\,Ax}{\Gamma''\vdash P}\,\wedge e_1 \quad \dfrac{}{\Gamma''\vdash R}\,Ax}{\Gamma''\vdash P\wedge R}\,\wedge_i}{\Gamma'' = P\wedge(Q\vee R), R \vdash (P\wedge Q)\vee(P\wedge R)}\,\vee i_2
}{P\wedge(Q\vee R) \vdash (P\wedge Q)\vee(P\wedge R)}\,\vee e
$$

si se dan P y Q. ME QUEDO CON (P∧Q)

SE Q VALEN P y R. ME QUEDO CON P∧R

P vale. Averiguo si vale Q o R.

X:)

$$
\dfrac{
\dfrac{}{\Gamma \vdash (P\wedge Q)\vee(P\wedge R)}\,Ax
\qquad
\dfrac{\dfrac{\dfrac{}{\Gamma,(P\wedge Q)\vdash P\wedge Q}\,Ax}{\Gamma,(P\wedge Q)\vdash P}\,\wedge e_1 \quad \dfrac{\dfrac{\dfrac{}{\Gamma,(P\wedge Q)\vdash (P\wedge Q)}\,Ax}{\Gamma,(P\wedge Q)\vdash Q}\,\wedge e_2}{\Gamma,(P\wedge Q)\vdash Q\vee R}\,\vee i_1}{\Gamma,(P\wedge Q)\vdash P\wedge(Q\vee R)}\,\wedge_i
\qquad
\dfrac{\dfrac{\dfrac{}{\Gamma,(P\wedge R)\vdash (P\wedge R)}\,Ax}{\Gamma,(P\wedge R)\vdash P}\,\wedge e_1 \quad \dfrac{\dfrac{\dfrac{}{\Gamma,(P\wedge R)\vdash (P\wedge R)}\,Ax}{\Gamma,(P\wedge R)\vdash R}\,\wedge e_2}{\Gamma,(P\wedge R)\vdash (Q\vee R)}\,\vee i_2}{\Gamma,(P\wedge R)\vdash P\wedge(Q\vee R)}\,\wedge_i
}{(P\wedge Q)\vee(P\wedge R) \vdash P\wedge(Q\vee R)}\,\vee e
$$

VALEN P y Q. TENGO P, BUSCO Q

VALEN P y R. TENGO P, BUSCO R.

P vale en ambos.

Debo ver si valen Q o R.

Xii)

$$
\dfrac{
\dfrac{}{\neg P\vee Q, P \vdash \neg P\vee Q}\,Ax
\qquad
\dfrac{\dfrac{\dfrac{}{\Gamma'\vdash P}\,Ax \quad \dfrac{}{\Gamma'\vdash \neg P}\,Ax}{\Gamma'\vdash \bot}\,\neg e}{\Gamma' = \neg P\vee Q, P, \neg P \vdash Q}\,\bot e
\qquad
\dfrac{}{\neg P\vee Q, P, Q \vdash Q}\,Ax
}{\dfrac{\neg P\vee Q, P \vdash Q}{\neg P\vee Q \vdash P\Rightarrow Q}\,\Rightarrow i}
$$

Fuerzo Bot.

O vale ¬P o Q. Necesito dos casos.

Xiii)

$$
\dfrac{\dfrac{}{\Gamma'\vdash P\Rightarrow Q}\,Ax \quad \dfrac{}{\Gamma'\vdash P}\,Ax}{\Gamma' = P\Rightarrow Q, P\Rightarrow\neg Q, P \vdash Q}\,\Rightarrow e
\qquad
\dfrac{\dfrac{}{\Gamma''\vdash P\Rightarrow\neg Q}\,Ax \quad \dfrac{}{\Gamma''\vdash P}\,Ax}{\Gamma'' = P\Rightarrow Q, P\Rightarrow\neg Q, P \vdash \neg Q}\,\Rightarrow e
$$

$\neg e$

$$P \Rightarrow Q,\ P \Rightarrow \neg Q,\ P \vdash \bot$$

$\neg i$

$$P \Rightarrow Q,\ P \Rightarrow \neg Q \vdash \neg P$$

VALE P, BUSCO METER P A CONTEXTO

X iv)

Ax: $\Gamma'' \vdash P \Rightarrow (Q \Rightarrow R)$ Ax: $\Gamma'' \vdash P$ $\Rightarrow e$

$\Gamma'' \vdash (Q \Rightarrow R)$ Ax: $\Gamma'' \vdash Q$ $\Rightarrow e$

Ax: $\Gamma'' \vdash \neg R$ $\Gamma'' \vdash R$ $\neg e$

$$\Gamma'' = P \Rightarrow (Q \Rightarrow R),\ P,\ \neg R,\ Q \vdash \bot$$

$\neg i$

$$\Gamma' = P \Rightarrow (Q \Rightarrow R),\ P,\ \neg R \vdash \neg Q$$

VALE P

NO VALE R.

