Exam of Fundamentals of Mathematics II

April 1st, 2022

1. Consider the following curve in 3-dimensional space parametrized by $t \in (-\infty, \infty)$:

$$\sigma(t) = (e^t - e^{-t} + 1, e^t + e^{-t} - 1, 3e^t + e^{-t} + 1).$$

   (a) Find parametric and cartesian equations for the tangent line to the curve for $t = 0$.

   (b) Find the equation of the normal plane to the curve for $t = 0$.

   (c) Check if the curve $\sigma(t)$ lies in a plane in $\mathbb{R}^3$. If it does, find the equation of this plane.

2. Consider the function $f(x, y, z) = 2z^2 - xy$

   (a) Consider the point $P = (1, 1, 1)$. Find the direction from this point in which the function f grows faster.

   (b) Consider the isosurface of the function f passing through the point $P = (1, -1, 1)$. Find the equation of the tangent plane to this surface at the point P.

   (c) Do the isosurfaces through $(1, 1, 1)$ and $(3, 1, 1)$ have the same shape? Explain why.

3. Compute the maxima and the minima of the function $f(x, y) = x^2 + y^2 + 2y - 1$ on the closed disc $x^2 + y^2 = 1$.

4. Compute the volume limited by the two surfaces $z = 4 - x^2 - y^2$ and $z = 1$.

Every problem has a maximum score of 2.5