1) $f(x)=\frac{x^2+\ln x}{\sin x} = \frac{x^2}{\sin x}+\frac{\ln x}{\sin x}$

$\frac{d}{dx}x^2=2x$ $\qquad \frac{d}{dx}\ln x=\frac{1}{x}$ $\qquad \frac{d}{dx}\sin x=\cos x$

$\frac{d}{dx}f=\frac{2x\cdot\sin x + x^2\cdot\cos x}{\sin^2 x}+\frac{\frac{1}{x}\cdot\sin x+\ln(x)\cdot\cos x}{\sin^2 x}$

2) $f(x)=\ln(tg\frac{x}{2})$

$\frac{d}{dx}\, tg\, 0{,}5x = \frac{1}{\cos^2(0{,}5x)}\cdot 0{,}5$ $\qquad \frac{d}{dx}\ln x=\frac{1}{x}$

$f'=\frac{1}{tg\frac{x}{2}}\cdot\frac{0{,}5}{\cos^2\frac{x}{2}}$

3) $f(x)=lg(x^3\cdot e^{2x})=lg(x^3)+lg(e^{2x})$

$\frac{d}{dx}lg\, x=\frac{1}{x\cdot\ln(10)}$ $\qquad \frac{d}{dx}x^3=3x^2$ $\qquad \frac{d}{dx}e^{2x}=e^{2x}\cdot 2$ $\qquad \frac{d}{dx}lg(e^x)=\frac{d}{dx}x\cdot lg(e)$

$f'(x)=3\cdot\frac{1}{x\cdot\ln(10)}+2(\cancel{1}\cdot lg(e)+\cancel{x\cdot 0})$

4) $f(x)=lg\sqrt{\frac{x^2+1}{x^3+5}}=\frac{0}{5}\log_{10}\left(\frac{x^2+1}{x^3+5}\right)$

$\frac{d}{dx}lg=\frac{1}{x\cdot\ln(10)}$

$\frac{d}{dx}\frac{x^2}{x^3+5}=\frac{2x(x^3+5)\cancel{+(1x^2)}(3x^2)}{(x^3+5)^2}$

$\frac{d}{dx}\frac{1}{x^3+5}=\frac{0+1(3x^2)}{(x^3+5)^2}$

$\frac{d}{dx}\frac{x^2+1}{x^3+5}=\frac{2x(x^3+5)+3x^2\cdot x^2+3x^2}{(x^3+5)^2}$

$f'(x)=\frac{1}{2}\cdot\frac{1}{\frac{x^2+1}{x^3+5}\cdot\ln 10}\cdot\frac{2x(x^3+5)+3x^2\cdot x^2+3x^2}{(x^3+5)^2}$

1a) $f(x) = \ln(\sin x)$ $\qquad x_0 = \pi/6$

$f'(x) = \frac{1}{\sin x} \cdot \cos x$

$f(x_0) \doteq -0{,}69$

$f'(x_0) = \sqrt{3}$

$y \doteq \sqrt{3}\left(x - \pi/6\right) + (-0{,}69)$

$y \doteq \sqrt{3}\,x - 1{,}6$

$$\frac{d}{dx} 3x^2 = 6x$$

$$\frac{d}{dx} \cancel{+}4\sin(2x) = 4\cdot\cos(2x)\cdot 2 = 8\cos(2x)$$

$$\frac{d}{dx} \cancel{t}\ x\cdot e^{3x} = 1\cdot e^{3x} + x(3\cdot e^{3x}) = e^{3x} + 3xe^{3x}$$

$$\cancel{\frac{d}{dx}\frac{3x^2}{x\cdot e^{3x}} = \frac{d}{dx}\frac{3x}{e^{3x}}} \qquad \cancel{3(3e^{3x}) - 3x} \quad \cancel{\frac{3(e^{3x}) - 3x(3e^{3x})}{(e^{3x})^2 = e^{6x}}}$$

$$\cancel{\frac{d}{dx}\frac{-4\sin(2x)}{x e^{3x}} = \frac{-8\cos(2x)\cdot xe^{3x} - 4\sin(2x)(e^{3x}(3x+1))}{x^2\cdot e^{3x}}}$$