(Q = V)

(Q⇒R), necesito Q xq gracias a él incluyo R.

Ejercicio 12

Probar que los siguientes secuentes son válidos:

- I. $(P \wedge \neg Q) \Rightarrow R, \neg R, P \vdash Q$
- II. $\neg P \Rightarrow Q \vdash \neg Q \Rightarrow P$
- III. $P \vee Q \vdash R \Rightarrow (P \vee Q) \wedge R$
- IV. $(P \vee (Q \Rightarrow P)) \wedge Q \vdash P$
- V. $P \Rightarrow Q, R \Rightarrow S \vdash (P \wedge R) \Rightarrow (Q \wedge S)$
- VI. $P \Rightarrow Q \vdash ((P \wedge Q) \Rightarrow P) \wedge (P \Rightarrow (P \wedge Q))$
- VII. $P \Rightarrow (Q \wedge R) \vdash (P \Rightarrow Q) \wedge (P \Rightarrow R)$
- VIII. $(P \Rightarrow Q) \wedge (P \Rightarrow R) \vdash P \Rightarrow (Q \wedge R)$
- IX. $P \vee (P \wedge Q) \vdash P$
- X. $P \Rightarrow (Q \vee R), Q \Rightarrow S, R \Rightarrow S \vdash P \Rightarrow S$
- XI. $(P \wedge Q) \vee (P \wedge R) \vdash P \wedge (Q \vee R)$

i)

Ax: $\Gamma' \vdash P$ Ax: $\Gamma' \vdash \neg Q$ $\wedge i$

Ax: $\Gamma' \vdash (P \wedge \neg Q) \Rightarrow R$ $\Gamma' \vdash P \wedge \neg Q$ $\Rightarrow e$

$\Gamma' \vdash R$ Ax: $\Gamma' \vdash \neg R$ $\neg e$

$$\Gamma' = (P \wedge \neg Q) \Rightarrow R,\ \neg R,\ P,\ \neg Q \vdash \bot$$

PBC

$$(P \wedge \neg Q) \Rightarrow R,\ \neg R,\ P \vdash Q$$

No vale R.

VALE P.

No vale Q ~> Busco meter en contexto

ii)

$$\dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma' \vdash \neg P \Rightarrow Q}\,Ax \qquad \dfrac{}{\Gamma' \vdash \neg P}\,Ax}{\Gamma' \vdash Q}\,{\Rightarrow_e} \qquad \dfrac{}{\Gamma' \vdash \neg Q}\,Ax}{\Gamma' = \neg P \Rightarrow Q, \neg Q, \neg P \vdash \bot}}{\neg P \Rightarrow Q, \neg Q \vdash P}\,PBC}{\neg P \Rightarrow Q \vdash \neg Q \Rightarrow P}\,{\Rightarrow_i}$$

($\vee \Rightarrow$?) si no tengo $\neg P$ es trivial $\vee$.

No vale P hace valer Q ~> necesito $\neg P$

iii)

$$\dfrac{\dfrac{\dfrac{}{\Gamma' \vdash P \vee Q}\,Ax \qquad \dfrac{}{\Gamma' \vdash R}\,Ax}{\Gamma' = P \vee Q, R \vdash (P \vee Q) \wedge R}\,\wedge_i}{P \vee Q \vdash R \Rightarrow (P \vee Q) \wedge R}\,{\Rightarrow_i}$$

iv)

$$\dfrac{\dfrac{}{\Gamma \vdash (P \vee (Q \Rightarrow P))}\,Ax \qquad \dfrac{}{\Gamma, P \vdash P}\,Ax \qquad \dfrac{\dfrac{}{\Gamma'' \vdash Q \Rightarrow P}\,Ax \qquad \dfrac{\dfrac{}{\Gamma'' \vdash (P \vee (Q \Rightarrow P)) \wedge Q}\,Ax}{\Gamma'' \vdash Q}\,\wedge_{e_2}}{\Gamma'' = \Gamma, (Q \Rightarrow P) \vdash P}\,{\Rightarrow_e}}{\Gamma = (P \vee (Q \Rightarrow P)) \wedge Q \vdash P}\,\vee_e$$

