2023/3/28

CSE3081 알고리즘 설계와 분석
HW1 답

1. 가. $(n+1)\log_2(n+1) - n$ 10

   나. $O(n\log n)$ 5

2. 가. $n - \log_2(n+1)$ 10

   나. $O(n)$ 5

3. 가. $\sum_{k=1}^{i} k = \frac{i(i+1)}{2}$ 2

   나. $\left\lfloor \frac{-1+\sqrt{1+8n}}{2} \right\rfloor$ 4

   다. 나.답의 2배 2

   라. $O(\sqrt{n})$ 2

4. 2와 12, 4와 8 10

5. 1, 3 10

6. 2, 6, 12 10

1. 번 풀이

$\sum_{i=1}^{k} i \cdot 2^{i-1}$ 을 I라 하자.

$$
\begin{array}{rl}
I = & 1\cdot 2^0 + 2\cdot 2^1 + 3\cdot 2^2 + \cdots + k\cdot 2^{k-1} \\
-)\ 2I = & \qquad 1\cdot 2^1 + 2\cdot 2^2 + \cdots + (k-1)\cdot 2^{k-1} + k\cdot 2^k \\
\hline
-I = & 2^0 + 2^1 + 2^2 + \cdots + 2^{k-1} - k\cdot 2^k
\end{array}
$$

$$\therefore\ I = k\cdot 2^k - \frac{2^k - 1}{2-1} = k\cdot 2^k - 2^k + 1$$

$k = \log_2(n+1)$ 이므로

$$I = \log_2(n+1)\, 2^{\log_2(n+1)} - 2^{\log_2(n+1)} + 1$$

$$= (n+1)\log_2(n+1) - (n+1) + 1$$

$$= (n+1)\log_2(n+1) - n$$

2. 번 풀이

$\sum_{i=1}^{k} (k-i)2^{i-1}$ 을 I라 하자.

$$
\begin{array}{rl}
I = & (k-1)2^0 + (k-2)2^1 + \cdots + 1\cdot 2^{k-2} + 0\cdot 2^{k-1} \\
-)\ 2I = & \qquad (k-1)2^1 + \cdots + 2\cdot 2^{k-2} + 1\cdot 2^{k-1} \\
\hline
-I = & (k-1) - (2^1 + 2^2 + \cdots + 2^{k-1})
\end{array}
$$

$$= k-1 - \frac{2\cdot(2^{k-1}-1)}{2-1} = k-1-2^k+2$$

$$\therefore\ I = 2^k - k - 1$$

$k = \log_2(n+1)$ 이므로

$$I = 2^{\log_2(n+1)} - \log_2(n+1) - 1 = n - \log_2(n+1)$$

```
c = 0;
i = 1; j = 1; m = 0 // n > 0 or n ≥ 1
while (j <= n) {
    i++;
    j = j + i;
    m = m + 2;
    c++;
}
```

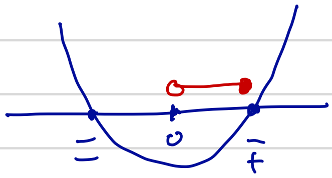
	$\le n$	$\le n$	$\le n$		$\le n$	$> n$
j_{before}	1	$1+2$	$1+2+3$	$\cdots$	$1+2+\cdots+p$	$1+2+\cdots+(p+1)$
i	2	3	4	$\cdots$	$p+1$	
j_{after}	$1+2$	$1+2+3$	$1+2+3+4$	$\cdots$	$1+2+\cdots+(p+1)$	
c	1	2	3	$\cdots$	p	

Find p such that $\frac{p(p+1)}{2} \le n$ and $\frac{(p+1)(p+2)}{2} > n$.

$$p^2+p-2n \le 0 \cdots ① \quad \& \quad p^2+3p+2-2n > 0 \cdots ②$$

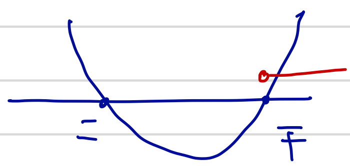
From ①,

$$p = \frac{-1 \pm \sqrt{8n+1}}{2} \rightarrow p \le \frac{-1+\sqrt{8n+1}}{2}$$

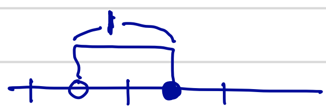


From ②,

$$p = \frac{-3 \pm \sqrt{8n+1}}{2} \rightarrow p > \frac{-3+\sqrt{8n+1}}{2}$$



$$\therefore \frac{-3+\sqrt{8n+1}}{2} < p \le \frac{-3+\sqrt{8n+1}}{2} + 1$$



$$\Rightarrow p = \left\lfloor \frac{-3+\sqrt{8n+1}}{2} \right\rfloor + 1 \quad \left(\ne \left\lceil \frac{-3+\sqrt{8n+1}}{2} \right\rceil \right)$$