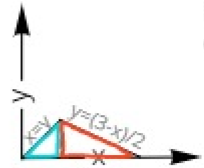
F25P2J 9.hf 2021.04.25.

Südi Tamás

1.a)

f = 2x+y
A = (0,0)
B = (1,1)
C = (3,0)



$$\int_0^1\int_0^x 2x+y\,dy\,dx = \int_0^1\left[2xy+\frac{y^2}{2}\right]_0^x dx$$

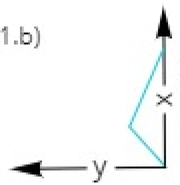
$$= \int_0^1 \frac{5}{2}x^2\,dx = \left[\frac{5x^3}{6}\right]_0^1 = \frac{5}{6}$$

$$\int_1^3\int_0^{\frac{3-x}{2}} 2x+y\,dy\,dx = \int_0^1\left[y\left(2x+\frac{y}{2}\right)\right]_0^{\frac{3-x}{2}} dx = \int_1^3 \frac{3-x}{2}\left(2x+\frac{3-x}{4}\right)$$

$$= \int_1^3 \frac{-7}{8}x^2+\frac{9}{4}x+\frac{9}{8} = \left[\frac{-7}{8\cdot 3}x^3+\frac{18}{8\cdot 2}x^2+\frac{9}{8}x\right] = \frac{11}{3}$$

$$\sum \frac{5+22}{6} = \frac{27}{6} = \underline{\underline{4{,}5}}$$

1.b)



x=y
y =(3-x)/2
2y=3-x
2y-3=-x
x=3-2y

$$\int_0^1\int_y^{3-2y} 2x+y\,dx\,dy = \int_0^1\left[x^2+xy\right]_y^{3-2y} dy =$$

$$= \int_0^1 9+4y^2-12y+3X-2y^2-y^2-y^2\,dy = \int_0^1 -9y+9 = -9\int_0^1 y-1\,dy$$

$$= -9\left[y\left(\frac{y}{2}-1\right)\right]_0^1 = \underline{\underline{4{,}5}}$$

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3)

$$f(x,y) = x^2 y$$

$$x = r \cdot \cos\theta$$

$$y = r \cdot \sin\theta$$

$$0 \le r \le 1$$

$$0 \le \theta \le \pi/4$$

$$\iint_V r^2 \cos^2\theta \cdot r \sin\theta \cdot \underbrace{r}_{\det J}\, dV$$

$$= \iint_V r^4 \cdot \cos^2\theta \cdot \sin\theta \, dv$$

$$u^2 \cdot -du$$

$$= \left[\frac{r}{5}\right]_0^1 \cdot \left[\frac{-\theta^3}{3}\right]_{u(0)=1}^{u(\frac{\pi}{4})=\frac{\sqrt{2}}{2}}$$

$$= \frac{1}{5} \cdot \left[\frac{-\left(\frac{\sqrt{2}}{2}\right)^3}{3} - \frac{-1^3}{3}\right] = \frac{1}{15}\left(1 - \frac{2\sqrt{2}}{8}\right) = \frac{4-\sqrt{2}}{60}$$

$$= 0.043\ldots$$