VALE Q. si vale Q vale P.

v)

$$\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma' \vdash P \Rightarrow Q}\,Ax \qquad \dfrac{\dfrac{}{\Gamma' \vdash P \wedge R}\,Ax}{\Gamma' \vdash P}\,\wedge_{e_1}}{\Gamma' \vdash Q}\,{\Rightarrow_e} \qquad \dfrac{\dfrac{}{\Gamma' \vdash R \Rightarrow S}\,Ax \qquad \dfrac{\dfrac{}{\Gamma' \vdash P \wedge R}\,Ax}{\Gamma' \vdash R}\,\wedge_{e_2}}{\Gamma' \vdash S}\,{\Rightarrow_e}}{\Gamma' = P \Rightarrow Q, R \Rightarrow S, (P \wedge R) \vdash (Q \wedge S)}\,\wedge_i}{P \Rightarrow Q, R \Rightarrow S \vdash (P \wedge R) \Rightarrow (Q \wedge S)}\,{\Rightarrow_i}$$

si vale P, vale Q. (necesito P y R en contexto, vale)

P vale y R vale.

Si vale R, vale S.

vi)

$$
\dfrac{\dfrac{\dfrac{}{P\Rightarrow Q,\ P\wedge Q \vdash P\wedge Q}\,Ax}{P\Rightarrow Q,\ P\wedge Q \vdash P}\,\wedge_{e1}}{P\Rightarrow Q \vdash (P\wedge Q)\Rightarrow P}\,\Rightarrow_i
\qquad
\dfrac{\dfrac{\dfrac{}{P\Rightarrow Q, P\vdash P}\,Ax \quad \dfrac{\dfrac{}{P\Rightarrow Q, P\vdash P\wedge Q}\,Ax}{P\Rightarrow Q, P\vdash Q}\,\wedge_{e2}}{P\Rightarrow Q,\ P \vdash P\wedge Q}\,\wedge_i}{P\Rightarrow Q \vdash P\Rightarrow(P\wedge Q)}\,\Rightarrow_i
$$

$$
\wedge_i:\quad P\Rightarrow Q \vdash ((P\wedge Q)\Rightarrow P)\wedge(P\Rightarrow(P\wedge Q))
$$

Si vale P vale Q. Necesito P sí o sí

vii)

$$
\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma' \vdash P\Rightarrow(Q\wedge R)}\,Ax \quad \dfrac{}{\Gamma'\vdash P}\,Ax}{\Gamma'\vdash Q\wedge R}\,\Rightarrow_e}{\Gamma' = P\Rightarrow(Q\wedge R),\ P\vdash Q}\,\wedge_{e1}}{P\Rightarrow(Q\wedge R) \vdash P\Rightarrow Q}\,\Rightarrow_i
\qquad
\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma'' \vdash P\Rightarrow(Q\wedge R)}\,Ax \quad \dfrac{}{\Gamma''\vdash P}\,Ax}{\Gamma''\vdash Q\wedge R}\,\Rightarrow_e}{\Gamma'' = P\Rightarrow(Q\wedge R),\ P\vdash R}\,\wedge_{e2}}{P\Rightarrow(Q\wedge R) \vdash P\Rightarrow R}\,\Rightarrow_i
$$

→ Q APARECE FÁCIL.
CON P ES FÁCIL

$$
\wedge_i:\quad P\Rightarrow(Q\wedge R) \vdash (P\Rightarrow Q)\wedge(P\Rightarrow R)
$$

Si vale P, valen Q y R. Necesito P seguro

viii)

$$
\dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma'\vdash P\Rightarrow Q}\,Ax \quad \dfrac{}{\Gamma'\vdash P}\,Ax}{\Gamma' = \Gamma, P \vdash Q}\,\Rightarrow_e \qquad \dfrac{\dfrac{}{\Gamma''\vdash P\Rightarrow R}\,Ax \quad \dfrac{}{\Gamma''\vdash P}\,Ax}{\Gamma'' = \Gamma, P \vdash R}\,\Rightarrow_e}{\Gamma, P \vdash Q\wedge R}\,\wedge_i}{\Gamma = (P\Rightarrow Q)\wedge(P\Rightarrow R) \vdash P\Rightarrow(Q\wedge R)}\,\Rightarrow_i
$$

P vale seguro. Necesito en CONTEXTO P. Los demás salen de él.