$$\cancel{f'(x) = \frac{3(e^{3x}) - 3x(3e^{3x})}{e^{6x}} + \frac{-8\cos(2x)\cdot x\cdot e^{3x} + 4\sin(2x)(e^{3x}(3x+1))}{x^2\cdot e^{3x}}}$$

$$6x^2\cancel{\cdot e^{3x}} \quad + \quad 3x^2e^{3x} - 3x^2(3x\cdot e^{3x})$$

$$\frac{6x(x\cdot e^{3x}) - 3x^2(e^{3x} + 3x\cdot e^{3x})}{x^2\cdot e^{6x}} - 4\,\frac{2\cos(2x)(x\cdot e^{3x}) - \sin(2x)(e^{3x} + 3x\cdot e^{3x})}{x^2\cdot e^{6x}}$$

$$\frac{6x^2\cdot e^{3x} - 3x^2e^{3x} - 9x^2\cdot e^{3x}}{x^2\cdot e^{6x}} - 4\,\frac{2x\cdot e^{3x}\cdot\cos(2x) - \sin(2x)e^{3x} - \sin(2x)(3x\cdot e^{3x})}{x^2\cdot e^{6x}}$$

$$3x^2\cancel{e^{3x}} - \underbrace{3x^2(3x\cdot\cancel{e^{3x}})}_{-9x^3}$$

$$\frac{6x^2\cdot\cancel{e^{3x}} - 3x^2\cancel{e^{3x}} - 9x^2\cancel{e^{3x}} - 8x\cdot\cancel{e^{3x}}\cdot\cos(2x) + 4\sin(2x)\cancel{e^{3x}} + \sin(2x)(12x\cdot\cancel{e^{3x}})}{x^2\cdot e^{6x} = x^2\cdot e^{3x}\cdot\cancel{e^{3x}}} \qquad \div\frac{e^{3x}}{e^{3x}}$$

$$3x^2 - 9x^3$$

$$\frac{\cancel{6x^2 - 3x^2 - 9x^2} - 8x\cos(2x) + \sin(2x)(4+12x)}{x^2\cdot e^{3x}} = \frac{\overset{3x^2-9x^3}{\cancel{6x^2}} - 8x\cos(2x) + \sin(2x)(4+12x)}{x^2\cdot e^{3x}}$$

F25P21 | 1. feladat | 2020. 11. 17

F25P21 ← 5

1) $f(x) = \ln\left(tg\frac{x}{2}\right) = \ln(tg\,0{,}5x)$

$$\frac{d}{dx}tg\,0{,}5x = 0{,}5\cdot\frac{1}{\cos^2(0{,}5x)} = \frac{1}{2\cos^2(0{,}5x)}$$

$$\frac{d}{dx}\ln(x) = \frac{1}{x}$$

$$f'(x) = \frac{1}{tg\frac{x}{2}}\cdot\frac{1}{2\cos^2\left(\frac{x}{2}\right)}$$

2) $f(x) = \ln^2(th\,x)$

$$\frac{d}{dx}th\,x = \frac{1}{ch^2(x)} \qquad \frac{d}{dx}\ln x = \frac{1}{x}$$

$$\frac{d}{dx}(\ln x)^2 = 2(\ln x)\cdot\frac{1}{x}$$

$$f'(x) = 2\ln(th\,x)\cdot\frac{1}{th\,x}\cdot\frac{1}{ch^2(x)}$$

3) $f(x) = lg\sqrt{\frac{x^2+1}{x^2+5}}$

$$g(x) = \sqrt{\frac{x^2+1}{x^2+5}} \qquad h(x) = \frac{x^2+1}{x^2+5} = \frac{x^2}{x^2+5} + \frac{1}{x^2+5}$$

