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Problem-1 State 6:

- To the left: $u_{left}(6) = (0 \cdot y^0) + (10 \cdot y^1) = \underline{10y}$
- To the right: $u_{right}(6) = (0 \cdot y^0) + (0 \cdot y^1) + (0 \cdot y^2) + (1 \cdot y^3) = \underline{y^3}$

$$u_{left}(6) = u_{right}(6)$$

$$\leftrightarrow 10y = y^3$$

$$10 = y^2$$

$$\boxed{y = \sqrt{10}}$$

Problem-2 (2.1) $\boxed{P(X_1 = 1) = \frac{2}{3}}$

$\boxed{P(X_1 = 2) = \frac{1}{3}}$

$$P(X_2 = 1) = P(X_1 = 1)\,P(X_2 = 1 \mid X_1 = 1) + P(X_1 = 2)\,P(X_2 = 1 \mid X_1 = 2) =$$
$$= \frac{2}{3} \cdot \frac{2}{3} + \frac{1}{3} \cdot \frac{1}{2} = \boxed{\frac{11}{18} = P(X_2 = 1)}$$

$$P(X_2 = 2) = P(X_1 = 1)\,P(X_2 = 2 \mid X_1 = 1) + P(X_1 = 2)\,P(X_2 = 2 \mid X_1 = 2) =$$
$$= \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{3} \cdot \frac{1}{2} = \boxed{\frac{7}{18} = P(X_2 = 2)}$$

(2.2) $$P(X_\infty = 1) = P(X_{\infty-1} = 2)\,P(X_\infty = 1 \mid X_{\infty-1} = 2) + P(X_{\infty-1} = 1) \cdot$$
$$\cdot P(X_\infty = 1 \mid X_{\infty-1} = 1)$$

$$\leftrightarrow P(X_\infty = 1) = P(X_\infty = 2)\,P(X_i = 1 \mid X_{i-1} = 2) + P(X_\infty = 1)\,P(X_i = 1 \mid X_{i-1} = 1)$$

$$\frac{1}{3} P(X_\infty = 1) = \frac{1}{2} P(X_\infty = 2)$$

$$P(X_\infty = 1) + P(X_\infty = 2) = 1$$

$$P(X_\infty = 1) = \frac{3}{5}$$

$$P(X_\infty = 2) = \frac{2}{5}$$