ix)

$$\dfrac{\dfrac{}{\Gamma \vdash P \vee (P \wedge Q)}\,\text{Ax} \qquad \dfrac{}{\Gamma, P \vdash P}\,\text{Ax} \qquad \dfrac{\dfrac{}{\Gamma, (P \wedge Q) \vdash P \wedge Q}\,\text{Ax}}{\Gamma, (P \wedge Q) \vdash P}\,\wedge e_1}{\Gamma = P \vee (P \wedge Q) \vdash P}\,\vee e$$

O vale P o vale $P \wedge Q$ de P. Con P me BASTA (en AMBAS vale).

x)

$$\dfrac{\dfrac{\dfrac{}{\Gamma'' \vdash P \Rightarrow (Q \vee R)}\,\text{Ax} \quad \dfrac{}{\Gamma'' \vdash P}\,\text{Ax}}{\Gamma'' = \Gamma', P \vdash (Q \vee R)}\,\Rightarrow e \qquad \dfrac{\dfrac{}{\Gamma''' \vdash Q \Rightarrow S}\,\text{Ax} \quad \dfrac{}{\Gamma''' \vdash Q}\,\text{Ax}}{\Gamma''' = \Gamma', P, Q \vdash S}\,\Rightarrow e \qquad \dfrac{\dfrac{}{\Gamma'''' \vdash R \Rightarrow S}\,\text{Ax} \quad \dfrac{}{\Gamma'''' \vdash R}\,\text{Ax}}{\Gamma'''' = \Gamma', P, R \vdash S}\,\Rightarrow e}{\dfrac{\Gamma', P \vdash S}{\Gamma' = P \Rightarrow (Q \vee R), Q \Rightarrow S, R \Rightarrow S \vdash P \Rightarrow S}\,\Rightarrow i}\,\vee e$$

si vale P, vale Q o R.

si vale Q vale S. Necesito Q y R si o si.

si vale R vale S. En un O, o vale Q o R.

xi)

$$\dfrac{\dfrac{}{\Gamma' \vdash (P \wedge Q) \vee (P \wedge R)}\,\text{Ax} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma', (P\wedge Q) \vdash P \wedge Q}\,\text{Ax}}{\Gamma', (P \wedge Q) \vdash P}\,\wedge e_1 \quad \dfrac{\dfrac{\dfrac{}{\Gamma', (P\wedge Q) \vdash P \wedge Q}\,\text{Ax}}{\Gamma', (P \wedge Q) \vdash Q}\,\wedge e_2}{\Gamma', (P \wedge Q) \vdash (Q \vee R)}\,\vee i_1}{\Gamma', (P \wedge Q) \vdash P \wedge (Q \vee R)}\,\wedge i \qquad \dfrac{\dfrac{\dfrac{}{\Gamma', (P\wedge R) \vdash P \wedge R}\,\text{Ax}}{\Gamma', (P \wedge R) \vdash P}\,\wedge e_1 \quad \dfrac{\dfrac{\dfrac{}{\Gamma', (P\wedge R) \vdash P \wedge R}\,\text{Ax}}{\Gamma', (P \wedge R) \vdash R}\,\wedge e_2}{\Gamma', (P \wedge R) \vdash Q \vee R}\,\vee i_2}{\Gamma', (P \wedge R) \vdash P \wedge (Q \vee R)}\,\wedge i}{\Gamma' = (P \wedge Q) \vee (P \wedge R) \vdash P \wedge (Q \vee R)}\,\vee e$$

O valen $P \wedge Q$ o valen $P \wedge R$

Ejercicio 13

Probar que los siguientes secuentes son válidos:

- I. $\neg P \Rightarrow \neg Q \vdash Q \Rightarrow P$
- II. $\neg P \vee \neg Q \vdash \neg(P \wedge Q)$
- III. $\neg P, P \vee Q \vdash Q$
- IV. $P \vee Q, \neg Q \vee R \vdash P \vee R$
- V. $P \wedge \neg P \vdash \neg(R \Rightarrow Q) \wedge (R \Rightarrow Q)$
- VI. $\neg(\neg P \vee Q) \vdash P$
- VII. $\vdash \neg P \Rightarrow (P \Rightarrow (P \Rightarrow Q))$
- VIII. $P \wedge Q \vdash \neg(\neg P \vee \neg Q)$
- IX. $\vdash (P \Rightarrow Q) \vee (Q \Rightarrow R)$

i)

$$\dfrac{\dfrac{\dfrac{}{\Gamma' \vdash Q}\,\text{Ax} \qquad \dfrac{\dfrac{}{\Gamma' \vdash \neg P \Rightarrow \neg Q}\,\text{Ax} \quad \dfrac{}{\Gamma' \vdash \neg P}\,\text{Ax}}{\Gamma' \vdash \neg Q}\,\Rightarrow e}{\Gamma' = \neg P \Rightarrow \neg Q, Q, \neg P \vdash \bot}\,\neg e}{}\,\text{PBC}$$

$$\dfrac{\neg P \Rightarrow \neg Q,\ Q \vdash P}{\neg P \Rightarrow \neg Q \vdash Q \Rightarrow P}\ {\scriptstyle \Rightarrow i}$$

