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Q-1 ⓐ $\boxed{\text{(ii) and (iii)}}$

(i) is incorrect network because given measurements M_1 and M_2, number of stars N is independent of the focus F_1 and F_2.

ⓑ $\boxed{\text{(ii)}}$ – needs less parameters among correct networks, then simplest network.

ⓒ

M \ N	1	2	3
0	$e(1-f)+f$	f	f
1	$(1-f)(1-2e)$	$e(1-f)$	0
2	$e(1-f)$	$(1-f)(1-2e)$	$e(1-f)$
3	0	$e(1-f)$	$(1-f)(1-2e)$
4	0	0	$e(1-f)$

ⓓ $N=2$, $N=4$, $N\geq 6$

ⓔ If we assume that prior distributions are $p_2, p_4, p_{\geq 6}$ $\Rightarrow$

$\Rightarrow$ 1) posterior for $N=2$: $p_2 e^2(1-f)^2$

2) posterior for $N=4$: $p_4 e f$
at most

3) posterior for $N\geq 6$: $p_{\geq 6} f^2$
at most

If we guess that priors are approximately corresponding, then $\boxed{N=2 \text{ is most likely}}$ as f is much less than e.

Otherwise, it's impossible to compute without prior distribution $P(N)$.

Q-2 (a) $P(B|+j,+m) = \alpha P(B) \sum_e P(+e) \sum_a P(+a|B,+e) P(+j|+a) P(+m|+a)$

$$P(B|+j,+m) = \alpha P(B) \sum_e P(+e) \left(0.9 \cdot 0.7 \cdot \begin{pmatrix} 0.95 & 0.29 \\ 0.94 & 0.001 \end{pmatrix} + 0.05 \cdot 0.01 \cdot \begin{pmatrix} 0.05 & 0.71 \\ 0.06 & 0.999 \end{pmatrix} \right) = \alpha P(B) \left(0.002 \cdot \begin{pmatrix} 0.5985 \\ 0.1831 \end{pmatrix} + 0.998 \cdot \begin{pmatrix} 0.5922 \\ 0.0011 \end{pmatrix} \right) = \alpha \begin{pmatrix} 0.0005922426 \\ 0.0014918576 \end{pmatrix} \approx \begin{bmatrix} 0.2842 \\ 0.7158 \end{bmatrix}$$

$$\boxed{\begin{aligned} P(+b|+j,+m) &= 0.2842 \\ P(-b|+j,+m) &= 0.7158 \end{aligned}}$$

(b) 25 arithmetic operations including normalization step: 7 additions; 16 multiplications; 2 divisions.

In comparison with 27 arithmetic operations by enumeration algorithm: 7 additions; 18 multiplications; 2 divisions.

Q-3 (a) When eliminating D we generate a new factor f_2 as follows:

$$\boxed{f_2(A,+c,E,F) = \sum_d P(E|d)\,P(F|d)\,f_1(A,+c,d)}$$

This leaves us with the factors:

$$\boxed{P(A),\ P(+c),\ P(G|+c,F),\ f_2(A,+c,E,F)}$$

When eliminating G we generate a new factor f_3 as follows:

$$\boxed{f_3(+c,F) = \sum_g P(g|+c,F)}$$

This leaves us with the factors:

$$\boxed{P(A),\ P(+c),\ f_2(A,+c,E,F),\ f_3(+c,F)}$$

When eliminating F we generate a new factor f_4 as follows:

$$\boxed{f_4(A,+c,E) = \sum_f f_2(A,+c,E,f)\,f_3(+c,f)}$$

This leaves us with the factors;

$$\boxed{P(A),\ P(+c),\ f_4(A,+c,E)}$$

(b)

$$\boxed{P(A,E|+c) = \frac{P(A)\,P(+c)\,f_4(A,+c,E)}{\sum_{a,e} P(a)\,P(+c)\,f_4(a,+c,e)}}$$

(c) The largest factor generated is $\boxed{f_2(A,+c,E,F)}$, because it has 3 non-instantiated variables, then 8 entries (2^3).

(d)

Variable Eliminated	Factor Generated
B	$f_1(A, +c, D)$
G	$f_2(+c, F)$
F	$f_3(+c, D)$
D	$f_4(A, +c, E)$

Q-4 (a)

$+a\ +b\ -c\ -d$	~~$+a\ -b\ -c\ +d$~~
$\rightarrow$ $+a\ -b\ +c\ -d$	$+a\ +b\ +c\ -d$ $\leftarrow$
$\rightarrow$ ~~$-a\ +b\ +c\ -d$~~	~~$-a\ +b\ -c\ +d$~~
$\rightarrow$ ~~$-a\ -b\ +c\ -d$~~	~~$-a\ -b\ +c\ -d$~~ $\leftarrow$

I) $\boxed{P(+c) = \frac{5}{8} = 0.625}$ – shown with arrows

II) $\boxed{P(+c \mid +a, -d) = \frac{2}{3} \approx 0.667}$ – not used samples are crossed out

(b)

Sample	Weight
$-a\ +b\ +c\ -d$	$P(+b \mid -a)\, P(-d \mid +c) = \frac{1}{3} \cdot \frac{5}{6} = \frac{5}{18} \approx 0.278$
$+a\ +b\ +c\ -d$	$P(+b \mid +a)\, P(-d \mid +c) = \frac{1}{5} \cdot \frac{5}{6} = \frac{1}{6} \approx 0.167$
$+a\ +b\ -c\ -d$	$P(+b \mid +a)\, P(-d \mid -c) = \frac{1}{5} \cdot \frac{1}{8} = \frac{1}{40} = 0.025$
$-a\ +b\ -c\ -d$	$P(+b \mid -a)\, P(-d \mid -c) = \frac{1}{3} \cdot \frac{1}{8} = \frac{1}{24} \approx 0.042$

(c) $$P(-a \mid +b, -d) = \frac{\frac{5}{18} + \frac{1}{24}}{\frac{5}{18} + \frac{1}{6} + \frac{1}{40} + \frac{1}{24}} = \frac{5}{8} = 0.625$$

(d) $\boxed{P(D \mid A)}$. Because likelihood weighting conditions are only on upstream evidence.