Inteligencia Artificial

Redes Neuronales Feedforward

Ejemplo de implementación

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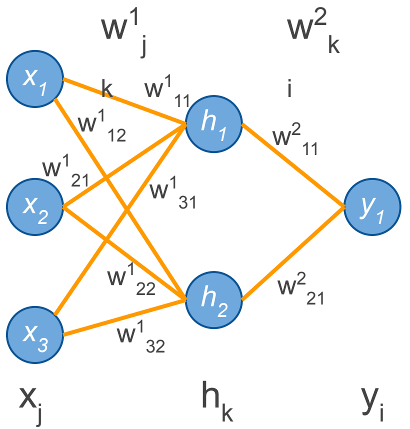
Ejemplo de implementación

- Implementación Matricial
  - *m* ejemplos
  - *n* entradas
  - *p* neuronas ocultas
  - *q* salidas



$$H = f(XW^1 + B^1) = f(Z)$$

$$XW^1 + B^1 = \begin{bmatrix} x_{11} & x_{12} & \dots & x_{1n} \\ x_{21} & x_{22} & \dots & x_{2n} \\ \vdots & & \ddots & \\ x_{m1} & x_{m2} & \dots & x_{mn} \end{bmatrix} \begin{bmatrix} w_{11}^1 & w_{12}^1 & \dots & w_{1p}^1 \\ w_{21}^1 & w_{22}^1 & \dots & w_{2p}^1 \\ \vdots & & \ddots & \\ w_{n1}^1 & w_{n2}^1 & \dots & w_{np}^1 \end{bmatrix} + \begin{bmatrix} b_1^1 & b_2^1 & \dots & b_p^1 \\ b_1^1 & b_2^1 & \dots & b_p^1 \\ \vdots & & \ddots & \\ b_1^1 & b_2^1 & \dots & b_p^1 \end{bmatrix}$$

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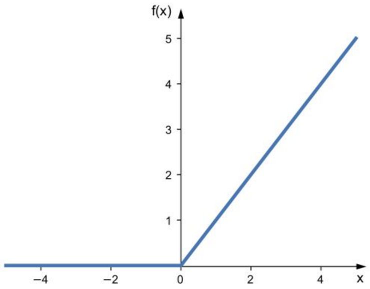
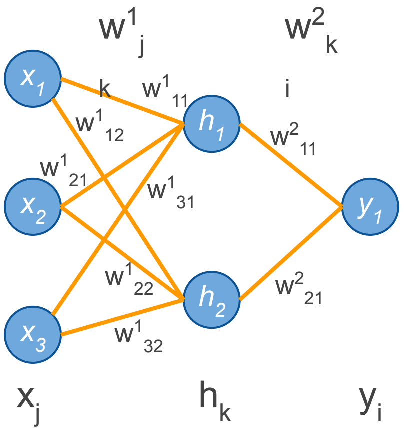
$$relu(x) = max(0, x)$$





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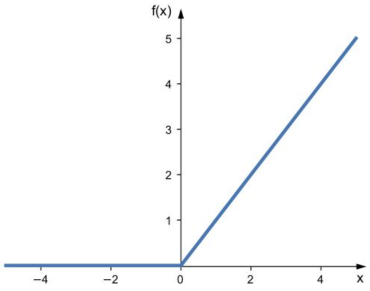