Si no vale P, no vale Q.

Busco $\neg P$ en contexto y Q. Para $\neg Q, Q^{\neg Ax}$

↳ $\neg P \Rightarrow \neg Q, \neg P$

ii)

$$\dfrac{\dfrac{}{\Gamma \vdash \neg P \vee \neg Q}{\scriptstyle Ax} \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma' \vdash P \wedge Q}{\scriptstyle Ax}}{\Gamma' \vdash P}{\scriptstyle \wedge e_1} \qquad \dfrac{}{\Gamma' \vdash \neg P}{\scriptstyle Ax}}{\Gamma' = \Gamma, \neg P, (P \wedge Q) \vdash \bot}{\scriptstyle \neg e}}{\Gamma, \neg P \vdash \neg(P \wedge Q)}{\scriptstyle \neg i} \qquad \dfrac{\dfrac{\dfrac{\dfrac{}{\Gamma'' \vdash P \wedge Q}{\scriptstyle Ax}}{\Gamma'' \vdash Q}{\scriptstyle \wedge e_2} \qquad \dfrac{}{\Gamma'' \vdash \neg Q}{\scriptstyle Ax}}{\Gamma'' = \Gamma, \neg Q, (P \wedge Q) \vdash \bot}{\scriptstyle \neg e}}{\Gamma, \neg Q \vdash \neg(P \wedge Q)}{\scriptstyle \neg i}}{\neg P \vee \neg Q \vdash \neg(P \wedge Q)}{\scriptstyle \vee e}$$

O vale $\neg P$ o vale $\neg Q$, después $\neg i$. DESP vale con $\neg e$ con $\neg P$ o $\neg Q$ y armo el $\wedge$.

iii)

$$\dfrac{\dfrac{}{\neg P, P \vee Q \vdash P \vee Q}{\scriptstyle Ax} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma' \vdash P}{\scriptstyle Ax} \quad \dfrac{}{\Gamma' \vdash \neg P}{\scriptstyle Ax}}{\Gamma' \vdash \bot}{\scriptstyle \neg e}}{\Gamma' = \neg P, P \vee Q, P \vdash Q}{\scriptstyle \bot e} \qquad \dfrac{}{\neg P, P \vee Q, Q \vdash Q}{\scriptstyle Ax}}{\neg P, P \vee Q \vdash Q}{\scriptstyle \vee e}$$

No vale P

Vale P o vale Q ~> Busco contradicción entre VALE P y NO vale P.

Vale Q. Vale mismo

iv)

$$\dfrac{\dfrac{}{\Gamma \vdash P \vee Q}{\scriptstyle Ax} \qquad \dfrac{\dfrac{}{\Gamma', P \vdash P}{\scriptstyle Ax}}{\Gamma', P \vdash P \vee R}{\scriptstyle \vee i_1} \qquad \dfrac{\dfrac{}{\Gamma'' \vdash \neg Q \vee R}{\scriptstyle Ax} \qquad \dfrac{\dfrac{\dfrac{}{\Gamma'', \neg Q \vdash \neg Q}{\scriptstyle Ax} \quad \dfrac{}{\Gamma'', \neg Q \vdash Q}{\scriptstyle Ax}}{\Gamma'', \neg Q \vdash \bot}{\scriptstyle \neg e}}{\Gamma'', \neg Q \vdash R}{\scriptstyle \bot e} \qquad \dfrac{}{\Gamma'', R \vdash R}{\scriptstyle Ax}}{\Gamma'' = \Gamma', Q \vdash R}{\scriptstyle \vee e}}{\Gamma = P \vee Q, \neg Q \vee R \vdash P \vee R}{\scriptstyle \vee e}$$

$P \vee Q, \neg Q \vee R, Q \vdash R$

O BIEN VALE $\neg Q$ ~> CONTRADICCIÓN

O BIEN VALE R ~> AX.