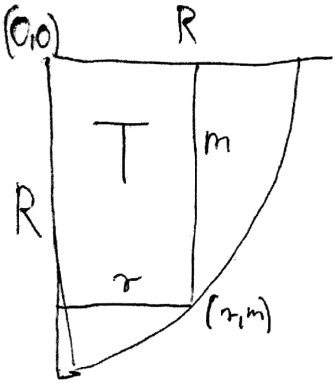
$$\left.\begin{aligned}\frac{d}{dx}\frac{x^2}{x^2+5} &= \frac{2x(x^2+5)-x^2(2x)}{(x^2+5)^2} = \frac{2x(x^2+5-x^2)}{(x^2+5)^2} = \frac{10x}{(x^2+5)^2}\\ \frac{d}{dx}\frac{1}{x^2+5} &= \frac{d}{dx}(x^2+5)^{-1} = -1(x^2+5)^{-2}\cdot 2x = \frac{-1\cdot 2x}{(x^2+5)^2} = \frac{-2x}{(x^2+5)^2}\end{aligned}\right\}+ = \frac{8x}{(x^2+5)^2} = h'(x)$$

$$g'(x) = \left(\frac{x^2+1}{x^2+5}\right)^{\frac{1}{2}} = \frac{1}{2}\left(\frac{x^2+1}{x^2+5}\right)^{-\frac{1}{2}}\cdot\frac{8x}{(x^2+5)^2}$$

$$f'(x) = \frac{1}{\sqrt{\frac{x^2+1}{x^2+5}}\cdot\log_e 10}\cdot\frac{1}{2}\left(\frac{x^2+1}{x^2+5}\right)^{-\frac{1}{2}}\cdot\frac{8x}{(x^2+5)^2}$$

F25P21

2020. 11. 17



$x^2+y^2=R^2$ $\quad /-$

$m^2=y$

$x=r$

$R=1$

$T=m\cdot r$

$r^2+m^2=R^2 \quad /-r^2$

$m^2=R^2-r^2 \quad /\sqrt{\ }$

$m=\sqrt{R^2-r^2}$

$T=r\sqrt{R^2-r^2}$

$T'=1\sqrt{R^2-r^2}+r\left(\cancel{\frac{1}{2}}(R^2-r^2)^{-\frac{1}{2}}\cdot(\cancel{2r})\right)$ $\quad R=1$

$T'=\sqrt{R^2-r^2}+\frac{-r^2}{(1-r^2)^{1/2}}$

$=\frac{1-r^2}{\sqrt{1-r^2}}+\frac{-r^2}{\sqrt{1-r^2}}=\frac{1-2r^2}{\sqrt{1-r^2}}=0$

$r=1=R$

$T=0$ ↯

$/\cdot\sqrt{1-r^2}$

$/+2r^2$

$1=2r^2 \quad /\div 2$

$r^2=\frac{1}{2} \quad /\sqrt{\ }$

$r=\sqrt{\frac{1}{2}}$

$r=\frac{1}{\sqrt{2}}$

$R^2=r^2+m^2$

$1=\frac{1}{2}+m^2$

$m^2=\frac{1}{2}$

$m=\frac{1}{\sqrt{2}}$

$T(r)\geq 0$

~~Mivel~~ $T(r=0)=T(r=R)=0$

~~ezért~~

$\forall T(x)>0 \quad 0<x<R$

$\Downarrow$

A T függvény globális maximumát a $]0;R[$ intervallumon veszi fel.

Mivel $T'(r)=0$ csak $r=\frac{1}{\sqrt{2}}$ esetén teljesül, ezért kizárásos alapon ez kell hogy legyen a globális maximum.

Ennek a hengernek a sugara $\frac{1}{\sqrt{2}}$ egység.