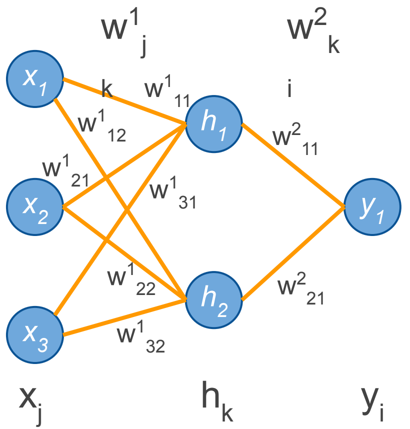

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$$Y = g(HW^2 + B^2)$$

g(x) = x *(función identidad)*

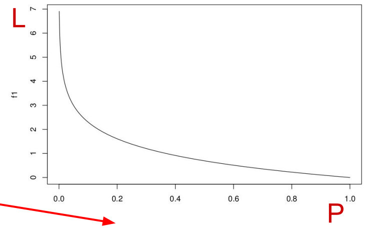
Ejemplo de implementación

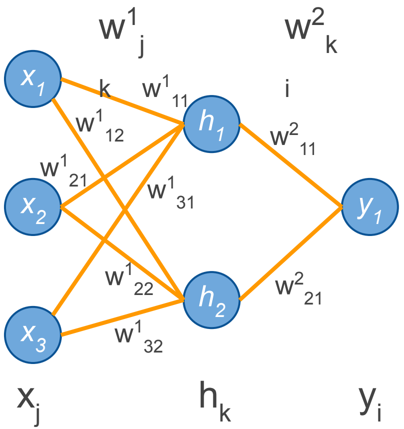
- Loss Function: Softmax

Score para la clase correcta (ejemplo m)

$$L = \frac{1}{m}\sum_{m} -log\left(\underbrace{\frac{e^{y_c{}^m}}{\sum_j e^{y_j}}}_{P}\right)$$

Score para la clase j (ejemplo m)





Ejemplo de implementación

- Aprendizaje: Backpropagation
  - Aplicando la *regla de la cadena*

$$\frac{\partial L}{\partial W^2} = \frac{\partial L}{\partial Y}\frac{\partial Y}{\partial W^2}$$

$$\frac{\partial L}{\partial B^2} = \frac{\partial L}{\partial Y}\frac{\partial Y}{\partial B^2}$$

$$\frac{\partial L}{\partial W^1} = \frac{\partial L}{\partial Y}\frac{\partial Y}{\partial H}\frac{\partial H}{\partial Z}\frac{\partial Z}{\partial W^1}$$

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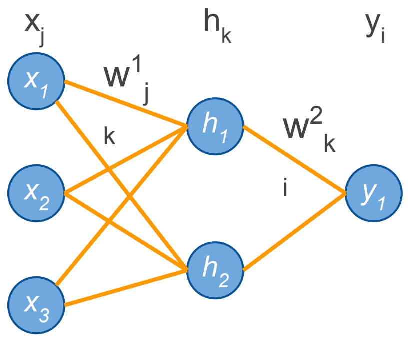
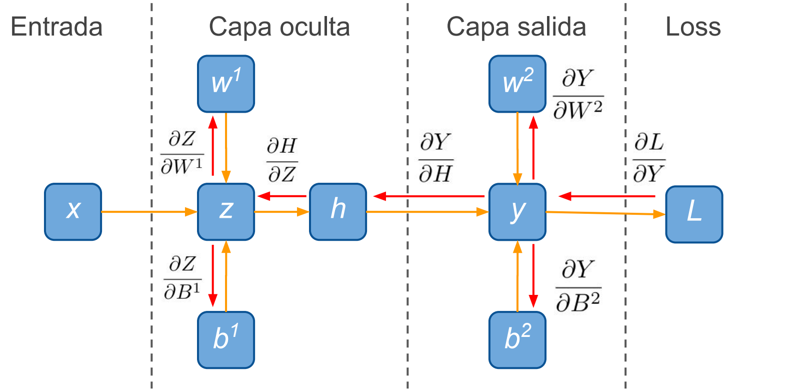




Ejemplo de implementación

- Aprendizaje: Backpropagation
  - Aplicando la *regla de la cadena*

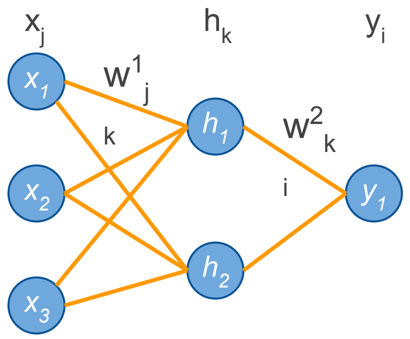
$$\frac{\partial L}{\partial W^2} = \frac{\partial L}{\partial Y}\frac{\partial Y}{\partial W^2}$$