O vale P o vale Q

o vale ¬Q ← Posible contradicción. o vale Q.

v)

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma' \vdash P \wedge \neg P$}
\RightLabel{$\wedge e_1$}
\UnaryInfC{$\Gamma' \vdash P$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma' \vdash P \wedge \neg P$}
\RightLabel{$\wedge e_2$}
\UnaryInfC{$\Gamma' \vdash \neg P$}
\RightLabel{$\neg e$}
\BinaryInfC{$\Gamma' \vdash \bot$}
\RightLabel{$\bot e$}
\UnaryInfC{$\Gamma' = P \wedge \neg P \vdash \neg(R \Rightarrow Q) \wedge (R \Rightarrow Q)$}
\end{prooftree}
$$

Busco bottom de una. No hay relación entre $P \wedge \neg P$ y lo q' concluye.

vi)

$$
\begin{prooftree}
\AxiomC{}
\UnaryInfC{$\neg(\neg P \vee Q) \vdash \neg P \vee Q$}
\AxiomC{$\neg(\neg P \vee Q), \neg P \vdash P$}
\AxiomC{$\neg(\neg P \vee Q), Q, \neg P \vdash \bot$}
\RightLabel{PbC}
\UnaryInfC{$\neg(\neg P \vee Q), Q \vdash P$}
\RightLabel{$\vee e$}
\TrinaryInfC{$\neg(\neg P \vee Q) \vdash P$}
\end{prooftree}
$$

O vale ¬P o vale Q.

$(P \wedge \neg Q)$

vii)

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma' \vdash \neg P$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma' \vdash P$}
\RightLabel{$\bot e$}
\BinaryInfC{$\Gamma' = \neg P, P \vdash Q$}
\RightLabel{$\Rightarrow i_3$}
\UnaryInfC{$\neg P \Rightarrow (P \Rightarrow (P \Rightarrow Q))$}
\end{prooftree}
$$

Contradicción

viii)

$$
\begin{prooftree}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma' \vdash P \wedge Q$}
\RightLabel{$\wedge e_1$}
\UnaryInfC{$\Gamma' \vdash P$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma' \vdash \neg P \vee \neg Q$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma', \neg P \vdash \neg P$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma'' \vdash P \wedge Q$}
\RightLabel{$\wedge e_2$}
\UnaryInfC{$\Gamma'' \vdash Q$}
\AxiomC{}
\RightLabel{Ax}
\UnaryInfC{$\Gamma'' \vdash \neg Q$}
\RightLabel{$\bot e$}
\BinaryInfC{$\Gamma'' = \Gamma', \neg Q \vdash \neg P$}
\TrinaryInfC{$\Gamma' \vdash \neg P$}
\RightLabel{$\neg e$}
\BinaryInfC{$\Gamma' = (P \wedge Q), \neg P \vee \neg Q \vdash \bot$}
\RightLabel{$\neg i$}
\UnaryInfC{}
\end{prooftree}
$$

$(P \wedge Q), \neg P \vee \neg Q, \neg Q$

o vale ¬P o vale ¬Q

$P \land Q \vdash \neg(\neg P \lor \neg Q)$

VALE P y Q.

ix)

$$\dfrac{\dfrac{}{\vdash Q \lor \neg Q}\,\text{LEM} \qquad \dfrac{\dfrac{\dfrac{}{Q \vdash Q \Rightarrow R}\,\text{Ax} \quad \dfrac{}{Q \vdash Q}\,\text{Ax}}{Q \vdash R}\,{\Rightarrow_e}}{Q \vdash (P \Rightarrow Q) \lor (Q \Rightarrow R)}\,\lor_{i_2}, \Rightarrow_i \qquad \dfrac{\dfrac{\dfrac{\dfrac{\dfrac{}{\neg Q, Q \vdash \neg Q}\,\text{Ax} \quad \dfrac{}{\neg Q, Q \vdash Q}\,\text{Ax}}{\neg Q, Q \vdash \bot}\,\neg_e}{\neg Q, Q \vdash R}\,\bot_e}{\neg Q \vdash Q \Rightarrow R}\,{\Rightarrow_i}}{\neg Q \vdash (P \Rightarrow Q) \lor (Q \Rightarrow R)}\,\lor_{i_2}}{\vdash (P \Rightarrow Q) \lor (Q \Rightarrow R)}\,\lor_e$$

Si vale P, vale Q.

Si vale Q vale R.

necesito Q en contexto. → No tengo nada contexto. FUERZO LEM.