F25P21

2020. november 24

(2/a) $\lim\limits_{x\to 0} \dfrac{\sin(x)-x}{\arcsin(x)-x} \overset{,,0"}{=} \underset{L'H}{=} \lim\limits_{x\to 0} \dfrac{\cos(x)-1}{\underbrace{\frac{1}{\sqrt{1-x^2}}}-1} = ,,\frac{0}{0}" \underset{L'H}{=} \lim\limits_{x\to 0} \dfrac{-\sin(x)\cdot(1-x^2)^{\frac{3}{2}}}{x} = \underline{\underline{-1}}$

$\frac{d}{dx}(1-x^2)^{-\frac{1}{2}} =$

$= -\frac{1}{2}(1-x^2)^{-\frac{3}{2}} \cdot (-2x)$

$= \dfrac{x}{(1-x^2)^{\frac{3}{2}}}$

(2/b) $\lim\limits_{x\to 0} \dfrac{e^x-e^{-x}-2x}{x^3} = ,,\frac{0}{0}" \underset{L'H}{=} \lim\limits_{x\to 0} \dfrac{e^x+e^{-x}-2}{3x^2} = ,,\frac{0}{0}" \underset{L'H}{=} \lim\limits_{x\to 0} \dfrac{e^x-e^{-x}}{6x} = ,,\frac{0}{0}"$

$\underset{L'H}{=} \lim\limits_{x\to 0} \dfrac{e^x+e^{-x}}{6} = \dfrac{2}{6} = \underline{\underline{\dfrac{1}{3}}}$

(3)

$f(x) := x^3 - x^2 - 2x$
$= x(x^2 - \cancel{2}x - 2)$
$= x(x+1)(x-2)$

$\underbrace{x_0 = 0,\ x_1 = -1,\ x_2 = +2}$ zérus~~pontok~~ helyek

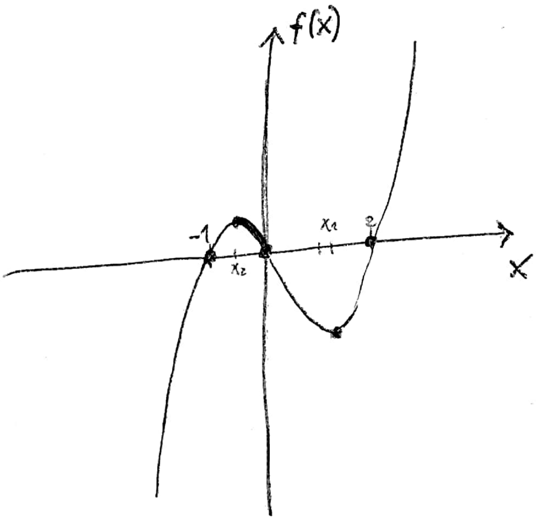
A függvény folytonos.

~~ÉK~~: ÉT: $\mathbb{R}$

Mivel:
- $\lim\limits_{x\to -\infty} f(x) = -\infty$
- $\lim\limits_{x\to \infty} f(x) = \infty$
- f folytonos

→ ∄ globális szélsőérték

Bolzano-tétel: ÉK: $\mathbb{R}$



$f(-\infty) = -\infty$
$f(-1) = 0$
$f(-0{,}55) = 0{,}63$
$f(0) = 0$
$f(1{,}22) = -2{,}11$
$f(2) = 0$
$f(\infty) = \infty$

$f'(x) = 3x^2 - 2x - 2 =$

$x_{1,2} = \dfrac{2 \pm \sqrt{4+4\cdot 2\cdot 3}}{2\cdot 3} = \dfrac{2 \pm \sqrt{28}}{2\cdot 3} = \dfrac{\cancel{2}\cdot 1 \pm \sqrt{7}}{\cancel{2}\cdot 3}$ ↗ $x_1 \approx 1{,}22$ ↘ $x_2 \approx -0{,}55$

$f''(x) = 6x - 2$
$f''(x_1) \approx 5{,}32 > 0 \to$ lok. min
$f''(x_2) \approx -5{,}3 < 0 \to$ lok. max

	$(-\infty,-1)$	$(-1, x_2)$ ~~$-0{,}55$~~	$(x_2, 0)$	$(0, x_1)$	$(x_1, 2)$	$(2, \infty)$
$f(x)$	↗; −	↗; +	↘; +	↘; −	↗; −	↗; +
$f'(x)$	↘; +	↘; −	↗; −	−	+	+

F25P29 11. hét 2020. 12. 01

A/1)

