Computational Logic
Practicing for the final exam (solutions to appear)

Exercise 1 *Let α and β be any sentences, determine whether or not the following statement is true. Explain your answer.*

$$\textit{If } \alpha \to \beta \textit{ is a tautology, then } \beta \textit{ is tautology.}$$

Exercise 2 *Let α and β be any sentences. Determine whether or not the following statement is true. Explain your answer.*

$$(\neg\alpha \lor \beta) \to \neg(\alpha \land \neg\beta) \textit{ is a tautology.}$$

Exercise 3 *Let α and β be any sentences, determine whether or not the following statement is true. Explain your answer.*

$$\textit{If } \alpha \leftrightarrow \beta \textit{ is a tautology, then } \alpha \textit{ and } \beta \textit{ are tautologies.}$$

Exercise 4 *Let α and β be any sentences, determine whether or not the following statement is true. Explain your answer.*

$$\textit{If } \alpha \to \beta \textit{ is a tautology, then } \alpha \textit{ is a contradiction.}$$

Exercise 5 *Determine whether the set*

$$\{A \to (C \lor B), A \land \neg E, (\neg E \land \neg D) \to \neg A, \neg D \to (B \leftrightarrow \neg C)\}$$

is satisfiable.

Exercise 6 *Determine whether the following sentences are a logical consequence of the set*

$$\{(A \land \neg B) \to \neg C, (\neg A \land B) \to C, \neg A \lor B\}.$$

(1) $\neg B$

(2) C

Exercise 7 *Let α be a sentence, prove or refute the following statement:*

α is a tautology if and only if α is a logical consequence of any set of sentences.

Exercise 8 *Using the Induction Principle for Sentences, prove that the number of right parentheses (i.e., ')') of a sentence is equal to the number of binary connectives.* **Warning:** You have to suppose that external parentheses are not removed.

Exercise 9 For each of the following sentences, obtain a logically equivalent sentence whose connectives are only the negation and the conjunction.

1. $\neg(\neg A \leftrightarrow \neg B) \vee C$
2. $\neg(A \rightarrow \neg B) \vee (C \leftrightarrow \neg A)$

Exercise 10 For each of the following sentences, obtain a logically equivalent sentence whose connectives are only the negation and the conditional.

1. $\neg(A \leftrightarrow \neg B) \wedge C$
2. $(\neg A \leftrightarrow B) \vee (C \wedge \neg A)$

Exercise 11 Symbolize the following sentences.

1. Spain's queen is Letizia or Bad Gyal.
2. If he is neither a singer nor an actor, then he will be out of the show.
3. Unless he gives me a good reason, either I will break up with him, or I will cheat on him.
4. I want to love you, but I will love you only if you finish telling such jokes.
5. For Novak to lose in Australia, it is necessary that Novak be ill.

Use the following **Symbolization key:**
A: Spain's queen is Letizia
B: Spain's queen is Bad Gyal
C: He is out of the show.
D: He is a singer.
E: He is an actor.
X: I want to love you.
Y: I will love you.
Z: You finish telling such jokes.
H: He gives me a good reason.
I: I will break up with him.
J: I will cheat on him.
K: Novak loses in Australia.
L: Novak is ill.

Exercise 12 Symbolize the following argument.

I will go to the concert only if the singer is performing the song "All too well". I will go to the concert if and only if I get tickets. For me to get tickets, it is necessary that I be rich. I am not rich. Therefore, I will not go to the concert.

Use the following **Symbolization key:**
A: I will go to the concert.
B: The singer is performing the song "All too well".
C: I get tickets.
D: I am rich.

Exercise 13 *Give a proof for the following argument (natural deduction):*

$$P \wedge (Q \vee R), P \to \neg R \vdash Q \vee E.$$

Exercise 14 *Give a proof for the following argument (natural deduction):*

$$\vdash \neg(A \wedge B) \to (\neg A \vee \neg B).$$

Exercise 15 *Give a proof for the following argument (natural deduction):*

$$\vdash (A \to B) \vee (B \to A).$$

Exercise 16 *Use the principle of induction for sentences of a first-order language $\mathcal{L}$ to prove that given a sentence of $\mathcal{L}$, a variable x, and a term t, φ has the same number of connectives than $\varphi(\frac{x}{t})$.*

Exercise 17 *For each of the following sentences, determine whether the sentence is closed or open and explain why.*

(A) $\forall x Px \wedge \exists y Rbx$.

(B) $\forall x \exists y \forall z (Rza \to (Px \vee Px))$.

Exercise 18 *Use three substitutions to obtain $\forall x Rdx$ from $\forall x Ryx$.*

Exercise 19 *For each of the following sentences, obtain $\varphi(\frac{x}{y})(\frac{y}{d})$*

(A) φ is $\forall x(x = y \to (Px \wedge a = y))$.

(B) φ is $Pa \wedge \neg((Rxy \vee Py) \wedge \exists y Px)$.

