Fundamentals of Math I — **AI Degree**

First Partial Exam — **November 11, 2021**

Full Name: ....

NIU: ....

Exercise 1. (2 points + 2 points)

Consider the matrices

$$A = \begin{pmatrix} 1 & 2 & -1 & 2 & 3 \\ -1 & -2 & 2 & 0 & -3 \\ 2 & 4 & -3 & 2 & 6 \end{pmatrix} \in M_{3\times 5}(\mathbb{R}), \qquad B = \begin{pmatrix} 1 \\ -1 \\ a \end{pmatrix} \in M_{3\times 1}(\mathbb{R}),$$

where $a \in \mathbb{R}$.

(a) Find the Reduced Row Echelon Form of the augmented matrix $[A \mid B]$.

(b) Find the values of a for which the system of linear equations $AX = B$ is compatible, and give the solution of the system for these values of a, determining the degree of freedom of the system and the parametric form of the solutions.

Exercise 2. (2 points + 1 point + 1 point)

Consider the matrix

$$A = \begin{pmatrix} 1 & -1 & -2 & -3 \\ -2 & 1 & 4 & 6 \\ -4 & 1 & 8 & 12 \end{pmatrix} \in M_{3\times 4}(\mathbb{R}),$$

and the corresponding linear map $f_A \colon \mathbb{R}^4 \to \mathbb{R}^3$ given by left multiplication by A.

(a) Find the dimension and a basis of $\mathrm{Ker}(f_A)$ and $\mathrm{Im}(f_A)$.

(b) Enlarge the basis of $\mathrm{Im}(f_A)$ to a basis of $\mathbb{R}^3$.

(c) Enlarge the basis of $\mathrm{Ker}(f_A)$ to a basis of $\mathbb{R}^4$.

Theory. ($0,7$ points + $0,7$ points + $0,6$ points)

For each of the following assertions, say if the assertion is true or false. Justify your answer in each case.

(a) Any homogeneous system of linear equations is compatible.

(b) Let $AX = B$ be a system of linear equations with $A \in M_n(\mathbb{R})$ and $B \in M_{n\times 1}(\mathbb{R})$. If the system has a unique solution, then A is invertible.

(c) The ordered family $\mathcal{B} = [(12, -13, 121), (15, 58, 301)]$ is a basis of the vector space $\mathbb{R}^3$.

All answers must be carefully explained. You must specify the theoretical results used in your arguments and procedures.