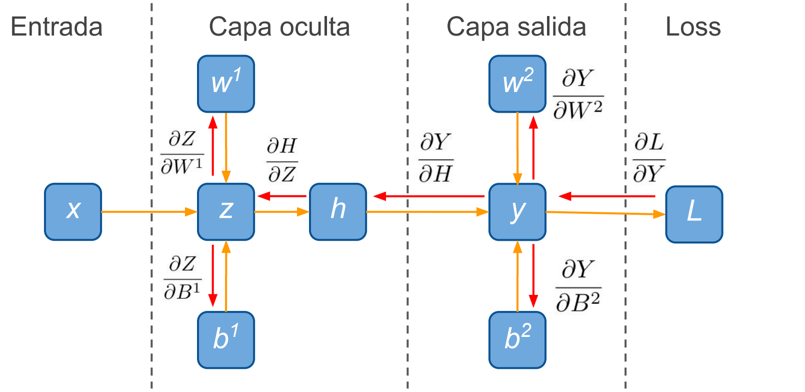
$$\frac{\partial L}{\partial Y} = \begin{cases} \frac{1}{m}(P_i - 1) & \text{(clase correcta)} \\ \frac{1}{m}(P_i) & \text{(otras clases)} \end{cases}$$

$$\frac{\partial Y}{\partial W^2} = H$$

$$L = \frac{1}{m}\sum_{m} -log\left(\frac{e^{y_c{}^m}}{\sum_j e^{y_j{}^m}}\right)$$

$$P_i = \frac{e^{y_i}}{\sum_j e^{y_j}}$$





Ejemplo de implementación

- Aprendizaje: Backpropagation
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$$\frac{\partial L}{\partial B^2} = \frac{\partial L}{\partial Y}\frac{\partial Y}{\partial B^2}$$

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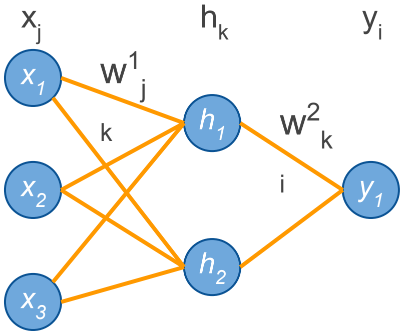
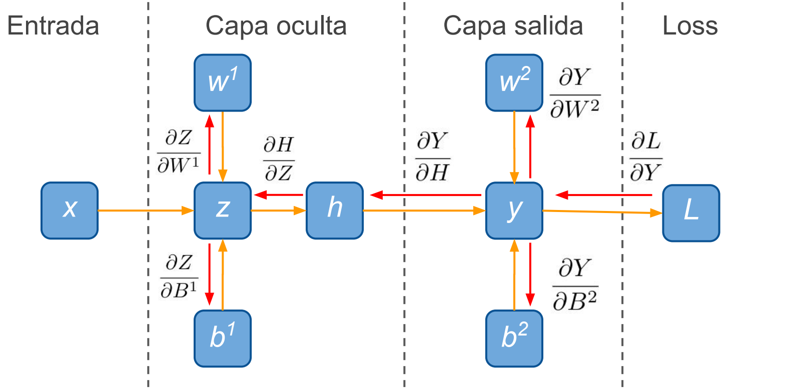
$$\frac{\partial Y}{\partial B^2} = 1$$

(se suma verticalmente a lo largo de m)

$$L = \frac{1}{m}\sum_m -log\left(\frac{e^{{y_c}^m}}{\sum_j e^{{y_j}^m}}\right)$$

$$P_i = \frac{e^{y_i}}{\sum_j e^{y_j}}$$





Ejemplo de implementación

- Aprendizaje: Backpropagation
    - Aplicando la *regla de la cadena*

$$\frac{\partial L}{\partial W^1} = \frac{\partial L}{\partial Y}\frac{\partial Y}{\partial H}\frac{\partial H}{\partial Z}\frac{\partial Z}{\partial W^1}$$