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} ctg(x)\,dx = \overbrace{\ln|\sin x| + C}^{F(x)}$$

$$\to \frac{\cos(x)}{\sin(x)} \to \cos(x)\cdot\frac{1}{\sin x}$$

$$F\left(\frac{3\pi}{2}\right) - F\left(\frac{\pi}{2}\right)$$

$$= \ln\left|\sin\frac{3\pi}{2}\right| - \ln\left|\sin\frac{\pi}{2}\right| = \underline{\underline{0}}$$

A/2)

$$\int_0^1 x\cdot e^{2x}/2\,dx = \frac{1}{2}\int_0^1 x\cdot e^{2x}$$

$$\int e^{2x} = \frac{e^{2x}}{2} + C$$

$$x\cdot\frac{e^{2x}}{2} - \underbrace{\int \cancel{1}\cdot\frac{e^{2x}}{2}}_{} + C$$

$$\frac{1}{2}\int e^{2x} = \frac{1}{2}\cdot\frac{e^{2x}}{2} + C$$

$$F(x) = \frac{1}{2}\left[x\cdot\frac{e^{2x}}{2} - \frac{1}{2}\left(\frac{e^{2x}}{2}\right)\right] + C$$

$$= \frac{e^{2x}}{4}\left(x - \frac{1}{2}\right)$$

$$F(1) - F(0) = \cancel{0{,}92 + 0{,}12} \approx \underline{\underline{1{,}05}}$$

11. hét

2020. 12. 01

F25P29

Südi Tamás

A/1)

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} ctg(x)\,dx = \overbrace{\ln|\sin x| + C}^{F(x)}$$

$$\frac{\cos x}{\sin x} \to \cos x \cdot \frac{1}{\sin x}$$

$$F\left(\tfrac{3\pi}{2}\right) - F\left(\tfrac{\pi}{2}\right)$$

$$= \ln|\sin \tfrac{3\pi}{2}| - \ln|\sin \tfrac{\pi}{2}| = \underline{0}$$

A/2)

$$\int_0^1 x \cdot e^{2x}/2 \; dx = \frac{1}{2}\int_0^1 x \cdot e^{2x}\,dx \qquad \int e^{2x}dx = \frac{e^{2x}}{2} + C$$

$$\frac{1}{2}\left[x \cdot \frac{e^{2x}}{2} - \int \cancel{x} \cdot \frac{e^{2x}}{2}\right] + C$$

$$F(x) = \frac{1}{2}\left[x \cdot \frac{e^{2x}}{2} - \frac{1}{2}\frac{e^{2x}}{2}\right] + C$$

$$= \frac{e^{2x}}{4}\left(x - \frac{1}{2}\right) \qquad F(1) - F(0) \approx \underline{1{,}05}$$

B)

$$\int_2^3 \frac{4x^2+13x-9}{x^3+2x^2-3x}\,dx = \int_2^3 \frac{1(4x^2+13x-9)}{x(x^2+2x-3)}\,dx = \int_2^3 \frac{A}{x} + \frac{Bx+C}{x^2+2x-3}\,dx$$

$$4x^2+13x-9 = A(x^2+2x-3) + Bx^2 + Cx$$

$$4x^2+13x-9 = (A+B)\cdot x^2 + (2A+C)x - 3A$$

$-3A = -9 \to A = 3$

$A+B = 4 \to B = 1$

$2A + C = 13 \to C = 7$

$$\int_2^3 \frac{3}{x}\,dx + \int_2^3 \frac{1x+7}{x^2+2x-3}\,dx = 3\ln(x) + \int_2^3 \frac{x+7}{(x+3)(x-1)}\,dx \Rightarrow 3\ln(x) + \int_2^3 \frac{-1}{x+3}\,dx + \int_2^3 \frac{2}{x-1}\,dx$$

$$3 \cdot \ln|x|$$

$$= \frac{A}{x+3} + \frac{B}{x-1}$$

$$x+7 = A(x-1) + B(x+3)$$

$x=1$: $8 = 4B$, $B = 2$

$x=-3$: $4 = -4A$, $A = -1$

$$F(x) = 3\ln|x| - \ln|x+3| + 2\ln|x-1| + C$$

$$F(3) - F(2) \approx \underline{2{,}42}$$

11. hét

2020. 12. 01

F25P29

Südi Tamás

A/1)