Exercise 20 *Consider a first-order language $\mathcal{L}$ with two predicate symbols P and Q, two binary relations R and S, and two individual constants c and d. Consider the following structure for $\mathcal{L}$*

$$\mathcal{A} = \langle A, P^{\mathcal{A}}, Q^{\mathcal{A}}, R^{\mathcal{A}}, S^{\mathcal{A}}, c^{\mathcal{A}}, d^{\mathcal{A}} \rangle, \text{ where:}$$

$A = \{1, 2, 3, 4, 5, 6\}$,

$P^{\mathcal{A}} = \{1, 4, 5, 6\}$, $Q^{\mathcal{A}} = \{2, 5, 6\}$,

$R^{\mathcal{A}} = \{\langle 1, 1\rangle, \langle 2, 2\rangle, \langle 3, 3\rangle, \langle 4, 4\rangle, \langle 5, 5\rangle, \langle 6, 6\rangle\}$,

$S^{\mathcal{A}} = \{\langle 1, 2\rangle, \langle 2, 1\rangle, \langle 2, 3\rangle, \langle 3, 2\rangle\}$,

$c^{\mathcal{A}} = 5$, and $d^{\mathcal{A}} = 1$.

Which of the following sentences are true in $\mathcal{A}$? Explain your answers.

(A) $\forall x(Py \to \exists y(\neg Px \wedge Qy))$.

(B) $\forall x \forall y((Qx \vee \neg Sdy) \to \exists z \neg Qy)$.

(C) $\forall x \forall y(Rxx \wedge \neg Sxx)$.

Exercise 21 *If possible, find a structure for which all the following sentences are true:* $\exists x(Px \wedge Qx)$, $\forall x(\neg Px \vee Qx)$, $\exists x(Px \wedge \neg Qx)$, and $\exists x(\neg Px \wedge \neg Qx)$.

Exercise 22 *Consider a first-order language $\mathcal{L}$ with three predicate symbols C, T, B, two binary relations L and R, and ten individual constants $a, b, c, d, e, f, g, h, i$, and j. Consider the following symbolization:*

Cx: x is a circle.

Tx: x is a triangle.

Bx: x is blue.

Lxy: x is on the left of y.

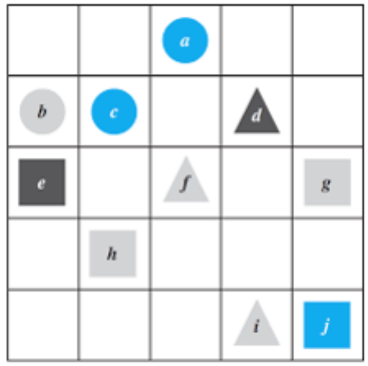
Rxy: x is on the right of y.

(A) $\forall x \exists y(Cy \to Rxy)$.

(B) $\neg(Td \leftrightarrow Bc)$.

(C) $\forall x \forall y(Ryx \to \exists z Rzy)$.

Using the following imagine, explain if each sentence above is true or false.



Exercise 23 *Consider the document "Preparing-for-Test-9.pdf", and the following line of the Barcelona underground:*

Exercise 24 *Propose a first-order language for representing and reasoning with knowledge about line L5 of the Barcelona underground. Include three relations (D, N, and Q), and one predicate symbol M. What could be the universe of discourse of a structure $\mathcal{A}$ in this example?*

Exercise 25 *We want to define M as the predicate for expressing that a station is adapted to people with reduced mobility. Find which stations of line L5 are adapted and use that knowledge to propose an interpretation of M in $\mathcal{A}$.*

Exercise 26 *Let us define two stations to be quite near if and only if they are on the same line, with at most three stations in between. Find the sentences* *to derive/obtain the complete interpretation of this relation Q. You can use the relations D and N.*

Exercise 27 *Using FOL, symbolize the following sentences (you have to define a first-order language and a structure for that language). Consider that the universe of discourse is the set of celestial bodies in the Solar System.*

1. The Earth is a planet.
2. The Moon is not a planet.
3. The Moon is a satellite.
4. The Earth orbits around the Sun.
5. Every planet is a satellite.
6. All the planets orbit around the Sun.

Exercise 28 *Using FOL, symbolize the following sentences (you have to define a first-order language and a structure for that language). Consider that the universe of discourse is the set of celestial bodies in the Solar System.*

1. Some planet orbits around the Moon.
2. There is at least one satellite in the Solar System.
3. No planet is a satellite.
4. No celestial body in the Solar System orbits around itself.
5. Celestial bodies in the Solar System do not orbit around satellites.
6. There is exactly one satellite in the Solar System.

Exercise 29 *Using FOL, symbolize the following sentences (you have to define a first-order language and a structure for that language).*

1. Everyone loves Taylor Swift.
2. Taylor Swift does not love everyone.
3. Everyone loves someone.
4. Taylor Swift loves exactly three people.
5. Taylor Swift loves at most three people.
6. Taylor Swift loves at least two people.

Exercise 30 *Using FOL, symbolize the following arguments (for each argument, you have to define a first-order language and a structure for that language).*

1. Willard is a logician. All logicians wear funny hats. So Willard wears a funny hat.

2. Nothing on my desk escapes my attention. There is a computer on my desk. As such, there is a computer that does not escape my attention.

3. All my dreams are black and white. Old TV shows are in black and white. Therefore, some of my dreams are old TV shows.

4. Neither Holmes nor Watson has been to Australia. A person could have seen a kangaroo only if they had been to Australia or to a zoo. Although Watson has not seen a kangaroo, Holmes has. Therefore, Holmes has been to a zoo.

5. No one expects the Spanish Inquisition. No one knows the troubles I've seen. Therefore, anyone who expects the Spanish Inquisition knows the troubles I've seen.

6. All babies are illogical. Nobody who is illogical can manage a crocodile. Berthold is a baby. Therefore, Berthold is unable to manage a crocodile.