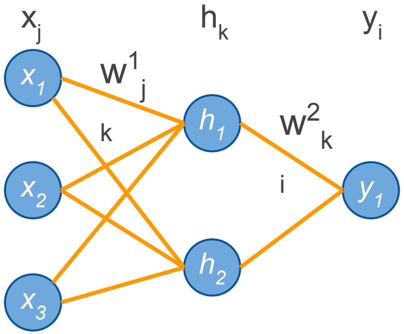
$$\frac{\partial Y}{\partial H} = W^2$$

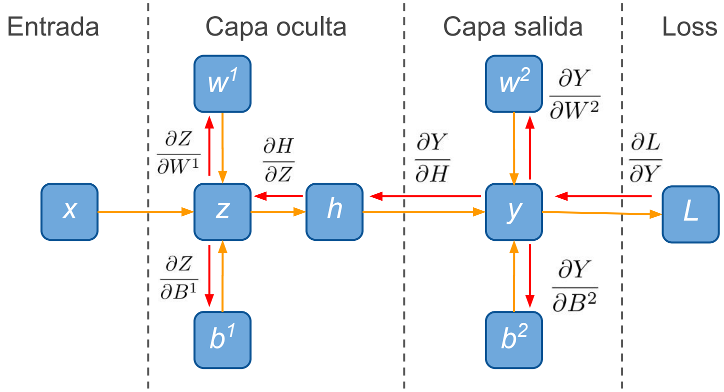
$$Y = HW^2 + B^2$$

$$\frac{\partial Z}{\partial W^1} = X$$

$$Z = XW^1 + B^1$$

$$\frac{\partial H}{\partial Z} = 1(z > 0)$$





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(se suma verticalmente a lo largo de m)

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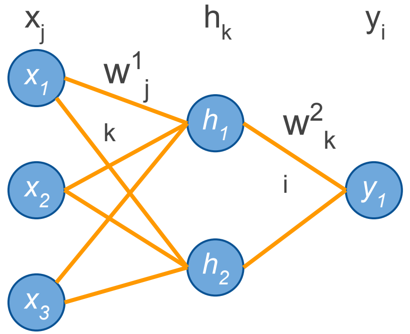
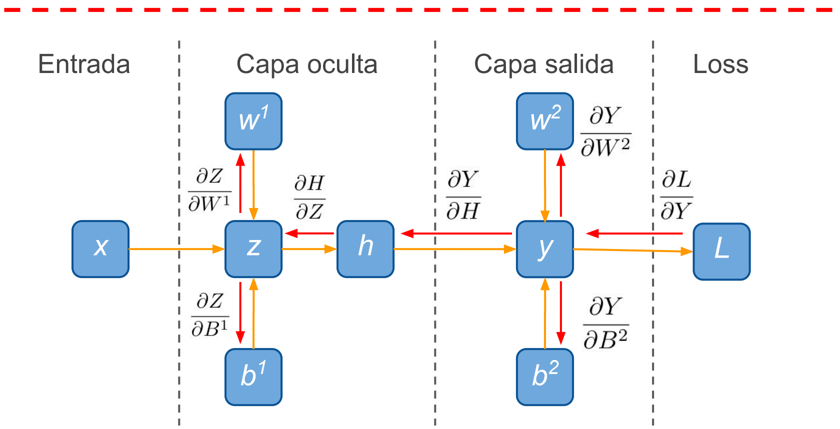


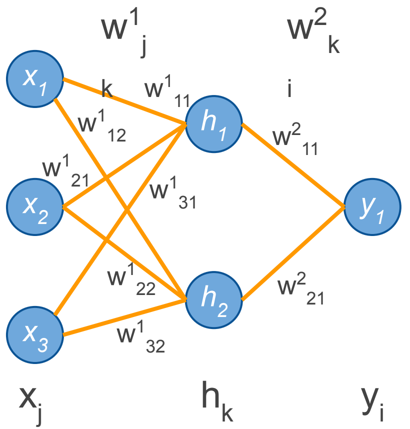


Ejemplo de implementación

- Aprendizaje: Backpropagation
  - Descenso por el gradiente

$$W_{t+1} = W_t - \varepsilon \nabla L_W$$

$$\begin{cases} W^2(t+1) = W^2(t) - \varepsilon \dfrac{\partial L}{\partial W^2} \\ B^2(t+1) = B^2(t) - \varepsilon \dfrac{\partial L}{\partial B^2} \\ W^1(t+1) = W^1(t) - \varepsilon \dfrac{\partial L}{\partial W^1} \\ B^1(t+1) = B^1(t) - \varepsilon \dfrac{\partial L}{\partial B^1} \end{cases}$$



Preguntas?
Opiniones?

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