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} ctg(x)\, dx = \overbrace{\ln|\sin x| + C}^{F(x)}$$

$$\frac{\cos x}{\sin x} \to \cos x \cdot \frac{1}{\sin x}$$

$$F\left(\frac{3\pi}{2}\right) - F\left(\frac{\pi}{2}\right)$$

$$= \ln|\sin \tfrac{3\pi}{2}| - \ln|\sin \tfrac{\pi}{2}| = \underline{0}$$

A/2)

$$\int_0^1 x \cdot e^{2x}/2 \; dx = \frac{1}{2} \int_0^1 x \cdot e^{2x}\, dx \qquad \int e^{2x} dx = \frac{e^{2x}}{2} + C$$

$$\frac{1}{2}\left[x \cdot \frac{e^{2x}}{2} - \int \cancel{x} \cdot \frac{e^{2x}}{2}\right] + C$$

$$F(x) = \frac{1}{2}\left[x \cdot \frac{e^{2x}}{2} - \frac{1}{2} \frac{e^{2x}}{2}\right] + C$$

$$= \frac{e^{2x}}{4}\left(x - \frac{1}{2}\right) \qquad F(1) - F(0) \approx \underline{1{,}05}$$

B)

$$\int_2^3 \frac{4x^2+13x-9}{x^3+2x^2-3x}\, dx = \int_2^3 \frac{(4x^2+13x-9)}{x(x^2+2x-3)} dx = \int_2^3 \frac{A}{x} dx + \frac{Bx+C}{x^2+2x-3}\, dx$$

$$4x^2+13x-9 = A(x^2+2x-3) + Bx^2 + Cx$$

$$4x^2+13x-9 = (A+B)\cdot x^2 + (2A+C)x - 3A$$

$-3A = -9 \to A = 3$

$A+B = 4 \to B = 1$

$2A + C = 13 \to C = 7$

$$\int_2^3 \frac{3}{x} dx + \int_2^3 \frac{1x+7}{x^2+2x-3} dx = 3\ln(x) + \int_2^3 \frac{x+7}{(x+3)(x-1)} dx \Rightarrow 3\ln(x) + \int_2^3 \frac{-1}{x+3} dx + \int_2^3 \frac{2}{x-1} dx$$

$$3 \cdot \ln|x|$$

$$= \frac{A}{x+3} + \frac{B}{x-1}$$

$$x+7 = A(x-1) + B(x+3)$$

$x=1$: $8 = 4B$, $B = 2$

$x=-3$: $4 = -4A$, $A = -1$

$$F(x) = 3\ln|x| - \ln|x+3| + 2\ln|x-1| + C$$

$$F(3) - F(2) \approx \underline{2{,}42}$$

3)

$$\int_0^2 2x\cdot e^x \, dx = 2\underbrace{\int_0^2 x\cdot e^x \, dx}$$

$$x\cdot e^x - \int 1\cdot e^x = x\cdot e^x - e^x$$

$$\frac{F(x)}{2} = x\cdot e^x - e^x$$

$$2\left(2\cdot e^2 - e^2 - 0\cdot e^0 - e^0\right) \approx \underline{\underline{16{,}78}}$$

F25P29

4)

$$\int_0^1 tg^2 x \, dx = \int_0^1 \frac{1}{\cos^2 x}\,dx + \int_0^1 \frac{-\cos^2 x}{\cos^2 x} = F(1) - F(0) \approx \underline{\underline{1{,}56}}$$

$$\left(\frac{\sin x}{\cos x}\right)^2 = \frac{\sin^2 x}{\cos^2 x} = \frac{1-\cos^2 x}{\cos^2 x}$$

$$1 = \sin^2 x + \cos^2 x$$

$$F(x) = \tan(x) - 1$$