Ayudantía 12 TY

Ecuaciones importantes

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \dots$$

Ecuación de onda $\rightarrow$ $\dfrac{\partial^2 f}{\partial t^2} - c^2 \nabla^2 f = F$

Ecuación de difusión $\rightarrow$ $\dfrac{\partial f}{\partial t} + k \nabla^2 f = F$

Ecuación de Poisson $\rightarrow$ $\nabla^2 f = F$ $\rightarrow$ normalmente densidad

(o laplace)

Forzante

Equació de onda (2D)

$$\frac{\partial^2 f}{\partial t^2} - c^2 \nabla^2 f = F$$

$$\iff \frac{f(x,y,t+1) - 2f(x,y,t) + f(x,y,t-1)}{dt^2} = c^2 \nabla^2 f + F$$

$$\iff \boxed{f(x,y,t+1) = 2f(x,y,t - f(x,y,t-1) + dt^2\left(c^2 \nabla^2 f + F\right)}$$

Ecuación de difusión

$$\frac{\partial f}{\partial t} - k\nabla^2 f = F$$

$$\Longleftrightarrow \frac{f_{(x,y,t+1)} - f_{(x,y,t)}}{dt} = k\nabla^2 f + F$$

$$\Longleftrightarrow \boxed{f_{(x,y,t+1)} = f_{(x,y,t)} + dt\left(k\nabla^2 f + F\right)}$$

Ecuación de Poisson

$$\nabla^2 f = F$$

$$\Rightarrow \frac{\partial^2 f}{\partial x^2} + \frac{\partial f}{\partial y^2} = F$$

$$\Leftrightarrow \frac{f_{(x+1,y)} - 2 f_{(x,y)} + f_{(x-1,y)}}{dx^2}$$

$$+ \frac{f_{(x,y+1)} - 2 f_{(x,y)} + f_{(x,y-1)}}{dy^2} = F$$

Asumiendo $dx = dy = dh$

$$\Rightarrow -4 f_{(x,y)} + f_{(x+1,y)} + f_{(x-1,y)} + f_{(x,y+1)} + f_{(x,y-1)} = F\,dh^2$$

$$\Rightarrow \boxed{f_{(x,y)} = \frac{1}{4}\left(f_{(x+1,y)} + f_{(x-1,y)} + f_{(x,y+1)} + f_{(x,y-1)} - F\,dh^2 \right)}$$

De aquí es directo

SOR:

$$\boxed{f_{(x,y)} = (1-\omega) f_{(x,y)} + \frac{\omega}{4}\left(f_{(x+1,y)} + f_{(x-1,y)} + f_{(x,y+1)} + f_{(x,y-1)} - F\,dh^2 \right)}$$

$\omega = 1 \longrightarrow$ dif. finitas

$\omega > 1 \longrightarrow$ SOR

$\omega < 1 \longrightarrow$ SUB-Relajación